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Precision Measurement and Calibration

Colorimetry

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Precision Measurement and Calibration

Selected NBS Papers on

Colorimetry

Isadore Nimeroff, Editor
Institute for Applied Technology
National Bureau of Standards
Washington, D.C. 20234

A compilation of previously published papers by the staff of the National Bureau of Standards, including selected abstracts by NBS and non-NBS authors. Issued in several volumes, see page iv.



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A b s t r a c t

This volume is one of an extended series which brings together the previously published papers, monographs, abstracts, and bibliographies by NBS authors dealing with the precision measurement of specific physical quantities and the calibration of the related metrology equipment. The contents have been selected as being useful to the standards laboratories of the United States in tracing to the NBS standards the accuracies of measurement needed for research work, factory production, or field evaluation.

Volume 9 treats the field of Colorimetry. It contains 37 reprints on Colorimetry and Spectrophotometry assembled in 8 sections: (1) Spectrophotometry, (2) Color Vision, (3) Color Specification, (4) Sources and Instrumentation, (5) Color Spacing and Metamerism, (6) Variabilities and Uncertainties, (7) Appearance and Gloss, (8) List of Publications, LP39 (in part), Colorimetry and Spectrophotometry Publications by the Staff of the National Bureau of Standards, January 1957 through August 1969.

Key Words: Appearance evaluation; color spacing; color vision; colorimetry; gloss; metamerism; spectrophotometry; standards; uncertainty; variability.

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Foreword

In the 1950's the tremendous increase in industrial activity, particularly in the missile and satellite fields, led to an unprecedented demand for precision measurement, which, in turn, brought about the establishment of hundreds of new standards laboratories. To aid these laboratories in transmitting the accuracies of the national standards to the shops of industry, NBS in 1959 gathered together and reprinted a number of technical papers by members of its staff describing methods of precision measurement and the design and calibration of standards and instruments. These reprints, representing papers written over a period of several decades, were published as NBS Handbook 77, Precision Measurement and Calibration, in three volumes: Electricity and Electronics; Heat and Mechanics; Optics, Metrology, and Radiation.

Some of the papers in Handbook 77 are still useful, but new theoretical knowledge, improved materials, and increasingly complex experimental techniques have so advanced the art and science of measurement that a new compilation has become necessary. The present volume is part of a new reprint collection, designated NBS Special Publication 300, which has been planned to fill this need. Besides previously published papers by the NBS staff, the collection includes selected abstracts by both NBS and non-NBS authors. It is hoped that SP 300 will serve both as a textbook and as a reference source for the many scientists and engineers who fill responsible positions in standards laboratories.

LEWIS M. BRANSCOMB, *Director.*

Preface

The general plan for this compilation has been reviewed by the Information Committee of the National Conference of Standards Laboratories. The plan calls for Special Publication 300 to be published in 11 volumes as presented on the inside of the front cover.

This division of subject matter has been chosen to assure knowledgeable selection of context rather than to attain uniform size. It is believed, however, that the larger volumes, of approximately 500 pages, will still be small enough for convenient handling in the laboratory.

The compilation consists primarily of original papers by NBS authors which have been reprinted by photoreproduction, with occasional updating of graphs or numerical data when this has appeared desirable. In addition, some important publications by non-NBS authors that are too long to be included, are represented by abstracts or references; the abstracts are signed by the individuals who wrote them, unless written by the author.

Each volume has a subject index and author index, and within each volume, contents are grouped by subtopics to facilitate browsing. Many entries follow the recent Bureau practice of assigning several key words or phrases to each document; these may be collated with titles in the index. Pagination is continuous within the volume, the page numbers in the original publications also being retained and combined with the volume page numbers, for example 100-10. The index notation 9-133 refers to volume 9, page 133 of this volume. A convenient list of SI (Système International) physical units and a conversion table are to be found inside the back cover.

The publications listed herein for which a price is indicated are available from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402 (foreign postage, one-fourth additional). Many documents in the various NBS non-periodical series are also available from the NBS National Technical Information Service, Springfield, Va. 22151. Reprints from the NBS Journal of Research or from non-NBS journals may sometimes be obtained directly from an author.

Suggestions as to the selection of papers which should be included in future editions will be welcome. Current developments in measurement technology at NBS are covered in annual seminars held at either the Gaithersburg (Maryland) or the Boulder (Colorado) laboratories. These developments are summarized, along with a running list of publications by NBS authors, in the monthly NBS Technical News Bulletin.

H. L. MASON,
*Office of Measurement Services,
NBS Institute for Basic Standards.*

Editor's Note

This volume deals with the analysis and interpretation of precision measurement data in the field of Colorimetry. It is a compendium of work in these fields published during the decade nominally 1957 to 1967 by personnel of the National Bureau of Standards. There is an attempt to include the high points of the research work, and to reference other works published during this period.

There are included in this volume reprints of 37 papers on Colorimetry with the associated fields of Spectrophotometry and Appearance Evaluation.

The Colorimetry papers are classified in seven sections. Section eight has a chronological listing of all papers published by personnel employed in Colorimetry at the National Bureau of Standards. The interpretation foreword that appears just before each of the papers sections helps to characterize that section.

Thanks are due to editors and publishers of journals and books, in which papers written by NBS researchers appear, for permission to reproduce these documents.

I. NIMEROFF, *Editor*.

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Foreword

Precise judging of color is possible through the standard observer system for colorimetry recommended since 1931 by the International Commission on Illumination. As this standard observer system is primarily a spectral specification of the spectral response characteristics of human observers with normal color vision, the development of precision spectrophotometry is essential to good colorimetry. This chapter, therefore, contains a selection of papers on Spectrophotometry published since Handbook 77 appeared.

In order to achieve precise spectral measurements, standards are required to aid in the calibration of spectrophotometers. The first two papers in this section describe the calibration and use of glass standards of spectral transmittance issued by the National Bureau of Standards. The first paper, by Keegan, Schleter, and Judd, is concerned with filters for checking the performance of systems that consist of a spectrophotometer equipped with tristimulus integrators. The spectral transmittance of the filters have been corrected for the known errors: wavelength scale, zero and 100-percent scales, slit-width, inertia, back-reflectance, and stray-energy.

The next two papers, written by Deane B. Judd, treat reflectance spectrophotometry and the terminology required in precision spectral measurement of opaque, reflecting materials. The paper on nomenclature aids in understanding reflectometry and in formulating proper conditions for measurement.

The final document in this section, by Schleter, describes, in some detail, the standards issued by the National Bureau of Standards for checking the calibration of spectrophotometers in the spectral range, 200 to 1000 nm. It discusses the standards found useful for checking wavelength scale and photometric scale of recording and non-recording spectrophotometers. Rather complete data for the absolute spectral reflectance (6° -hemispherical) of smoked magnesium oxide and the spectral reflectance factor of standards made of white structural glass, called Vitrolite, relative to a perfect diffuser and to MgO.

Glass Filters for Checking Performance of Spectrophotometer-Integrator Systems of Color Measurement

Harry J. Keegan, John C. Schleter, and Deane B. Judd

(January 16, 1962)

A set of five specially selected colored-glass filters to identify variables of malfunction of photoelectric recording spectrophotometers equipped with tristimulus integrators have been standardized on a number of spectrophotometers corrected for all known errors (wavelength, zero, 100 percent, slit-width, inertia, back-reflectance, and stray-energy). To these standardized spectrophotometric data definite amounts of these errors were deliberately introduced and converted to tristimulus values and chromaticity coordinates of the International Commission of Illumination system of colorimetry for Sources *A*, *B*, and *C*. Similar reductions show the effects of slit widths of 1, 5, 10, and 15 millimicrons ($m\mu$) on computed results both by the selected-ordinate method of 10, 30, and 100 ordinates, and by the weighted-ordinate methods of 1-, 5-, 10-, and 15- $m\mu$ intervals. Duplicate sets of these glasses have been evaluated by visual comparison with this set of master standards, and are available as part of the Standard Materials Program of the National Bureau of Standards. By comparing the certified values of luminous transmittance and chromaticity coordinates for a set of these glasses with the values obtained on a particular integrator-spectrophotometer combination, the type and extent of instrumental errors may be evaluated.

1. Introduction

In 1956, Dr. Nathaniel H. Pulling, Instrument Department, General Electric Co., West Lynn, Mass., proposed that the National Bureau of Standards develop a set of four or five non-light-scattering glasses to serve as standards of tristimulus values to check performance of colorimeters, particularly the automatic spectrophotometer equipped with integrator. At that time it was believed that such a set of filters might consist of a selenium red, a selenium yellow, a dense cobalt, and a nearly non-selective glass of 15 to 20 percent transmittance. Although the certified tristimulus values would be intended to be those corresponding to an extrapolation to "zero slit width" (the band pass of a spectrophotometer so small that further reduction will not affect the photometric value), the glasses might serve also to check the adjustment of an integrator attached to a spectrophotometer with 10- $m\mu$ slits by giving in the paper describing the development of the standards the tristimulus values of the master standards not only for zero slit width, but also those found for 4- $m\mu$ and 10- $m\mu$ slits.

Because there were no accepted means in industry for checking the performance of spectrophotometer-integrator systems, two meetings were held at the National Bureau of Standards with representatives of manufacturers of spectrophotometers, glass manufacturers, and industrial users of spectrophotometers with tristimulus integrators. At the meeting held on September 11, 1957, fifteen representatives of industry attended and confirmed the need for glass standards to check spectrophotometer-integrator systems. A second meeting on May 14, 1958 reviewed a selection of five types of filters and approved of their standardization. Typical spectrophotometric curves of the five glasses selected for study are shown in figure 1.

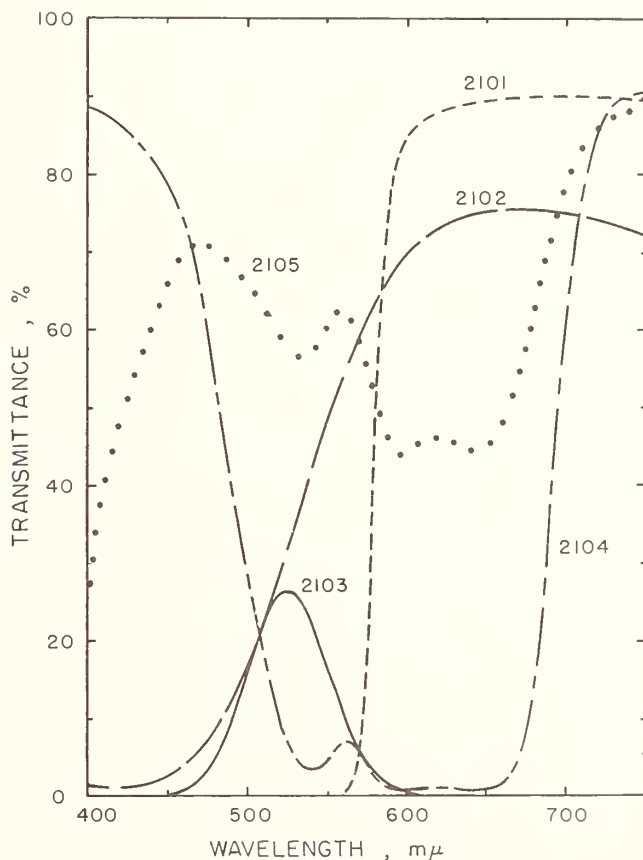


FIGURE 1. Spectral transmittance of the five NBS standard materials for checking spectrophotometer-integrator systems. 2101, Selenium Orange Red; 2102, Signal Yellow; 2103, Sextant Green; 2104, Cobalt Blue; 2105, Selective Neutral.

Materials. Two hundred 2×2-in. polished squares of each of five types of glass, each group of 200 squares being from the same melt, were purchased from Corning Glass Works. Our designations for the five types of glass, the Corning designation, and thickness are as follows:

2101, *Selenium orange red*, Corning 3480, 2.6 mm.

2102, *Signal yellow*, Corning 3307, 2.6 mm.

2103, *Serfant green*, Corning 4010, 4.4 mm.

2104, *Cobalt blue*, Corning 5551, 2.6 mm.

2105, *Selective neutral*, special glass developed by

Corning for this project, Corning 5045, 2.9 mm.

Three sets of these five glasses were arbitrarily selected and designated Master No. 1, Master No. 2, and Master No. 3. Later two sets of limit glasses for each of the five glasses were selected as described in 2.3 below.

2. Method

2.1. Spectrophotometry of Master Standards

Measurements of spectral transmittance were made on each of the three master standards and the two limit standards on the Cary Model 14 and the General Electric [1, 2]¹ recording spectrophotometers, with check measurements on Master No. 3 set on the Beckman DU indicating spectrophotometer and on the König-Martens visual spectrophotometer.

The methods of using three of these spectrophotometers have been previously described [3, 4, 5]. The methods of using the Cary spectrophotometer are described in detail in section 2.1.a.

Additional corrections made on the GE spectrophotometer data are explained in section 2.1.b.

Details of the measurements on the Beckman DU and the K-M spectrophotometers are explained in section 2.1.c.

a. Cary Recording Spectrophotometer

Measurements. Spectral-transmittance measurements were made on each of fifteen glass filters comprising three sets of five master filters designated Master No. 1, No. 2, and No. 3, on a Cary Model 14M (Serial No. 173) recording spectrophotometer at the National Bureau of Standards for the following conditions:

(1) Temperature control. Measurements were made under conditions of controlled temperature on both the laboratory (25 °C) and in the sample compartment of the spectrophotometer (22 °C). The same temperature-controlled water which cooled the sample compartment was circulated around the monochromator to assist in maintaining the wavelength stability of the instrument.

(2) Scanning rate and chart speed. The monochromator scan was driven at a speed of 0.25 mμ/sec and the chart speed of the recorder was 5 in./min. This combination of scanning rate and chart speed resulted in recordings having a wavelength scale of 1 mμ/chart division. The slow scanning rate was chosen to reduce the inertia effect in the recording mechanism during the recording of steep portions of the transmittance curves.

(3) Wavelength calibration. The wavelength scale of the spectrophotometer, as it is recorded on the chart at the above scan rate and chart speed, was calibrated by means of the emission lines of a mercury source. It was found that by applying a correction of +0.2 mμ over the range 380 to 770 mμ in reading the spectral-transmittance data of the filters from the recordings the wavelength-scale errors were satisfactorily taken into account.

(4) Slide wire. The recorder of the spectrophotometer was equipped with a density ($-\log T$) slide wire. The recorder covers the density range of 0 to 2 using two pens, one recording the density range 0 to 1 (100% T to 10% T); the other, the range 1 to 2 (10% T to 1% T). The transfer from the first pen to the second pen is automatic. For some of the filters, such as the 2101 orange-red, the density of the filter exceeds density 2. In these cases, photoetched nickel screens were introduced into the comparison beam of the spectrophotometer to extend the range. Two such screens were used, each having a density slightly greater than density 1, which thus allowed the measurements to be made to approximately density 4. Measurements were made of the spectral transmittance of the screens over the entire wavelength range.

(5) Zero curve. The zero-density curve (100% T) on the Cary can be adjusted by means of potentiometers so as to read approximately zero at all wavelengths regardless of the source-detector combination used. Since it is not possible to make the adjustment agree perfectly at all wavelengths, a zero-density curve was run for the entire wavelength range being used, before and after a series of measurements on the filters.

(6) Slit width. The physical slit width of the monochromator during the measurement of the filters was 0.1 mm or less for densities less than 2. From the dispersion curve of the monochromator supplied by the manufacturer, it was determined that the band pass of the monochromator for these slit widths varied from approximately 0.3 to 0.4 mμ for the wavelength range covered. As the screens are introduced into the comparison beam, the slit width of the monochromator increases. It is estimated that the band pass was approximately 1 mμ or less for the density range 0 to 3.

(7) Source-detector. Two source-detector combinations were used for these measurements of spectral transmittance. For the wavelength range 360 to 600 mμ, the "visible" tungsten source and 1P28 multiplier phototube were used. The "infrared" tungsten source and lead sulfide cell were used for the wavelength range 550 to 790 mμ.

Data reduction. The recordings of spectral transmittance of the filters and the screens were read at each 1-mμ interval from 360 to 790 mμ. The wavelength correction of +0.2 mμ was applied at each wavelength read. The zero-density curves run before and after the particular measurement of a filter or screen were read and averaged, and subtracted from the reading of the filter or screen. Where screens were used to increase the photometric range of the spectrophotometer, the density of the

¹ Figures in brackets indicate the literature references at the end of this paper.

screen at that wavelength was added to that recorded for the filter. In the wavelength region 550 to 600 $m\mu$, where data were taken both with the multiplier phototube and with the lead sulfide cell, an average of the two sets of data was tentatively used. After the corrections for the zero-density curve and the screens had been made, the data were coded for introduction into the IBM 704 high-speed digital computer for conversion from a logarithmic to a linear scale.

It was found that the data taken with the lead sulfide cell differed slightly but systematically from the data taken with the multiplier phototube in the wavelength region 550 to 600 $m\mu$ where the two sets of data overlapped. Although the average data in this wavelength region were used for some of the preliminary computations, corrected data from the lead sulfide cell were used for the finally adopted data. The correction to the lead sulfide cell data was made by taking the density difference between the multiplier phototube and the lead sulfide cell data at 600 $m\mu$ and applying this difference to the density data from the lead sulfide cell for the wavelength range 610 to 770 $m\mu$. The multiplier-phototube data were used for the wavelength range 380 to 600 $m\mu$. The corrected data for the lead sulfide cell were converted from a logarithmic to a linear scale and combined with the multiplier phototube data to form the adopted data for the Cary.

b. General Electric Spectrophotometer

Comparison of the spectral transmittances of the five glasses of master set No. 3 obtained on the GE-II² recording spectrophotometer, after routine zero, 100 percent, and wavelength-scale corrections had been applied, with the average spectral transmittances obtained for the same glasses on the Cary-14, the Beckman DU, and the König-Martens spectrophotometers showed small but regular discrepancies. Examination of these discrepancies suggested that they are ascribable to the combined effects of error sources already identified, but hitherto often regarded individually as yielding errors in general either nearly or completely negligible. These sources of error are of three kinds and will be hereafter identified as slit-width errors, inertia errors, and back-reflectance errors.

Slit-width errors. The slit function of the GE-II spectrophotometer is approximated closely by an isosceles triangle whose apex is at the wavelength, λ , and whose base extends from $\lambda-10$ $m\mu$ to $\lambda+10$ $m\mu$, giving a width at half height equal to 10 $m\mu$ [6]. By slit-width cams this width is maintained nearly constant independent of wavelength [7]. A simple formula for correction of slit-width errors may be derived by assuming the correction to be proportional to the second derivative of the spectral-transmittance (T_λ) function of the filter, and by approximating this second derivative by the difference between twice the reading, R'_λ , at wavelength λ and the sum of the readings $R'_{\lambda-10}$ and $R'_{\lambda+10}$, pre-

ceding and following this wavelength by 10 $m\mu$. The formula:

$$\begin{aligned} T_\lambda &= R'_\lambda + (2R'_\lambda - R'_{\lambda-10} - R'_{\lambda+10})/10 \\ &= 1.2R'_\lambda - 0.1R'_{\lambda-10} - 0.1R'_{\lambda+10} \end{aligned} \quad (1)$$

is equivalent to the first terms of the expansion form quoted by Gibson [6] from Forsythe [7] with K taken as 10 instead of 12. The terms, R'_λ , $R'_{\lambda-10}$, and $R'_{\lambda+10}$, refer to values obtained from the GE-II spectrophotometer with all corrections applied except that for slit-width error.

Since the 5 glasses of master set No. 3 had been measured at each millimicron throughout the visible spectrum by means of the Cary-14 spectrophotometer with slit widths not exceeding 1 $m\mu$ for transmittances greater than 1 percent, it was possible to compute the values of spectral transmittance that would have been read by an instrument having slit widths of any value greater than 1 $m\mu$. Such computations were made for a triangular slit function with a width of 10 $m\mu$ at half height, and the differences between the resulting values and those for the narrow slits of the Cary spectrophotometer were applied to the GE-II readings as slit-width corrections. It was noted, however (see columns 3 and 4 of table 1), that approximately the same corrections were obtainable from formula (1); so this formula is a convenient short way to state the sizes of the slit-width corrections applied, and is also a satisfactory simple statement of slit-width corrections to be applied to other specimens measured on the GE-II spectrophotometer.

Inertia errors. The recording mechanism of the automatic GE-II spectrophotometer is controlled by positive and negative impulses operating on the pen assembly (driving motor, gears, and pen holder) which has appreciable inertia and friction. This mechanism gives a dynamic evaluation of the transmittance at wavelength λ ; that is, the transmittance-indicating position of the pen at the time, t , when the specimen is being illuminated by light of wavelength, λ , is based on signals received at a time, $t-c$, somewhat before wavelength, λ , is reached. The pen position at time, t , is thus found by extrapolation over the time interval, c . This extrapolation process depends on the strength and frequency of the impulses relative to the inertia and friction of the pen assembly. On a rising curve the pen assembly may coast past the correct value because of inertia, or it may lag behind because of friction. The discrepancy between the true value of the spectral transmittance and the value recorded by extrapolation may be called the inertia error. Let us assume that if $R_o(t)$ is the correct reading at time, t , the actual reading $R(t)$, will, because of inertia error, be a function not only of $R_o(t)$, but also of the velocity of the pen assembly at time, $t-c$, just previous to time, t , of the following simple form:

$$R(t) = R_o(t) + k(dR/dt)_{t-c}$$

where c and k are constants to be evaluated from a consideration of $R-R_o$ for known specimens.

² "GE-II" indicates the second NBS-GE spectrophotometer, GE Serial No. 732986.

TABLE 1. Spectral transmittances of master set No. 3 orange-red glass filter 2101

Obtained by applying the routine zero, 100%, and wavelength-scale corrections to the readings of the GE-II spectrophotometer; the slit-width, inertia, and back-reflectance corrections; the results of applying these three corrections; and comparison of uncorrected and corrected GE-II results to the average spectral transmittances of the same glass filter obtained by three other spectrophotometers (Cary-14, Beckman DU, König-Martens)

Wavelength	Spectral transmittance GE-II with routine corrections	Corrections				Sum (4) (5) (6)	Spectral transmittance		Differences	
		Slit-width		Inertia	Back-reflectance		GE-II with all corrections	Mean of Cary, Beckman, and K-M	(8)-(9)	(2)-(9)
		Eq 1	Experimental							
1	2	3	4	5	6	7	8	9	10	11
<i>mμ</i>										
550	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
560	.05	-.96	-.19	.00	.00	-.19	.00	.00	.00	.05
570	9.70	-3.19	-3.93	-.68	-.03	-4.64	5.06	4.67	.39	5.03
580	51.30	+1.44	+1.99	-2.91	-.08	-1.00	50.30	48.44	1.86	2.86
590	78.51	+2.03	+1.82	-1.90	-.19	-0.27	78.24	77.12	1.12	1.39
600	85.43	+0.49	+0.44	-0.48	-.23	-.27	85.16	84.48	0.58	0.95
610	87.45	+ .11	+ .09	-.14	-.24	-.29	87.16	86.62	.54	.83
620	88.31	+ .03	+ .10	-.06	-.24	-.20	88.11	87.97	.14	.34
630	88.95	+ .01	+ .09	-.04	-.24	-.19	88.76	88.60	.16	.35
640	89.45	+ .02	+ .01	-.04	-.25	-.28	89.17	89.06	.11	.39
Average -----									0.49	1.22

The correction, $R_0 - R$, to be added to R to obtain the true value, R_0 , may be estimated with some reliability by taking $-k(R_\lambda - R_{\lambda-2c})$ from the recorded curve. This shift from the time scale to the wavelength scale is justified because the instrument scans the spectrum in the direction of increasing wavelength with a constant speed. The assumption that the inertia of the pen assembly has a significant regular influence on the reading $R(t)$ does not involve a decision between a tendency of the assembly to lag behind the correct reading or jump ahead of it. This decision has to be made in the course of evaluating the constant, k , from a consideration of $R - R_0$ for known specimens. If k is found to be greater than zero the inertia correction refers to a correction for lag; if less than zero, to a correction for lead.

The empirical evaluation was based partly on data taken several years ago by Keegan [8] (1956) and partly on the present data for the five glasses of master set No. 3. The best fit to the 1956 data was found by setting $c = 5 \text{ m}\mu$, and k either at -0.05 or -0.07 . The best fit to the present data was found for $c = 5 \text{ m}\mu$, $k = -0.07$. Note that both sets of data indicate k less than zero. The inertia correction for the GE-II spectrophotometer is thus a lead correction. If R_λ and $R_{\lambda-10}$ are the uncorrected readings of the spectrophotometric curve plotted by the instrument, the formula for spectral transmittance, T_λ , with this evaluation of constants becomes

$$\begin{aligned} T_\lambda &= R_\lambda - 0.07(R_\lambda - R_{\lambda-10}) \\ &= 0.93R_\lambda + 0.07R_{\lambda-10} \end{aligned} \quad (2)$$

on the assumption that all other corrections are negligible. Optimally this inertia correction should be made first, followed by the zero correction, the 100 percent correction, the wavelength-scale correction, the slit-width correction, and finally the back-reflectance correction, but in practice these corrections are sufficiently small that no significant addi-

tional error is introduced by changes in the order of applying them.

Back-reflectance errors. Through the specimen compartment of the GE-II spectrophotometer, two divergent beams pass from the decentered lenses to the entrance ports of the integrating sphere [2]. The axis of each beam makes an angle of about 6° with the optical axis of the instrument, one to the right, the other to the left. The specimen filter whose spectral transmittance is to be measured is inserted in one of these beams (the specimen beam) so that the axis of the beam is perpendicular to the faces of the filter. All of the flux reflected from the front face of the filter and a fraction of the flux reflected from the back face contributes to a reflected beam directed back toward the decentered lens. Since this reflected beam is also diverging, part of it may, and as a matter of fact does, reach the decentered lens transmitting the reference beam, as was pointed out by Middleton [9], and enters the sphere through the entrance port of the reference beam. This added flux, some of which has never passed through the specimen, contributes to the illuminance of the sphere during the specimen phase of the cycle and causes a spuriously high indication of the spectral transmittance of the specimen. This error is what is meant by back-reflectance error. It could be eliminated by insertion of an inconvenient vertical partition between the specimen compartment and the decentered lenses; but since the error for non-metallized glass filters is of the order of one or two tenths of one percent of the full scale, it has heretofore been considered negligible.

By obtaining the reading of the spectral transmittance of a clear plate first inserted perpendicular to the specimen beam, and second tilted until no part of the reflected beam crosses over to the other side of the instrument, it was found that the maximum error from this source for a glass of refractive index equal to 1.5 is 0.0026. By taking into account the fact that as the transmittance decreases from 0.923

to zero, the reflectance of the specimen filter declines from 0.079 to 0.040, and that the photometer scale is adjusted to be correct at 0.10, the following expression for the reflection correction accurate to 0.0001 was derived:

$$\text{Back-reflectance correction} = -0.0031 R_{\lambda}^2, \quad (3)$$

where R_{λ} is defined in formula (2).

This correction was applied to all of the data on the glasses of master set No. 3 obtained on the GE-II spectrophotometer.

To indicate the degree to which the three often neglected corrections (slit-width, inertia, back-reflectance) succeeded in accounting for the regular deviations of the results by the GE-II spectrophotometer from the mean of those by three other spectrophotometers (Cary-14, Beckman DU, König-Martens), table I has been prepared for the selenium orange-red glass (2101) of master set No. 3 for the wavelength range 550 to 640 m μ . It may be seen from columns 10 and 11 that, by applying these three, often neglected corrections, both the maximum difference and the average difference between the GE-II results and the mean of results by three other spectrophotometers have been reduced by more than a factor of 2. The GE-II reads higher on glass filter 2101 than the other spectrophotometers, even after these negative corrections have been applied. The causes of these residual deviations (column 10) are not known. The GE-II with all known corrections applied agrees notably better with the other spectrophotometers in its measurement of the other four glasses of master set No. 3.

c. Beckman DU and König-Martens Spectrophotometers

Check measurements of spectral transmittance of the five master standards of set No. 3 were made at certain wavelengths on the Beckman DU and the K-M spectrophotometers as follows:

On Beckman DU spectrophotometer

- (1) Glass 2101 on "absolute basis" 560 to 750 m μ at every 10 m μ .
- (2) Glass 2102 relative to spectrophotometric standard Corning HT yellow [5, 6] at wavelengths at which the standard is specified.
- (3) Glass 2103 on "absolute basis," 450 to 610 m μ at every 10 m μ .
- (4) Glass 2104 relative to spectrophotometric standard cobalt blue [5, 6] at wavelengths at which standard is specified.
- (5) Glass 2105 on "absolute basis," 400 to 750 m μ at every 10 m μ .

On K-M spectrophotometer

- (1) Glass 2101 at the wavelengths 560, 578 (Hg line), and 620 m μ .
- (2) Glass 2102 at wavelengths 436 (Hg line) and 620 m μ .
- (3) Glass 2103 at wavelengths 460, 520, 600, 610, and 620 m μ .
- (4) Glass 2104 at wavelengths 595 and 645 m μ .
- (5) Glass 2105 at wavelengths 470, 530, 595, 620, and 640 m μ .

Stray-energy filters used for the Beckman DU spectrophotometer were Corning 9863 for 320 to 400 m μ , Corning 2424 with blue-sensitive cell from 600 to 660 m μ and Corning 3965 with red-sensitive cell. Similarly appropriate stray-energy filters were used on the K-M.

2.2. Evaluation of Errors

In order to evaluate errors in the computation of tristimulus values and chromaticity coordinates due to (1) neglect of slit-width corrections, (2) the use of summation-intervals of various sizes in the weighted-ordinate method, and (3) the use of various numbers of ordinates in the selected-ordinate method, a computer program was prepared for the IBM 704 high-speed digital computer.

This program converts data of spectral transmittance into colorimetric terms for slit-widths of 1 (near-zero), 5, 10, and 15 m μ by (a) the weighted-ordinate method for 1-, 5-, 10-, and 15-m μ summation intervals, and (b) the selected-ordinate method for 10, 30, and 100 ordinates. The conversions can be made for any Planckian source from approximately 1,000 °K to 10,000 °K including CIE standard source, *A*, and for CIE sources *B*, and *C*. Published values of the tristimulus functions ($\bar{x}, \bar{y}, \bar{z}$) for each 1-m μ interval from 380 to 770 m μ and the wavelength of the selected ordinates for 10, 30, and 100 ordinates are used in the program [10].

The input data for this program are values of spectral transmittance for each 1-m μ interval from 360 to 790 m μ measured on an instrument with a band pass so small that further reduction will not affect the photometric value. The data must be corrected for wavelength, zero curve, and 100 percent curve instrumental errors before introduction into the program.

a. Neglect of Slit-Width Corrections

The effect of the change in slit width on the 1-m μ (near zero) slit-width, spectral-transmittance, input data were computed by using a triangular slit-width weighting function of the type:

$$T_{\lambda} = \left(\sum_{i=0}^n [(n-i)T_{\lambda-i} + (n-i)T_{\lambda+i}] - nT_{\lambda} \right) / n^2, \quad (4)$$

where T_{λ} is the spectral transmittance at wavelength λ , n is the nominal slit width, and i is a wavelength difference less than or equal to n . The spectral-transmittance data were computed for each 1-m μ interval from 380 to 770 m μ for $n=5, 10$, and 15 . The colorimetric coordinates X, Y, Z, x, y were computed for the spectral transmittance data for 1-, 5-, 10-, and 15-m μ slit widths using the weighted-ordinate method for a 1-m μ summation interval. The colorimetric coordinates indicate the effect to be expected when the glass filters are measured on spectrophotometers having slit widths larger than 1 m μ .

b. Use of Summation Intervals of Various Sizes in the Weighted-Ordinate Method

From the spectral-transmittance input data for 1-m μ slit width and the spectral-transmittance data computed from them for 5-, 10-, and 15-m μ slit widths, the colorimetric coordinates X, Y, Z, x, y were computed by the weighted-ordinate method from every 5th, 10th, and 15th value of spectral transmittance, tristimulus function, and spectral irradiance of the source. The resulting colorimetric coordinates indicated the change which would be expected by using summation intervals of 5, 10, and 15 m μ with the weighted-ordinate method.

c. Use of Various Numbers of Ordinates in the Selected-Ordinate Method

The colorimetric coordinates were then computed for the spectral transmittance data for the four slit widths by means of the selected-ordinate method for 10, 30, and 100 ordinates. The correct values of spectral transmittance at the wavelength indicated by the selected-ordinates used were computed by third-difference osculatory interpolation [11]. The resulting colorimetric coordinates indicated the change which could be expected by using the selected-ordinate method for 10, 30, and 100 ordinates.

2.3. Colorimetry of Duplicates

From the 200 squares of each type of glass, 100 were chosen by visual inspection for issuance as duplicate standards. The criteria were freedom from seeds, bubbles, striae, scratches, chips, and other visually detectable defects.

Although the 100 glass squares of each type of glass so chosen were taken from the same melt, visual inspection by diffused light from daylight fluorescent lamps revealed small color differences among them. By means of these differences it was possible to arrange the 100 glasses in an essentially one-dimensional sequence, and from this sequence the terminal glasses (called limit filters) were chosen to indicate the color range of the group as follows:

Orange red.....weak limit (WL) and strong limit (SL).
Yellow.....weak limit (WL) and strong limit (SL).
Green.....yellow limit (YL) and blue limit (BL).
Blue.....light limit (LL) and dark limit (DL).
Neutral.....yellow limit (YL) and blue limit (BL).

The limit filters so selected were measured on the General Electric, and Cary spectrophotometers, and on the Barnes, Gardner, Hunter, Colormaster, and Judd CDC (chromaticity difference colorimeter) colorimeters. An analysis of the data so obtained showed that smaller uncertainties would be obtained if no photoelectric colorimeter was used for the duplicate standards. Accordingly, measurements were made on the 100 glasses in each of the red, yellow, and blue sets by means of the CDC visual colorimeter.

The chromaticity-difference ($\Delta x, \Delta y$) data so obtained showed that the color differences among the yellow and blue glasses were ascribable to small

variations in the thicknesses of the glasses. The measured thicknesses of the glasses were used to find adopted values of X, Y, Z, x, y . As expected from the dependence of the color of selenium glasses on annealing temperature, the orange-red glasses showed no correlation of chromaticity with thickness. The values of luminous transmittance were, however, inferred from the measured chromaticity coordinates, on the assumption that all orange-red glasses contained the same absorbing material though produced in varying amounts because of differences in annealing temperature. This assumption was checked by measurement of the luminous transmittance of five of the duplicate orange-red glasses on the Martens photometer.

3. Results

3.1. Spectral Transmittances of Master Set No. 3

Tables 2 through 6 show for every 10 m μ the results of measuring the spectral transmittances of the five glasses of master set No. 3 and applying the routine corrections. Table 7 shows the values of spectral transmittance obtained at the wavelengths of the emission lines of mercury and helium.

TABLE 2. Spectral transmittance of glass filter 2101, master set No. 3, as measured on the indicated spectrophotometers

Wavelength	Cary Model 14	General Electric	Beckman DU	König-Martens
m μ				
380	0.000			
90	.000			
400	.000	0.000		
10	.000	.000		
20	.000	.000		
30	.000	.000		
40	.000	.000		
450	.000	.000		
60	.000	.000		
70	.000	.000		
80	.000	.000		
90	.000	.000		
500	.000	.000		
10	.000	.000		
20	.000	.000		
30	.000	.000		
40	.000	.000		
550	.000	.000		
60	.001	.000	0.001	0.001
70	.048	.097	.046	
80	.478	.513	.479	
90	.774	.785	.768	
600	.846	.854	.844	
10	.866	.874	.866	
20	.876	.883	.880	.880
30	.883	.889	.886	
40	.886	.894	.892	
650	.888	.897	.895	
60	.891	.900	.898	
70	.893	.901	.900	
80	.893	.903	.901	
90	.893	.903	.901	
700	.894	.903	.901	
10	.893	.903	.901	
20	.894	.902	.900	
30	.894	.901	.900	
40	.894	.900	.899	
750	.891	.899	.896	
60	.891			
70	.889			

3.2. Corrections Developed From Present Study

a. Lead Sulfide Cell Corrections for Cary Data

The results of the measurements, given above, for the Cary indicate the data obtained by the lead sulfide cell without corrections. The values corrected for discrepancies between the data taken with the multiplier phototube and the lead sulfide cell are higher by the factor 1.003 from 600 to 770 $m\mu$.

b. Slit Width, Inertia, and Back-Reflectance on GE Data

The data listed in tables 2 to 6 show for the General Electric recording spectrophotometer the results of the measurements taking into consideration only the routine corrections for 100 percent, zero, and wavelength scale. In table 8 are listed the results from the GE corrected for the above routine errors plus the corrections for inertia, slit width, and back reflectance.

3.3. Derivation of Adopted Values of Spectral Transmittance of Master Set No. 3

The finally adopted data of spectral transmittance for the five filters of master set No. 3 were derived as a weighted mean of three sets of measurements:

TABLE 3. Spectral transmittance of glass filter 2102, master set No. 3, as measured on the indicated spectrophotometers

Wavelength	Cary Model 14	General Electric	Beckman DU	König-Martens
$m\mu$				
380	0.024			
90	.017		0.018	
400	.013	0.013		
10	.011	.012		
20	.011	.011	.011	
30	.013	.012		
40	.017	.017		
450	.025	.025		
60	.038	.040		
70	.058	.060		
80	.087	.088		
90	.123	.127		
500	.171	.174		
10	.226	.229		
20	.287	.292	.287	
30	.353	.358	.352	
40	.419	.424		
550	.482	.488		
60	.539	.546	.540	
70	.591	.598		
80	.634	.640		
90	.669	.675		
600	.695	.700	.697	
10	.716	.720		
20	.731	.733	.732	0.729
30	.741	.742		
40	.749	.749	.750	
650	.752	.753		
60	.754	.755	.756	
70	.755	.756		
80	.753	.755		
90	.751	.753	.754	
700	.750	.750		
10	.746	.746		
20	.741	.742	.745	
30	.734	.736		
40	.729	.729		
750	.722	.722	.725	
60	.714			
70	.705			

(1) the Cary data with lead sulfide cell correction, (2) the GE with routine corrections and corrections for inertia, slit width, and back reflectance, and (3) the Beckman DU. Where measured data were lacking, as in the case with the Beckman DU, values of spectral transmittance were interpolated. The weights used for the data were 4 (Cary), 3 (GE), and 3 (Beckman DU) for the filters designated 2101, 2102, 2103, and 2105. In the case of the filter designated 2104, however, little data were taken by the Beckman DU and the assigned weights were 4 (Cary), 4 (GE), and 2 (Beckman DU). The adopted weighted mean data of spectral transmittance for the five filters of master set No. 3 are listed in table 9.

3.4. Derivation of Adopted Tristimulus Values and Chromaticity Coordinates

The spectral transmittance data listed in table 9 were processed by means of an IBM 704 high-speed digital computer which converts spectral-transmittance data into colorimetric terms for 10- $m\mu$ summation intervals by means of the tristimulus functions, $\bar{x}, \bar{y}, \bar{z}$, adopted by the CIE in 1931. What is desired are tristimulus values and chromaticity coordinates based on the CIE tristimulus functions

TABLE 4. Spectral transmittance of glass filter 2103, master set No. 3, as measured on the indicated spectrophotometers

Wavelength	Cary Model 14	General Electric	Beckman DU	König-Martens
$m\mu$				
380	0.000			
90	.000			
400	.000	0.000		
10	.000	.000		
20	.000	.000		
30	.000	.000		
40	.000	.000	0.000	
450	.002	.000	.002	
60	.007	.008	.007	0.008
70	.023	.024	.022	
80	.051	.053	.049	
90	.098	.102	.095	
500	.161	.164	.159	
10	.223	.225	.217	
20	.262	.259	.260	.263
30	.260	.257	.262	
40	.222	.218	.224	
550	.164	.159	.166	
60	.105	.103	.107	
70	.058	.058	.061	
80	.029	.028	.031	
90	.013	.012	.014	
600	.005	.005	.006	.005
10	.002	.000	.002	.002
20	.001	.000	.001	.001
30	.000	.000		
40	.000	.000		
650	.000	.000		
60	.000	.000		
70	.000	.000		
80	.000	.000		
90	.000	.000		
700	.000	.000		
10	.000	.000		
20	.000	.000		
30	.000	.000		
40	.000	.000		
750	.000	.000		
60	.000			
70	.000			

TABLE 5. *Spectral transmittance of glass filter 2104, master set No. 3, as measured on the indicated spectrophotometers*

Wavelength	Cary Model 14	General Electric	Beckman DU	König-Martens
<i>mμ</i>				
380	0.887			
90	.894		0.904	
400	.889	0.891		
10	.873	.880		
20	.861	.864		
30	.839	.844		
40	.815	.818		
450	.783	.784		
60	.740	.738		
70	.658	.649		
80	.532	.522		
90	.390	.388		
500	.284	.280		
10	.180	.180		
20	.103	.102	.104	
30	.048	.053		
40	.033	.037	.036	
550	.044	.048		
60	.069	.069	.070	
70	.060	.058		
80	.026	.025		
90	.009	.009		
600	.008	.008	.009	
10	.010	.010		
20	.011	.011	.012	
30	.010	.010		
40	.008	.008	.008	
650	.009	.008		
60	.017	.017		
70	.048	.050		
80	.145	.149	.150	
90	.360	.365	.350	
700	.606	.614		
10	.766	.776		
20	.849	.859	.856	
30	.887	.892		
40	.900	.904		
750	.904	.910	.907	
60	.904			
70	.906			

TABLE 6. *Spectral transmittance of glass filter 2105, master set No. 3, as measured on the indicated spectrophotometers*

Wavelength	Cary Model 14	General Electric	Beckman DU	König-Martens
<i>mμ</i>				
380	0.022		0.024	
90	.088		.089	
400	.247	0.284	.253	
10	.401	.418	.406	
20	.484	.496	.487	
30	.544	.557	.547	
40	.602	.615	.608	
450	.657	.664	.659	
60	.695	.701	.696	
70	.708	.713	.713	0.717
80	.700	.703	.705	
90	.680	.683	.684	
500	.658	.659	.662	
10	.624	.626	.629	
20	.590	.592	.594	
30	.565	.568	.566	.565
40	.570	.575	.568	
550	.603	.609	.605	
60	.621	.620	.624	
70	.583	.579	.587	
80	.509	.504	.514	
90	.446	.452	.452	
600	.444	.449	.445	
10	.455	.460	.458	
20	.459	.463	.462	.462
30	.453	.457	.457	
40	.443	.448	.446	.448
650	.448	.453	.450	
60	.479	.484	.478	
70	.540	.546	.534	
80	.628	.633	.623	
90	.717	.721	.714	
700	.788	.794	.788	
10	.831	.839	.832	
20	.857	.864	.860	
30	.872	.878	.874	
40	.880	.886	.883	
750	.886	.891	.889	
60	.890			
70	.895			

TABLE 7. *Supplementary spectral transmittance measurements of master set No. 3 on the indicated filters and spectrophotometers*

Wave-length	Element	Filter 2101			Filter 2102			Filter 2104			Filter 2105		
		Cary* Model 14	Beckman DU	König-Martens	Cary* Model 14	Beckman DU	König-Martens	Cary* Model 14	Beckman DU	König-Martens	Cary* Model 14	Beckman DU	König-Martens
<i>mμ</i>													
404.7	Mercury				0.012	0.013		0.882	0.893				
435.8	Mercury				.015	.015	0.015	.826	.833				
471.3	Helium				.061	.061		.646	.645				
491.6	Mercury							.365	.374				
501.6	Helium				.181	.179		.265	.270				
546.1	Mercury				.457	.456		.036	.038				
578.0	Mercury	0.374		0.393	.627	.625		.032	.032				
587.6	Helium				.663	.659							
595.0	(**)	.821	0.824					.007		0.008	0.440		0.441
645.0	(**)							.008		.008			
667.8	Helium							.038	.037				
706.5	Helium							.709	.726				

*The values given for the Cary are for comparison only. They are data taken for the following wavelengths: 405.0, 436.0, 471.0, 492.0, 502.0, 546.0, 578.0, 588.0, 595.0, 645.0, 668.0, and 706.0 *mμ*.

**Measured with the continuum of the tungsten source.

for 1 *mμ* summation intervals. These data were derived in the following manner. Tristimulus values and chromaticity coordinates were available from measurements of spectral transmittance made with the Cary for both 1- and 10-*mμ* summation intervals but for the Hardy values of the tristimulus functions.

The following derivation will serve as an example:

$$X'_1 - X'_{10} = \Delta X$$

$$X_{10} + \Delta X = X_1$$

where X'_1 and X'_{10} are the X tristimulus values com-

TABLE 8. *Corrected spectral transmittance of master set No. 3*

Measurements made on the General Electric spectrophotometer

Wavelength	Filter 2101	Filter 2102	Filter 2103	Filter 2104	Filter 2105
$m\mu$					
400	0.000	0.013	0.000	0.891	0.272
10	.000	.012	.000	.878	.415
20	.000	.011	.000	.863	.492
30	.000	.012	.000	.843	.552
40	.000	.016	.000	.819	.610
450	.000	.024	.000	.786	.661
60	.000	.038	.006	.743	.698
70	.000	.058	.022	.658	.712
80	.000	.086	.049	.532	.702
90	.000	.123	.097	.394	.684
500	.000	.170	.163	.289	.661
10	.000	.225	.223	.183	.627
20	.000	.287	.260	.107	.593
30	.000	.354	.261	.053	.566
40	.000	.419	.222	.036	.571
550	.030	.483	.163	.046	.606
60	.000	.541	.106	.071	.623
70	.051	.594	.061	.061	.583
80	.503	.637	.029	.025	.537
90	.782	.672	.012	.009	.449
600	.832	.696	.005	.008	.448
10	.872	.717	.000	.010	.459
20	.881	.731	.000	.011	.462
30	.888	.741	.000	.010	.457
40	.892	.747	.000	.008	.446
650	.895	.751	.000	.007	.449
67	.896	.753	.000	.015	.478
70	.898	.754	.000	.042	.539
80	.900	.753	.000	.131	.627
90	.900	.751	.000	.348	.714
700	.900	.749	.000	.606	.789
10	.900	.744	.000	.770	.835
20	.899	.740	.000	.855	.861
30	.898	.734	.000	.890	.875
40	.898	.727	.000	.901	.883
750	.896	.720	.000	.907	.889

puted from data taken on the Cary for the subscripted summation intervals, based on the tristimulus function of Hardy; X_{10} is the X tristimulus value computed from spectral transmittance data of the adopted weighted mean for 10 $m\mu$ summation intervals, based on the CIE tristimulus function; and X_1 is the derived X tristimulus value for the adopted weighted mean for 1 $m\mu$ summation interval based on the CIE tristimulus function. Similar derivations were made for the Y and Z tristimulus values and the chromaticity coordinates x, y, z were computed in the normal manner. The adopted tristimulus values and chromaticity coordinates are listed in table 10 for the filters of master set No. 3.

3.5. Estimates of Uncertainty Both for Master Set No. 3 and for the Duplicates

a. Master Set No. 3

The uncertainties of the tristimulus values of the five filters of master set No. 3 were estimated by computing the tristimulus values for each set of adopted values (one for each of the Cary, GE, and Beckman spectrophotometers) and taking the range of these computed values as the estimate. Table 11 shows these ranges, not only for the tristimulus values (X, Y, Z), but also for the chromaticity coordinates (x, y). These ranges correspond roughly to estimates of three times the standard deviations of the adopted values. [12].

TABLE 9. *Weighted mean spectral transmittance of master set No. 3*

Based on measurements from the Cary Model 14, General Electric, and Beckman DU spectrophotometers

Wavelength	Filter 2101	Filter 2102	Filter 2103	Filter 2104	Filter 2105
$m\mu$					
380	0.000	0.023	0.000	0.890	0.023
90	.000	.016	.000	.897	.086
400	.000	.013	.000	.892	.256
10	.000	.011	.000	.877	.407
20	.000	.011	.000	.863	.487
30	.000	.013	.000	.842	.547
40	.000	.017	.000	.818	.606
450	.000	.025	.002	.785	.659
60	.000	.038	.007	.742	.696
70	.000	.058	.022	.659	.711
80	.000	.087	.050	.533	.702
90	.000	.123	.097	.392	.682
500	.000	.170	.160	.286	.660
10	.000	.225	.221	.181	.626
20	.000	.287	.261	.105	.592
30	.000	.353	.261	.050	.566
40	.000	.420	.223	.035	.570
550	.000	.484	.164	.044	.605
60	.001	.540	.106	.069	.623
70	.049	.593	.059	.059	.584
80	.489	.637	.029	.025	.510
90	.776	.671	.013	.008	.449
600	.847	.696	.005	.008	.446
10	.869	.718	.002	.010	.458
20	.880	.732	.001	.011	.462
30	.887	.742	.000	.010	.456
40	.891	.750	.000	.008	.445
650	.893	.754	.000	.008	.449
60	.896	.756	.000	.016	.479
70	.898	.756	.000	.044	.539
80	.899	.755	.000	.140	.627
90	.899	.753	.000	.354	.716
700	.899	.752	.000	.607	.790
10	.899	.748	.000	.770	.834
20	.898	.743	.000	.854	.860
30	.898	.736	.000	.891	.874
40	.898	.730	.000	.902	.883
750	.895	.724	.000	.907	.889
60	.896	.716	.000	.907	.893
70	.894	.707	.000	.906	.897

TABLE 10. *Adopted tristimulus values X, Y, Z and chromaticity coordinates x, y, z for master set No. 3*

Source A						
Filter	X	Y	Z	x	y	z
2101	68.665	36.874	0.033	0.6504	0.3493	0.0003
2102	70.395	55.365	2.301	.5497	.4323	.0180
2103	3.197	9.074	1.302	.2355	.6685	.0959
2104	6.714	5.812	23.493	.1864	.1614	.6522
2105	54.893	53.882	23.069	.4163	.4087	.1750
Source B						
2101	50.977	28.293	0.028	0.6429	0.3568	0.0004
2102	56.302	50.760	4.369	.5053	.4555	.0392
2103	3.483	10.782	2.195	.2116	.6550	.1334
2104	12.832	7.975	59.742	.1593	.0990	.7417
2105	51.390	55.598	54.736	.3179	.3434	.3386
Source C						
2101	45.019	25.312	0.025	0.6399	0.3598	0.0004
2102	51.458	48.886	5.566	.4859	.4616	.0526
2103	3.556	11.304	2.642	.2032	.6459	.1510
2104	17.216	9.099	84.340	.1556	.0822	.7622
2105	51.830	56.142	75.444	.2826	.3061	.4113

TABLE 11. Range (ΔX , ΔY , ΔZ) for the tristimulus values of the indicated filters of master set No. 3; also ranges (Δx , Δy , Δz) for the chromaticity coordinates

Filter	Source A			Source B			Source C		
	ΔX	ΔY	ΔZ	ΔX	ΔY	ΔZ	ΔX	ΔY	ΔZ
2101	0.62	0.49	0.00	0.53	0.42	0.00	0.49	0.49	0.00
2102	.11	.02	.02	.07	.02	.05	.06	.01	.05
2103	.13	.13	.03	.12	.11	.08	.11	.10	.10
2104	.15	.23	.14	.18	.27	.35	.20	.28	.50
2105	.19	.20	.16	.20	.22	.43	.21	.22	.61
	Δx	Δy	Δz	Δx	Δy	Δz	Δx	Δy	Δz
2101	0.0009	0.0009	0.0000	0.0010	0.0010	0.0000	0.0011	0.0011	0.0000
2102	.0002	.0003	.0001	.0001	.0004	.0006	.0001	.0005	.0006
2103	.0053	.0040	.0036	.0048	.0035	.0046	.0046	.0041	.0051
2104	.0019	.0034	.0057	.0008	.0025	.0032	.0005	.0019	.0024
2105	.0003	.0004	.0007	.0004	.0007	.0012	.0004	.0008	.0012

b. Duplicates

The ranges found for the chromaticity coordinates of the duplicates by measurement of the limit filters were smaller than the uncertainties of the chromaticity coordinates of the master standard No. 3 for the green filters (2103) and the selective neutral filters (2105). Since the uncertainties of the certified values of these filters would not be significantly reduced by individual measurements of these duplicates, they have been certified as having precisely the same values as their respective master standards from set No. 3.

The ranges found for the chromaticity coordinates of the duplicates of the yellow (2102) and blue (2104) filters, however, were notably larger than the uncertainties of the master standards, and that of the orange-red (2101) duplicates was comparable to the uncertainty of the orange-red master standard. On this account, the chromaticity coordinates of these duplicates were measured relative to the master standard by the CDC visual colorimeter [13] either for source A or C (A for 2101 and 2104, C for 2102) whichever made the chromaticity differences most readily perceptible. The certified values of luminous transmittance for all three sources, and those of the chromaticity coordinates for the sources not used in the CDC measurements were inferred by interpolation from computations for $T^{1/2}$, T , and T^2 for the corresponding master standard.

The uncertainties of the certified values of the duplicates are, of course, larger than those of the

corresponding values for the corresponding standard from master set No. 3 (see ranges given in table 11). The estimated uncertainties for the duplicates are given in table 12. They were estimated as the square root of the sum of the squares of the uncertainties for the master standard and of the additional uncertainties introduced by relating the duplicates to the master standards.

4. Use of the Set of Color Standards

The purchaser of a duplicate set of five glass filters for checking the performance of spectrophotometer-integrator systems of color measurement will receive a report giving the tristimulus values, X, Y, Z , and chromaticity coordinates, x, y , for each filter for each of CIE sources A, B, and C. It is presumed that he bought them because he has at his disposal a spectrophotometer-integrator system and wished to check its performance, to identify any sources of error, and to correct them, or correct for them, if possible.

4.1. Calculation of Par Values (X_0 , Y_0 , Z_0 , x_0 , y_0) for a Given Spectrophotometer-Integrator System in Perfect Adjustment

It should be noted that values certified for each duplicate set refer to slit widths so small that further reduction would not change the values, and that the integrations have been carried out by summation over wavelength intervals so small that further reduction of the interval would not change the values. The spectrophotometer-integrator system at the disposal of the purchaser, however, is characterized by slit widths which the purchaser will have evaluated, and the integrator is characterized by a system which the purchaser will know; that is, it will be by continuous integration equivalent to a summation interval approaching zero, or by a weighted ordinate summation of known wavelength interval, or by a selected-ordinate summation of a known number of ordinates, or some other known system. If this spectrophotometer-integrator system is characterized either by slit widths significantly different from zero, or by an

TABLE 12. Estimated uncertainties in the tristimulus values X , Y , Z and chromaticity coordinates x , y certified for the duplicates

Filter	Tristimulus values			Chromaticity coordinates	
	X	Y	Z	x	y
	%	%	%		
2101	0.6	0.5	0.0	0.001	0.001
2102	.2	.2	.1	.001	.001
2103	.2	.2	.1	.005	.004
2104	.2	.3	.5	.003	.004
2105	.4	.3	.7	.001	.001

integrator-summation interval significantly different from zero, or has other permanent defects (such as back reflectance) arising from its design, the system cannot be expected to yield the certified values (X, Y, Z, x, y) for the standard glass filters even if it is in perfect adjustment.

The first step, therefore, is to compute the changes in the certified values for the filters expected to be introduced by use of slit widths or summation intervals significantly greater than zero, and the changes expected to be introduced by other permanent defects (such as back reflectance) arising from the design of the spectrophotometer-integrator system. To assist the purchaser to calculate from the certified value (X, Y, Z, x, y) the values, X_0, Y_0, Z_0, x_0, y_0 , for the five standard filters that are par for this system, the influence of making the slit width and summation intervals greater than zero have been computed. Table 13 shows the influence of substituting for slits approaching zero width, 5-, 10-, and 15-m μ slits with triangular slit functions. Note

that the influence is far from linear with slit widths, but varies approximately as its square. Table 14 shows the influence of substituting 5-, 10-, and 15-m μ summation intervals for intervals approaching zero, and table 15 shows the influence of substituting 100, 30, and 10 selected ordinates for weighted-ordinate summation over intervals of 1 m μ . Note again that these influences are not linear with the dependent variables; so the influences have to be found from tables 14 and 15 by graphical interpolation. Finally, table 16 shows the changes introduced by back-reflectance errors computed from formula 3. These changes for a spectrophotometer-integrator system based on the GE spectrophotometer may be found by evaluating the maximum error, E , in transmittance so introduced and by multiplying the changes in table 16 by the ratio, $E/0.026$.

The entries in tables 13, 14, 15, and 16, and the values found from them, are the built-in errors $\Delta X, \Delta Y, \Delta Z, \Delta x, \Delta y$; the par values X_0, Y_0, Z_0, x_0, y_0 , are found as $X_0 = X + \Delta X$; $x_0 = x + \Delta x$, and so on.

TABLE 13. Influence of substituting for slits approaching zero width, 5- 10- and 15-m μ slits with triangular slit functions

Filter	Source A					Source B					Source C				
	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy
2101															
5m μ	-0.032	+0.008	0.000	-0.00016	-0.00016	-0.018	+0.019	0.000	-0.00023	+0.00023	-0.013	+0.022	0.000	-0.00027	+0.00027
10m μ	-.133	+.035	.000	-.00066	+.00065	-.074	+.077	+.001	-.00097	+.00096	-.054	+.090	.000	-.00110	+.00110
15m μ	-.299	+.078	.000	-.00148	+.00148	-.167	+.173	+.001	-.00217	+.00216	-.121	+.202	+.001	-.00247	+.00246
2102															
5m μ	-.011	-.008	+.005	-.00002	-.00001	-.008	-.007	+.010	-.00005	-.00004	-.007	-.006	+.013	-.00006	-.00006
10m μ	-.044	-.034	+.018	-.00009	-.00006	-.033	-.027	+.041	-.00020	-.00015	-.028	-.023	+.054	-.00027	-.00023
15m μ	-.099	-.076	+.040	-.00020	-.00013	-.074	-.060	+.091	-.00047	-.00035	-.063	-.052	+.122	-.00062	-.00053
2103															
5m μ	+.009	-.002	+.004	+.00047	-.00068	+.009	-.005	+.010	+.00034	-.00083	+.008	-.006	+.013	+.00030	-.00093
10m μ	+.037	-.008	+.016	+.00193	-.00281	+.035	-.021	+.041	+.00141	-.00342	+.035	-.024	+.055	+.00123	-.00381
15m μ	+.083	-.018	+.037	+.00432	-.00630	+.079	-.046	+.092	+.00316	-.00769	+.078	-.054	+.125	+.00274	-.00855
2104															
5m μ	+.006	+.012	-.010	+.00013	+.00030	.000	+.013	-.026	+.00004	+.00018	-.002	+.013	-.036	+.00001	+.00014
10m μ	+.026	+.050	-.041	+.00054	+.00124	+.004	+.055	-.106	+.00014	+.00075	-.007	+.056	-.147	+.00007	+.00058
15m μ	+.060	+.114	-.091	+.00123	+.00280	+.010	+.125	-.237	+.00033	+.00169	-.014	+.127	-.331	+.00017	+.00132
2105															
5m μ	+.009	+.003	-.009	+.00006	+.00001	+.002	.000	-.023	+.00005	+.00004	.000	-.001	-.032	+.00005	+.00005
10m μ	+.037	+.012	-.039	+.00025	+.00006	+.009	+.001	-.096	+.00022	+.00019	-.002	-.003	-.133	+.00020	+.00021
15m μ	+.082	+.027	-.088	+.00056	+.00014	+.021	+.003	-.218	+.00051	+.00043	-.006	-.006	-.302	+.00045	+.00049

TABLE 14. Influence of substituting 5-, 10-, and 15-m μ summation intervals for intervals approaching zero

Filter	Source A					Source B					Source C				
	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy
2101															
5m μ	+0.028	+0.015	0.000	0.00000	0.00000	+0.020	+0.012	0.000	0.00000	0.00000	+0.019	+0.010	0.000	0.00000	0.00000
10m μ	+.037	+.028	.000	-.00005	+.00005	+.027	+.021	.000	-.00004	+.00004	+.025	+.019	.000	-.00004	+.00004
15m μ	-.141	-.145	-.001	+.00044	-.00043	-.144	-.128	.000	+.00040	-.00040	-.118	-.106	.000	+.00037	-.00037
2102															
5m μ	+.004	+.001	.000	+.00001	-.00001	+.003	-.001	-.001	+.00003	.00000	+.003	-.001	-.003	+.00003	.00000
10m μ	-.001	-.002	-.002	+.00001	.00000	-.002	-.004	-.004	+.00003	+.00001	-.003	-.004	-.006	+.00004	+.00002
15m μ	+.019	-.007	-.006	+.00012	-.00007	+.010	-.008	-.011	+.00013	-.00002	+.022	-.004	-.014	+.00019	-.00006
2103															
5m μ	-.001	-.004	-.001	+.00003	-.00002	-.001	-.005	-.001	+.00002	-.00002	-.002	-.006	-.001	+.00003	-.00003
10m μ	+.002	.000	-.003	+.00003	+.00008	+.002	-.001	-.005	+.00017	+.00014	+.001	-.001	-.008	+.00018	+.00018
15m μ	+.005	-.005	-.003	+.00042	-.00021	+.006	-.005	-.004	+.00037	-.00014	+.004	-.007	-.005	+.00036	-.00015
2104															
5m μ	+.002	+.009	+.003	-.00003	+.00019	.000	+.013	+.002	-.00002	+.00014	+.001	+.014	+.001	-.00002	+.00012
10m μ	+.002	+.010	-.015	+.00007	+.00030	-.004	+.012	-.041	+.00003	+.00019	-.006	+.013	-.057	+.00001	+.00016
15m μ	+.005	+.003	-.005	+.00013	+.00007	+.010	+.004	+.030	+.00004	.00000	+.019	+.006	+.075	+.00002	-.00002
2105															
5m μ	-.006	-.007	-.004	+.00001	.00000	-.007	-.007	-.008	.00000	.00000	-.006	-.007	-.012	.00000	.00000
10m μ	-.016	-.012	-.033	+.00007	+.00010	-.023	-.013	-.079	+.00009	+.00016	-.026	-.013	-.108	+.00008	+.00017
15m μ	+.005	-.009	-.046	+.00019	+.00009	-.004	-.008	-.078	+.00015	+.00014	+.004	-.010	-.081	+.00015	+.00009

TABLE 15. Influence of substituting 100, 30, and 10 selected ordinates for weighted-ordinate summation over intervals of 1 μ

Filter	Source A					Source B					Source C				
	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy
2101															
100	-0.003	+0.001	-0.035	+0.00020	+0.00013	+0.012	+0.009	-0.028	+0.00022	+0.00014	-0.033	-0.011	-0.026	+0.00018	+0.00019
30	-0.003	.000	-0.035	+0.00021	+0.00012	+0.010	+0.012	-.028	+0.00019	+0.00017	-.071	-.030	-.026	+0.00015	+0.00022
10	+0.086	+0.027	-.035	+0.00034	-.00001	-.084	+0.042	-.028	-.00048	+0.00084	+0.045	-.089	-.026	+0.00129	-.00092
2102															
100	-.016	+0.005	-.014	-.00002	+0.00013	+0.017	+0.007	-.027	+0.00017	+0.00008	+0.004	-.003	-.025	+0.00015	+0.00007
30	+0.198	+0.039	-.042	+0.00071	-.00035	+0.155	+0.014	-.099	+0.00107	-.00015	-.011	+0.011	-.118	+0.00044	+0.00062
10	+0.525	+0.216	-.130	+0.00147	-.00037	+0.791	+0.118	-.267	+0.00418	-.00155	+0.144	+0.104	-.354	+0.00186	+0.00145
2103															
100	-.027	.000	+0.010	-.00170	+0.00083	.000	-.004	+0.025	-.00030	-.00106	+0.018	.000	+0.043	+0.00035	-.00028
30	+0.354	+0.024	+0.038	+0.01836	-.01820	+0.133	.000	+0.047	+0.00569	-.00703	+0.001	+0.022	+0.048	-.00075	-.00136
10	+1.420	+0.295	+0.032	+0.06604	-.05697	+1.084	+0.113	-.097	+0.04855	-.03451	-.132	+0.123	-.199	-.00517	+0.01484
2104															
100	-.023	-.127	-.014	+0.00022	-.00280	-.103	-.099	-.012	-.00085	-.00097	-.148	-.083	-.044	-.00096	-.00055
30	-1.103	-.406	-.020	-.02370	-.00463	-.495	-.175	+0.017	-.00490	-.00138	-.218	-.156	-.013	-.00144	-.00113
10	-3.410	-.936	+0.052	-.08221	-.00767	-2.498	-.836	+0.278	-.02600	-.00690	-.070	-.841	+0.324	+0.00019	-.00722
2105															
100	+0.913	-.011	+0.010	+0.00006	-.00012	+0.002	-.022	+0.020	+0.00001	-.00014	-.029	-.024	+0.023	-.00012	-.00008
30	-.195	-.138	+0.020	+0.00050	-.00008	-.151	-.042	+0.092	-.00074	-.00005	-.136	-.020	+0.135	-.00071	-.00008
10	-.500	-.356	+0.098	-.00142	-.00035	-.716	-.117	+0.300	-.00341	+0.00041	-.148	-.023	+0.439	-.00123	-.00058

TABLE 16. Changes introduced by back-reflectance errors computed from formula 3

Filter	ΔX	ΔY	ΔZ	Δx	Δy
Source A					
2101	+0.180	+0.093	0.000	+0.00003	-0.00002
2102	+0.149	+0.106	+0.001	+0.00007	-.00004
2103	+0.001	+0.004	+0.001	-.00004	+0.00004
2104	+0.012	+0.003	+0.052	-.00001	-.00021
2105	+0.087	+0.091	+0.048	-.00005	-.00001
Source B					
2101	+0.132	+0.070	0.000	+0.00002	-0.00003
2102	+0.117	+0.092	+0.001	+0.00009	-.00003
2103	+0.002	+0.006	+0.001	-.00003	+0.00004
2104	+0.027	+0.007	+0.137	-.00001	-.00013
2105	+0.085	+0.096	+0.113	-.00006	-.00003
Source C					
2101	+0.116	+0.063	0.000	+0.00002	-0.00002
2102	+0.116	+0.087	+0.001	+0.00011	-.00002
2103	+0.001	+0.006	+0.001	-.00002	+0.00005
2104	+0.037	+0.009	+0.195	-.00001	-.00010
2105	+0.087	+0.098	+0.154	-.00005	-.00003

4.2. Influence of Various Maladjustments

The spectrophotometer-integrator system of the purchaser is subject to a variety of malfunctions. Some of them are the result of maladjustments, such as failure of the zero and 100 percent points of the photometric scale to be set correctly, or errors in the adjustment of the wavelength scale; others are the result of wear or the accumulation of dust on the surfaces of optical parts which can introduce stray energy; and still others will be of obscure origin, such as temporary misalignment of optical parts due to development of unforeseen thermal gradients, or faulty linkage between spectrophotometer and integrator. By introduction of the five standard glass filters at regular intervals into the schedule of runs, it should be possible to detect the appearance of any significant malfunction and in some cases to identify its cause. To assist the purchaser in his interpreta-

tion of deviations of the values read, X_r, Y_r, Z_r, x_r, y_r , from the corresponding par values, X_0, Y_0, Z_0, x_0, y_0 , the pattern of influence of several sorts of malfunction on the readings for the five glasses of the set has been computed. These patterns include the influence of uniform displacement of the wavelength scale, of the zero and 100 percent points on the photometric scale, of stray energy, of inertia error, and back-reflectance error.

a. Wavelength Scale

Table 17 shows the changes in tristimulus values, X, Y, Z , and chromaticity coordinates, x, y , that would be introduced by a uniform displacement of the wavelength scale by $\pm 1 \mu$ and $\pm 2 \mu$. It will be noted that standard filters 2101 and 2103 (orange-red and green) provide sensitive indications of displacement of wavelength scale. Since the slope of the transmittance curve with wavelength is appreciable for filter 2101 only between 570 and 590 μ (see table 2), this filter responds to wavelength scale displacement only in this spectral region. Filter 2103, however, responds to wavelength scale displacements in two regions: 470 to 520 μ and 530 to 590 μ . For both filters the changes in tristimulus values and chromaticity coordinates are substantially linear with wavelength error.

b. Zero of Photometric Scale

If the instrument by maladjustment wrongly reads zero when the specimen transmittance is really greater than zero, this maladjustment may be called positive displacement of the instrument zero. If the instrument indicates a transmittance of greater than zero when the specimen transmittance is actually zero, this maladjustment may be called negative displacement of the instrument zero. Table 18 shows the changes that would be introduced by a uniform displacement of the instrument zero of the photometric scale by ± 1.0 and ± 0.5 percent independent of wavelength. In computing the influence of positive displacements of the instrument zero, negative

TABLE 17. *Changes in tristimulus values, X, Y, Z, and chromaticity coordinates, x, y, that would be introduced by a uniform displacement of the wavelength scale by $\pm 1 \text{ m}\mu$ and $\pm 2 \text{ m}\mu$*

Filter	Source A					Source B					Source C				
	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy
2101															
+2m μ	+1.582	+1.517	+0.003	-0.00404	+0.00402	+1.435	+1.395	+0.003	-0.00474	+0.00471	+1.363	+1.332	+0.003	-0.00501	+0.00499
+1m μ	+0.794	+0.751	+0.001	-0.00199	+0.00198	+0.717	+0.687	+0.002	-0.00233	+0.00232	+0.680	+0.655	+0.001	-0.00246	+0.00245
-1m μ	-0.794	-0.751	-0.001	+0.00199	-0.00198	-0.717	-0.687	-0.002	+0.00233	-0.00232	-0.680	-0.655	-0.001	+0.00246	-0.00245
-2m μ	-1.582	-1.517	-0.003	+0.00404	-0.00402	-1.435	-1.395	-0.003	+0.00474	-0.00471	-1.363	-1.332	-0.003	+0.00501	-0.00499
2102															
+2m μ	+0.527	+0.801	+0.153	-0.00222	+0.00124	+0.512	+0.878	+0.308	-0.00306	+0.00093	+0.508	+0.900	+0.397	-0.00343	+0.00062
+1m μ	+0.261	+0.402	+0.076	-0.00112	+0.00064	+0.253	+0.442	+0.150	-0.00155	+0.00051	+0.251	+0.453	+0.194	-0.00173	+0.00037
-1m μ	-0.261	-0.402	-0.076	+0.00112	-0.00064	-0.253	-0.442	-0.150	+0.00155	-0.00051	-0.251	-0.453	-0.194	+0.00173	-0.00037
-2m μ	-0.527	-0.801	-0.153	+0.00222	-0.00124	-0.512	-0.878	-0.308	+0.00306	-0.00093	-0.508	-0.900	-0.397	+0.00343	-0.00062
2103															
+2m μ	-0.250	-0.347	+0.122	-0.01060	-0.00229	-0.238	-0.325	+0.243	-0.01057	-0.00720	-0.226	-0.306	+0.311	-0.01052	-0.00941
+1m μ	-0.124	-0.166	+0.061	-0.00524	-0.00098	-0.117	-0.153	+0.120	-0.00525	-0.00337	-0.111	-0.143	+0.153	-0.00524	-0.00445
-1m μ	+0.124	+0.166	-0.061	+0.00524	+0.00098	+0.117	+0.153	-0.120	+0.00525	+0.00337	+0.111	+0.143	-0.153	+0.00524	+0.00445
-2m μ	+0.250	+0.347	-0.122	+0.01060	+0.00229	+0.238	+0.325	-0.243	+0.01057	+0.00720	+0.226	+0.306	-0.311	+0.01052	+0.00941
2104															
+2m μ	-0.070	-0.274	-0.506	+0.00253	-0.00388	-0.193	-0.403	-1.112	+0.00102	-0.00297	-0.258	-0.458	-1.484	+0.00079	-0.00256
+1m μ	-0.036	-0.136	-0.246	+0.00118	-0.00193	-0.094	-0.200	-0.535	+0.00049	-0.00148	-0.124	-0.227	-0.712	+0.00038	-0.00128
-1m μ	+0.036	+0.136	+0.246	-0.00118	+0.00193	+0.094	+0.200	+0.535	-0.00049	+0.00148	+0.124	+0.227	+0.712	-0.00038	+0.00128
-2m μ	+0.070	+0.274	+0.506	-0.00253	-0.00388	+0.193	+0.403	+1.112	-0.00102	+0.00297	+0.258	+0.458	+1.484	-0.00079	+0.00256
2105															
+2m μ	-0.114	-0.240	+0.158	-0.00024	-0.00122	-0.061	-0.252	+0.486	-0.00072	-0.00193	-0.011	-0.251	+0.728	-0.00078	-0.00215
+1m μ	-0.081	-0.139	+0.080	-0.00017	-0.00086	-0.041	-0.142	+0.345	-0.00042	-0.00100	-0.004	-0.141	+0.366	-0.00044	-0.00110
-1m μ	+0.081	+0.139	-0.080	+0.00017	+0.00086	+0.041	+0.142	-0.345	+0.00042	+0.00100	+0.004	+0.141	-0.366	+0.00044	+0.00110
-2m μ	+0.114	+0.240	-0.158	+0.00024	+0.00122	+0.061	+0.252	-0.486	+0.00072	+0.00193	+0.011	+0.251	-0.728	+0.00078	+0.00215

TABLE 18. *Changes that would be introduced by a uniform displacement of the zero of the photometric scale by ± 1.0 and ± 0.5 percent independent of wavelength*

Filter	Source A					Source B					Source C				
	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy
2101															
+1.0%	-0.234	-0.202	0.000	+0.00048	-0.00048	-0.202	-0.184	0.000	+0.00059	-0.00059	-0.189	-0.176	0.000	+0.00064	-0.00063
+0.5%	-0.118	-0.103	0.000	+0.00025	-0.00025	-0.102	-0.095	0.000	+0.00031	-0.00031	-0.096	-0.090	0.000	+0.00033	-0.00033
-0.5%	+0.207	+0.315	+0.177	-0.00235	+0.00068	+0.241	+0.357	+0.424	-0.00522	-0.00009	+0.265	+0.372	+0.588	-0.00729	-0.00095
-1.0%	+0.411	+0.627	+0.352	-0.00464	+0.00133	+0.479	+0.711	+0.844	-0.01025	-0.00017	+0.527	+0.741	+1.169	-0.01427	-0.00186
2102															
+1.0%	-0.400	-0.452	-0.335	+0.00200	+0.00049	-0.434	-0.499	-0.816	+0.00411	+0.00272	-0.472	-0.518	-1.136	+0.00541	+0.00448
+0.5%	-0.199	-0.225	-0.166	+0.00099	+0.00024	-0.216	-0.248	-0.406	+0.00203	+0.00134	-0.235	-0.258	-0.565	+0.00267	+0.00221
-0.5%	+0.197	+0.223	+0.166	-0.00097	-0.00023	+0.213	+0.246	+0.402	-0.00198	-0.00131	+0.232	+0.255	+0.559	-0.00258	-0.00214
-1.0%	+0.392	+0.444	+0.330	-0.00192	-0.00046	+0.425	+0.489	+0.801	-0.00391	-0.00258	+0.462	+0.507	+1.114	-0.00510	-0.00421
2103															
+1.0%	-0.524	-0.682	-0.191	-0.01614	+0.02066	-0.513	-0.729	-0.400	-0.01128	+0.02325	-0.511	-0.743	-0.521	-0.00970	+0.02556
+0.5%	-0.214	-0.367	-0.088	-0.00804	+0.01051	-0.299	-0.385	-0.233	-0.00685	+0.01381	-0.297	-0.390	-0.306	-0.00584	+0.01518
-0.5%	+0.530	+0.453	+0.170	+0.01768	-0.02164	+0.476	+0.444	+0.413	+0.01095	-0.02405	+0.470	+0.441	+0.575	+0.00893	-0.02720
-1.0%	+1.056	+0.901	+0.339	+0.03265	-0.03996	+0.946	+0.884	+0.821	+0.02029	-0.04455	+0.936	+0.878	+1.143	+0.01650	-0.05023
2104															
+1.0%	-0.984	-0.919	-0.123	-0.01780	-0.01742	-0.828	-0.905	-0.259	-0.00650	-0.00903	-0.778	-0.895	-0.344	-0.00428	-0.00673
+0.5%	-0.518	-0.473	-0.061	-0.00918	-0.00868	-0.433	-0.463	-0.129	-0.00340	-0.00455	-0.406	-0.457	-0.171	-0.00224	-0.00340
-0.5%	+0.513	+0.468	+0.060	+0.00857	+0.00810	+0.429	+0.458	+0.128	+0.00328	+0.00439	+0.402	+0.453	+0.169	+0.00217	+0.00331
-1.0%	+1.021	+0.932	+0.120	+0.01659	+0.01569	+0.854	+0.911	+0.255	+0.00645	+0.00863	+0.800	+0.901	+0.336	+0.00429	+0.00652
2105															
+1.0%	-0.557	-0.467	-0.127	-0.00059	+0.00002	-0.484	-0.451	-0.311	-0.00055	-0.00015	-0.469	-0.445	-0.435	-0.00048	-0.00017
+0.5%	-0.277	-0.232	-0.063	-0.00029	+0.00001	-0.241	-0.225	-0.154	-0.00028	-0.00008	-0.234	-0.222	-0.217	-0.00024	-0.00008
-0.5%	+0.275	+0.231	+0.063	+0.00029	-0.00002	+0.238	+0.222	+0.154	+0.00026	+0.00007	+0.231	+0.219	+0.214	+0.00024	+0.00009
-1.0%	+0.547	+0.459	+0.125	+0.00058	-0.00003	+0.474	+0.442	+0.305	+0.00053	+0.00013	+0.459	+0.436	+0.426	+0.00047	+0.00017

values of computed spectral-transmittance readings were counted as equal to zero on the assumption that no spectrophotometer-integrator system will be designed actually to subtract substantial amounts from the tristimulus sums to correspond to such negative values. Note that standard filters 2102 (yellow) and 2104 (blue) both are sensitive to this type of maladjustment. This corresponds to the fact that for both filters the spectral transmittance approaches but does not become less than 1 percent. Note also that filters 2101 (orange red) and 2103 (green) though sensitive to negative displacements of the instrument

zero are less so for positive displacements. Filter 2101, in particular, shows almost no change in chromaticity coordinates, x, y , for positive displacements. Positive displacements only serve to move the apparent cutoff of the curve of spectral transmittance slightly toward longer wavelength, but negative displacements cause the system to respond as if the filter transmitted 0.5 or 1 percent throughout the whole short-wave and middle-wave part of the visible spectrum (380 to 580 m μ) where the transmittance is actually almost precisely zero. The errors introduced by negative displacements are in fact equivalent

TABLE 19. *Changes that would be introduced by a uniform displacement of the 100% of the photometric scale by ± 1.0 and ± 0.5 percent independent of wavelength*

Filter	Source A					Source B					Source C				
	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy
2101															
+1.0%	-0.676	-0.363	0.000	0.00000	0.00000	-0.502	-0.279	0.000	0.00000	0.00000	-0.444	-0.249	0.000	0.00000	0.00000
+ .5%	- .340	- .182	.000	.00000	.00000	- .252	- .140	.000	.00000	.00000	- .223	- .125	.000	.00000	.00000
- .5%	+ .344	+ .185	.000	.00000	.00000	+ .255	+ .141	.000	.00000	.00000	+ .225	+ .126	.000	.00000	.00000
-1.0%	+ .691	+ .371	.000	.00000	.00000	+ .513	+ .284	+ .001	.00000	.00000	+ .452	+ .254	.000	.00000	.00000
2102															
+1.0%	- .696	- .547	- .022	.00000	.00000	- .556	- .501	- .043	.00000	.00000	- .508	- .483	- .055	.00000	.00000
+ .5%	- .350	- .275	- .011	.00000	.00000	- .280	- .252	- .022	.00000	.00000	- .255	- .243	- .028	.00000	.00000
- .5%	+ .352	+ .278	+ .012	.00000	.00000	+ .282	+ .255	+ .022	.00000	.00000	+ .258	+ .245	+ .028	.00000	.00000
-1.0%	+ .709	+ .558	+ .024	.00000	.00000	+ .567	+ .512	+ .045	.00000	.00000	+ .518	+ .492	+ .056	.00000	.00000
2103															
+1.0%	- .032	- .089	- .013	.00000	.00000	- .034	- .107	- .023	.00000	.00000	- .035	- .112	- .026	.00000	.00000
+ .5%	- .016	- .045	- .006	.00000	.00000	- .017	- .054	- .011	.00000	.00000	- .017	- .056	- .013	.00000	.00000
- .5%	+ .016	+ .046	+ .007	.00000	.00000	+ .018	+ .054	+ .011	.00000	.00000	+ .018	+ .056	+ .014	.00000	.00000
-1.0%	+ .032	+ .092	+ .013	.00000	.00000	+ .035	+ .109	+ .022	.00000	.00000	+ .036	+ .114	+ .028	.00000	.00000
2104															
+1.0%	- .067	- .058	- .232	.00000	.00000	- .127	- .079	- .589	.00000	.00000	- .171	- .090	- .833	.00000	.00000
+ .5%	- .033	- .029	- .117	.00000	.00000	- .064	- .040	- .296	.00000	.00000	- .086	- .045	- .419	.00000	.00000
- .5%	+ .034	+ .029	+ .117	.00000	.00000	+ .065	+ .040	+ .300	.00000	.00000	+ .086	+ .046	+ .422	.00000	.00000
-1.0%	+ .069	+ .059	+ .236	.00000	.00000	+ .130	+ .080	+ .602	.00000	.00000	+ .174	+ .092	+ .849	.00000	.00000
2105															
+1.0%	- .541	- .531	- .227	.00000	.00000	- .506	- .548	- .539	.00000	.00000	- .511	- .554	- .743	.00000	.00000
+ .5%	- .272	- .267	- .114	.00000	.00000	- .254	- .275	- .270	.00000	.00000	- .257	- .279	- .374	.00000	.00000
- .5%	+ .275	+ .270	+ .116	.00000	.00000	+ .257	+ .278	+ .274	.00000	.00000	+ .259	+ .281	+ .377	.00000	.00000
-1.0%	+ .552	+ .543	+ .232	.00000	.00000	+ .517	+ .559	+ .550	.00000	.00000	+ .521	+ .565	+ .785	.00000	.00000

lent to admixture of the corresponding amounts of the source color, and positive displacements are equivalent to subtraction of corresponding amounts of the source color, unless (as in filters 2101 and 2103) there are substantial spectral regions within which the filters have zero transmittance. Except for this difference between positive and negative displacements caused by counting negative values of transmittance arbitrarily as zero, the changes are substantially linear with displacement of the zero.

c. 100 Percent Point of Photometric Scale

Table 19 shows the changes corresponding to a displacement of the 100 percent point of the photometric scale by ± 0.5 and ± 1.0 percent independent of wavelength. As is obvious, such displacements have no influence on chromaticity coordinates whatsoever. The changes produced in the tristimulus values, except for rejection errors, are precisely linear with the displacement. Standard filter 2105 (selective neutral), since it has the highest average transmittance, is the most sensitive of the five to this maladjustment.

d. Stray-Energy Errors

The basic idea of a spectrophotometer is to irradiate the specimen with flux confined to a narrow spectral band and to compare the flux leaving the specimen to that incident on it. No spectrophotometer succeeds perfectly in confining the flux incident on the specimen to the narrow spectral band intended; the specimen is always irradiated by energy outside this band, called stray energy (cosmic rays, radiant flux from the room leaking into the instrument, radiant flux from the instrument source reaching the specimen by multiple reflection within

the instrument without passing through the dispersing elements, radiant flux reaching the specimen passing through the dispersing elements after having been scattered by irregularities of the optical surfaces, or after having been scattered by dust particles on them, and so on).

Spectrophotometers, like the GE or Cary-14, having two dispersing systems are likely to show negligible stray-energy errors; those, like the Beckman DU or König-Martens, having but a single prism or grating are likely to show important stray-energy errors, particularly if the integrating attachment prevents the employment of stray-energy filters. For such instruments it is particularly important to have a means of checking for the amount of stray-energy errors, because the accidental displacement of a baffle within the instrument, or accumulation of dust on the optical surfaces, may introduce large errors following continued use of a system initially free of significant stray-energy error.

The most frequent kind of stray-energy error arises from the incidence of radiant flux having the spectral distribution of the instrument source. So as to assist purchasers of a set of standard filters to use them for the detection of stray-energy errors of this most frequent sort, calculations have been made on the assumption that the stray energy and the dispersed energy alike have the spectral distribution of CIE source A (color temperature 2,854 °K), and that the specimen receives a constant small stray irradiance of this sort regardless of nominal wavelength.

If H_λ is the irradiance of the detector from dispersed flux from source A, and if the spectral sensitivity of the detector is S_λ , then, in the absence of stray energy the response of the detector for the blank beam will be $kH_\lambda S_\lambda \Delta\lambda$, where $\Delta\lambda$ is the spec-

tral band width, and k is the constant of proportionality. Similarly, the response of the detector with a specimen of spectral transmittance, T_λ , inserted into the beam will be $kH_\lambda S_\lambda T_\lambda \Delta\lambda$. The reading, R_λ , of the spectrophotometer will be the ratio: $kH_\lambda S_\lambda T_\lambda \Delta\lambda / kH_\lambda S_\lambda \Delta\lambda$, and this ratio is seen to be equal to T_λ ; so $R_\lambda = T_\lambda$, as intended.

If now there is stray energy of spectral irradiance, fH_λ , where f is a small fraction chosen arbitrarily to be within the range of usual stray energy, then the reading, $R_{s\lambda}$, of the spectrophotometer may be expressed as:

$$R_{s\lambda} = \frac{H_\lambda S_\lambda T_\lambda \Delta\lambda + f \Sigma H_\lambda S_\lambda T_\lambda \Delta\lambda}{H_\lambda S_\lambda \Delta\lambda + f \Sigma H_\lambda S_\lambda \Delta\lambda} \quad (5)$$

The summations correspond to the fact that the stray energy is undispersed, and must be evaluated over the entire range for which the product, $H_\lambda S_\lambda$, is significantly different from zero. It will be noticed from eq 5 that if the fraction, f , of the stray energy is sufficiently small, then the spectrophotometer will read spectral transmittance correctly ($R_{s\lambda} = T_\lambda$); but for all spectral regions in which $H_\lambda S_\lambda \Delta\lambda$ is small compared to $f \Sigma H_\lambda S_\lambda \Delta\lambda$, the reading, $R_{s\lambda}$, will approach the transmittance of the filter for the aspect

TABLE 20. Spectral sensitivities for two types of detector: photomultiplier type and lead sulfide type

Wave-length	Photo-multiplier sensitivity	Wave-length	Lead sulfide sensitivity
$m\mu$		μ	
300	340	0.3	39.5
310	450	.4	51.0
320	545		
330	630	.5	62.5
340	655	.6	74.0
		.7	85.0
350	665	.8	96.0
360	665	.9	107.5
370	660		
380	655	1.0	122
390	642	1.1	122
		1.2	124
400	620	1.3	124
410	595	1.4	146
420	575		
430	555	1.5	152
440	525	1.6	154
		1.7	164
450	500	1.8	177
460	475	1.9	188
470	440		
480	410	2.0	195
490	375	2.1	197
		2.2	204
500	348	2.3	198
510	318	2.4	186
520	285		
530	255	2.5	171
540	220	2.6	154
		2.7	107
550	187	2.8	48
560	151	2.9	26
570	122		
580	96	3.0	13
590	72.5	3.1	6
		3.2	2
600	51.0	3.3	0
610	35.0		
620	20.0		
630	9.5		
640	4.0		
650	1.6		
660	0.8		
670	0.4		
680	0.2		
690	0.1		

TABLE 21. Values of transmittance T_{ad} of the five filters of master set No. 3 for the aspect of source-A energy detected by photomultiplier and lead sulfide detectors

Filter	2101	2102	2103	2104	2105
Photomultiplier	0.076	0.233	0.086	0.410	0.543
Lead sulfide	0.868	0.664	0.200	0.605	0.845

of source A responded to by the detector, T_{ad} , or $R_{s\lambda} \rightarrow T_{ad}$.

The summations have been computed for the five standard filters over the spectral range 0.3 to 3.3 μ for two spectral-sensitivity functions given in table 20, one intended to be representative of detectors of the photomultiplier type; the other, of the lead sulfide type. Table 21 gives the values of T_{ad} for the five filters evaluated by these two types of detectors, and figures 2 to 6 show the adopted values of spectral transmittance for the filters of master set No. 3 compared to values of $R_{s\lambda}$ for $f=0.01$. It will be noted that the curves of $R_{s\lambda}$ approach the value of T_{ad} in the spectral regions where the detector response is relatively low.

Table 22 shows the changes in tristimulus values, X , Y , Z , and chromaticity coordinates, x , y , caused by the introduction of the amounts of stray energy corresponding to $f=0.001$, 0.005, and 0.010. It will

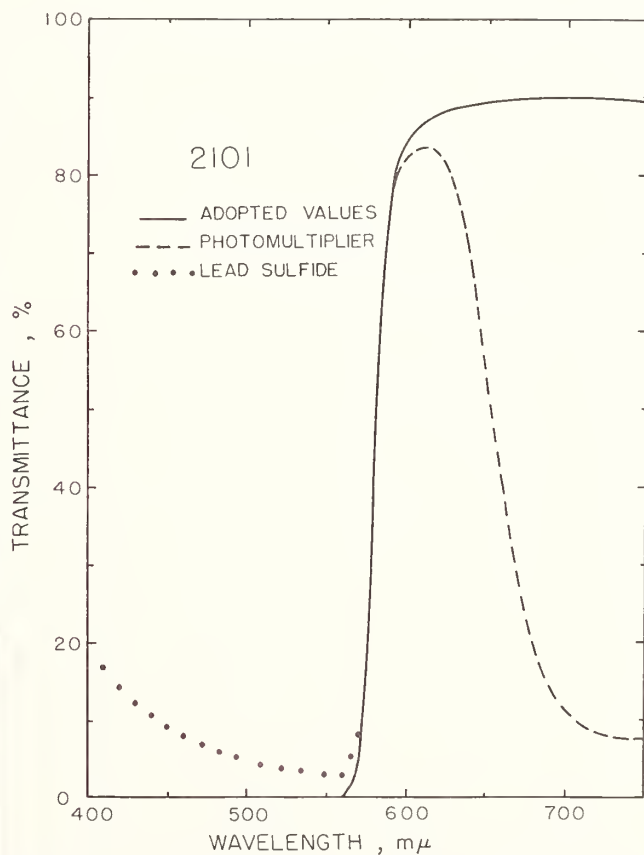


FIGURE 2. Adopted values of spectral transmittance for master set No. 3, 2101 orange-red glass compared to values of $R_{s\lambda}$ for $f=0.01$ for photomultiplier and lead sulfide systems.

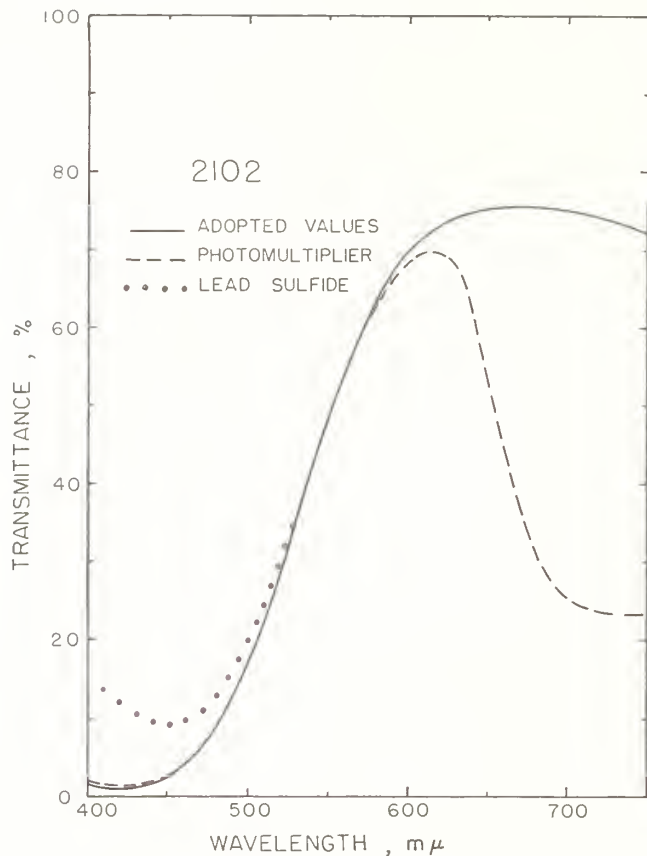


FIGURE 3. Adopted values of spectral transmittance for master set No. 3, 2102 yellow glass compared to values of R_{sk} for $f=0.01$ for photomultiplier and lead sulfide systems.

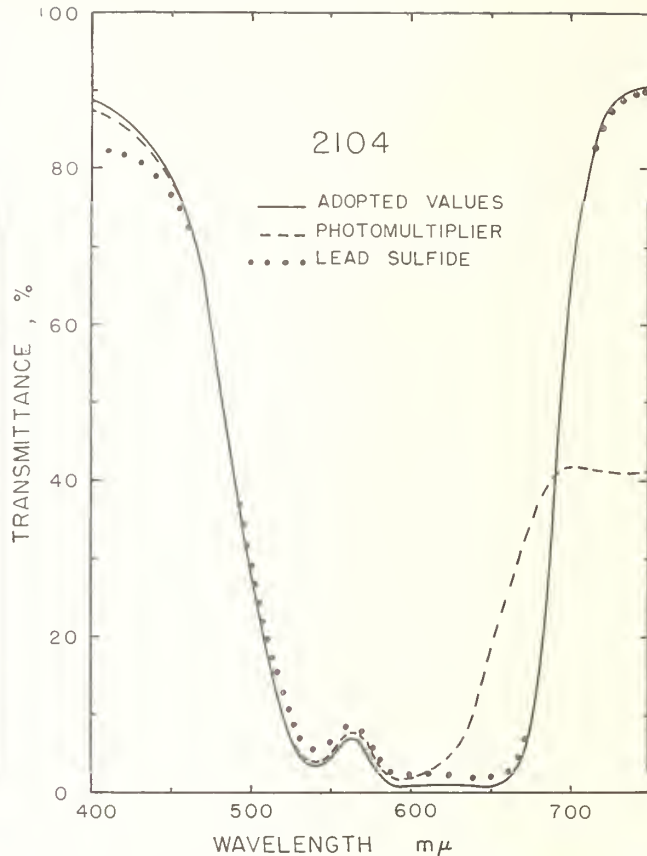


FIGURE 5. Adopted values of spectral transmittance for master set No. 3, 2104 cobalt blue glass compared to values of R_{sk} for $f=0.01$ for photomultiplier and lead sulfide systems.

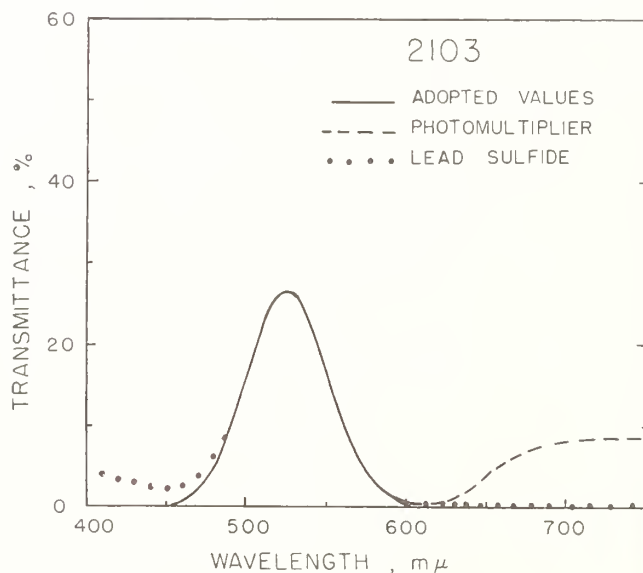


FIGURE 4. Adopted values of spectral transmittance for master set No. 3, 2103 sextant green glass compared to values of R_{sk} for $f=0.01$ for photomultiplier and lead sulfide systems.

be noted that standard filter 2101 (orange red) is sensitive to stray energy with a photomultiplier-type detector by change in its X tristimulus value and to stray energy with a lead sulfide detector by change in its Z tristimulus value, as is also filter 2102 (yellow). Filter 2103 (green) also serves for both types of detector, the lead sulfide detector through the x chromaticity coordinate, and the photomultiplier through the y coordinate. Filter 2104 (blue) indicates by its X tristimulus value the presence of stray energy with the photomultiplier type of detector. For the lead sulfide detector these changes are closely linear with fraction, f , of stray energy; for the photomultiplier type of detector they vary approximately as $f^{2/3}$.

e. Inertia Errors

Table 23 shows the corresponding changes introduced by inertia errors computed from formula 2. All standard filters are influenced appreciably by inertia errors; filters 2101 and 2102, in X and Y ; filter 2103, in x ; and filters 2104 and 2105 in Z .

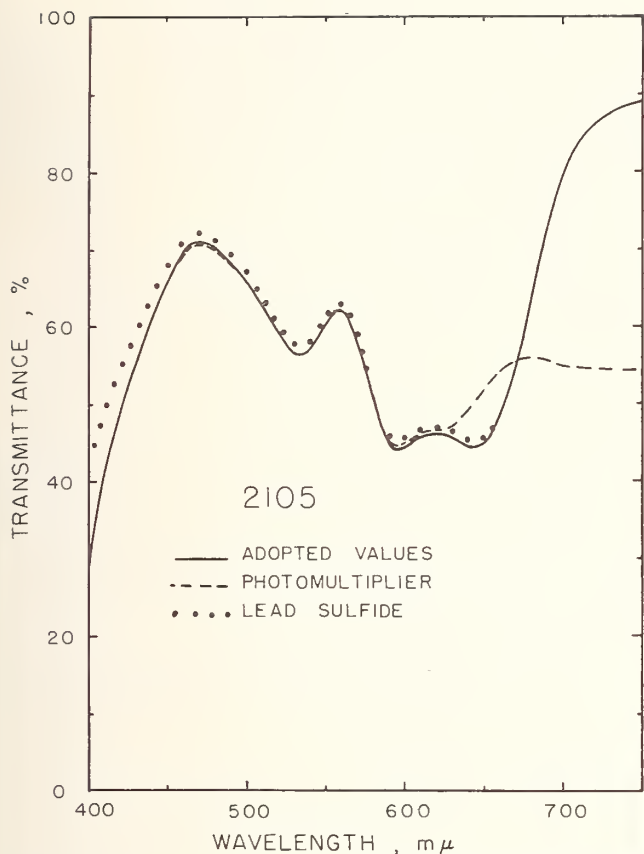


FIGURE 6. Adopted values of spectral transmittance of master set No. 3, 2105 selective neutral glass compared to values of R_{SA} for $f=0.01$ for photomultiplier and lead sulfide systems.

4.3. Diagnosis of Maladjustments

Table 24 identifies which of the certified variables (X , Y , Z , x , y) of which standard filters serves best to detect one or another type of maladjustment or malfunction of the spectrophotometer-integrator system. It thus summarizes the uses of each filter and indicates the degree to which the choice of filter has been found justified. This table also suggests a possible simple systematic way to diagnose the simple ills of a spectrophotometer-integrator system from the differences between the actual instrument readings, X_r , Y_r , Z_r , x_r , y_r , and the corresponding par values X_0 , Y_0 , Z_0 , x_0 , y_0 .

TABLE 22. Changes in tristimulus values, X , Y , Z , and chromaticity coordinates, x , y , caused by the introduction of the amounts of stray energy corresponding to $f=0.010$, 0.005 , and 0.001
MPT (photomultiplier-type detector); PbS (lead sulfide type detector)

Filter	Source A					Source B					Source C				
	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy	ΔX	ΔY	ΔZ	Δx	Δy
2101 MPT	0.010 -8.834	-3.583	+0.032	-0.00837	+0.00799	-5.766	-2.367	+0.081	-0.00830	+0.00712	-4.826	-1.991	+0.114	-0.00846	+0.00663
	.005 -5.831	-2.317	+0.016	-0.00557	+0.00538	-3.734	-1.501	+0.040	-0.00533	+0.00477	-3.102	-1.252	+0.056	-0.00534	+0.00446
	.001 -1.903	-0.729	+0.004	-0.00188	+0.00184	-1.161	-0.449	+0.010	-0.00170	+0.00157	-0.946	-0.367	+0.014	-0.00167	+0.00145
PbS	0.010 +1.172	+1.846	+2.954	-0.02419	-0.00217	+2.040	+2.302	+7.532	-0.06101	-0.02120	+2.625	+2.490	+10.669	-0.08629	-0.03697
	.005 +.599	+0.935	+1.551	-0.01289	-0.00132	+1.058	+1.170	+3.968	-0.03401	-0.01216	+1.369	+1.267	+5.626	-0.04953	-0.02165
	.001 +.122	+0.190	+0.324	-0.00273	-0.00031	+0.219	+0.238	+0.832	-0.00752	-0.00276	+0.285	+0.258	+1.181	-0.01128	-0.00502
2102 MPT	0.010 -5.661	-2.439	+0.075	-0.01043	+0.00859	-3.714	-1.682	+1.98	-0.01027	+0.00646	-3.110	-1.444	+0.284	-0.01020	+0.00517
	.005 -3.691	-1.538	+0.039	-0.00684	+0.00575	-2.373	-1.032	+1.03	-0.00653	+0.00437	-1.972	-0.876	+0.147	-0.00640	+0.00358
	.001 -1.237	-0.490	+0.008	-0.00232	+0.00200	-0.757	-0.310	+0.020	-0.00208	+0.00151	-0.617	-0.257	+0.029	-0.00197	+0.00126
PbS	0.010 +.391	+0.523	+2.097	-0.00963	-0.00595	+1.036	+0.770	+5.419	-0.02198	-0.02123	+1.474	+0.886	+7.709	-0.02937	-0.03241
	.005 +.207	+0.268	+1.108	-0.00511	-0.00322	+0.549	+0.396	+2.872	-0.01194	-0.01164	+0.783	+0.456	+4.090	-0.01618	-0.01800
	.001 +.048	+0.059	+0.239	-0.00111	-0.00071	+0.122	+0.086	+0.622	-0.00264	-0.00260	+0.173	+0.099	+0.886	-0.00363	-0.00407
2103 MPT	0.010 +.921	+0.337	+0.023	+0.04156	-0.03474	+0.609	+0.201	+0.067	+0.02434	-0.02135	+0.318	+0.160	+0.098	+0.01962	-0.01848
	.005 +.604	+0.220	+0.012	+0.02811	-0.02332	+0.390	+0.132	+0.034	+0.01594	-0.01356	+0.258	+0.105	+0.051	+0.01271	-0.01142
	.001 +.222	+0.081	+0.003	+0.01071	-0.00880	+0.135	+0.048	+0.008	+0.00368	-0.00464	+0.112	+0.038	+0.012	+0.00443	-0.00372
PbS	0.010 +.448	+0.258	+0.600	+0.01138	-0.04275	+0.592	+0.245	+1.590	+0.00417	-0.07071	+0.694	+0.249	+2.278	+0.00197	-0.08774
	.005 +.249	+0.130	+0.316	+0.00592	-0.02330	+0.306	+0.124	+0.839	+0.00212	-0.03964	+0.361	+0.126	+1.204	+0.00093	-0.04997
	.001 +.046	+0.021	+0.057	+0.00118	-0.00445	+0.055	+0.020	+0.152	+0.00045	-0.00769	+0.066	+0.020	+0.219	+0.00022	-0.00984
2104 MPT	0.010 +3.887	+1.821	-0.118	+0.06822	+0.02202	+2.625	+1.344	-0.330	+0.02434	+0.01171	+2.208	+1.187	-0.481	+0.01549	+0.00836
	.005 +2.377	+1.075	-0.060	+0.04417	+0.01335	+1.595	+0.780	-0.167	+0.01505	+0.00680	+1.338	+0.684	-0.243	+0.00947	+0.00479
	.001 +.613	+0.268	-0.012	+0.01218	+0.00347	+0.416	+0.194	-0.033	+0.00401	+0.00169	+0.350	+0.170	-0.049	+0.00250	+0.00118
PbS	0.010 +1.300	+1.575	-0.375	+0.02157	+0.03040	+0.917	+1.609	-1.199	+0.00864	+0.01809	+0.712	+1.606	-1.813	+0.00572	+0.01412
	.005 +.648	+0.792	-0.212	+0.01121	+0.01594	+0.448	+0.809	-0.667	+0.00437	+0.00928	+0.339	+0.808	-1.005	+0.00287	+0.00720
	.001 +.127	+0.157	-0.050	+0.00228	+0.00330	+0.085	+0.161	-0.152	+0.00087	+0.00189	+0.062	+0.161	-0.229	+0.00057	+0.00146
2105 MPT	0.010 +.469	+0.188	-0.041	+0.00161	-0.00048	+0.343	+0.125	-0.092	+0.00139	-0.00002	+0.294	+0.102	-0.124	+0.00119	+0.00010
	.005 +.198	+0.083	-0.020	+0.00068	-0.00018	+0.162	+0.060	-0.044	+0.00065	-0.00000	+0.143	+0.051	-0.060	+0.00058	+0.00005
	.001 -.096	-0.034	-0.004	+0.00030	-0.00016	-0.038	-0.016	-0.010	-0.00011	+0.00004	-0.026	-0.011	-0.014	-0.00006	+0.00002
PbS	0.010 +.962	+0.875	+0.733	-0.00081	-0.00131	+1.048	+0.907	+1.949	-0.00117	-0.00262	+1.161	+0.923	+2.809	-0.00117	-0.00307
	.005 +.490	+0.445	+0.391	-0.00047	-0.00073	+0.541	+0.463	+1.042	-0.00067	-0.00147	+0.604	+0.471	+1.503	-0.00067	-0.00172
	.001 +.105	+0.096	+0.191	-0.00012	-0.00018	+0.119	+0.099	+0.243	-0.00017	-0.00036	+0.134	+0.102	+0.350	-0.00017	-0.00042

TABLE 23. *Changes introduced by inertia errors computed from formula 2*

Filter	ΔX	ΔY	ΔZ	Δx	Δy
Source A					
2101	+0.647	+0.546	+0.001	-0.00122	+0.00122
2102	+ .206	+ .297	+ .045	- .00074	+ .00046
2103	- .107	- .120	+ .035	- .00468	+ .00065
2104	- .035	- .120	- .155	+ .00065	- .00196
2105	- .062	- .097	+ .070	- .00019	- .00046
Source B					
2101	+0.564	+0.483	+0.001	-0.00138	+0.00137
2102	+ .196	+ .320	+ .090	- .00099	+ .00039
2103	- .101	- .106	+ .067	- .00440	- .00089
2104	- .068	- .167	- .332	+ .00028	- .00138
2105	- .032	- .097	+ .207	- .00036	- .00077
Source C					
2101	+0.528	+0.455	+0.001	-0.00144	+0.00143
2102	+ .192	+ .326	+ .116	- .00109	+ .00031
2103	- .097	- .098	+ .084	- .00428	- .00154
2104	- .086	- .186	- .440	+ .00023	- .00116
2105	- .010	- .095	+ .305	- .00036	- .00085

TABLE 24. *Identification of the variable (X, Y, Z, x, y) and standard filter giving detection, at optimum or nearly optimum sensitivity, of various kinds of malfunction*

Malfunction \ Filter	2101	2102	2103	2104	2105	See table
Wavelength-scale	$XYxy$	-----	x	Zy	-----	17
Zero point of photometric scale	$x(*)$	$z(C)$	y	$xy(A)$	-----	18
100% point of photometric scale	XY	XY	-----	Z	XYZ	19
Stray-energy photomultiplier	X	Xx	x	x	-----	22
Stray-energy lead sulfide	Zx	Zx	y	y	-----	22
Inertia	XY	XY	x	Z	Z	23

(*) Use this variable only if x_r is less than x_0 .

(C) Use this variable only if the integrations are carried out for source C.

(A) Use this variable only if the integrations are carried out for source A.

If it were known that the malfunctions of the spectrophotometer-integrator system were confined to the five listed in table 24, and if the precision of the system were substantially perfect, it would be possible to evaluate easily the extent of each of these types of malfunction from the differences between the actual instrument readings and the par values. Note that for each source used in integration the information regarding malfunction of the instrument obtained in this way has no less than 15 degrees of freedom (three tristimulus values for each of the five filters). If the uniform displacement of the wavelength scale were designated Δ_λ , the displacement of the 100 percent point by Δ_{100} , the stray-energy fraction, f , by Δ_f , and the inertia constant by Δ_i , then we may write an expression for, say, $X_r - X_0$ for standard filter 2101 (orange red) that states merely that the difference between X_r and X_0 is made up of contributions from the only types of malfunction that the system has, thus:

$$X_r - X_0 = \Delta_\lambda \frac{\partial X}{\partial \Delta_\lambda} + \Delta_0 \frac{\partial X}{\partial \Delta_0} + \Delta_{100} \frac{\partial X}{\partial \Delta_{100}} + \Delta_f \frac{\partial X}{\partial \Delta_f} + \Delta_i \frac{\partial X}{\partial \Delta_i} \quad (6)$$

Note that 14 other such expressions can be written. The five variables are thus over-determined, and a least-square analysis should result in precise values of the five unknowns even with some lack of precision in the instrument reading. Note that the partial derivatives of eq 6 have been sufficiently well evaluated in tables 17, 18, 19, 22, and 23. The values of Δ_λ , Δ_0 , Δ_{100} , Δ_f , and Δ_i that might be found in this way are in the same units in which the partial derivatives, read from the tables, are expressed. Thus, if $\partial x / \partial \Delta_\lambda$ is read from table 17 as amount per millimicron, Δ_λ will be in $m\mu$, and if $\partial X / \partial \Delta_i$ is read from table 23, which is based on $k = -0.07$, Δ_i evaluates k of eq 2 in multiples of 0.07.

Since actual spectrophotometer-integrator systems are beset with more complicated malfunctions than the five simple types listed in table 24, and since the precision may be low enough so that not all of the differences like $X_r - X_0$ will be significant, it is presumed that this least-square solution for unknowns from 15 observation equations will usually not be worth doing. Table 24 suggests the following simplified procedure:

1. Determine the displacement of the 100 percent point on the photometric scale, Δ_{100} , from the average of the 8 variables specified in table 19 as responsive to this type of malfunction:

$$\Delta_{100} = \frac{1}{8} \left[\Sigma^{1,2,5} \frac{(X_r - X_0)}{\partial X / \partial \Delta_{100}} + \Sigma^{1,2,5} \frac{(Y_r - Y_0)}{\partial Y / \partial \Delta_{100}} + \Sigma^{4,5} \frac{(Z_r - Z_0)}{\partial Z / \partial \Delta_{100}} \right] \quad (7)$$

The partial derivatives are obtained from table 19. The first two summations are to be taken for standard filters 2101, 2102, and 2105; the third summation, for standard filters 2104 and 2105.

2. Tentatively (see next paragraph) determine wavelength-scale displacement Δ_λ as the solution of the following two simultaneous equations:

$$\left. \begin{aligned} \text{For filter 2103, } x_r - x_0 &= \Delta_\lambda (\partial x / \partial \Delta_\lambda) + \Delta_0 (\partial x / \partial \Delta_0) \\ \text{For filter 2101, } y_r - y_0 &= \Delta_\lambda (\partial y / \partial \Delta_\lambda) + \Delta_0 (\partial y / \partial \Delta_0) \end{aligned} \right\} \quad (8)$$

where the partial derivatives are read from tables 17 and 18. Incidental to this solution for Δ_λ there will also be found a solution for Δ_0 , but this should be set aside.

The idea behind this recommendation for finding the wavelength-scale displacement is that the x -coordinate of filter 2103 and the y -coordinate of filter 2101 are the most sensitive indications of wavelength-scale displacement. Note also that by using chromaticity coordinates, no account need be taken even of very large displacements in the 100 percent point found from eq 6, because such displacements have no influence on the chromaticity coordinates. This plan is strictly applicable to spectrophotometer-integrator systems having negligible stray-energy and inertia errors, and should apply very well to

systems involving double dispersion. It is believed to be worth trying for other systems as well.

3. Determine the displacement of the zero-point from each of four equations, two of them in x -coordinate for standard filters 2101 and 2102, and two in y -coordinate for standard filters 2103 and 2104.

For filters 2101 and 2102:

$$\Delta_0 = [(x_r - x_0) - \Delta_\lambda (\partial x / \partial \Delta_\lambda)] / (\partial x / \partial \Delta_0)$$

For filters 2103 and 2104:

$$\Delta_0 = [(y_r - y_0) - \Delta_\lambda (\partial y / \partial \Delta_\lambda)] / (\partial y / \partial \Delta_0)$$

where the partial derivatives are found from tables 17 and 18, the value of Δ_λ is that found from eq 8. The average of the four values of zero-point displacement so found is close to the best available from the actual and par values for the five standard filters; the deviations of the individual values from this average is an indication of the extent to which other types of malfunction are afflicting the system.

4. Analogous procedures might be used for systems afflicted with stray energy (single-dispersion systems) in which the partial derivatives would be read from table 22, and for systems afflicted with inertial effects in which the partial derivatives would be read from table 23. It seems likely, however, to be more useful to evaluate stray energy by comparison of the actual curves of spectral transmittance obtained on the spectrophotometer of the system with the applicable curve of $R_{s\lambda}$ found from eq 5; see figures 2 through 6.

5. Summary

A reexamination has been made of the fundamental measurement of spectral transmittance of nonscattering transparent colored media. A search made to resolve errors often considered as negligible or self-compensating showed the significance of errors that may arise from lack of consideration of stray energy, slit width, back reflectance, and recorder inertia.

The effect of type of integration of the spectrophotometer data in the conversion to colorimetric terms showed that summations by the 10 $m\mu$ weighted ordinate method or by the 30-selected-ordinate method are sufficient for most colorimetric work.

The effect of stray energy on both spectrophotometric and colorimetric data is illustrated.

We acknowledge the assistance of Victor R. Weidner in checking most of the tabulations of data in the tables of this report, Mrs. Iola Smith for the visual comparisons of the limit samples of these glasses on the Judd chromaticity-difference colorimeter as well as for the checking of the Y scale of the red duplicates on the Walker-Haupt transmittance photometer, and Miss Marion Belknap for the spectrophotometric measurements on the Beckman DU and the König-Martens spectrophotometers.

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(Paper 66A3-154)

Recalibration of the NBS Glass Standards of Spectral Transmittance¹

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(July 10, 1963)

In 1934, Gibson, Walker, and Brown developed sets of four colored glass filters to serve as working standards of spectral transmittance for checking the reliability of spectrophotometers. Several sets of these glasses were measured carefully and reserved and designated as future reference standards. Duplicate standards evaluated by comparison with the reference standards are available by purchase to the public. The current set of reference standards was established in the years 1945 to 1947, and one of these reference standards (selenium-red) was recalibrated in 1952. This paper reports a recalibration, made in 1961 and 1962, of all four glasses (selenium-red, carbon-yellow, copper-green, cobalt-blue) on three spectrophotometers (Cary 14, Beckman DU, König-Martens). Except for the cobalt-blue standard, the values of spectral transmittance found differ from those previously assigned by amounts differing at some wavelengths by as much as or slightly more than the uncertainties estimated for the present values, though not by amounts exceeding the combined uncertainties of the present and previous determinations. The indicated changes for these three standards are fairly regular, however, and support the view that the selenium-red and carbon-yellow standards are changing chiefly by formation of a reflectance-reducing film on the surfaces. The indicated rate of upward drift is slow, and suggests that it takes about 10 years for the drift to exceed the assigned uncertainty.

1. Introduction

In 1934, Gibson, Walker, and Brown [1]² described a set of four colored glass filters which they had developed to serve as standards of spectral transmittance to check the reliability of spectrophotometers. Several sets of these glasses were carefully measured at a number of wavelengths by using mercury, helium and incandescent sources and were reserved and designated as future reference standards. Duplicate standards evaluated by comparison with the reference standards were made available to the public by purchase. Although nearly nonselective filters are best suited to the detection of errors in the photometric scale, the filters chosen have spectral transmittances that vary markedly with wavelength and thus permit detection of slit-width errors and stray-energy errors as well as photometric-scale errors; thus a single selective filter may afford checks of the photometric scale over a large fraction of the whole scale [2, 3]. Table 1 identifies the glasses that have served as reference standards, and figure 1 shows the approximate spectral transmittances of these filters. The reports issued with the sets of duplicate standards gave the assigned values of transmittance at about 20 wavelengths between 390 and 750 nm³ together with the estimated uncertainties and the temperature coefficients.

TABLE 1. Identification of the glasses used as reference standards

No.	Designation	1934-1940	1945-1947
1	Selenium-red	Jena OG 3-8 Jena OG 3-10	Jena OG 3-21
2	Carbon-yellow	Corning HT Yellow α^{16} HT Yellow α^7	Corning HT Yellow C-14
3	Copper-green	Jena BG 14-1 Jena BG 14-10	Jena BG 14-19
4	Cobalt-blue	Corning G 55A α^{11} G 55A α^3	Corning G 55A β^9

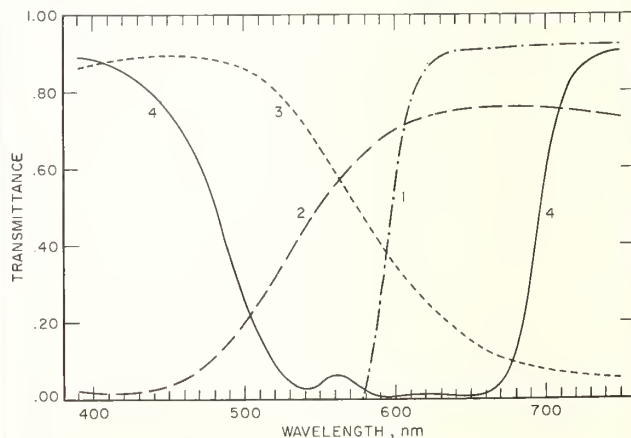


FIGURE 1. Typical spectral transmittance curves of glass duplicate standards issued by the NBS for checking the reliability of spectrophotometers.

The four types are: 1. Selenium-red, 2. Carbon-yellow, 3. Copper-green, and 4. Cobalt-blue.

¹ An abridged version of this paper will appear in the J. Opt. Soc. Am. 54 January 1964.

² Figures in brackets indicate the literature references at the end of this paper.

³ Nanometers, 10⁻⁹ meters, formerly called millimicrons.

TABLE 2. Estimated uncertainty in the value of absorbance, U_a , and the change d , in absorbance for temperature change of 15°C , for the selected wavelengths for incandescent (Inc.) and line (Hg and He) sources used for the four reference standards

Wavelength	Source	Selenium-red		Carbon-yellow		Copper-green		Cobalt-blue	
λ in nm		U_a	d	U_a	d	U_a	d	U_a	d
390	Inc.			± 0.052	-0.0005	± 0.002	0.000	± 0.002	0.000
404.7	Hg			.069	-.0025	.002	.000	.002	.000
420	Inc.			.046	-.0025				
435.8	Hg			.014	-.0015	.001	.000	.002	.000
471.3	He			.011	+.001	.001	.000	.004	.000
491.6	Hg							.005	+.0005
501.6	He			.006	+.0035	.002	.000	.007	+.001
520	Inc.			.004	+.0035			.010	+.0005
530	Inc.			.003	+.003	.002	.000		
540	Inc.							.008	-.003
546.1	Hg			.003	+.0025	.003	.000	.008	-.004
560	Inc.			.002	+.002	.003	.000	.007	-.003
570	Inc.	0.000							
578	Hg	$\pm .06$	$+.017$.003	+.0015	.004	-.0005	.008	-.004
587.6	He	.04	+.11	.003	+.0015				
600	Inc.	.008	+.022	.002	+.001	.006	-.0005	.024	-.005
620	Inc.	.003	+.002	.002	+.0005	.005	-.001	.017	-.003
640	Inc.	.002	.000	.003	+.0005	.007	-.001	.023	-.005
660	Inc.	.002	.000	.003	.000				
667.8	He					.007	-.001	.027	+.004
680	Inc.							.033	+.006
690	Inc.	.002	.000	.003	.000			.026	+.004
706.5	He							.004	+.002
710	Inc.					.012	-.001		
720	Inc.	.002	.000	.003	-.0005			.003	+.0005
750	Inc.	.002	.000	.003	-.0005	.015	-.0005	.002	.000

TABLE 3. Previously adopted value of transmittance, T , and estimated uncertainty in the values of transmittance, U , at the respective wavelengths for the four reference standards

Wave-length	Selenium-red (1952)*		Carbon-yellow (1946)*		Copper-green (1946)*		Cobalt-blue (1946)*	
λ in nm	T	U	T	U	T	U	T	U
390			0.025	± 0.003	0.862	± 0.003	0.895	± 0.004
404.7			.020	.003	.877	.003	.884	.005
420			.019	.002				
435.8			.0240	.0007	.893	.004	.806	.003
471.3			.081	.002	.894	.004	.612	.006
491.6							.344	.004
501.6			.208	.003	.859	.005	.245	.004
520			.316	.003			.091	.002
530			.379	.003	.760	.004		
540							.0308	.0006
546.1			.479	.003	.671	.004	.0335	.0006
560			.557	.003	.585	.004	.064	.001
570	0.000							
578	.0042	± 0.0005	.636	.003	.473	.004	.0273	.0005
587.6	.118	.01	.668	.003				
600	.550	.02	.699	.003	.350	.003	.0074	.0002
620	.852	.007	.731	.004	.256	.003	.0100	.0002
640	.904	.005	.747	.005	.187	.003	.0074	.0002
660	.914	.005	.754	.005				
667.8					.122	.002	.034	.002
680							.14	.01
690	.919	.005	.755	.005			.34	.02
706.5							.713	.007
710					.074	.002		
720	.918	.005	.748	.005			.845	.005
750	.917	.005	.730	.005	.057	.002	.901	.005

*Date of adoption of values.

Table 2 gives for each wavelength the light source used and the estimated uncertainties and a multiple of the temperature coefficients of the reference standards expressed on the absorbance scale. Absorbance is defined as the logarithm of the reciprocal of the internal transmittance T_i . Since internal transmittance for these glasses is closely equal to transmittance T multiplied by a constant (about 1/0.9) to correct for reflection losses [8], differences in absorbance are closely equal to differences in the logarithm of the reciprocal of transmittance T un-

corrected for reflection losses. Accordingly the uncertainties U_a in absorbance entered in table 2 are computed from the uncertainties U in transmittance as:

$$U_a = [\log_{10} (T+U) - \log_{10} (T-U)]/2$$

Similarly, the temperature coefficients δ of absorbance are computed from the relation:

$$-\log_{10} T_\theta = -\log_{10} T_{25} + b\delta(\theta - 25).$$

where θ is the temperature of the glass in degrees C, b is the thickness of the glass in mm, and $-\log_{10} T_{25}$ is the absorbance measurement made at 25 °C. The temperature coefficient δ is thus the change in $-\log_{10} T$ (closely equal to the change in $-\log_{10} T_i$) for a temperature change of 1 °C, and a thickness of 1 mm. These glass reference standards are measured at 25 °C and are used over a temperature range of ± 15 °C. For convenience, because the temperature coefficient is relatively small compared with the value of absorbance, a quantity d equal to 15 times the temperature coefficient is used to designate the change in absorbance for a temperature change of 15 °C, as follows:

$$-\log_{10} T_{\theta} = -\log_{10} T_{25} + bd(\theta - 25)/15.$$

This relationship is considered valid for values of θ between 0 and 50 °C.

The current set of reference standards was measured in the years 1945 to 1947. A detailed analysis of the data obtained for one of the reference standards (cobalt-blue, Corning G 55A β^9) was published by Gibson and Balcom [4] in 1947. The uncertainties, U , were estimated as the huge error (4.9 times the probable error) of the mean values of transmittance. In 1950, Gibson and Belknap [5] published a study of the permanence of samples of the carbon-yellow, the copper-green, and the cobalt-blue glasses exposed under clear glass to south skylight and sunlight for periods up to three years. Expressed in terms of transmittance change ΔT , these samples were found to change but slightly, excepting only the copper-green glass below 500 nm, where there appears to be a definite solarization effect.

No such study was made for ordinary use of these glasses in the laboratory. It was assumed that if the glasses would change only slowly with extensive exposure to sunlight under glass, they would not change significantly in many years of ordinary use. A redetermination of the spectral transmittance of the current reference standard for selenium-red glass was carried out in 1952 and resulted in slight changes in the assigned values, but it could not be determined with certainty that the transmittance had changed. The 1946 and 1952 data are given in table 3.

Meanwhile, interest in the duplicate standards was increasing. Table 4 shows the number of duplicate standards of each type of glass issued in the years 1933–1942, 1943–1946, and 1947–1962.

TABLE 4. Numbers of duplicate standards issued

Years	Calibration of duplicate standards in charge of:	Selenium-red	Carbon-yellow	Copper-green	Cobalt-blue	Total
1933–1942	Mabel E. Brown and Geraldine W. Haupt	7	20	10	15	52
1943–1946	Margaret M. Balcom and Lois A. Peterson	0	11	7	16	34
1947–1962	Marion A. Belknap	20	161	113	99	393
	Totals.....	27	192	130	130	479

A set of these glass standards, issued in 1958 to the Frankford Arsenal, was submitted in October 1961 for recalibration. A routine comparison of them with the same reference standards used in the original calibration showed that the transmittances for each of the four glasses bore a significantly different relation to the corresponding reference standard than in 1958. It was presumed that this changed relation referred primarily to the duplicate standards, whose uses and possible exposure to radiant energy at the Frankford Arsenal were unspecified, rather than to the reference standards known to be irradiated only by incandescent-lamp light for short periods of time at infrequent intervals; and this presumption was subsequently proved to be correct. Nevertheless, check data obtained in 1952, 1953, 1959, and 1960 for the reference standards were reviewed. Although no single set of data proved that any of the reference standards had certainly changed, all four sets of data considered together showed some evidence of a slow continuing drift in spectral transmittance of the selenium-red, the carbon-yellow and the copper-green reference standards. The cobalt-blue glass alone showed no change in spectral transmittance. An extensive recalibration of all four reference standards was then undertaken. It is the purpose of the present paper to describe this recalibration, and to discuss its results.

2. Method of Spectrophotometry

The recalibration of the four reference standards of spectral transmittance was carried out by means of the Cary 14 recording spectrophotometer, the Beckman DU manual spectrophotometer [6], and the König-Martens visual spectrophotometer [7]. Table 5 lists the spectrophotometers used in the several calibrations of reference standards of spectral transmittance. The procedural details of the present recalibration are given below.

2.1. Cary Recording Spectrophotometer

Two sets of measurements of each of the four reference standards of spectral transmittance were made on the Cary recording photoelectric spectrophotometer (Model 14M, Serial No. 173) by means of the absorbance ($-\log T$) scale. At the same time measurements were made of a clear borosilicate crown glass, 1.0 mm thick, of known index of refraction and Abbe value ($n_D = 1.517$, $\nu = 64.5$).⁴ This index and Abbe value are the same as those found for the borosilicate crown glass, 1.0 mm thick, whose spectral transmittances were known from a previous study [8]. These known spectral transmittances were used as a photometric-scale correction for transmittances of the reference standards greater than 40 percent. For each wavelength the ratio R_{λ} of the reference-standard transmittance to

⁴ We are indebted to Irving Malitson for these measurements of index of refraction and Abbe value

TABLE 5. *Spectrophotometers used in the several calibrations of reference standards of spectral transmittance*

Spectrophotometer	Date of calibration			
König-Martens visual.....	1930-1940	1940-1950	1950-1960	1960-
Gibson photoelectric and thermoelectric.	1930-1940	1940-1950		
Hilger photographic sector-photometer.	1930-1940			
Beckman DU photoelectric ("BQ-1")			1950-1960	1960-
Beckman DU photoelectric ("BQ-2")		1940-1950		
Cary 14M recording photoelectric.....				1960-

that of the borosilicate crown glass was computed from the absorbance readings. The value T_λ of the transmittance of the reference standard was computed as:

$$T_\lambda = R_\lambda T_{0\lambda},$$

where $T_{0\lambda}$ is the known spectral transmittance of the borosilicate crown glass [8]. For T_λ less than 40 percent the value corresponding to the reading of the absorbance scale was accepted without correction. The wavelength calibration of the instrument was carried out as described by Keegan, Schleter, and Judd [9]. All measurements refer to the specimen at 25 °C.

2.2. Beckman DU Spectrophotometer

The spectral transmittances of the reference standards were measured on the Beckman DU ("BQ-1") spectrophotometer with incandescent source after calibration of wavelength scale at emission lines of mercury, helium, hydrogen, neon, and cesium. Frequent checks of the wavelength calibration were made by means of the mercury line at 546.1 nm. Each standard was measured in two positions in the sample holder for each set of readings. The holder was positioned so as to place the standard near the exit slit and away from the phototube. As recommended by Gibson and Balcom [4], the measurements were made with no lens over the exit slit. The spectral transmittance of each standard was measured at selected wavelengths by ratio to a blank beam set to 100 percent transmittance as a reference. The shift from the blue-sensitive to red-sensitive phototube was made at 620 nm. The selector switch was shifted to 0.1 for transmittance readings below 10 percent to give "times-10" readings at narrower slitwidths. A purple stray-energy filter was used for all measurements taken at 390 nm. The sample compartment was surrounded by a water jacket maintained at 25 °C by a constant-temperature water bath. The numbers of sets of readings made on the reference standards are as follows:

Selenium-red		Carbon-yellow	Copper-green	Cobalt-blue
Below 600nm	Above 600nm	Full spectra	Full spectra	Full spectra
2	3	7	3	2

The extra set of readings above 600 nm for the sele-

mium-red standard was obtained by ratio of readings for the standard to those for a clear glass (Corning 9700).

2.3. König-Martens Spectrophotometer

The spectral transmittances of the four reference standards were measured on the König-Martens visual spectrophotometer [7] by using emission lines of a mercury source and a helium source and, at some wavelengths, by using an incandescent source. As explained on pages 464-465 of reference [5], it was early found advisable, because of the low luminosities of the Hg and He sources at 471.3, 491.6, 667.8, and 706.5 nm, to use the incandescent source at these wavelength settings, and this has been done in all later work on the König-Martens visual spectrophotometer. However, for the present standardization of the NBS reference standards, measurements were made with both incandescent and line sources at these four wavelengths. The wavelength scale was checked by settings on certain of the mercury and helium lines each time the instrument was used and each time the width of the ocular slit was changed. The widths of the collimator and ocular slits were 0.2 mm for the readings taken by means of the incandescent source, and were varied from 0.2 to 0.4, or 0.5 mm for the line sources. Accurately calibrated sector disks (transmittances approximately 1, 10, 50, and 60 percent) were used for measurement of low transmittances or for transmittances near to those of the sectors to increase the accuracy of the readings. Stray-energy filters were used at wavelengths near the two ends of the visible spectrum and at wavelengths of low transmittance. Each set of readings consisted of 20 readings of angle on the Martens photometer; first, 10 for the standard inserted in one beam, and second, 10 for it in the other beam. The transmittance was computed as the cotangent of the first angle multiplied by the tangent of the second. At least two sets of readings were taken for each standard at each selected wavelength between 404.7 and 660 nm and a few additional readings between 660 and 706.5 nm because of the added uncertainty of the settings in this region of low luminosity. All readings were made with the standards in a holder maintained at 25° C by means of a constant-temperature water bath.

3. Reduction of Data and Estimates of Uncertainty

The spectral transmittances assigned to the reference standards are weighted means of the transmittances found individually by the three spectrophotometers used. The weights assigned to the individual values of transmittance for a particular standard at a particular wavelength were based on the known relative capabilities of the three instruments. For example, the values obtained by means of the König-Martens visual spectrophotometer in spectral regions yielding observing fields of low luminance (such as near the extremes of the visible

spectrum) were given less weight because of the resulting unreliability of the visual settings. On the other hand values of spectral transmittance obtained by means of an emission line in a spectral region of rapid variation of transmittance with wavelength were given more weight.

In tables 6, 7, 8, and 9, are given for each of the four reference standards and for each of the three instruments at the respective wavelengths, the average value of the transmittance so measured, the assigned weight, the range in transmittance, the adopted weighted mean, and the estimated uncertainty in the value of transmittance.

The year or years of measurement on the indicated spectrophotometers are shown in the parenthesis under the name of the instrument. The uncertainty

of the assigned value of transmittance at a particular wavelength was made the same as that assigned previously (see table 3); with exceptional cases where it was agreed that a change was necessary. In these cases the uncertainty was estimated from the range of the individual values obtained on the three instruments averaged over a spectral region variable in extent from 0 to 50 nm and centered on that wavelength. Some smoothing of these values was resorted to whenever no reason was apparent for a rapid change in uncertainty with wavelength. In no case was the uncertainty allowed to be less than one-half the range of the spectrophotometric data. These ranges correspond roughly to estimates of three times the standard deviation of the adopted value [10].

TABLE 6. Spectral transmittance, T , of selenium-red reference standard Jena OG 3-21, 1.7 mm, as measured on indicated spectrophotometers; assigned weight, W , range, R , adopted weighted mean, and estimated uncertainty of value of transmittance, U

Wave-length λ in nm	König-Martens (1962)*		Beckman DU (1961-1962)*		Cary Model 14 (1962)*		Range	Adopted weighted mean	Esti- mated uncer- tainty
	T	W	T	W	T	W	R	T	U
390 to 570	0.000	-----	0.000	-----	0.000	-----	0.000	0.000	-----
578 587.6	.0041 .115	4 4	.0033 .102	2 3	.0034 .110	4 3	.0008 .013	.0037 .110	± 0.0005 .01
600	.564	3	.556	4	.561	3	.008	.560	.01
620	.858	4	.856	4	.861	2	.005	.857	.006
640	.906	4	.903	4	.909	2	.006	.905	.004
660	.913	4	.914	4	.920	2	.007	.915	.004
690	.917	3	.918	5	.925	2	.008	.919	.004
720	.918	2	.919	6	.926	2	.008	.920	.004
750	.919	1	.919	7	.925	2	.006	.920	.004

*Date of measurement.

TABLE 7. Spectral transmittance, T , of carbon-yellow reference standards Corning IIT yellow C14, 2.0 mm, as measured on indicated spectrophotometers; assigned weight, W , range, R , adopted weighted mean and estimated uncertainty of transmittance, U

Wave-length λ in nm	König-Martens (1962)*		Beckman DU (1961-1962)*		Cary Model 14 (1962)*		Range	Adopted weighted mean	Esti- mated uncer- tainty
	T	W	T	W	T	W	R	T	U
390	-----	-----	0.025	5	0.025	5	0.000	0.025	± 0.003
404.7	0.019	3	.019	4	.019	3	.000	.019	.003
420	-----	-----	.019	5	.019	5	.000	.019	.002
435.8	.0244	4	.0234	2	.0248	4	.0014	.0244	.0008
471.3	.081	2	.080	4	.082	4	.002	.081	.002
501.6	.209	3	.206	4	.209	3	.003	.208	.003
520	.316	3	.318	4	.320	3	.004	.318	.003
530	.381	3	.380	4	.382	3	.002	.381	.003
546.1	.480	3	.480	4	.484	3	.004	.481	.003
560	.558	3	.560	4	.562	3	.004	.560	.003
578	.634	3	.639	4	.642	3	.008	.638	.004
587.6	.671	3	.669	4	.674	3	.005	.671	.004
600	.702	4	.702	4	.704	2	.002	.702	.004
620	.736	4	.734	4	.739	2	.005	.736	.004
640	.750	4	.751	4	.756	2	.006	.752	.005
660	.756	4	.758	4	.763	2	.007	.758	.005
690	.760	3	.759	4	.764	3	.005	.761	.005
720	-----	-----	.752	6	.756	4	.004	.754	.005
750	-----	-----	.733	6	.736	4	.003	.734	.005

*Date of measurement.

TABLE 8. Spectral transmittance, T , of copper-green reference standard Jena BG 14-19, 2.0 mm, as measured on indicated spectrophotometers; assigned weight, W , range, R , adopted weighted mean, and estimated uncertainty of transmittance, U

Wave-length	König-Martens (1962)*		Beckman DU (1962)*		Cary Model 14 (1962)*		Range	Adopted weighted mean	Estimated uncertainty
λ in nm	T	W	T	W	T	W	R	T	U
390			0.866	6	0.864	4	0.002	0.865	± 0.003
404.7	0.876	2	.878	4	.878	4	.002	.878	.003
435.8	.893	3	.891	4	.892	3	.002	.892	.003
471.3	.893	2	.893	4	.892	4	.001	.893	.003
501.6	.862	3	.861	4	.860	3	.002	.861	.003
530	.759	3	.765	4	.762	3	.006	.762	.004
546.1	.672	3	.678	4	.677	3	.006	.676	.004
560	.593	3	.594	4	.589	3	.005	.592	.004
578	.477	3	.481	4	.476	3	.005	.478	.004
600	.348	3	.358	4	.352	3	.010	.353	.005
620	.258	3	.257	4	.258	3	.001	.258	.003
640	.187	3	.188	4	.187	3	.001	.187	.003
667.8	.123	3	.123	4	.122	3	.001	.123	.002
710	.078	1	.075	5	.075	4	.003	.075	.002
750			.057	5	.057	5	.000	.057	.002

* Date of measurement.

TABLE 9. Spectral transmittance, T , of cobalt-blue reference standard Corning G 55 A³, 3.0 mm, as measured on indicated spectrophotometers; assigned weight, W , range, R , adopted weighted mean, and estimated uncertainty of transmittance, U

Wave-length	König-Martens (1962)*		Beckman DU (1962)*		Cary Model 14 (1962)*		Range	Adopted weighted mean	Estimated uncertainty
λ in nm	T	W	T	W	T	W	R	T	U
390			0.894	6	0.892	4	0.002	0.893	± 0.004
404.7	0.887	2	.880	4	.880	4	.007	.881	.005
435.8	.808	3	.808	4	.805	3	.003	.807	.003
471.3	.613	2	.618	4	.612	4	.006	.615	.006
491.6	.344	2	.351	4	.342	4	.009	.346	.004
501.6	.248	3	.249	4	.244	3	.005	.247	.004
520	.091	4	.092	3	.089	3	.003	.091	.002
540	.0314	4	.0308	3	.0310	3	.0006	.0311	.0006
546.1	.0339	4	.0334	3	.0344	3	.0010	.0339	.0006
560	.064	4	.064	3	.065	3	.001	.064	.001
578	.0290	4	.0281	3	.0274	3	.0016	.0282	.0008
600	.0073	4	.0072	3	.0073	3	.0001	.0073	.0004
620	.0100	4	.0104	3	.0104	3	.0004	.0102	.0004
640	.0075	6	.0082	2	.0074	2	.0008	.0076	.0004
667.8	.032	4	.032	4	.034	2	.002	.032	.002
680	.127	2	.132	5	.139	3	.012	.133	.01
690	.310	2	.338	5	.352	3	.042	.337	.02
706.5	.708	2	.703	4	.714	4	.011	.708	.007
720			.844	7	.852	3	.008	.846	.005
750			.902	7	.909	3	.007	.904	.005

* Date of measurement.

In table 10 the change in transmittance between the present value and the previously adopted value of spectral transmittance is given for each of the four reference standards at the respective wavelengths.

Figures 2, 3, 4, and 5 show as circles these differences in spectral transmittance assigned by the present recalibration and that assigned by the

previous calibration for the selenium-red, the carbon-yellow, the copper-green, and the cobalt-blue standards. They also show as broken lines centered about the abscissa the estimates of the uncertainties of the values of spectral transmittance resulting from the present recalibration.

TABLE 10. Change in spectral transmittance, ΔT , for the four reference standards between the present adopted weighted mean and the previously adopted values

Wave-length	Seleni- um- red	Carbon- yellow	Copper- green	Cobalt- blue
λ in nm	ΔT (1962-1952)	ΔT (1962-1946)	ΔT (1962-1946)	ΔT (1962-1946)
390	-----	0.000	+0.003	-0.002
404.7	-----	-.001	+.001	-.003
420	-----	.000	-----	-----
435.8	-----	+.0004	-.001	+.001
471.3	-----	.000	-.001	+.003
491.6	-----	-----	-----	+.002
501.6	-----	.000	+.002	+.002
520	-----	+.002	+.002	.000
530	-----	+.002	-----	-----
540	-----	-----	-----	+.0003
546.1	-----	+.002	+.005	+.0004
560	-----	+.003	+.007	.000
570	0.000	-----	-----	-----
578	-.0005	+.002	+.005	+.0009
587.6	-.008	+.003	-----	-----
600	+.010	+.003	+.003	-.0001
620	+.005	+.005	+.002	+.0002
640	+.001	+.005	.000	+.0002
660	+.001	+.004	-----	-----
667.8	-----	-----	+.001	-.002
680	-----	-----	-----	-.003
690	.000	+.006	-----	-.002
706.5	-----	-----	-----	-.005
710	-----	-----	+.001	-----
720	+.002	+.006	-----	+.001
750	+.003	+.004	.000	+.003

4. Discussion

It will be noted from figure 2 that three of the ten newly assigned values of spectral transmittance for the selenium-red reference standard differ from the previous assigned values by amounts approaching the estimated uncertainty. Because the previously assigned values are uncertain by about the same amount, this recalibration by itself fails to prove that the selenium-red standard has changed since 1952. The results of the two spectrophotometers (König-Martens and Beckman DU "BQ-1") common to the 1952 determination and the present determination, however, both indicate that a change has occurred. Furthermore, comparison of the 1947 with the 1952 determination shows similar changes. It seems likely that indications of increases in transmittance between 620 and 750 nm correspond to formation of a reflectance-reducing film on the surfaces. Several of the duplicates of this reference standard show visible evidence of such films. The decreases in transmittance indicated near the cut-off of the transmittance curve may not be real. (See fig. 2, wavelengths 570 and 587.6 nm). In any case, there is no established cause for such a change.

Figure 3 gives an indication of formation of a reflectance-reducing film on the surfaces of the carbon-yellow reference standard similar to that by figure 2 for the selenium-red standard. Note that the indicated discrepancies between the present and the previous assignments of spectral transmittance are roughly proportional to the spectral transmittance.

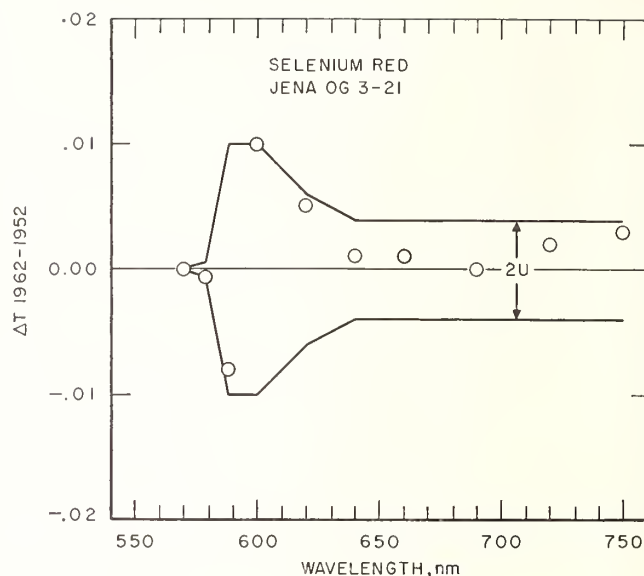


FIGURE 2. Selenium-red reference standard.

Differences (shown as circles) between spectral transmittances assigned in 1962 and in 1952 compared to estimates (shown as broken lines) of the uncertainty of the 1962 recalibration.

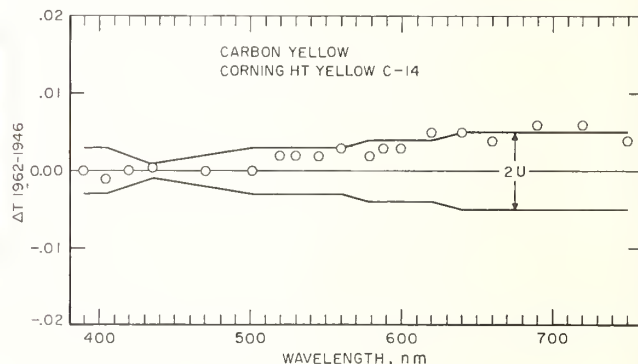


FIGURE 3. Carbon-yellow reference standard.

Differences (shown as circles) between spectral transmittances assigned in 1962 and in 1946 compared to estimates (shown as broken lines) of the uncertainty of the 1962 recalibration.

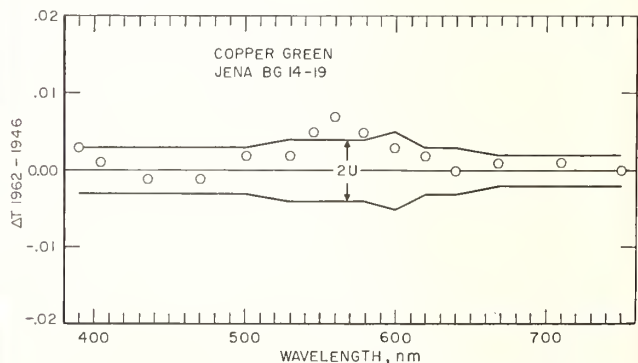


FIGURE 4. Copper-green reference standard.

Differences (shown as circles) between spectral transmittances assigned in 1962 and in 1946 compared to estimates (shown as broken lines) of the uncertainty of the 1962 recalibration.

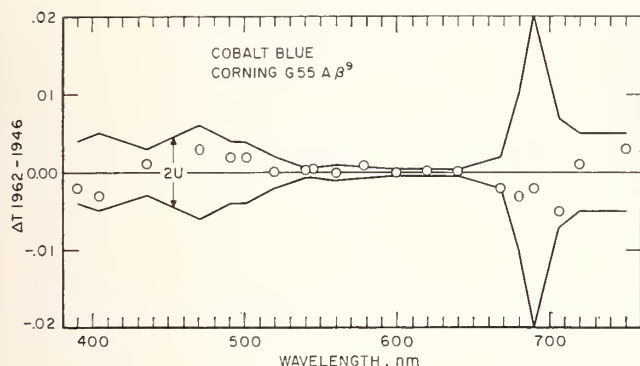


FIGURE 5. Cobalt-blue reference standard.

Differences (shown as circles) between spectral transmittances assigned in 1962 and in 1946 compared to estimates (shown as broken lines) of the uncertainty of the 1962 recalibration.

tances themselves, (see fig. 1), as would be expected from the formation of a reflectance-reducing film.

Figure 4 indicates some increases (390, 405, 500–620 nm) and some decreases (436, 471 nm) in assigned values of spectral transmittance. This indicated pattern is not consistent with the hypothesis that a reflectance-reducing film has been forming on the copper-green reference standard. If the indicated changes are real, and this has not been proved, they must be ascribed to a cause other than film formation (such as a slow chemical change in the glass), or to a combination of film formation with other cause or causes unknown.

Figure 5 gives no plausible suggestion that the cobalt-blue reference standard is changing even slowly. In table 10 the indicated changes are either small compared to the estimated uncertainty of the present determinations, or irregular.

In summary, it may be stated that three of the reference standards of spectral transmittance are probably subject to a slight impermanence, the maximum changes in a 10-year period being of the

order of magnitude of the uncertainties in their calibration. Owners of duplicate standards not subjected to unusually severe usage (exposure to high-energy particles, ultraviolet energy, high temperatures, or chemical fumes) may find in figures 2 to 5 a reasonably valid basis for revising the adopted transmittance for their duplicate standards. In the absence of information to the contrary, the changes may be assumed as a first approximation to be proportional with time, and the time for recently issued duplicates (1952 to 1961) should be counted from 1952 for the selenium-red standards, and from 1946 for the carbon-yellow, the copper-green, and the cobalt-blue standards.

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(Paper 67A6-243)

REFLECTANCE SPECTROPHOTOMETRY†

By

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A reflectance spectrophotometer is used to determine, for each part of the spectrum in turn, the ratio of the spectral radiant flux reflected from a light-diffusing specimen to that reflected from a light-diffusing standard substituted for the specimen. This ratio has traditionally been called *relative spectral directional reflectance*, *relative* because the specimen is evaluated in terms of a standard, such as magnesium oxide, *spectral* because the measurement is repeated for the various parts of the spectrum, and *directional* because, in general, the flux accepted for measurement is not all of the diffused or reflected flux but merely that leaving the specimen or standard in particular directions. A reflectance spectrophotometer thus consists of a device to irradiate the specimen or standard in particular directions, a detector of radiant flux by which the flux leaving the specimen or standard in particular directions is measured, and a spectrometer by means of which radiant flux is dispersed into a spectrum either before or after incidence on the specimen, permitting measurements to be made by means of a series of spectral bands, each with a relatively narrow wavelength range.

If the measured values of relative spectral directional reflectance are plotted versus wavelength, frequency, or wave number, at sufficiently small intervals, these values define what is often imprecisely called the spectral reflectance curve of the specimen. This curve gives basic information about the specimen. If it extends over the range of the visible spectrum it constitutes the basic information from which the color of a nonfluorescent specimen may be predicted for any known light source. From the spectral reflectance curve of an opaque specimen of a nonfluorescent material may be computed the ratio of the absorption coefficient to the scattering coefficient of the material as a function of wavelength. From two spectral reflectance curves of a translucent specimen of the same nonfluorescent material, one obtained with a black backing, the other with a white, may be computed both the absorption coefficient and the scattering coefficient of the material as a function of wavelength.

The absorption and scattering coefficients as functions of wavelength characterize the material and often permit the identification of the various constituents, such as dyes or pigments, and the determination of the concentrations of those constituents. Reflectance spectrophotometry may thus serve as a method of qualitative and quantitative analysis of a light-diffusing material supplementary to other methods.

From the absorption and scattering coefficients of a material may be computed the spectral reflectance curve of a specimen of any thickness of the material, and from the coefficients of a number of constituents may be computed the spectral reflectance curve of a material made up by mixture of these constituents in any proportion. By reverse application of this possibility, the proportions of a given group of colorant

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constituents required to produce any given color may be computed. Reflectance spectrophotometry thus permits formulation of colorants by calculation to produce a color match. See also COLOR DESIGNATION AND SPECIFICATION.

THEORY

The only theoretical model currently in significant use is the two-constant model developed by Kubelka and Munk (1). Slightly improved conformity to the facts is available from the six-constant model by Duntley (2), and from the rigorous model developed by Chandrasekhar (3) and Giovanelli (4); but the essential relations are given with satisfactory approximation by the two-constant theory (5), except for specimens of reflectance approaching zero.

Kubelka-Munk Model

The following conditions are assumed: (1) The extent of the horizontal layer is so large relative to its thickness that flux diffused horizontally out of the specimen through its edges can be neglected compared to what goes up or down. (2) The material of the layer is macroscopically uniform in spite of the microscopic inhomogeneities that must be present so as to cause diffusion of any radiant flux passing through the layer. (3) The flux incident on the top of the layer is perfectly diffused; that is, viewed from any point on the upper surface of the layer the source has constant radiance regardless of direction of view. (4) The optical medium in contact with the top of the specimen has the same index of refraction as the medium of the layer. Papers, textiles, or cold-water paint measured in air closely comply with this requirement. Paint film, plastic, and ceramic specimens immersed in a liquid of the same index of refraction as the specimen also comply.

Figure 1 shows a colorant layer of thickness X in optical contact with a backing of reflectance R_g , which is the ratio of the upward-proceeding flux entering the specimen through the bottom face to the downward-proceeding flux penetrating that face. Diffused flux is incident on the top of the layer. A fraction of the incident flux is reversed in direction and leaves through the upper face of the layer. This fraction is the reflectance, R , of the combination of colorant layer and backing. The absorption coefficient, K , is the fraction of the incident flux absorbed in passing diffusely through an elementary layer of thickness, dx . K is twice the fraction absorbed in passing perpendicularly through the layer (8). The scattering coefficient, S , is the fraction of the

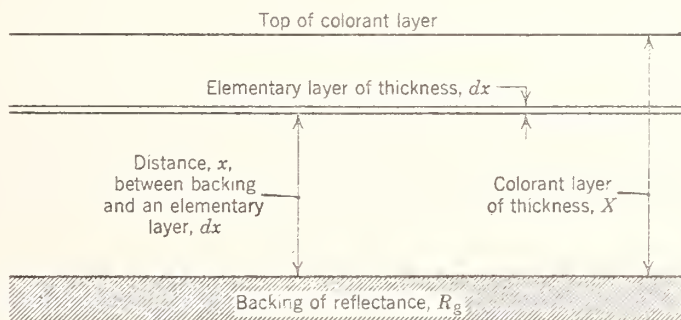


Fig. 1. Analysis of the reflection and transmission of light by a light-scattering layer.

REFLECTANCE SPECTROPHOTOMETRY

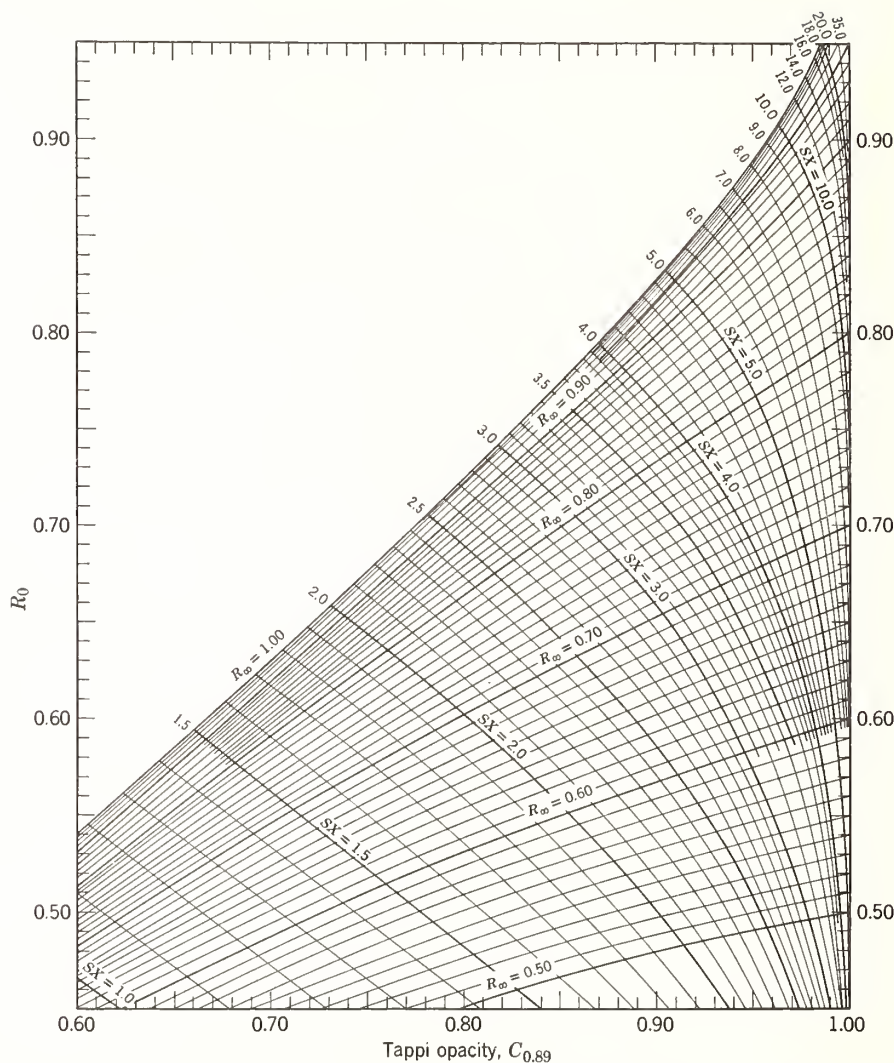


Fig. 2. Reflectance-opacity diagram based on the Kubelka-Munk model. It shows the interrelation of reflectance, R_0 , reflectivity, R_∞ , and Tappi opacity, $C_{0.89} = R_0/R_{0.89}$. The chart gives (1) the dependence of reflectance, Tappi opacity, and printing opacity on thickness; (2) interconversion of Tappi opacity and printing opacity; and (3) scattering power, SX , from reflectance and Tappi opacity.

incident flux backscattered by an elementary layer of thickness dx . To relate reflectance R with the scattering coefficient, S , and absorption coefficient, K , characterizing the material of the colorant layer, note that the change, dj , in the upward-proceeding flux, j , consists of $(S + K)j dx$ lost by scattering and absorption from the upward-proceeding flux and of $Si dx$ gained by backscattering of the flux incident on the elementary layer from the top; that is,

$$dj = -(S + K)j dx + Si dx \quad (1)$$

$$-di = -(S + K)i dx + Sj dx \quad (2)$$

Equation 2 refers to the analogous changes in the downward-proceeding flux, i . By solving these differential equations simultaneously and applying the boundary conditions, that is, for $x = 0$, $j/i = R_g$, and for $x = X$, $j/i = R$, Kubelka and Munk (1) obtained the following expression for reflectance:

$$R = \frac{(R_g - R_\infty)/R_\infty - R_\infty(R_g - 1/R_\infty) \exp(2bSX)}{R_g - R_\infty - (R_g - 1/R_\infty) \exp(2bSX)} \quad (3)$$

where R_∞ is the reflectance of the colorant layer for indefinitely great thickness ($X \rightarrow \infty$), and b is an abbreviation for $[(K^2/S^2) + 2K/S]^{1/2}$. They also showed that, with a an abbreviation for $(S + K)/S$,

$$R_\infty = a - b = 1 + K/S - [(K^2/S^2) + 2K/S]^{1/2} \quad (4)$$

whence

$$K/S = (1 - R_\infty)^2/2R_\infty \quad (5)$$

They also showed that

$$1/R_\infty = a + b = 1 + K/S + [(K^2/S^2) + 2K/S]^{1/2} \quad (6)$$

whence

$$a = (1/R_\infty + R_\infty)/2 \quad (7)$$

$$b = (1/R_\infty - R_\infty)/2 \quad (8)$$

Equation 3 is very general and, combined with Equation 5, summarizes all of the implications of the two-constant model for the behavior of light-scattering layers. This summary is inconvenient, however, because it does not permit explicit solution for the absorption coefficient, K , and scattering coefficient, S . Figure 2 is a reflectance-opacity chart, prepared (6) from Equation 3, that permits graphical solution for scattering coefficient S , after which K may be found from Equation 5. The abscissa is the ratio, R_0/R_{89} , of the reflectance R_0 of the layer with a black backing ($R_g = 0$) to the reflectance R_{89} , of the layer combined with backing for which $R_g = 89\%$. In paper technology this ratio is called contrast ratio, and is used as a measure of the opacity of the layer. The ordinate is reflectance, R_0 , of the layer with a black backing. By determining R_0 and R_{89} for the layer, and by plotting the corresponding point on this chart, the implied value of reflectance, R_∞ , producible by increasing indefinitely the thickness of the layer, and the implied value of the scattering coefficient, S , for the material of the layer may be read.

Following the lead of Steele (7), who expressed Equation 3 in terms of hyperbolic functions, Kubelka (8) developed a form of Equation 3 involving the hyperbolic cotangent which could be solved explicitly for the scattering coefficient, S . Equations 9-16 show the relationships between reflectance of layer-backing combinations, R , reflectance of backing, R_g , reflectance of an infinitely thick layer of the same material, R_∞ , the thickness of the material, X , and the scattering coefficient of the material, S (8). The second expression of Equation 12 was derived by Nordham and Göttsching (9).

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<i>Characteristic</i>	<i>Equation</i>
$R = f(SX, R_g, R_\infty)$	$R = \frac{1 - R_g(a - b \operatorname{ctgh} bSX)}{a - R_g + b \operatorname{ctgh} bSX}$ (9)

$R_0 = f(SX, R_\infty)$	$R_0 = 1/(a + b \operatorname{ctgh} bSX)$ (10)
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$SX = f(R, R_g, R_\infty)$	$SX = \frac{1}{b} \left(\operatorname{Ar} \operatorname{ctgh} \frac{a - R}{b} - \operatorname{Ar} \operatorname{ctgh} \frac{a - R_g}{b} \right)$ (11)
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$SX = f(R_0, R_\infty)$	$SX = \frac{1}{b} \left(\operatorname{Ar} \operatorname{ctgh} \frac{1 - aR_0}{bR_0} \right)$
	$= 2.303 \frac{R_\infty}{1 - R_\infty^2} \log \left[\frac{R_\infty(1 - R_0R_\infty)}{R_\infty - R_0} \right]$ (12)

$R = f(R_0, R_g, R_\infty)$	$R = \frac{R_0 - R_g(2aR_0 - 1)}{1 - R_0R_g}$ (13)
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$R_0 = f(R, R_g, R_\infty)$	$R_0 = (R - R_g)/[1 - R_g(2a - R)]$ (14)
-----------------------------	---

$R = f(R_1, R_0, R_g)$	$R = \frac{R_0(1 - R_g) + R_1R_g(1 - R_0)}{1 - R_0R_g}$ (15)
------------------------	---

$R_\infty = f(R, R_0, R_g)$	$a = \frac{1}{2} \left[R + \frac{R_0 - R + R_g}{R_0R_g} \right]$ (16)
-----------------------------	---

where $\operatorname{ctgh} u$, hyperbolic cotangent of u , is the abbreviation for $(e^u + e^{-u})/(e^u - e^{-u})$; $\operatorname{Ar} \operatorname{ctgh}$ is the inverse of the hyperbolic cotangent. If $v = \operatorname{ctgh} u$, then $u = \operatorname{Ar} \operatorname{ctgh} v$.

INSTRUMENTATION

Table 1 summarizes the characteristics of the five commercially available spectrophotometers. A discussion of each follows.

General Electric Spectrophotometer

The General Electric spectrophotometer in its 1953 redesigned form (10) consists of a double-dispersion prism monochromator combined with a polarization photometer operating on the null principle. A schematic diagram is shown in Figure 3. Dispersed radiant flux is directed alternately onto the test specimen and the comparison surface at 6° off the perpendicular, and the flux diffused in all directions, except those of the incident flux, is caught in a white-lined integrating sphere where it is sensed by a "cesium oxide" phototube. The flux incident on the comparison surface is controlled by a Rochon prism. If this prism is rotated to the balance position, the flux reflected by the comparison surface will be the same as that reflected from the specimen, the irradiance of the sphere will be constant, and the phototube current will be steady. If the Rochon prism is not set at the balance position, the phototube current will have an ac component which is used to drive the Rochon prism toward the balance position by a servo motor, the direction of drive being determined by the phase of the ac component. A cam translates rotation of the Rochon prism into terms of the ratio of the flux reflected by the specimen to that reflected by the comparison surface, and a recording mechanism plots this ratio as a function of wavelength from 380 to 700 m μ .

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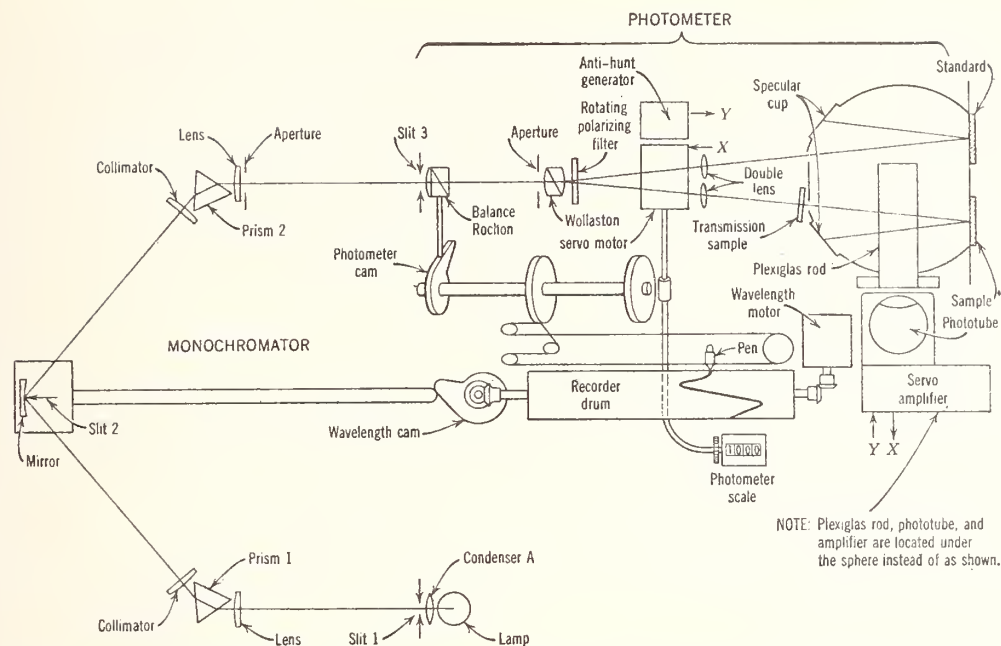


Fig. 3. Schematic diagram of General Electric recording spectrophotometer. Courtesy General Electric Company.

or nanometers, nm. This optical null method has the advantage of not requiring that the response of the phototube be linearly related to the incident flux. The flux incident on specimen and standard, however, is polarized in a plane that rotates through 180°

Table 1. Characteristics of Five Reflectance Spectrophotometers Commercially Available in 1964

Characteristic	General Electric	Beckman non-recording DU	Beckman DK-2A	Cary 14		Bausch & Lomb Spectronic 505
				dispersed incident flux	undispersed incident flux	
sources	incandescent	incandescent and quartz-mercury arc	incandescent and hydrogen discharge	incandescent and hydrogen discharge	incandescent	incandescent quartz-mercury arc and hydrogen discharge
angular condition incidence	6°	0°	0° or diffuse	0°	diffuse	25°
collection	diffuse	35-55°	diffuse or 0°	diffuse	0°	diffuse
detector	cesium oxide	cesium oxide and cesium antimony	photomultiplier and lead sulfide	photomultiplier	photomultiplier and lead sulfide	photomultiplier
photometric scale	optical	electrical	electrical	electrical	electrical	electrical
dispersion	double monochromator	single monochromator	single monochromator	double monochromator	double monochromator	single monochromator
stray-energy correction	not needed	provided by filters	needed but not provided	not needed	not needed	needed but not provided
spectral band pass, maximum, mμ	10	15	15	9	9	5
dispersing elements	2 prisms	1 prism	1 prism	1 prism, 1 grating	1 grating, 1 prism	2 gratings
wavelength range, mμ	380-700	210-1000	210-2500	260-800	350-2500	310-730
time for 1 scan, min	4-6	100-200	5-30	5	30	1-10

REFLECTANCE SPECTROPHOTOMETRY

as the incident flux is shifted from standard to specimen and back to the standard. On this account the balancing mechanism fails to function for samples (such as shiny textiles) whose reflectance for plane-polarized flux varies significantly with orientation of the plane of polarization. Special optical devices (11) are available to correct this defect. The use of two prisms in series, the exit slit of the first serving as the entrance slit for the second, reduces stray energy to negligible amounts. The spectral pass band is $10\text{ m}\mu$; therefore slit-width corrections are sometimes required, as are also photometric-scale and wavelength-scale corrections. Exclusion of the flux specularly reflected from glossy specimens is provided for at the option of the operator.

Beckman Spectrophotometers

The Beckman model DU spectrophotometer, which is nonrecording (12), widely used for measurement of transparent specimens, is available with a reflectance attachment by which the specimen is irradiated along the perpendicular and the flux diffused

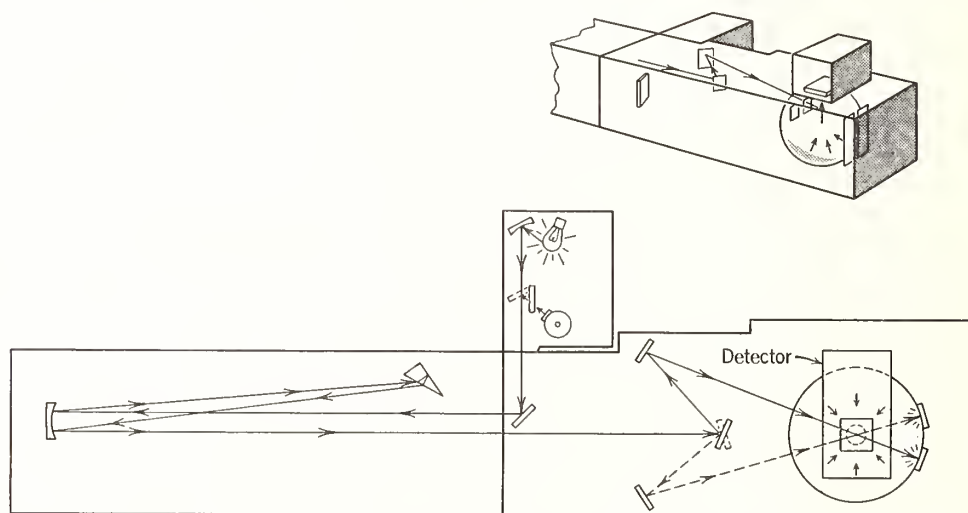


Fig. 4. Schematic diagram of Beckman DK-2A spectro-reflectometer. The Beckman DU nonrecording spectrophotometer corresponds to the left half of the diagram. Courtesy Beckman Instruments, Inc.

by the specimen between 35 and 55° is collected for measurements. Two vacuum phototubes are used. One is a long-wave sensitive "cesium oxide" tube; the other is a short-wave sensitive "cesium antimony" tube. The photoelectric current from the irradiated phototube produces a voltage drop across the load resistor which is balanced by a potentiometer. Accuracy of the photometric scale requires that the relation between the response of the phototube and the incident flux be linear, and this condition is found to be well fulfilled. Radiant flux from the source is dispersed by passage twice through a 30° quartz prism, and is then incident onto the specimen; see the left half of Figure 4. This instrument has the usual amount of stray energy present for single-dispersion spectrophotometers, and filters for the reduction of stray energy often must be used. The spectral band pass varies with wavelength from about 3 to $15\text{ m}\mu$. The useful wavelength range is approximately $210\text{--}1000\text{ m}\mu$.

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The Beckman Model DK-2A spectro-reflectometer, shown schematically in Figure 4, consists of a combination of the nonrecording Beckman DU spectrophotometer with an attachment for irradiating alternately specimen and standard, and with another attachment for recording the ratio of the flux diffused by the specimen to that diffused by the standard as a function of wavelength. The irradiating beam falls on the specimen when the rocking mirror is in one of its two positions, and on the standard when it is in the other; see the right half of Figure 4. The direction of irradiation is perpendicular to the surface of specimen or standard, and, except for the near-perpendicular directions, the flux diffused is caught by a white-lined sphere. Alternatively, the specimen may be tilted so as to be irradiated a few degrees off the perpendicular with the result that any flux specularly reflected from a glossy specimen may be included in the measurement at the option of the operator. Because stray-energy filters are not provided, the recorded curve is significantly in error for some specimens. Two detectors are supplied, a photomultiplier tube and a lead sulfide cell, as well as two sources, an incandescent lamp and a hydrogen discharge lamp. The useful wavelength range is approximately 210–2500 $m\mu$. Provision is also made for interchanging source and detector, in which case the specimen is irradiated diffusely by undispersed flux and is viewed along the perpendicular. This arrangement is used for measurement of fluorescent specimens.

Cary Model 14 Spectrophotometer

The Cary 14 spectrophotometer is supplied with a reflectance attachment which may be used to irradiate the specimen and standard either with dispersed flux or with undispersed flux. Figure 5 shows the former method. Flux leaves the incandescent lamp, passes through the entrance slit, is dispersed into a spectrum by double passage through a 30° quartz prism, and one spectral band is selected by the exit slit, which also forms the entrance slit for a grating. The grating disperses into another spectrum the flux of this band plus any stray flux outside it so that only the flux of the desired

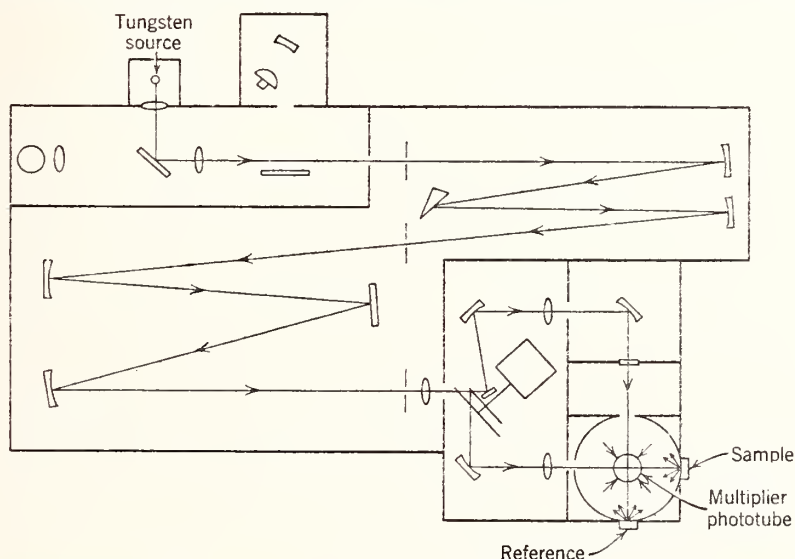


Fig. 5. Schematic diagram of the Cary 14 recording spectrophotometer used to irradiate the sample with dispersed flux.

REFLECTANCE SPECTROPHOTOMETRY

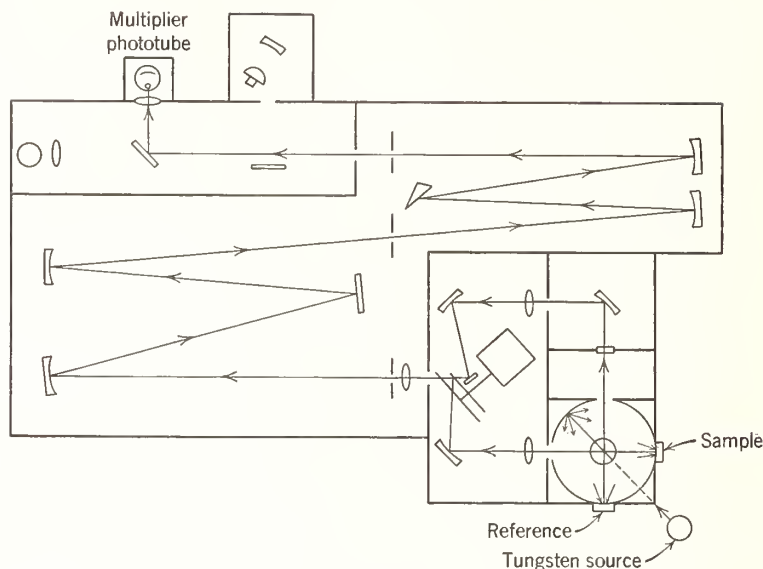


Fig. 6. Schematic diagram of the Cary 14 recording spectrophotometer used to irradiate the sample with undispersed flux by interchanging source and detector.

band is transmitted by the exit slit of the grating. This flux then is chopped and directed alternately to the specimen and standard in the perpendicular direction. An integrating sphere, white-lined with magnesium oxide, catches all of the flux diffused from the specimen or standard except that escaping out of the holes through which the incident beams enter the sphere. The recorder indicates as a function of wavelength the ratio of the flux diffused by the specimen to that diffused by the standard. If the photomultiplier tube and tungsten source are interchanged as in Figure 6, the specimen and standard are irradiated almost perfectly diffusely and are viewed along the perpendicular. This portion of the diffused flux is then chopped and undergoes double dispersion so that the photomultiplier tube receives alternately for each spectral band the flux diffused by the specimen and that diffused by the standard. A hydrogen discharge lamp is provided for use in the short-wave portion of the spectrum, and a lead sulfide cell is provided for the long-wave portion of the spectrum. The useful wavelength range is approximately 350–2500 $m\mu$. The use of undispersed flux to irradiate the specimen often causes undesirable heating of the specimen but it must be used with specimens of fluorescent material, or else the flux leaving the specimen by fluorescence is wrongly recorded at the exciting wavelength rather than correctly at its own wavelengths. Undispersed flux is also used to irradiate the specimen for long-wave measurements requiring the lead sulfide detector, because only by this method does the instrument focus sufficient flux onto the small sensitive area of the detector. The spectral width of the pass band is automatically adjusted to yield an optimum value of the detector response; therefore it varies for each specimen up to maximum values ranging from 3 to 9 $m\mu$, depending on the wavelength region.

Bausch and Lomb Spectrophotometer

The Bausch & Lomb Spectronic 505 spectrophotometer (13) uses two gratings to increase dispersion, but since it uses only two slits, the recordings for some speci-

mens are in significant error because of stray energy. Dispersed flux is incident on specimen and standard at 25° from the perpendicular, and the diffused flux is collected by an integrating sphere lined with barium sulfate. The detector is a photomultiplier tube, and the useful range is 310–730 $m\mu$. The scanning speed is automatically controlled, slowing down when the rate of change of reflectance with wavelength is high. Three sources are supplied, incandescent, quartz-mercury arc, and hydrogen discharge lamp.

OPERATING PROCEDURE

Preparation of Specimen

For applicability of the Kubelka-Munk analysis the specimen must conform approximately to the conditions given above. If the thickness of a translucent specimen is larger than one-tenth of the minimum diameter of the spot viewed by the collection device of the spectrophotometer, loss of flux through the edges begins to become important because of violation of condition 1. Inhomogeneities within the specimen, such as may be caused by the floating of one pigment component in a paint film to the surface, can lead to erroneous evaluation of absorption and scattering coefficients of the material because of violation of condition 2. None of the five spectrophotometers has both diffuse irradiation and diffuse viewing as required by condition 3, but the Kubelka-Munk model still serves as a useful approximation for analysis of data obtained by any of these reflectance spectrophotometers. Glossy-finish paint, plastic, or ceramic films may be usefully measured in contact with air in spite of their violation of condition 4. In this case the measurements should be corrected for the effect of external and internal reflection from the front face, such as by the relation developed by Saunderson (14):

$$R' = \rho + (1 - \rho)(1 - \rho_i)R/(1 - \rho_i R) \quad (17)$$

where R' is the instrumental reading expressed relative to the perfect diffuser, R is the reflectance to which the Kubelka-Munk model applies, ρ is the external reflectance of the surface (0.04–0.05), and ρ_i is the internal reflectance (0.4–0.6).

Calibration

The first step is calibration of the wavelength scale carried out either by means of emission lines of known wavelength in the spectra of mercury, hydrogen, helium, neon, cesium, rubidium, potassium, sodium, or other elements (15), or by obtaining the absorption spectra of rare-earth glasses (didymium, holmium) showing series of sharp absorption bands at known wavelengths (16).

The second step is calibration of the photometric scale carried out by obtaining at least a "zero" reading and a "100%" reading. The zero reading is obtained either by inserting a black-lined cavity into the specimen holder or by blocking off the beam either incident on or leaving the specimen, whichever is possible. The 100% reading is obtained by inserting a reflectance standard, usually magnesium oxide, in the specimen position. A magnesium oxide reflectance standard may be prepared by allowing the smoke from magnesium ribbon or turnings burning in air to deposit on a metal surface until an opaque layer about 1 mm thick is formed (17). The Kubelka-Munk model applies to reflectances on an absolute scale rather than to reflectances expressed relative to magnesium oxide. Table 2 shows as a function of wavelength the re-

REFLECTANCE SPECTROPHOTOMETRY

Table 2. Spectral Reflectance of Magnesium Oxide for Diffuse Irradiation and Hemispherical Viewing Relative to the Perfect Diffuser in Percent

Wavelength, $m\mu$	Middleton and Sanders (best coating)	Budde	Average
360	96.9		96.9
370	97.0		97.0
380	97.1	96.9	97.0
390	97.2	97.4	97.3
400	97.3	97.7	97.5
410	97.4	97.9	97.7
420	97.5	98.0	97.8
430	97.6	98.0	97.8
440	97.7	98.0	97.8
450	97.8	98.0	97.9
460	97.8	98.1	98.0
470	97.9	98.2	98.0
480	97.9	98.2	98.0
490	98.0	98.1	98.0
500	98.0	98.0	98.0
510	98.0	98.0	98.0
520	98.0	97.9	98.0
530	98.0	97.9	98.0
540	98.0	98.0	98.0
550	98.0	98.0	98.0
560	98.0	98.0	98.0
570	98.0	98.0	98.0
580	97.9	98.0	98.0
590	97.9	98.0	98.0
600	97.9	98.0	98.0
610	97.9	98.0	97.9
620	97.8	98.0	97.9
630	97.8	97.9	97.8
640	97.7	97.9	97.8
650	97.7	97.9	97.8
660	97.6	97.9	97.8
670	97.6	97.8	97.7
680	97.4	97.8	97.6
690	97.3	97.7	97.5
700	97.2	97.7	97.4
710		97.7	97.4
720		97.6	97.3
730		97.6	97.3
740		97.6	97.3
750		97.5	97.2
760		97.5	97.2
770		97.4	97.1

reflectance (diffuse incidence and diffuse collection) of a layer of magnesium oxide sufficiently thick to be opaque first as determined by Middleton and Sanders (18) for their best coating, second as determined by Budde (19) from 380 to 770 $m\mu$, and finally, a set of average values. Alternatively the 100% reading is evaluated by obtaining a reading for a white ceramic standard calibrated in terms of magnesium oxide for the angular conditions of the instrument used (15). Further checks of the photometric scale can be made by means of glass standards of spectral transmittance (20) issued by the National Bureau of Standards.

Reflectance Spectrum of Specimen

The reflectance spectrum of the specimen is obtained by placing the specimen in the specimen position and by placing in the reference position the same reference standard used in that position in the calibration of the photometric scale.

Data Handling

Obtaining the spectral directional reflectance of a specimen relative to magnesium oxide R/R_{MgO} from the zero reading, R'' , the 100% reading, R''_{100} , and the specimen reading, R'' , involves the following steps: (1) Apply the wavelength-scale corrections. (2) Apply the photometric-scale corrections by computing the ratio R'/R_{MgO} at each corrected wavelength as follows:

$$R'/R_{\text{MgO}} = (R'' - R''_0)/(R''_{100} - R''_0) \quad (18)$$

(3) Apply slit-width corrections wherever they are significant. These corrections are approximately proportional to the second derivative of the curve of reflectance plotted against wavelength and to the square of the spectral band width. For a band of $10 \text{ m}\mu$ (wavelength separation at half-height for triangular slit function) the slit-width correction ΔR_{sw} (21) is closely $(2R''_{\lambda} - R''_{\lambda-10} - R''_{\lambda+10})/10$, where R'' is the reflectance reading at wavelength λ to which the correction is to be added, and $R''_{\lambda-10}$ and $R''_{\lambda+10}$ are similar uncorrected reflectance readings for wavelengths below and above by $10 \text{ m}\mu$. Usually these corrections will be found to be less than the errors of measurement. Compute

$$R/R_{\text{MgO}} = R'/R_{\text{MgO}} + \Delta R_{\text{sw}} \quad (19)$$

To obtain the spectral directional reflectance R of the specimen relative to the perfect diffuser, compute

$$R = R_{\text{MgO}}(R/R_{\text{MgO}}) \quad (20)$$

where R/R_{MgO} is obtained from Equation 19, and R_{MgO} is obtained from a separate determination, such as given in Table 2. Although the values given in Table 2 refer to diffuse irradiation and diffuse collection, goniophotometric studies of magnesium oxide and magnesium carbonate (22) indicate that they apply within less than 1% to either of the two angular conditions, normal-diffuse or diffuse-normal. Thus the values in Table 2 apply to all of the spectrophotometers summarized in Table 1 except the Beckman DU, whose angular conditions approximate normal 45° . For normal- 45° multiply the values of Table 2 by about 1.02.

QUALITATIVE APPLICATIONS

According to the Kubelka-Munk model the absorption coefficient K and scattering coefficient S as functions of wavelength characterize the reflectance-producing behavior of any mixture of colorants (pigments or dyes) in any medium, such as paint vehicle, plastic, paper, and textile. These coefficients may be computed as the concentration-weighted average of those of the colorants and medium:

$$K = c_m K_m + c_1 K_1 + c_2 K_2 + \dots + c_n K_n \quad (21)$$

$$S = c_m S_m + c_1 S_1 + c_2 S_2 + \dots + c_n S_n \quad (22)$$

REFLECTANCE SPECTROPHOTOMETRY

where the sum of the concentrations of the medium and n colorants is unity ($c_m + c_1 + c_2 + \dots + c_n = 1$).

If, as is often so, the medium, for example, white paint, white paper, or textile, is the source of substantially all of the scattering, and has added to it relatively small concentrations (c_n much smaller than c_m) of colorants that are the source of substantially all of the absorption (that is, if K_m and S_n approach zero), then the ratio of absorption coefficient to scattering coefficient for the colorant-medium combination becomes

$$K/S = (c_1K_1 + c_2K_2 + \dots + c_nK_n)/c_mS_m \quad (23)$$

If the specimen contains only one colorant in concentration c_1 ($c_n = 0$ for $n \neq 1$), it is evident that K/S at any one wavelength is proportional to c_1 ; and, in general, if the ratios of concentrations of the colorants are kept constant at any values, K/S for the specimen is proportional to the total concentration $\Sigma c = c_1 + c_2 + \dots + c_n$ of the colorants; thus

$$K/S = Q\Sigma c \quad \text{or} \quad \log (K/S) = \log Q + \log \Sigma c \quad (24)$$

where Q is an abbreviation for $(c_1K_1 + c_2K_2 + \dots + c_nK_n)/c_mS_m\Sigma c$. Note that Q contains all the terms (K_n and S_m) that are functions of wavelength. It follows from Equation 24 that if $\log (K/S)$ is plotted against wavelength, the same shape of curve results regardless of the size of Σc . An increased concentration of colorants results simply in the curve being displaced vertically, and leaves the shape of the curve as an undisguised characteristic of the colorant-medium combination. On this account, $\log (K/S)$ is often called the optical signature. If, as is usually approximately so for wavelength regions of considerable extent, the scattering coefficient S_m of the medium varies negligibly with wavelength, the optical signature is written from the colorants alone.

The reflectance spectrophotometer is widely used for opaque specimens to measure R_∞ from which, by Equation 5, the optical signature may be plotted:

$$\log (K/S) = \log \frac{(1 - R_\infty)^2}{2R_\infty} \quad (25)$$

From the optical signature of any opaque specimen containing only one colorant, the identity of the colorant can usually be found by comparing the optical signature with those of known commercially important colorants. A considerable variation in concentration can be tolerated before the signature becomes illegible because of inability of the instrument to detect small departures from the reflectance of the medium, as for low values of Σc , or small departures from zero, as for high values of Σc . If there are two colorants in the specimen, the identification of both is often possible by searching for signatures of two known colorants whose weighted average agrees with that of the specimen. If more than two colorants are present, the optical signature of the specimen may still provide important clues to their identity as a supplement to other methods of qualitative analysis.

QUANTITATIVE APPLICATIONS

1. *Determination of Absorption and Scattering Coefficients.* Given for a translucent specimen of thickness X and for any one wavelength the spectral directional reflectance

R_0 for black backing ($R_g = 0$), and R for white backing (R_g much greater than zero), the absorption coefficient K and scattering coefficient S of the material of the specimen at that wavelength can be found by the following method. Find a from Equation 16, then b from its definition as $(a^2 - 1)^{1/2}$, then SX from either Equation 11 or Equation 12. Divide SX by X to find S . Then find R_∞ as $a - b$, then the ratio K/S from Equation 5, then multiply this ratio by S to find K . Alternatively, if $R_g = 89\%$, SX can be found from Figure 2 by plotting R_0 against R_0/R_{89} .

2. *Prediction of Reflectance as a Function of Thickness.* Given the absorption coefficient K and scattering coefficient S of a material, the reflectance R_0 for black backing possessed by a specimen of thickness X can be calculated. Compute a and b from their definitions in terms of K and S , then compute R_0 from Equation 10.

3. *Predictions of Thickness Required for Hiding.* Given the absorption coefficient K and scattering coefficient S of a material, it is possible to find the thickness X required to make the ratio R_0/R_{80} equal to 0.98 (usual definition of hiding). An iterative solution may be obtained by computing a and b from their definitions in terms of K and S , substituting them into Equations 9 and 10, and trying various values of X until one is found yielding R_0/R_{80} equal to 0.98. An explicit solution is found by writing from Equations 9 and 10 the expression for R_0/R_{80} , setting this equal to 0.98, then solving for $\text{ctgh } bSX$. The solution is given as part of a tentative standard method of evaluating the hiding power of paints (23).

4. *Quantitative Determination of a Single Constituent.* Given at some one wavelength the spectral directional reflectance R_∞ of an opaque specimen containing a mixture in unknown concentration c_u of a single known colorant in a known medium, the unknown concentration, c_u , can be determined. Obtain the absorption and scattering coefficients, K_m and S_m , for the known medium at that wavelength as in application 1; similarly obtain K_{m1} and S_{m1} of the combination of the known medium with a known small concentration (say 0.001) of the known colorant added. Equations 21 and 22 become

$$K_{m1} = (1 - c_1)K_m + c_1K_1 \quad (26)$$

$$S_{m1} = (1 - c_1)S_m + c_1S_1 \quad (27)$$

whence

$$K_1 = [K_{m1} - (1 - c_1)K_m]/c_1 \quad (28)$$

$$S_1 = [S_{m1} - (1 - c_1)S_m]/c_1 \quad (29)$$

From the measured value of R'_∞ for the specimen of unknown concentration c_u compute by Equation 5 the value of K'/S' . By Equations 21 and 22 this ratio must equal

$$\frac{K}{S} = \frac{(1 - c_u)K_m + c_uK_1}{(1 - c_u)S_m + c_uS_1} \quad (30)$$

By substituting for K_1 and S_1 the expressions given in Equations 28 and 29 and solving for c_u/c_1 the following solution is obtained:

$$\frac{c_u}{c_1} = \frac{S_m[(K/S) - (K_m/S_m)]}{S_m[(K/S) - (K_m/S_m)] + S_{m1}(K_{m1}/S_{m1}) - (K/S)} \quad (31)$$

Equation 31 yields a well-determined value of the unknown concentration c_u , unless at the wavelength chosen, K/S for the unknown specimen approaches K_m/S_m for the

medium. In practice, the determination of c_u is often much simpler. If the medium supplies substantially all of the scattering, and the colorant added supplies substantially all of the absorption, as in Equation 23, $K_m = S_1 = 0$; and if both the known concentration c_1 and the unknown c_u are negligible compared to unity, S_m may be set equal to S_{m1} and Equation 31 becomes

$$c_u/c_1 = (K/S)/(K_{m1}/S_{m1}) = \frac{(1 - R_\infty^2)/2R_\infty}{(1 - R_{\infty,m1}^2)/2R_{\infty,m1}} \quad (32)$$

whence it is seen that it is necessary only to prepare an opaque specimen with a known concentration c_1 , and measure the reflectance $R_{\infty,m1}$.

5. *Reflectance Computed for Known Colorants.* Given a combination of n known colorants (specified by K_n and S_n) with a known medium (specified by K_m and S_m) in known concentrations c_n , it is possible to find the reflectance R_∞ of an opaque layer of the combination, and the reflectance R of a layer of any thickness X backed by a backing of any reflectance, R_g . K/S can be found for the combination by Equation 23, R_∞ by Equation 4, and R by Equation 9.

6. *Colorant Formulation for the Nonmetameric Match of a Given Color.* Given the reflectance R_∞ as a function of wavelength for an opaque specimen made of a combination of known medium and three known colorants specified by K_m , K_1 , K_2 , and K_3 and by S_m , S_1 , S_2 , and S_3 , all known functions of wavelength, one can determine the concentrations, c_1 , c_2 , and c_3 , of the three colorants by which a duplicate of the opaque specimen may be made. This application is an extension of application 4 from one to three unknowns, and is the reverse of application 5 for $n = 3$. From Equations 21 and 22, K/S takes the form of

$$\frac{K}{S} = \frac{c_m K_m + c_1 K_1 + c_2 K_2 + c_3 K_3}{c_m S_m + c_1 S_1 + c_2 S_2 + c_3 S_3} \quad (33)$$

where $c_m = 1 - c_1 - c_2 - c_3$. Since there are three unknowns, select three well-separated wavelengths. If there are three minima in the curve of R_∞ plotted against wavelength, the choice of wavelengths near these minima will usually be good. Compute for these three wavelengths K/S by Equation 5 from the given values of R_∞ , and form for each wavelength an equation setting K/S , thus found, equal to the right-hand member of Equation 33, and solve for c_1 , c_2 , and c_3 which, of course, do not depend upon wavelength. The same method is applicable to any number n of colorants. The solutions are usually found by automatic digital computation. Compute K/S by Equation 33 for all wavelengths by using the computed values of the concentrations, and compare with the values of K/S found from the given values of R_∞ . If the colorants have been correctly identified and evaluated, these two evaluations of K/S should agree. Many colorant formulations are based on Equation 23 where applicable. The advantage is that the individual colorants then need only be specified by K_i/S_m , $i = 1, 2, \dots, n$. An analog computer, known as the Colorant Mixture Computer (COMIC), has been developed (24) to solve by trial and error equations of the type of Equation 23. The values of K_i/S_m (or analogous functions) are predetermined for each colorant at 10 wavelengths. The concentrations c_i are controlled by potentiometers set by the operator. The differences between the computed values of K/S and the given values of K/S at the 10 wavelengths appear as dots on an oscilloscope screen. The operator adjusts the values of c_i until the dots form a horizontal line showing that a nonmetameric match has been achieved.

7. *Colorant Formulation for the Metameric Match of a Given Color.* Given the values of reflectance R_∞ of an opaque specimen of an unknown material, the values of K_i for three substantially nonscattering colorants, and the values of S_m for a substantially nonabsorbing medium, all as functions of wavelength, it is possible to determine the concentrations c_i yielding in an opaque specimen a color match with the opaque specimen of unknown material. Since only by luck would the colorants be identical to those of the unknown material, in general there will be no selection of concentrations c_i by which the given values of R_∞ can be duplicated at all wavelengths. The selection to be sought is therefore that yielding the same tristimulus values, X , Y , and Z , as those implied by R_∞ . Tristimulus values may be defined for any set of reflectances R for the wavelength interval 380–780 m μ as follows:

$$X = \sum_{380}^{780} E_\lambda R(\lambda) \bar{x}(\lambda) \Delta\lambda \quad (34)$$

$$Y = \sum_{380}^{780} E_\lambda R(\lambda) \bar{y}(\lambda) \Delta\lambda \quad (35)$$

$$Z = \sum_{380}^{780} E_\lambda R(\lambda) \bar{z}(\lambda) \Delta\lambda \quad (36)$$

where E_λ is the spectral irradiance of the source, and $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$, are the tristimulus values of a spectrum of unit irradiance defining the 1931 CIE standard observer; see COLOR DESIGNATION AND SPECIFICATION. There is no known explicit solution for c_i , but the trial-and-error solution may proceed by making a preliminary selection of c_1 , c_2 , and c_3 . Find K/S for each wavelength by Equation 23, and the corresponding R'_∞ from Equation 4. The condition to be satisfied is

$$\sum_{380}^{780} E_\lambda R'_\infty(\lambda) \bar{x}(\lambda) \Delta\lambda = \sum_{380}^{780} E_\lambda R_\infty(\lambda) \bar{x}(\lambda) \Delta\lambda \quad (37)$$

and analogous conditions involving $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$. If the preliminary selection fails, as it almost certainly will, make other selections until Equation 37 is satisfied. This kind of an iterative solution may be found by means of an automatic digital computer in a few minutes. The solution may also be easily found in the COMIC (Colorant Mixture Computer) (24,25) by means of an attachment developed for this purpose.

For analogous treatments of specimens failing to follow the Kubelka-Munk model, and for discussion of diverse special problems arising in instrumental colorant formulation, see References 26 to 34.

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Terms, Definitions, and Symbols in Reflectometry

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Angular conditions of incidence are described as hemispherical, conical, or directional; the same adjectives are used to describe the angular conditions of collection. This classification of angular conditions leads to nine kinds of reflectance; symbols for them are proposed in which 2π , g , and θ_0 , ϕ_0 refer to hemispherical, conical, and directional incidence, 2π , g' , and θ_r , ϕ_r refer to the corresponding kinds of collection. Use of the perfectly reflecting mirror and of the perfectly reflecting diffuser as reference standards in reflectometry is discussed. Three of the nine reflectance ratios, specimen to perfect diffuser, in which the collection is directional have already been named radiance [luminance] factor. It is proposed to differentiate them by angular condition of incidence. It is also proposed to name the other six ratios: reflectance factor qualified by the same adjectives identifying the type of incidence and collection as are used for reflectance. The interrelationships of these 18 concepts are shown both by formulas for computing one from another and by diagrams indicating the process (integration, summation, averaging, equality, reflectance of perfect diffuser, and reciprocity) by which values of one concept may be computed from those of another.

INDEX HEADINGS: Photometry; Reflectance; Radiometry.

IF radiant energy is incident on an element of surface at an energy-flow rate Φ_0 , and is reflected by the element of surface at a rate Φ_r , the ratio Φ_r/Φ_0 is called the reflectance of the surface. In general, this ratio depends on the wavelength and state of polarization of the incident energy. Furthermore, if the detector of radiant flux employed has, like the human eye, a spectral sensitivity not independent of wavelength, the ratio of fluxes so evaluated is still called reflectance with a qualifying adjective (such as luminous, if the detector evaluates the flux in accord with the standard luminous-efficiency function). If, however, attention is confined to incident energy of fixed wavelength composition, or to detectors whose spectral sensitivities are strictly independent of wavelength, there still remain many problems of terminology and symbols. These problems arise because of the various angular conditions of incidence and collection used in reflectometry, and because most reflectometers do not measure the ratio of fluxes directly but employ specular surfaces or diffusing surfaces of known reflection characteristics as standards.

In a series of conferences among the staff of the National Bureau of Standards during the summer of 1965, a set of terms and symbols was developed to indicate the various types of angular conditions used in reflectometry. All of us agreed to use this one set of terms and symbols to facilitate communication among us. These terms and symbols do not contradict those currently being considered in the nomenclature committees of various international bodies (International Commission on Illumination, CIE; International Standards Organization, ISO; International Union of Pure and Applied Physics, IUPAP) nor of the nomenclature committees of the national bodies, American Standards Association, Optical Society of America, and the Illuminating Engineering Society, but rather are extensions of those concepts and symbols.

The purpose of this paper is to present the NBS terminology and symbols for reflectometry, to give the arguments that shaped them, to give careful definitions of the terms, and to demonstrate the convenience of the symbols by using them to indicate the rather complicated interrelations among the concepts defined. We hope that this extension of accepted terminology, or some revision of it, will find acceptance in the

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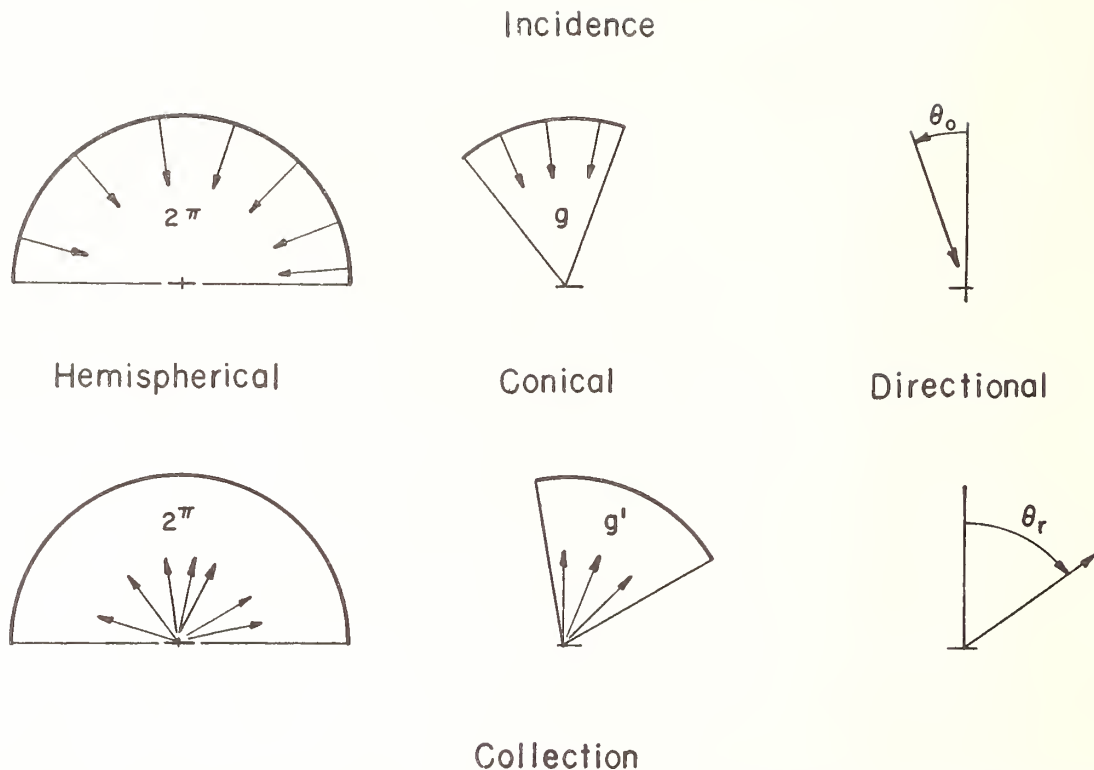


FIG. 1. The upper diagrams indicate hemispherical (2π), conical (g), and directional (θ_0) incidence, respectively, and the lower diagrams indicate hemispherical (2π), conical (g'), and directional (θ_r) collection. The indications are for a particular pair of azimuths: ϕ_0 and $\phi_0 + \pi$, for incidence; and ϕ_r and $\phi_r + \pi$, for collection. The arrows in the upper diagrams all point toward the center of the specimen represented by a short horizontal line; those in the lower diagrams all point away from that center.

various national societies concerned with radiative transfer of which the Optical Society of America is one, and eventually also within international standardizing bodies. A preliminary version of these terms and symbols has been under consideration by CIE Committee E-1.1, Nomenclature, since December 1965, and has received active support from the Swedish National Committee of the CIE.

REFLECTANCE

Our point of departure lies in the customary definition of reflectance as "ratio of reflected radiant flux to incident flux" given, for example, in the book, *The Science of Color*, by the Committee on Colorimetry of the Optical Society of America.¹ This definition might be interpreted to mean "ratio of *all* of the reflected flux to incident flux," but we have not so interpreted it. Instead, we have taken a cue from the note appended to the definition currently being considered by Committee E-1.1 on Nomenclature, International Commission on Illumination:

"20-065, *Reflectance*: Ratio of the reflected radiant or luminous flux to the incident flux."

"Note 1: Where mixed reflection occurs, the (total) reflectance may be divided into two parts, direct reflectance and diffuse reflectance, corresponding, respectively, to the modes of reflection."

Note that the ratio of the flux specularly reflected to the incident flux is still called direct reflectance even though not all of the reflected flux is included. We have accordingly agreed among ourselves to remove this ambiguity by the following more explicit definition of reflectance.

Reflectance: Ratio of some specified portion of the reflected flux to incident flux. Symbol: ρ .

Note 1: When qualified by one of the adjectives, hemispherical, conical, or directional, the term refers to all of the reflected flux, to the part within a finite solid angle less than $2\pi sr$, or confined to essentially one direction, respectively. If no qualifying adjective is used, reflectance for hemispherical collection is meant. The qualifying adjective that specifies the angles of collection may be preceded by the adjective, hemispherical, conical, or directional to indicate whether the incident flux is uniformly distributed over the hemisphere, uniformly distributed over a finite solid angle less than $2\pi sr$, or confined to essentially one direction, respectively.

¹ Committee on Colorimetry, Optical Society of America, *The Science of Color* (New York: Thomas Y. Crowell Company, 1953), p. 379. Currently available from Optical Society of America, 1155 Sixteenth St., N.W., Washington D. C. 20036.

TABLE I. Values of the nine kinds of reflectance for the perfect mirror and for the perfect diffuser.

Kind of reflectance	Reflectance symbol	Evaluation of quantity	
		For perfect mirror	For perfect diffuser
Bi-hemispherical	$\rho(2\pi:2\pi)$	1	1
Hemispherical-conical	$\rho(2\pi:g')$	$(1/\pi) \int_{\omega'} \cos\theta_r d\omega'$	$(1/\pi) \int_{\omega'} \cos\theta_r d\omega'$
Hemispherical-directional	$\rho(2\pi:\theta_r,\phi_r)$	$(1/\pi) \cos\theta_r d\omega'$	$(1/\pi) \cos\theta_r d\omega'$
Conical-hemispherical	$\rho(g:2\pi)$	1	1
Bi-conical	$\rho(g:g')$	$\frac{\int_{\text{overlap}} \cos\theta_r d\omega'}{\int_{\omega} \cos\theta_r d\omega}$	$(1/\pi) \int_{\omega'} \cos\theta_r d\omega'$
Conical-directional	$\rho(g:\theta_r,\phi_r)$	$(\cos\theta_r d\omega') / \int_{\omega} \cos\theta_r d\omega$ for θ_r, ϕ_r within mirror image of g ; 0, for θ_r, ϕ_r not within	$(1/\pi) \cos\theta_r d\omega'$
Directional-hemispherical	$\rho(\theta_0,\phi_0:2\pi)$	1	1
Directional-conical	$\rho(\theta_0,\phi_0:g')$	1, for $\theta_0 = \text{some } \theta_r$ and $\phi_0 = \text{some } \phi_r + \pi$; 0, for $\theta_0 \neq \text{any } \theta_r$ or $\phi_0 \neq \text{any } \phi_r + \pi$	$(1/\pi) \int_{\omega'} \cos\theta_r d\omega'$
Bi-directional	$\rho(\theta_0,\phi_0:\theta_r,\phi_r)$	1, for $\theta_0 = \theta_r$ and for $\phi_0 = \phi_r + \pi$ 0, for $\theta_0 \neq \theta_r$, or for $\phi_0 \neq \phi_r + \pi$	$(1/\pi) \cos\theta_r d\omega'$

Note 2: The symbol ρ may be followed by a two-part notation in parenthesis, the two parts being separated by a colon; the first part identifies the angles of incidence; the second, those of collection.

Figure 1 illustrates what is meant by hemispherical, conical, and directional incidence and collection, and Table I lists the terminology and symbols proposed for nine kinds of reflectance implied by these terms. Hemispherical incidence is notated 2π for the solid angle subtended by a hemisphere. Conical incidence is notated g for the geometry of incidence. If the cone to be specified is a right-circular cone, g may be written: $\alpha_0, \theta_0, \phi_0$, where α_0 is the half-angle of the cone, θ_0 and ϕ_0 are angles specifying the direction of its axis; thus θ_0 is the angle that the axis of the cone makes with the normal to the element of surface and may be called zenith angle or polar angle or colatitude; ϕ_0 is the azimuth angle or longitude. Most actual reflectometers irradiate the specimen along angles bounded by a right circular cone. If a reflectometer has more complicated angular conditions of incidence (single cone of non-circular cross-section or annular, which is the difference between two cones, or the sum of two or more cones), the symbol g must stand for these angular conditions of incidence, however complicated they may be. For these complicated cases, as well as for the simple cases, g must be explicitly defined. Directional incidence is notated simply by the zenith angle θ_0 and the azimuth ϕ_0 of this direction. If the direction of incidence is perpendicular to the surface of the specimen, $\theta_0 = 0$ and

no value of azimuth angle exists; then the notation (θ_0, ϕ_0) may be abbreviated as (0) .

The first column of Table I gives the adjectives by which the nine kinds of reflectance are differentiated. The second column gives the corresponding symbols. Some idea of the relations between the nine kinds of reflectance is indicated in columns 3 and 4 which give their values for the perfect mirror and the perfect diffuser, respectively.

By perfect mirror is meant a surface such that, when irradiated within an elemental solid angle centering on the direction θ_0, ϕ_0 from specimen to source, it reflects all of the incident energy within an elemental solid angle, $d\omega = \sin\theta_0 d\theta_0 d\phi_0$, centered on the direction θ_r, ϕ_r , where $\theta_r = \theta_0$ and $\phi_r = \phi_0 + \pi$. For these viewing angles, the bi-directional reflectance of a perfect mirror is unity; but for any other angles, zero. For hemispherical collection, the reflectance of the perfect mirror is likewise unity; this is indicated in the third column for bi-hemispherical reflectance, conical-hemispherical reflectance, and directional-hemispherical reflectance.

The directional-conical reflectance of a perfect mirror is unity if the direction θ_0, ϕ_0 coincides with any part of the mirror image of g' ; otherwise it is zero. The conical-directional reflectance is similarly zero if the direction, θ_r, ϕ_r , fails to coincide with any part of the mirror image of g ; otherwise it is $\cos\theta_r d\omega' / \int_{\omega} \cos\theta_r d\omega$, which approaches zero. The bi-conical reflectance of the perfect mirror is the integral of $\cos\theta_r d\omega'$ for the range of overlap between g and the mirror image of

TABLE II. Interrelationships among the nine kinds of reflectance.

Term	Symbol	Expression of relationship
Bi-hemispherical reflectance	$\rho(2\pi:2\pi)$	$= \int_{2\pi} \frac{\rho(2\pi:\theta_r, \phi_r)}{d\omega'} d\omega' = (1/\pi) \int_{2\pi} \rho(\theta_0, \phi_0:2\pi) \cos\theta_0 d\omega$
Hemispherical-conical reflectance	$\rho(2\pi:g')$	$= \int_{\omega'} \rho(2\pi:\theta_r, \phi_r) d\omega' = (1/\pi) \int_{2\pi} \rho(\theta_0, \phi_0:g') \cos\theta_0 d\omega$
Hemispherical-directional reflectance	$\rho(2\pi:\theta_r, \phi_r)$	$= (1/\pi) \int_{2\pi} \rho(\theta_0, \phi_0:\theta_r, \phi_r) \cos\theta_0 d\omega$
Conical-hemispherical reflectance	$\rho(g:2\pi)$	$= \int_{2\pi} \frac{\rho(g:\theta_r, \phi_r)}{d\omega'} d\omega'$ $= \int_{\omega} \rho(\theta_0, \phi_0:2\pi) \cos\theta_0 d\omega / \int_{\omega} \cos\theta_0 d\omega$
Bi-conical reflectance	$\rho(g:g')$	$= \int_{\omega'} \frac{\rho(g:\theta_r, \phi_r)}{d\omega'} d\omega'$ $= \int_{\omega} \rho(\theta_0, \phi_0:2\pi) \cos\theta_0 d\omega / \int_{\omega} \cos\theta_0 d\omega$
Conical-directional reflectance	$\rho(g:\theta_r, \phi_r)$	$= \int_{\omega} \rho(\theta_0, \phi_0:\theta_r, \phi_r) \cos\theta_0 d\omega / \int_{\omega} \cos\theta_0 d\omega$
Directional-hemispherical reflectance	$\rho(\theta_0, \phi_0:2\pi)$	$= \int_{2\pi} \frac{\rho(\theta_0, \phi_0:\theta_r, \phi_r)}{d\omega'} d\omega'$
Directional-conical reflectance	$\rho(\theta_0, \phi_0:g')$	$= \int_{\omega'} \frac{\rho(\theta_0, \phi_0:\theta_r, \phi_r)}{d\omega'} d\omega'$
Bi-directional reflectance	$\rho(\theta_0, \phi_0:\theta_r, \phi_r)$	$= \rho(c, d:a, b), \quad \theta_0 = a, \quad \phi_0 = b, \quad \theta_r = c, \quad \phi_r = d$

g' , divided by the integral of $\cos\theta_0 d\omega$ over the solid angle ω specified by g .

By perfect diffuser is meant a surface that, when irradiated according to any angular mode whatsoever, reflects all of the incident energy in accord with Lambert's law; that is, so that the radiance of the surface is constant, independent of the angle of view. For hemispherical collection the reflectance of the perfect diffuser is therefore unity, and column 4 of Table I shows this for bi-hemispherical, conical-hemispherical, and directional-hemispherical reflectance. Since it is impossible to discriminate between the perfect mirror and the perfect diffuser by any observations with hemispherical incidence, the expressions for bi-hemispherical, hemispherical-conical, and hemispherical-directional reflectances of the perfect diffuser are the same as those for the perfect mirror. Since the perfect diffuser distributes the reflected flux independent of angular conditions of incidence, the three expressions for the reflectance of the perfect diffuser for hemispherical incidence apply to conical and directional incidence also.

If reflectance of the perfect mirror is unity for a given set of angular conditions, the ratio of the flux reflected by any specimen to that reflected by the perfect mirror will be numerically identical to the re-

flectance, itself, of the specimen. The perfect mirror thus may serve as a reflectance standard with a value of unity for these kinds of reflectance and actual mirrors may serve as standards with appropriate values less than unity. Table I shows that all kinds of reflectance involving hemispherical collection are eligible as are also some choices of angles for directional incidence. The other four kinds of angular conditions require evaluation of the reflectance of the specular surface used as standard for the particular condition: hemispherical-conical, hemispherical-directional, bi-conical, and conical-directional conditions; on this account mirrors are inconvenient standards for these kinds of reflectance. Only the kinds of conditions for which the mirror surface has zero reflectance, however, are actually ineligible: some choices of conical-directional, directional-conical, and bi-directional.

If reflectance of the perfect diffuser is unity for a given set of angular conditions, the ratio of the flux reflected by any specimen to that reflected by the perfect diffuser will be numerically identical to the reflectance, itself, of the specimen. The perfect diffuser may thus serve as a reflectance standard with a value of unity for these kinds of reflectance. Table I shows that these kinds of reflectance are bi-hemispherical, conical-hemispherical, and directional-hemispherical.

Any diffusing surface may serve as a reflectance standard provided that its reflectance for the particular angular conditions has been evaluated. For example, magnesium oxide is often used as a reflectance standard with the value, unity, assigned because, for a considerable wavelength range, it approximates rather closely the perfect diffuser. Note from Table I that the ratio of reflectance of the perfect mirror to that of the perfect diffuser is unity for five kinds of reflectance, but for conical-directional and bi-directional conditions, the ratio is either zero or indefinitely great depending on the choice of angles.

If the bi-directional reflectances of a specimen are known for all angles of incidence and collection, it is possible to compute values of all of the other eight kinds of reflectance of that specimen from those values. From the expressions representing these eight kinds of reflectance as functions of the bi-directional reflectances, relationships among these eight kinds may be derived. Table II gives the most important of these relationships. The bi-directional reflectance of a diffusing specimen approaches zero and cannot be measured directly. The ratio $\rho(\theta_0, \phi_0; \theta_r, \phi_r)/d\omega'$, however, can be found as the limit of $\rho(\theta_0, \phi_0; g')/\omega'$ as ω' approaches zero. On the other hand, the bi-directional reflectance of a glossy specimen is a finite number between 0 and 1. It may be measured by using a source of small solid-angular subtense and a detector of equal, or slightly greater, solid-angular subtense so placed as to receive all of the specularly reflected beam. McNicholas² pointed out in 1928 that in any measurement of bi-directional reflectance, by virtue of the Helmholtz reciprocity relation, the same value is obtained by interchanging the source and detector positions. The last equality in Table II expresses this reciprocity relation. It has sometimes been assumed erroneously that, because of this relation, the value of bi-conical reflectance is likewise unchanged if the source and collection angles are interchanged. The falsity of this assumption becomes conspicuously evident when the solid-angular subtense of g is allowed to approach an elemental solid angle, and that of g' to approach 2π , in which case we have to ask whether the value of directional-hemispherical reflectance, $\rho(\theta_0, \phi_0; 2\pi)$, is made equal to that of hemispherical-directional reflectance, $\rho(2\pi; \theta_r, \phi_r)$, by setting θ_r and ϕ_r equal to the values previously assigned to θ_0 and ϕ_0 . Note, however, that the former is a finite quantity, while the latter approaches zero. If the specimen is the perfect diffuser, the former, as shown in Table I, is equal to 1; the latter is $(1/\pi) \cos \theta, d\omega'$. There is no reciprocal relation between these two kinds of reflectance.

RADIANCE FACTOR AND REFLECTANCE FACTOR

Although the motive for measuring the ratio of the flux reflected by the specimen to that which would be

reflected into the same solid angle with the same conditions of incidence by an approximation to the perfect diffuser is often to obtain an evaluation of the reflectance, this ratio is important in its own right and has received the name radiance factor (symbol β) whenever the collection is accomplished within an elemental solid angle. The following definition of radiance [luminance] factor is that currently given by Committee E-1.1 on Nomenclature, International Commission on Illumination (CIE); but we have added the notes.

Radiance [luminance] factor, (at a given point on the surface of a non-self-radiating body, in a given direction, under specified conditions of irradiation): ratio of the radiance [luminance], in that direction, of the body to that of a perfect reflecting diffuser identically irradiated [illuminated]. Symbol: β .

Note 1: When qualified by the adjective, hemispherical, conical, or directional, radiance factor refers to incident flux uniformly distributed over the hemisphere, uniformly distributed over a finite solid angle less than $2\pi sr$, or confined to essentially one direction, respectively.

Note 2: The symbol β may be followed by a two-part notation in parenthesis, the two parts being separated by a colon; the first part identifies the angles of incidence; the second, the angle of view. These notes provide names and symbols for three of the nine ratios of flux reflected by the specimen to that reflected by the perfect diffuser: hemispherical radiance factor, conical radiance factor, and directional radiance factor. If the collection is over a greater than elemental solid angle, the name radiance factor is not appropriate. We propose the name reflectance factor for these ratios as follows:

Reflectance factor (at a point of a surface, for the part of the reflected radiation contained in a given cone, and for incident radiation of given spectral and geometric distributions):

Ratio of the radiant flux reflected in the directions delimited by the cone to that reflected in the same directions by a perfect reflecting diffuser identically irradiated. Symbol: R .³

Note 1: The cross-section of the cone need not be circular nor even boundable by a single closed curve. It may, for example, be annular, or what in the ordinary restricted meaning of the term, cone, would be described as two or more cones.

Note 2: If the solid angle of the cone approaches zero, or $2\pi sr$, the reflectance factor approaches radiance factor β , or reflectance for hemispherical collection, respectively.

Note 3: The adjectives, hemispherical, conical, and directional recommended for *reflectance* to designate the geometrical modes of incidence and collection are also appropriate for *reflectance factor*; and the corresponding notations: 2π , g , or θ_0, ϕ_0 , respectively, are appropriate for use in parentheses following the symbol,

² H. J. McNicholas, J. Res. Natl. Bur. Std. (U. S.) 1, 29 (1928).

³ C. S. McCamy, Phot. Sci. Eng. 10, 314 (Nov.-Dec. 1966).

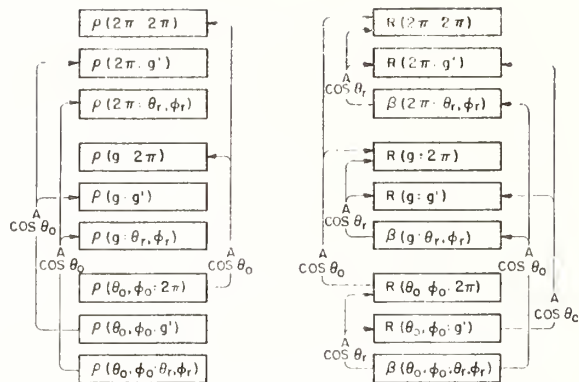


FIG. 2. The symbols refer to the nine kinds of reflectance ρ , the six kinds of reflectance factor (R with finite solid-angular subtense of collection), and the three kinds of radiance factor (β with elemental solid-angular subtense of collection). Each arrow starts from the symbol for the concept whose average value corresponds to another of the concepts, and the arrow head indicates the symbol for that concept. In the labeling of the arrows, A stands for average, and the appropriate weighting factor (either $\cos \theta_0$ or $\cos \theta_r$) is also shown. All six kinds of reflectance factors can be expressed as averages of directional radiance factor; but only two of the eight other kinds of reflectance can be expressed as averages of bi-directional reflectance.

R , with the notation for incidence preceding a colon, and that for collection: 2π , or g' , following it.

Note 4: If the directions of incidence are confined to a solid angle ω not approaching $2\pi sr$, and the cone of collection has a solid angle ω' not approaching $2\pi sr$, the term, bi-conical reflectance factor, is appropriate. The relation between bi-conical reflectance factor and conical-hemispherical reflectance is as follows: If several choices of the cone of collection are made such that no two of the cones overlap and such that together they sum to $2\pi sr$, then the appropriately weighted average of the several bi-conical reflectance factors is numerically equal to the conical-hemispherical reflectance for the same incidence.

By dividing each of the expressions in Table II by their values for the perfect diffuser, given in Table I, it is possible to derive expressions indicating the chief interrelationships among the six kinds of reflectance factor and the three kinds of radiance factor, and also their relationships to the various kinds of reflectance. Table III gives the most important of these expressions. Many of these relationships have already been published. McNicholas² published the first one listed in Table II, the relationship between bi-hemispherical reflectance, $\rho(2\pi:2\pi)$, and directional-hemispherical reflectance, $\rho(\theta_0, \phi_0:2\pi)$; he also published the first one listed in Table III, between bi-hemispherical reflectance, $\rho(2\pi:2\pi)$, and hemispherical radiance factor, $\beta(2\pi:\theta_r, \phi_r)$, and the one listed third from the bottom in Table III, that between directional-hemispherical reflectance, $\rho(\theta_0, \phi_0:2\pi)$, and directional radiance factor, $\beta(\theta_0, \phi_0:\theta_r, \phi_r)$. Brandenburg and Neu⁴ have recently

drawn attention again to this last relationship. Doubtless others of these relationships have also already been published; the multiplicity of terms used to refer to them discourages a literature search. The derivations are all straight-forward, but without systematic terminology it has been very hard for an author to communicate what it is that he has derived. It will be noted from Table III that, without exception, these expressions correspond to averaging either with $\cos \theta_0$ or with $\cos \theta_r$ as the weighting factor. To compute hemispherical and conical reflectance factor from radiance factor the weighting factor is $\cos \theta_r$. To pass from directional incidence to conical and hemispherical incidence requires, of course, the weighting factor, $\cos \theta_0$. These twelve averages are shown diagrammatically by arrows in Fig. 2. The notation A refers to average; below this the weighting factor is given. Figure 2 also shows in a similar way the kinds of reflectance that are connected by weighted averages, but note that there are only six such averages, those with weighting factors of $\cos \theta_r$.

Figure 3 shows most of the other interrelations among the nine kinds of reflectance, the six kinds of reflectance factor, and the three kinds of radiance factor. To pass from reflectance to reflectance factor or radiance factor for the same angular conditions of incidence and collection, and the reverse, requires application of a factor equal to the reflectance of the

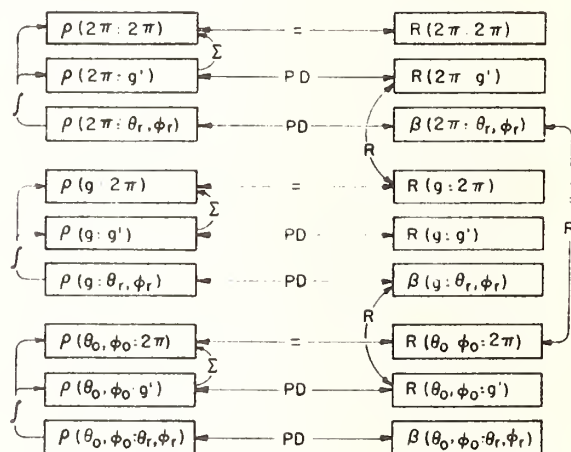


FIG. 3. Symbols shown are the same as those in Fig. 2. The arrows, like those in Fig. 2, pass from the symbols for concepts whose values may be processed so as to compute a value for the concept whose symbol is indicated by the arrow head. The arrows are labeled to indicate the process of computation: integral sign for integration, summation sign for summation, equal sign for numerical equality, PD for multiplication or division by the reflectance of the perfect diffuser, and R for numerical equivalents that, because of the Helmholtz reciprocity relation, hold when the angular conditions of incidence and collection are interchanged. There exist three reciprocal relations that cannot be shown by this diagrammatic method:

$$\begin{aligned}\rho(\theta_0=m, \phi_0=n; \theta_r=u, \phi_r=v) &= \rho(u, v; m, n) \\ \beta(\theta_0=m, \phi_0=n; \theta_r=u, \phi_r=v) &= \beta(u, v; m, n) \\ R(g=a; g'=b) &= R(b; a).\end{aligned}$$

The first of these three is the only reciprocal relation among the kinds of reflectance.

⁴ W. M. Brandenburg and J. T. Neu, J. Opt. Soc. Am. **56**, 97 (1966).

TABLE III. Interrelationships among the six kinds of reflectance factor, the three kinds of radiance factor, and the nine kinds of reflectance.

Term	Symbol	Expression of relationship
Bi-hemispherical reflectance factor	$R(2\pi:2\pi)$	$=\rho(2\pi:2\pi) = (1/\pi) \int_{2\pi} \beta(2\pi:\theta_r, \phi_r) \cos\theta_r d\omega'$
Hemispherical-conical reflectance factor	$R(2\pi:g')$	$= \int_{\omega'} \beta(2\pi:\theta_r, \phi_r) \cos\theta_r d\omega' / \int_{\omega'} \cos\theta_r d\omega'$
Hemispherical radiance factor	$\beta(2\pi:\theta_r, \phi_r)$	$= (1/\pi) \int_{2\pi} \beta(\theta_0, \phi_0:\theta_r, \phi_r) \cos\theta_0 d\omega$
Conical-hemispherical reflectance factor	$R(g:2\pi)$	$=\rho(g:2\pi) = (1/\pi) \int_{2\pi} \beta(g:\theta_r, \phi_r) \cos\theta_r d\omega'$
Bi-conical reflectance factor	$R(g:g')$	$= \int_{\omega'} \beta(g:\theta_r, \phi_r) \cos\theta_r d\omega' / \int_{\omega'} \cos\theta_r d\omega'$
Conical radiance factor	$\beta(g:\theta_r, \phi_r)$	$= \int_{\omega} \beta(\theta_0, \phi_0:\theta_r, \phi_r) \cos\theta_0 d\omega / \int_{\omega} \cos\theta_0 d\omega$
Directional-hemispherical reflectance factor	$R(\theta_0, \phi_0:2\pi)$	$=\rho(\theta_0, \phi_0:2\pi) = (1/\pi) \int_{2\pi} \beta(\theta_0, \phi_0:\theta_r, \phi_r) \cos\theta_r d\omega'$
Directional-conical reflectance factor	$R(\theta_0, \phi_0:g')$	$= \int_{\omega'} \beta(\theta_0, \phi_0:\theta_r, \phi_r) \cos\theta_r d\omega' / \int_{\omega'} \cos\theta_r d\omega'$
Directional radiance factor	$\beta(\theta_0, \phi_0:\theta_r, \phi_r)$	$=\pi\rho(\theta_0, \phi_0:\theta_r, \phi_r)/\cos\theta_r d\omega'$

perfect diffuser for those angular conditions; see Table I. Since, for hemispherical collection, the perfect diffuser has a reflectance of 100%, the relationship is that of numerical equality; this is indicated on Fig. 3 by a double-headed arrow marked with an equal sign. For the other two types of collection, the double arrow is marked with "PD" for perfect diffuser. For conical collection the factor is $(1/\pi) \int_{\omega'} \cos\theta_r d\omega'$; and for directional collection the factor is $(1/\pi) \cos\theta_r d\omega'$, as shown in Table I. To pass from reflectance for directional collection to that for conical and hemispherical collection, requires as indicated in Table II, integration. This is indicated in Fig. 3 by single-headed arrows marked with an integral sign. To pass from reflectance for conical collection to that for hemispherical collection requires summation for the same choices of solid angles as are indicated in note 4 to the definition of reflectance factor. These three relationships are indicated on Fig. 3 by single-headed arrows marked with a summation sign. The corresponding relationships for reflectance factor are averages, but, to prevent Fig. 2 from becoming too confusing, arrows for these relationships have been left off of Fig. 2.

Although the Helmholtz reciprocity relation applies in reflectance only to the bi-directional angular condition, there are three pairs of such relations involving reflectance factor and radiance factor, indicated on Fig. 3 by double-headed arrows marked with R for reciprocity. For example, one double-headed arrow indicates that conical radiance factor $\beta(g:\theta_r, \phi_r)$ of any

specimen is numerically the same as directional-conical reflectance factor $R(\theta_0, \phi_0:g')$ of that specimen, provided that $g=g'$, $\theta_0=\theta_r$, and $\phi_0=\phi_r$. Table III shows that making these substitutions in either of the expressions makes it identical to the other. Similarly, the numerical identity of hemispherical radiance factor $\beta(2\pi:\theta_r, \phi_r)$ and directional-hemispherical reflectance factor $R(\theta_0, \phi_0:2\pi)$ provided $\theta_0=\theta_r$ and $\phi_0=\phi_r$ is obvious by inspection of Table III. The other reciprocal relation indicated in Fig. 3, that between hemispherical-conical reflectance factor $R(2\pi:g')$ and conical-hemispherical reflectance factor $R(g:2\pi)$ cannot be seen by inspection of Table III, because the expression for the former is in terms of $\beta(2\pi:\theta_r, \phi_r)$, while the latter is in terms of $\beta(g:\theta_r, \phi_r)$; but substitution of the expressions given in Table III for these kinds of radiance factor in terms of directional radiance factor $\beta(\theta_0, \phi_0:\theta_r, \phi_r)$ readily shows the two to be equivalent provided that $g=g'$. A similar substitution shows that $R(g=a:g'=b)=R(b:a)$. McNicholas² has already drawn attention to the reciprocal relation, shown in Fig. 3, between hemispherical radiance factor, $\beta(2\pi:\theta_r, \phi_r)$, and directional-hemispherical reflectance factor, $R(\theta_0, \phi_0:2\pi)$. The numerical equality, shown in Fig. 3, between the latter and directional-hemispherical reflectance, $\rho(\theta_0, \phi_0:2\pi)$, permitted McNicholas to write:

$$\rho(\theta_0, \phi_0:2\pi) = \beta(2\pi:\theta_r, \phi_r)$$

for θ_r and ϕ_r set equal to the values previously assigned to θ_0 and ϕ_0 , respectively.

SUMMARY

Names and symbols have been proposed for nine kinds of reflectance corresponding to the nine principal kinds of angular conditions. Names (reflectance factor, and radiance factor, with modifiers like those used for reflectance) and corresponding symbols have also been proposed for the ratios of the nine kinds of reflectance to that of the perfect diffuser. The convenience of the symbols has been demonstrated by using them to indicate the rather complicated, and little appreciated, relationships among the eighteen concepts defined (nine

kinds of reflectance, six kinds of reflectance factor, and three kinds of radiance factor); see Tables II and III. These relationships have also been classified as integrals, summations, averages, ratio to perfect diffuser, and reciprocity relations, and have been shown diagrammatically in Figs. II and III. We submit that all eighteen of these concepts are important in reflectometry, and will eventually be given suitable names and symbols. Our proposal is a systematic extension of existing terminology and is presented as a suggestion to authors and nomenclature committees dealing with reflectometry.

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
WASHINGTON, D. C. 20234

Letter
Circular
LC-1017

Re-issued
January 1967

STANDARDS FOR CHECKING THE
CALIBRATION OF SPECTROPHOTOMETERS
(200 to 1000 nm)

Contents

1. Introduction.
 2. Checking the wavelength scale.
 - 2.1 Non-recording spectrophotometers.
 - 2.2 Recording spectrophotometers.
 - 2.2.1 General Electric recording spectrophotometers.
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 3. Checking the photometric scale.
 - 3.1 Glass standards of spectral transmittance.
 - 3.2 Solution standards of spectral transmittancy.
 4. Reference standards of spectral reflectance factor relative to freshly prepared magnesium oxide.
 - 4.1. For the General Electric recording spectrophotometer.
 - 4.2. For the Beckman quartz (model DU, non-recording) spectrophotometer.
 5. References.
1. Introduction.

In continuation of a type of activity carried on for many years at the National Bureau of Standards, there is described in this letter circular the various types of standards that are available for issuance by the Bureau for the purpose of checking or maintaining the over-all reliability of spectrophotometers in the ultraviolet, visible and near-infrared regions of the spectrum. Certain other information of similar purpose is also included.

Experience has shown that spectrophotometers can easily get out of adjustment. Although repeated trials may give the same values over and over again, indicating high sensitivity and precision, and the instrument may otherwise appear to be functioning perfectly, gross errors in wavelength may nevertheless be rendering the values obtained highly unreliable. Other causes of error may likewise be present and unsuspected, particularly with the photoelectric spectrophotometers now comprising so large a percentage of the total in use. The use of the various standards described herein has been found of considerable assistance in detecting and eliminating errors that would otherwise be present, or in confirming that the instrument is in fact giving reliable results.

Reference is made in this letter circular to the NBS test fee schedules currently in effect. Those relating to spectrophotometry are designated as 202.105, Spectrophotometric Standards, and 202.106, Spectrophotometric Measurements. These schedules are obtainable from the National Bureau of Standards.

2. Checking the Wavelength Scale.

Most of the present-day spectrophotometers have a direct-reading wavelength scale; that is, the scale, instead of being divided in uniform linear or circular measure, is divided and engraved directly in nanometers (formerly called millimicrons). This greatly facilitates setting the instrument at any desired wavelength. The accuracy of many of these direct-reading wavelength scales is remarkably good, when put in the best average adjustment, considering the difficulties of quantity production of such scales. When so adjusted, it is not uncommon to find them in error by not more than 1 nm throughout the ultraviolet and visible spectrum. However, if one wishes the uncertainties in his wavelength settings to be of the order of 0.1 nm, a careful check of these direct-reading scales is necessary.

Certain sources and wavelengths that have proved especially suitable for the calibration of various types of spectrophotometers are listed in tables 1a and 1b which are similar in scope and purpose to table 1 of Circular 484 (1)*.

*Numbers in parentheses refer to the References, Section 5.

All values of tables 1a and 1b are consistent with those published in the M.I.T. wavelength tables (2). Which of these sources to use, and which wavelengths of the several sources are the most suitable, will depend on the type of instrument. Furthermore, the procedure will vary importantly depending on whether the instrument is a non-recording or a recording spectrophotometer.

Table 1a. Wavelengths of Sources Suitable for Calibration of Spectrophotometers.

<u>Wavelength</u> (nm)	<u>Note</u>	<u>Wavelength</u> (nm)	<u>Note</u>	<u>Wavelength</u> (nm)	<u>Note</u>	<u>Wavelength</u> (nm)	<u>Note</u>
Mercury Arc in Quartz (Same wavelengths in glass above 300 nm)							
205.29		253.48	3*	296.73	*	398.40	
222.47		253.65		302.15		400.63	
223.41		257.63		302.35	6*	404.66	*
225.88		260.32		302.56		407.78	*
230.21		265.20		302.75		435.83	*
232.32		265.37	4*	312.57	*	491.60	
235.25		265.51		313.15	7*	546.07	*
237.83	*	269.95		313.18		576.96	
239.94	1*	275.28	*	334.15	*	579.07	9*
239.97		275.97		349.28		623.44	
244.69		280.35		365.01		671.62	
246.41		280.45	5*	365.48	8*	690.72	
248.20		284.78		366.29		1014.0	*
248.27	2*	289.36	*	366.33		1128.7	
248.38		292.54		390.64			

Helium Discharge Tube in Glass

318.77		396.47		447.15	*	667.81	*
361.36		402.62		471.31	*	706.52	*
363.42		412.08		492.19		728.13	
370.50		414.38		501.57	*	1083.0	*
381.96		438.79		504.77			
388.86	*	443.75		587.56	*		

* These lines have been found most useful on the Beckman DU spectrophotometer.

1. A value of 239.95 is recommended for the unresolved pair.
2. A value of 248.3 is recommended when the 3 lines are unresolved.
3. The intensity of 253.48 is negligible compared to that of 253.65. The latter value should be used when the lines are unresolved.
4. The 265.20 line is somewhat stronger than the others and a value of 265.3 is recommended when the three lines are unresolved.
5. These two lines are of approximately the same intensity and a value of 280.40 is recommended for the unresolved pair.
6. The two shorter lines are considerably stronger than the other two. It is probable that a value of 302.25 should be used for the unresolved lines.
7. A value of 313.16 is recommended for the unresolved pair.
8. With the arc used on the Beckman DU spectrophotometer the ratio of intensities for 365.01 : 365.48 : 366.33 is 100 : 48 : 36, approximately. The intensity of the 366.29 line appears negligible relative to that of 366.33.
9. These two lines are of approximately the same intensity and a value of 578.0 is recommended for the unresolved pair.

Table 1b. Wavelengths of Sources Suitable for Calibration of Spectrophotometers

Neon Discharge Tube		Aluminum Spark in Air	Hydrogen Arc
<u>Wave- length</u> (nm)	<u>Relative Intensity</u>	<u>Wavelength</u> (nm)	<u>Wavelength</u> (nm)
585.25	5	216.88	434.05
588.19	4	217.40	486.13
594.48	8	220.46	656.28
597.55	2	221.00	
603.00	2	226.35	
607.43	8	226.91	
609.62	13	236.71	Sodium Arc
614.31	25	237.21	<u>Wavelength</u>
616.36	6	237.31	(nm)
621.73	4	237.34	589.00
626.65	11	237.84	589.59
630.48	4	256.80	
633.44	20	257.51	Cesium Arc
638.30	23	257.54	<u>Wavelength</u>
640.22	100	263.16	(nm)
650.65	39	265.25	852.11
653.29	8	266.04	894.35
659.90	12	281.62	
667.83	23	308.22	
671.70	14	309.27	
692.95	23	358.69	
702.41	2	394.40	
703.24	45	396.15	
705.91	--		
717.39	5		
724.52	17		
743.89	4		
748.89	--		
753.89	--		
754.40	--		

2.1 Non-Recording Spectrophotometers.

The best procedure for checking the wavelength scale of a non-recording spectrophotometer is by direct use of a source of radiant energy having spectral lines of suitable intensity and adequately spaced throughout the spectral range of interest. Various sources are available and can be recommended for such purpose. How many sources, or how many wavelengths, to use in such a calibration depends, of course, on the desires of the individual investigator.

In this connection it should be noted that the number of significant figures of importance in spectrophotometry (including "absorption spectroscopy") is of a different order of magnitude than that used in emission spectroscopy or in standard wavelength tables. In the visible spectrum with the usual type of spectrophotometer it seems impossible to maintain the wavelength calibration with uncertainties less than about 0.1 nm. While the uncertainty may be less in the ultraviolet with a prism instrument, there seems no purpose served in giving standard wavelengths to better than 0.01 nm for spectrophotometric calibration.

Two suitable sources for wavelength calibration are the mercury lamp and the helium lamp. A mercury lamp in a quartz envelope is by far the best single source for wavelength calibration from 205 to 1014 nm. A mercury lamp in a glass envelope provides the same spectral lines except that below about 300 nm they are not transmitted by the glass envelope.

The helium lines are especially well placed for wavelength calibration in the visible spectrum, and the strong lines at 388 and 1083 nm are also often very useful. Many other sources, flame or arc, are available for visual wavelength calibration (2, 3) but most of these are too unstable for accurate calibration with a photoelectric detector.

These same sources and many others are also useful for the wavelength calibration of spectrographs used in photographic spectrophotometry. Between 200 and 400 nm the series of doublets obtained from the aluminum spark in air is very useful because they are so readily recognized.

Not all of the lines for any of the sources are given in tables 1a and 1b but only those that are considered especially suitable for the purpose. Furthermore, not even all of those listed for any one source may be suitable for any one particular instrument. The mercury arc in quartz is an example. All of the lines listed (and still others) can be used for wavelength calibration of a photographic spectrophotometer over the range of sensitivity of the plate used. The lines from 404.7 to 690.7 nm can be used for visual calibration of a spectrophotometer. But not all of the lines are suitable for calibration of a photoelectric instrument, and those that prove adequate will depend on the sensitivity and slit widths characteristic of any particular instrument. One must be very careful that other lines are not included, in addition to the one on which the settings are supposedly being made, of sufficient intensity to affect the wavelength setting.

Special attention should perhaps be called to the use of a cesium arc at 852.1 and 894.3 nm (4). From tables 1a and 1b it is apparent that there are few suitable lines between 706.5 and 1014.0 nm, particularly from steady sources necessary or desirable in the calibration of photoelectric spectrophotometers. The neon discharge tube gives many lines between 750 and 1000 nm (2) but these have not been found satisfactory in the calibration of photoelectric spectrophotometers. In the orange and red the neon lines are useful for visual calibration and many of these can be used to calibrate photoelectric spectrophotometers (5) if the sensitivity is such that very narrow slits can be used. The relative intensities (6) given in table 1b will help in case of overlapping.

The best technique to use in wavelength calibration of non-recording spectrophotometers, given a suitable source, will vary from instrument to instrument and method to method. A few general principles can be given here, however.

In photographic spectrophotometry it usually is sufficient to photograph a known spectrum at the top and bottom of the plate, unless the source used for the absorption spectra itself carries such known reference lines. A few of these reference lines will then serve to correlate that particular plate with whatever complete calibration curve has previously been established by more extensive measurements with the various sources.

On visual and photoelectric non-recording spectrophotometers, it usually is necessary, for highest precision, to have a basic reference line to which all of the other wavelengths are compared by direct check. At the Bureau the Hg yellow lines have proved most suitable for the König-Martens visual spectrophotometer (7). At the slit widths used the overlapping of the two lines gives a central brighter "line" taken as 578.0 nm with a luminous background against which the slit jaws are readily seen. A luminous background, or slight illumination of the ocular slit, always facilitates calibration when an eyepiece is used. Visual calibration without an eyepiece is usually less precise unless very narrow slits are used.

Two techniques have been used at the Bureau in the calibration of non-recording photoelectric spectrophotometers. On the Gibson spectrophotometer (8) the slits are always 0.1 mm wide or greater and the most reliable calibration is obtained by plotting galvanometer deflections at closely adjacent wavelengths. The most probable value for the wavelength reading is given by the intersection of the two straight lines resulting from a plot of the data for any given line, the correction being given by the difference between this value and the true wavelength. This is illustrated in reference (1).

On the Beckman DU spectrophotometer the same method has been used (9), but at the Bureau it has seemed preferable and is much more rapid, to calibrate with a narrow slit and record the wavelength dial reading for the maximum left deflection of the galvanometer needle as the wavelength dial is slowly turned. The most suitable reference line on two of the Bureau's instruments has proved to be the Hg green line at 546.07 nm (5).

2.2 Recording Spectrophotometers.

The initial wavelength calibration of a recording spectrophotometer, such as the manufacturer must carry out in connection with cutting his cams or preparing his reading scale, is not here considered, but only the check of such a calibration by the user of the instrument.

Such a user can, of course, follow the procedure prescribed above for checking the wavelength calibration of non-recording spectrophotometers. However, there are two important reasons for following a different procedure for recording spectrophotometers. For such instruments it is desirable to have a calibration that is made with the instrument operating. It is further desirable in most kinds of work to have this calibration appear on the graph sheet so that difficulties connected with positioning of the sheet, expansion or contraction of the paper with humidity or temperature, or instrumental variations can be eliminated.

Wavelength calibrations of this kind can be made if a material is available having a number of strong and narrow absorption or transmission bands suitably spaced over the spectral range of interest. Two materials have been used or suggested for this purpose: (a) Glasses containing rare-earth oxides, such as didymium glasses and holmium oxide glasses, have been used for many years at the National Bureau of Standards (10, 11), (b) quartz-Polaroid combinations have been proposed (12) and may prove useful for such work.

The use of a didymium glass or a holmium oxide glass in this manner would not in general be accurate unless it is calibrated at nearly the same slit widths as are to be used. Most of the absorption bands that are usable for the purpose are multiple bands and the wavelengths of maximum absorption often depend on the slit widths. This has been illustrated in previous publications (1, 10).

While the use of a didymium glass or a holmium oxide glass for checking the wavelength calibrations of a recording spectrophotometer is highly recommended, there are two other uses of these glasses which are not recommended. First, these glasses are not well suited for checking the photometric scale of any spectrophotometer, recording or non-recording. Transmittances at the peaks of the absorption bands are too dependent on slit widths, and transmittances on the steep parts of the curve are too dependent on slight wavelength errors, both as illustrated in Fig. 1 of reference (10) and in Fig. 8 of reference (1). Second, the use of these glasses to check the wavelength calibration of a non-recording spectro-

photometer is considered much inferior from the standpoints of time, convenience and reliability to the direct use of line sources as described in Sec. 2.1. The National Bureau of Standards has consistently refused to accept didymium glasses or holmium oxide glasses for calibration for either of these two purposes.

2.2.1. General Electric Recording Spectrophotometer.

The NBS didymium glass standards were carefully calibrated by point-by-point measurements on the König-Martens visual and Gibson photoelectric spectrophotometers with slit widths approximating the 10- and 20- nanometer slits used on the NBS General Electric spectrophotometer. Some of these values have been published (13).

The most suitable didymium glass for the purpose, considering type of curve and availability, is a Corning 5120 glass of 3.0 mm thickness. While it is not known how much the wavelengths of maximum absorption of this 5120 glass might vary from melt to melt, glasses from at least three melts have been measured, and there has never been any certain variation among the samples tested. For much work it is probably safe to use the values given in table 2.

Table 2. Wavelengths of maximum absorption for Corning 5120 glasses of 3.0 mm thickness as obtained at the National Bureau of Standards for the slit widths indicated.

<u>Wavelength of Maximum Absorption (nm)</u>	<u>Approximate Spectrum Interval Transmitted by Slits (nm)</u>
441.0	10
475.5	10
528.7	10
585.0	10
684.8	10
743.5	10
745.5	20
808.	20
883.	20
1067.	20

For those who wish greater certainty, however, the Bureau has obtained a supply of Corning 5120 glass in 2-inch polished squares and of 3.0 mm thickness. These are measured and the values reported in accordance with NBS test fee schedule 202.105, items d to f. The measurements consist of recording a curve of the test glass on the same sheet as the curve of the NBS standard glass and deriving values of the wavelengths of minimum transmittance of the former relative to

those of the latter. The over-all uncertainties of the values so reported are considered to be not greater than ± 1 nm from 441.0 nm to 743.5 nm, and not greater than ± 2 nm from 745 to 1067 nm.

Methods of use of a calibrated didymium glass on a G. E. recording spectrophotometer are described in references (1), (10), and (13).

2.2.2. Cary Model 14 recording spectrophotometer.

The NBS holmium oxide glass standards were carefully calibrated by using a Cary Model 14 recording spectrophotometer, the wavelength indicating dial of which had previously been calibrated by means of a number of sources having wavelengths throughout the ultraviolet and visible spectral regions. Measurements of the wavelengths of minimum transmittance were made as functions of slit width over the range 0.06 to 5.0 nanometers of spectral width. Eleven sharp absorption bands were found to be sufficiently symmetrical that the wavelengths of minimum transmittance indicated by the recorder remained constant for slit widths up to about 2 nanometers.

The most suitable holmium oxide glass for the purpose is a Corning 3130 glass of approximately 2.5 mm thickness. It is not known how much the wavelengths of minimum transmittance of this 3130 glass might vary from melt to melt. It is known that, for some 3130 glasses, the absorption of the base glass prevents the use of the glass for wavelength calibration in the ultraviolet near 241 nanometers. For much work it is probably safe to use the values given in the following table.

Wavelengths of minimum transmittance for Corning 3130 glasses of 2.5 mm thickness as obtained at the National Bureau of Standards for slit widths less than 2 nm.

Useful wavelengths
between 240 and 370 nm

241.5
279.3
287.6
333.8
360.8

Useful wavelengths
between 360 and 650 nm

360.8
385.8
418.5
453.4
459.9
536.4
637.5

For those who require that the base glass of the standard transmit sufficiently for the standard to be useful at 241 nanometers, the Bureau has obtained a supply of Corning 3130 glass in 2-inch polished squares and of 2.5 mm thickness. These are measured and the values reported in accordance with NBS test fee schedule 202.105, items g to i.

The measurements consist of recording a curve of the test glass on the same sheet as the curve of the NBS standard glass and deriving values of the wavelengths of minimum transmittance of the former relative to those of the latter. The over-all uncertainties of the values so reported are considered to be not greater than ± 0.5 nm. The present supply of holmium oxide glass contains striae, and in some cases strains, which have not appreciably altered the wavelengths of minimum transmittance.

The methods of use of a calibrated holmium oxide glass on a Cary Model 14 recording spectrophotometer are similar to those described in references (1), (10), and (13) relating to the method of use of a didymium glass on a G. E. recording spectrophotometer.

3. Checking the Photometric Scale.

A check of the photometric scale of a spectrophotometer independent of all other sources of error is difficult or impossible to make on most spectrophotometers. Useful for this purpose would be a series of samples whose respective transmittances do not vary with wavelength, which will not displace the beam, which do not reflect strongly, and for which the transmittances can be independently determined with high accuracy. However, no such glasses or other materials are available.

On a visual spectrophotometer such a check is possible by means of rapidly rotating sectors. If such sectors are properly made, the angle of the total opening relative to 360° , that is, the effective transmittance of the sector, can be measured on a circular dividing engine with uncertainties only in the fifth decimal place. Of course, the use of such rotating sectors is based on the validity of Talbot's law. Throughout the spectrum, no certain deviations from this relation are known. As a matter of fact, average values obtained over a period of years in measuring the "transmittances" of accurately calibrated rotating sectors on the NBS König-Martens spectrophotometer prove both the validity of Talbot's law at various wavelengths and the reliability of the instrument over most of the photometric scale, or else there is a remarkable balancing of errors.

3.1 Glass Standards of Spectral Transmittance.

Usually on photoelectric spectrophotometers the use of rotating sectors to check the reliability of the photometric scale either is impossible or is attended with too much uncertainty for one reason or another.

Accordingly, shortly after the advent of commercial photo-electric spectrophotometers, the National Bureau of Standards instituted the service of issuing glass standards of spectral transmittance (14). To date, about 700 of these filters have been issued with accompanying certificates. Further information about these filters, particularly with reference to their permanence, may be found in several publications (15, 16).

All of the filters offered for this purpose have transmittances varying through the visible spectrum from high (0.75 or more) to low (0.06 or less). In one sense these might be considered inferior to strictly neutral filters in that a deviation from the true value may be attributed to causes other than inaccuracy of the photometric scale. On the other hand, they are superior to the neutral filters in detecting stray-energy, slit-width, and gross wavelength errors.

The four types of glass filters are designated as "carbon yellow", "cobalt blue", "copper green", and "selenium orange". They are available in either discs approximately 30 mm in diameter or 2-inch squares and are approximately 3, 3, 2, and 2.5 mm thick, respectively. They are issued in accordance with NBS test fee schedule 202.105, item a. The transmittances reported are for 25°C and are usually obtained by measuring, wavelength by wavelength on the Beckman DU spectrophotometer, the ratio of transmittance of test glass to standard. The standards have been calibrated (5, 16) by extensive measurements on the Beckman DU, König-Martens, Gibson, and Cary Model 14 spectrophotometers, from which also are derived the effects of temperature change on the transmittances and the major part of the uncertainties reported for the values.

To give the reader a better idea of the transmittances he may expect on the standards issued, there are given in table 3 the transmittances of the respective NBS standards at the wavelengths used and reported. The transmittances of the standards issued will not in general be identical with those of table 3 but will not be greatly different from them.

Transmittances of these filters at wavelengths other than those given will be determined on request from 365 nm to 1000 nm in accordance with item c of NBS test fee schedules 202.105. Values will be obtained for temperatures of 25°C. The effect of change of temperature has not been determined at the Bureau for these glasses outside the range from 390 to 750 nm. It is known, however, that for all four types of glass the temperature effects are very small from 750 to 1000 nm, and are probably negligible for the usual room temperature variations. On the other hand, temperature effects are always large for these kinds of glass when the transmittance curve is decreasing rapidly towards shorter wavelengths (1, Figs. 29 and 30), so that increasingly large temperature effects may be expected for these filters in the ultraviolet.

Table 3. Values of Spectral Transmittance of NBS Glass Standards for Checking the Photometric Scale of Spectrophotometers. The transmittances of glasses issued by the Bureau will not, in general, be identical with those of table 3 but will not be greatly different from them.

<u>Wavelength</u> (nm)	<u>Transmittance for glasses designated as</u>			
	carbon yellow	cobalt blue	copper green	selenium orange
390	0.025	0.895	0.862	
404.7	.020	.884	.877	
420	.019	----	----	
435.8	.0240	.806	.893	
471.3	.081	.612	.894	
491.6	----	.344	----	
501.6	.208	.245	.859	
520	.316	.091	----	
530	.379	----	.760	
540	----	.0308	----	
546.1	.479	.0335	.671	
560	.557	.064	.585	
570	----	----	----	
578	.636	.0273	.473	0.0042
587.6	.668	----	----	.118
600	.699	.0074	.350	.55
620	.731	.0100	.256	.852
640	.747	.0074	.187	.904
660	.754	----	----	.914
667.8	----	.034	.122	----
680	----	.14	----	----
690	.755	.34	----	.919
706.5	----	.713	----	----
710	----	----	.074	----
720	.748	.845	----	.918
750	.730	.901	.057	.917

3.2. Solution Standards of Spectral Transmittancy.

The photometric scale of spectrophotometers may be checked, if one prefers, by means of solutions of known spectral transmittancies, instead of by means of the standard glasses. For this purpose the following solutions are recommended:

1. The aqueous solutions of copper sulphate and cobalt ammonium sulphate used in the series of filters developed at the National Bureau of Standards for reproducing the colors of sunlight and daylight and for the determination of color temperatures (17).

The published values for absorbancy and transmittancy are given in tables 4 and 5 herein, together with the composition and certain other pertinent information. Many additional details are given in M114 (17) including the changes in absorbancy (A_s) with temperature. Both solutions obey Beer's law over a considerable range of concentrations. The values given for the eight Hg and He wavelengths are considered the most reliable, with an uncertainty in A_s not exceeding 0.001 for the particular chemicals used. Spectrophotometric reproducibility of the chemicals is also considered in the paper. By increasing the thickness or concentration a wide range of the photometric scale can be covered, except at the shorter wavelengths.

2. An aqueous solution of potassium chromate, K_2CrO_4 (0.04g/l) in 0.05N KOH. This solution has been studied by many investigators here and abroad, and is considered one of the most suitable as a standard of spectral transmittancy and absorbancy in the ultraviolet. The absorption in the violet is also useful because the copper and cobalt solutions have too little absorption in this region to be of much value. The most recent determination is given in NBS Research Paper 2331 (18), which also notes most of the previous work. This solution has also been used in a recent comparative survey of photoelectric spectrophotometers (19). The values adopted in RP2331 are given in table 6. These data were obtained from solutions prepared in the following ways:

(1) A solution of K_2CrO_4 , stock material, reagent grade, 0.0400 gram per liter, in 0.05N KOH.

(2) A solution of K_2CrO_4 of the same concentration and alkalinity as (1) but prepared from 0.0303 gram of $K_2Cr_2O_7$ which when converted gave 0.0400 gram of K_2CrO_4 per liter.

At wavelengths greater than 260 nm both solutions were found to remain stable (in transmittancy) for 5 or 6 years when stored in ordinary storeroom glass bottles. "Flaking" may occur during this time and any sediment should be allowed to settle to the bottom of the bottle. It is recommended that alkali-resistant ware, now available, be used for storing the solutions. At wavelengths less than 260 nm, it is recommended that solutions not over 6 months old prepared from chemicals of the highest purity be used.

Table 4. Values of Spectral Absorbancy, A_s , and Transmittancy, T_s for Standard Copper Sulphate Solution as Specified, Unfiltered, Thickness 10.00 mm. Temperature 25°C, Having the Following Composition:

Copper Sulphate ($\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$)* 20.000 grams
 Sulphuric Acid (specific gravity 1.835) 10.0 cc
 Water (distilled) to make 1000. cc

<u>Wavelength</u> (nm)	<u>A_s</u>	<u>T_s^{**}</u>	<u>Wavelength</u> (nm)	<u>A_s</u>	<u>T_s^{**}</u>
350	0.0090	0.979	600	0.0680	0.855
60	.0063	.986	10	.0885	.816
70	.0046	.989	20	.1125	.772
80	.0035	.992	30	.143	.719
90	.0028	.994	40	.180	.661
400	.0023	.995	650	.224	.597
10	.0019	.996	60	.274	.532
20	.0016	.996	70	.332	.466
30	.0014	.997	80	.392	.406
40	.0012	.997	90	.459	.348
450	.0011	.997	700	.527	.297
60	.0011	.997	10	.592	.256
70	.0012	.997	20	.656	.221
80	.0014	.997	30	.715	.193
90	.0018	.996	40	.768	.171
500	.0026	.994	750	.817	.152
10	.0038	.991	Hg 404.7	.0021	.995
20	.0055	.987	Hg 435.8	.0013	.997
30	.0079	.982	Hg 491.6	.0019	.996
40	.0111	.975	He 501.6	.0028	.994
550	.0155	.965	Hg 546.1	.0135	.969
60	.0216	.951	Hg 578.0	.0368	.919
70	.0292	.935	He 587.6	.0487	.894
80	.0390	.914	He 667.8	.319	.480
90	.0518	.888			

*Analysis showed the copper sulphate to have 99.7 percent of the theoretical copper content.

**These values of T_s are derived from the values of A_s .

$$A_s = -\log_{10} T_s$$

Table 5. Values of Spectral Absorbancy, A_s , and Transmittancy, T_s for Standard Cobalt Ammonium Sulphate Solution, Unfiltered, Thickness 10.00 mm, Temperature 25°C having the following composition:*

Cobalt ammonium sulphate ($\text{CoSO}_4 \cdot (\text{NH}_4)_2\text{SO}_4 \cdot 6\text{H}_2\text{O}$)** 14.481 grams
 Sulphuric acid (specific gravity 1.835) 10.0 cc
 Water (distilled) to make 1000. cc

Wavelength (nm)	A_s	T_s ***	Wavelength (nm)	A_s	T_s ***
350	0.0038	0.991	600	0.0137	0.969
60	.0040	.991	10	.0124	.972
70	.0050	.989	20	.0115	.974
80	.0065	.985	30	.0112	.975
90	.0088	.980	40	.0110	.975
400	.0125	.972	650	.0105	.976
10	.0168	.962	60	.0097	.978
20	.0224	.950	70	.0087	.980
30	.0340	.925	80	.0076	.983
40	.0522	.887	90	.0066	.985
450	.0773	.837	700	.0054	.988
60	.1031	.789	10	.0046	.989
70	.1213	.756	20	.0038	.991
80	.1349	.733	30	.0032	.993
90	.1472	.713	40	.0030	.993
500	.1635	.686	750	.0028	.994
10	.1742	.670	Hg 404.7	.0144	.967
20	.1689	.678	Hg 435.8	.0437	.904
30	.1452	.716	Hg 491.6	.1497	.708
40	.1113	.774	He 501.6	.1661	.682
550	.0775	.837	Hg 546.1	.0901	.813
60	.0496	.892	Hg 578.0	.0219	.951
70	.0308	.932	He 587.6	.0167	.962
80	.0207	.953	He 667.8	.0089	.980
90	.0158	.964			

* These data apply accurately also from 400 to 750 $m\mu$, to a similar solution made up with 10.3 grams of cobalt sulphate ($\text{CoSO}_4 \cdot 7\text{H}_2\text{O}$).

**Chemical analysis showed a cobalt (plus nickel) content of 100.0 percent of the theoretical, the ratio of nickel to cobalt (metals) being 1 to 200.

***These values of T_s are derived from the values of A_s .
 $A_s = -\log_{10} T_s$.

The data of table 6 are based on extensive measurements made with the Hilger sector-photometer photographic spectrophotometer, and the Beckman DU photoelectric spectrophotometer, supplemented with data obtained with the Brackett quartz photoelectric spectrophotometer and, above 400 nm, with data obtained on the König-Martens visual spectrophotometer and the General Electric recording spectrophotometer. The temperatures were kept close to 25°C. In the values of T_s given in table 6 there is considerable uncertainty in the third decimal.

4. Reference Standards of Spectral Reflectance Factor Relative to Freshly Prepared Magnesium Oxide.

In 1957 the International Commission on Illumination (CIE) adopted a resolution concerning the use of a perfect diffuser for colorimetry (20). The resolution reads, "For colorimetric specification of opaque specimens the perfect diffuser is recommended for ultimate adoption as the reference standard." They note that this action will result in a "recommended set of absolute spectral reflectances of magnesium oxide or other working standards". The absolute spectral reflectance values of MgO adopted by NBS are based on work reported by Goebel et al (21). These adopted values are given in table 7.

As a working standard of spectral reflectance factor nothing has as yet been found more suitable than freshly prepared magnesium oxide. Its (total) luminous reflectance is high, 0.97 or 0.98, and nothing has been found of certainly higher reflectance. Its luminous factor, R_0 , 45, is 1.00, and its spectral selectivity throughout the visible spectrum appears to vary by less than 1 percent. These data are based on work by Priest (22), McNicholas (23), and Preston (24) and are summarized in National Bureau of Standards letter circular LC-547 (25). More recent work by Benford and others (26, 27) and by Middleton and Sanders (28, 29) closely confirm these results and extend the data into the ultraviolet and infrared.

Other agencies besides the Bureau have also recommended MgO for the same or similar purposes. In 1931, the International Commission on Illumination adopted a resolution which may be translated as follows: "For the colorimetric measurement of opaque materials the luminance of the specimen studied ought to be expressed as a function of the luminance of a surface of the oxide of magnesium considered under the same conditions of illumination and observation" (30). In 1944, this method was incorporated in ASTM Standard Method of test for spectral characteristics and color of objects and materials (31).

Table 6. Values of Spectral Transmittancy, T_s , and Absorbancy, A_s , for Standard Potassium Chromate Solution, Unfiltered, Thickness 10.00 mm, Temperature 25°C, having the following composition:
0.0400 gram per liter of potassium chromate (K_2CrO_4) in 0.05 normal potassium hydroxide solution*

Wavelength (nm)	T_s	A_s^{**}	Wavelength (nm)	T_s	A_s^{**}
220	0.358	0.446	335	0.600	0.222
25	.601	.221	40	.483	.316
30	.674	.171	45	.373	.428
			50	.276	.559
35	.616	.210	55	.199	.701
40	.507	.295			
45	.402	.396	60	.148	.830
50	.319	.496	65	.116	.936
53.6	.279	.554	70	.103	.987
			75	.102	.991
55	.268	.572	80	.117	.932
60	.233	.633			
65	.202	.695	85	.150	.824
70	.180	.745	90	.202	.695
75	.175	.757	95	.294	.532
			400	.402	.396
80	.194	.712	04.7	.515	.288
85	.257	.590			
90	.373	.428	10	.632	.199
95	.533	.273	20	.751	.124
96.7	.598	.223	30	.824	.084
			35.8	.859	.066
300	.709	.149	40	.882	.054
02.2	.771	.113			
05	.834	.079	50	.927	.033
10	.895	.048	60	.960	.018
13.2	.905	.043	70	.980	.009
			80	.991	.004
15	.900	.046	90	.997	.001
20	.864	.064	500	1.000	.000
25	.804	.095			
30	.710	.149			
34.2	.620	.208			

* This solution of potassium hydroxide can be prepared with sufficient accuracy by dissolving 3.3 grams of potassium hydroxide sticks (85% KOH) of reagent quality in sufficient distilled water to make 1 liter.

**These values of A_s are derived from the values of T_s ,
 $A_s = -\log_{10} T_s$

Note: Distilled water only was used in the reference cell.

Table 7 Absolute 6°-Hemispherical Reflectance, ρ

of Smoked MgO 1-mm thick

Wave- length (nm)	ρ	Wavelength (nm)	ρ	Wavelength (nm)	ρ	Wavelength (nm)	ρ
400	0.989	600	0.992	800	0.983	1000	0.978
10	.990	10	.991	10	.983	10	.978
20	.990	20	.991	20	.982	20	.977
30	.991	30	.991	30	.982	30	.977
40	.991	40	.990	40	.981	40	.977
50	.992	50	.990	50	.981	50	.976
60	.992	60	.990	60	.981	60	.976
70	.993	70	.989	70	.981	70	.976
80	.993	80	.989	80	.980	80	.976
90	.993	90	.988	90	.980		
500	.993	700	.988	900	.980		
10	.993	10	.988	10	.980		
20	.993	20	.987	20	.980		
30	.993	30	.987	30	.979		
40	.993	40	.986	40	.979		
50	.993	50	.986	50	.979		
60	.992	60	.985	60	.979		
70	.992	70	.985	70	.979		
80	.992	80	.984	80	.978		
90	.992	90	.984	90	.978		

While the characteristics above noted make fresh MgO an excellent working standard, it has other characteristics that are undesirable and that make the calibration and use of a secondary working standard a very advisable procedure. An MgO surface is extremely fragile and thus is not very suitable for continued handling. A more serious defect is that its spectral reflectance may change by slight but definite amounts within a short time (sometimes in a day) after preparation, the reflectance decreasing below 550 nm. Furthermore, the nature and extent of the changes seem somewhat variable (1, Fig. 33; 28). A third reason for use of a secondary working standard is that slight variations in reflectance (0.1 or 0.2 percent) have been noted for different preparations of freshly prepared MgO. By calibrating the secondary working standard against several different MgO preparations a more representative standard is obtained than would be any single MgO surface by itself.

White structural glass by the name of Vitrolite, with one surface polished, has proved the most suitable for secondary standards of spectral directional reflectance, although the material is not uniformly good for this purpose and must be selected with care. A considerable supply of suitable Vitrolite has been obtained by the National Bureau of Standards, and standards are calibrated and issued for either the General Electric recording spectrophotometer or the Beckman DU spectrophotometer.

4.1 For the General Electric Recording Spectrophotometer.

The Vitrolite standards issued for use with the General Electric recording spectrophotometer are about 4 inches square and 5/16 inch thick. They are covered in NBS test fee schedule 202.105, items j and k.

In the present model of the spectrophotometer, the design is such that the radiant energy is incident in a slightly diverging beam whose axis is at 6° to the perpendicular to the sample. The specular component of the reflected energy is thus diverted away from the entrance aperture towards a port on the side. This port may be filled with MgO or with a black material, so that for plane glossy surfaces the measurement can be made with the specular component "included" or "excluded".

To give one a better idea of the spectral reflectance factor relative to the perfect diffuser and relative to MgO for the Vitrolite standards thus issued, there is given in table 7 two sets of values from 400 to 1080 nm with specular component excluded that apply to one of the NBS standards. Values reported under test fee items j and k with specular component excluded will probably be closely similar to these. With specular component included, the values are greater by roughly 0.04.

Only one Vitrolite working standard is needed for the measurement of spectral reflectance factor on the General Electric recording spectrophotometer. This calibrated Vitrolite standard and the samples to be tested are in turn placed at the sample aperture of the integrating sphere, and any highly reflecting substance such as MgO , BaSO_4 , or MgCO_3 may be used at the comparison aperture provided the material to be tested does not reflect more than the comparison material.

The reflectance factors of the test samples relative to the perfect diffuser and relative to freshly prepared MgO are obtainable by use of correction factors derived from the ratios between the standard Vitrolite values and the values for the Vitrolite read from the curve sheet. This is explained in detail in the report accompanying each standard.

By this procedure only one Vitrolite standard is necessary and the Bureau does not issue these standards in pairs, as some have requested. For transmittance measurements any two nearly identical white surfaces are suitable and no standard reflecting surface is necessary.

4.2 For the Beckman Quartz (DU, Non-Recording) Spectrophotometer.

The Vitrolite standards issued for use with the Beckman DU spectrophotometer are about 1 1/2 by 2 inches and 5/16 inch thick. They are covered in NBS test fee schedule 202.105, items L to n. In this case the radiant energy is incident upon the sample in a nearly parallel beam whose axis is perpendicular to the sample. The reflected energy accepted for measurement is taken in an annular "beam" whose axis is closely 45° to the perpendicular in all directions, but the parts of which may vary in direction roughly from 35° to 55° . The specular component is thus excluded from the measurements and the values obtained relative to magnesium oxide and reported do not differ greatly from those shown in the columns headed R' in table 8.* At 400 nm and below, these values as obtained on NBS standard V2-B4 for the Beckman spectrophotometer are:

<u>Wavelength</u> (nm)	<u>Spectral Reflectance Factor</u> R'
350	0.753
360	.812
370	.826
380	.809
390	.852
400	.868

* See note on page 22.

Table 8. Spectral Reflectance Factor of NBS Standard Vitrolite V1-G4 Relative to a Perfect Diffuser, R, and Relative to Freshly Prepared Magnesium Oxide, R', for Excluded Specular Component of Reflected Radiant Energy on a General Electric Recording Spectrophotometer.

Wavelength (nm)	For Visible Spectrum (400 to 750 nm) (black velvet port) (10 nm slits)		Wavelength (nm)	For Near Infrared Spectrum (730 to 1080 nm) (black cavity port) (20 nm slits)	
	R	R'		R	R'
400	0.865	0.875	730	0.850	0.861
10	.856	.865	40	.846	.858
20	.850	.859			
30	.850	.858	750	.844	.856
40	.848	.856	60	.841	.854
			70	.838	.851
450	.856	.863	80	.835	.849
60	.864	.871	90	.834	.848
70	.868	.874			
80	.870	.876	800	.831	.845
90	.872	.878	10	.829	.843
			20	.826	.841
500	.874	.880	30	.824	.839
10	.877	.883	40	.820	.836
20	.879	.885			
30	.881	.887	850	.818	.834
40	.882	.888	60	.816	.832
			70	.814	.830
550	.882	.888	80	.811	.828
60	.881	.888	90	.809	.826
70	.880	.887			
80	.878	.885	900	.808	.824
90	.876	.883	10	.806	.822
			20	.805	.821
600	.873	.880	30	.802	.819
10	.869	.877	40	.800	.817
20	.866	.874			
30	.863	.871	950	.800	.817
40	.860	.869	60	.799	.816
			70	.797	.814
650	.859	.868	80	.796	.814
60	.858	.867	90	.796	.814
70	.857	.867			
80	.856	.866	1000	.795	.813
90	.856	.866	10	.794	.812
			20	.793	.812
700	.855	.865	30	.792	.811
10	.855	.865	40	.792	.811
20	.853	.864			
30	.852	.863	1050	.791	.810
40	.850	.862	60	.791	.810
			70	.791	.810
750	.850	.862	80	.791	.810

It is important to note that the values of spectral reflectance factor relative to MgO obtained and reported for use on the Beckman DU spectrophotometer are not valid for use on the General Electric spectrophotometer, and vice versa, because of the notably different irradiation-reception conditions of the two types of instrument. Similarly the values reported for either instrument should not be used for other types of instrument, unless the geometrical conditions are sufficiently similar as to make the values valid for such purpose.*

* In any comparison of results of reflectance factor measurements on instruments with differing geometries of irradiation and reception, it should be remembered that the reflectance factor of MgO also varies with the geometrical conditions. See second paragraph of Section 4, above.

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Letter Circular 1017

(Supersedes LC 929 dated 11/26/48)

Prepared by: KSG:HJK:JCS/

Colorimetry and Spectrophotometry Section

Metrology Division

Institute for Basic Standards

National Bureau of Standards

2. Color Vision

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Foreword

In attempting to develop precision methods for colorimetry we need a complete understanding of how the human visual system functions to produce color vision. Toward this end, the two papers that begin this section on color vision were written by Judd. These papers review the work performed by researchers on color vision throughout the world in the century 1860-1960 and at the National Bureau of Standards in the half-century 1916-1966.

Since 1931, the International Commission on Illumination (CIE) has been recommending a tristimulus system for color measurement of small, foveal fields (nominally 2°). In recent years a need was felt for a similar system that is useful for larger viewing fields (nominally 10°). Typical of the work done internationally for this purpose is that reported in the next three papers of this section. The large-field system was adopted by the CIE in 1964.

Although known for a long time, the effect of changes in color induced by changes in background were studied at the National Bureau of Standards by a guest worker from Japan, H. Takasaki. The next two papers describe the "crispening effects" found for grays with change in gray background and chromatic colors with change of background color.

Color-vision deficiency is studied in the expectation that anomalous color vision might help to understand normal color vision. The last two papers reflect some of the studies performed on the topic commonly called "color blindness". The last paper describes a set of twenty-two colors to provide maximum contrast for persons with deficient color vision.

FUNDAMENTAL STUDIES OF COLOR VISION FROM 1860 TO 1960

By DEANE B. JUDD

NATIONAL BUREAU OF STANDARDS

What I call fundamental studies of color vision got their start with Maxwell's determination (1860) of the color-matching functions of the normal human eye. Figure 1 (Judd, 1961) compares Maxwell's determinations for himself and one other observer (K) with modern color-matching functions. The open points and the lines drawn through them come from the 1931 CIE standard observer; the points marked otherwise are from Maxwell. Except for the extremes of the spectrum the discrepancies are only slightly more than can be explained by individual differences among observers with normal color vision. These curves, R , G , and B , indicate the radiant flux of spectrum primaries [630, 528, 457 nanometers (nm)] required in an additive mixture to match unit radiant flux for each part of the spectrum in turn. Note that at the red primary the R -curve is unity; the other two, zero. This means simply that one unit of radiant flux at this wavelength color-matches itself, and the same for the other two primaries, of course. Note also that some of the radiant fluxes indicated by the curves are less than zero. To speak of a negative radiant flux makes no sense physically; there is no such thing. But in color matching there is a useful meaning, universally recognized since Maxwell's time, of a negative radiant flux. It means simply that the negative amount of the primary, instead of being added to the other two primaries, is added to the part of the spectrum whose color is being determined. For example, at 500 nm Figure 1 shows that by adding about 0.2 unit of radiant flux of the red spectrum primary (630 nm) to one unit of radiant flux at 500 nm, there is produced a blue-green color that can be matched by about 0.4 unit of the green primary (528 nm) added to about 0.2 unit of the blue primary (457 nm).

The functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ not only show how much of the primaries is required to match each part of the spectrum in turn, but they also indicate how much of the primaries is required to match any known combination of the various parts of the spectrum. On this account these functions are known as color-matching functions. By means of these functions, successful predictions may be made of whether any two lights of known spectral distributions $P_{\lambda,1}$ and $P_{\lambda,2}$ will, or will not, look alike. The conditions for color match are:

$$\int_0^\infty (P_{\lambda,1} - P_{\lambda,2}) \bar{r}(\lambda) d\lambda = 0,$$

$$\int_0^\infty (P_{\lambda,1} - P_{\lambda,2}) \bar{g}(\lambda) d\lambda = 0,$$

$$\int_0^\infty (P_{\lambda,1} - P_{\lambda,2}) \bar{b}(\lambda) d\lambda = 0.$$

These conditions for color match follow directly from Grassmann's law (1853) of additivity, well verified (König, 1887; v. Kries, 1905) for the macular regions of the retina. They state merely that if for each spectral component of a compound light the equivalent is given in amounts of the three primaries, the sum of these equivalents will color-match the sum of the spectral components making up the compound light. If the conditions for a color match are satisfied because the two lights are physically identical ($P_{\lambda,1} = P_{\lambda,2}$) throughout the visible spectrum, the match is said to be isomeric (following Ostwald's terminology) or spectral (Nimer-

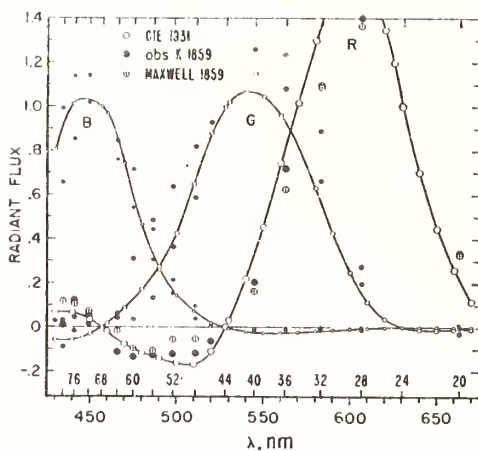


FIG. 1.—Radiant flux of the spectrum primaries, 630.2, 528.1, and 456.9 nm, required in an additive mixture to match unit radiant flux of the spectrum. 1931 CIE standard observer, \circ — \circ ; Maxwell's observer K, \bullet ; Maxwell (observer J), \odot . Note that Maxwell's data expressed in units of radiant flux are in substantial agreement with the 1931 CIE standard observer. The numbers above the base line refer to wavelengths designated on Maxwell's instrumental scale, nonlinear because of prismatic dispersion.

off, 1965). In this trivial case, the color-matching functions are not needed. If the two lights have different spectral distributions, the color match is said to be metameric; and in this usual case the color-matching functions are crucial.

Because of this generality of their application, studies yielding color-matching functions might be said to be fundamental. There are, however, two other reasons for regarding them as fundamental.

Types of Color Vision.—Table 1 summarizes the principal types of human color vision (Judd, 1943).

Normal color vision requires three color-matching functions for its definition, as shown in Figure 1. On this account normal color vision is classed as trichromatic. Trichromatic observers can make light-dark, yellow-blue, and red-green discriminations. There are three types of abnormal trichromatic color vision, known as protanomaly, deuteranomaly, and tritanomaly (rare).

Partially color-blind vision requires but two color-matching functions for its definition, and is known as dichromatic vision. There are three types of dichromatic vision, known as protanopia, deuteranopia, and tritanopia. A dichromat can match any color by a suitable additive combination of but two primary lights; and he is able to make discriminations of but two sorts as indicated in Table 1.

Totally color-blind vision requires but one color-matching function for its definition, and is known as monochromatic vision. A monochromat can match any color by a suitable amount of any light taken as a primary simply by adjusting the primary light to the same brightness, because a monochromat cannot make any discrimination other than light-dark.

Since the color-matching functions provide a basis for classifying human vision into the three types, trichromatic, dichromatic, and monochromatic, this is another sense in which we may regard them as fundamental.

Theories of Color Vision.—Major attention between 1860 and 1960 has been devoted to three categories of color-vision theory: three-components theory, opponent-colors theory, and stage theory. All of these theories, and indeed any tenable theory of color vision, must explain in detail why precisely the conditions for color match expressed by color-matching functions hold. The color-matching

TABLE 1
TYPES OF HUMAN COLOR VISION

Types	Light-dark	Response Yellow-blue	Red-green	Maximum luminosity at λ (nm)	Type designation
Trichromatism	Yes	Yes	Yes	555	Normal
	Yes	Yes	Weak	540	Protanomaly
	Yes	Yes	Weak	560	Deuteranomaly
	Yes	Weak	Yes	560	Tritanomaly
Dichromatism	Yes	Yes	—	540	Protanopia
	Yes	Yes	—	560	Deuteranopia
	Yes	—	Yes	560	Tritanopia
Monochromatism	Yes	—	—	510	Cone blindness
	Yes	—	—	560	—
	Yes	—	—	540	—
	Yes	—	—	540	—

functions thus play the role of quantitative boundary conditions for any theory of color vision. This is the major reason for regarding them as fundamental.

Three-components theory: This theory was briefly stated in 1807 by Thomas Young, and was elaborated by Helmholtz about 50 years later. It assumes the existence of three independent response mechanisms in the normal eye: one predominantly sensitive to long-wave light and yielding the response *red*; a second predominantly sensitive to middle-wave light and yielding the response *green*; and a third sensitive to short-wave light and yielding the response *violet*. Young thought that the long-wave and short-wave extremes of the spectrum could excite, respectively, the red and violet responses to the exclusion of the other two. Let us see how color-matching functions yield precisely the response curves implied by this assumption.

The key to this problem is choice of primaries. In Figure 1 are shown color-matching functions expressed relative to the spectrum primaries (630, 528, and 457 nm) chosen by Maxwell. From Grassmann's law of additivity it can be shown that precisely the same decisions as to color-match, or failure to color-match, can be obtained from functions R' , G' , and B' , as from R , G , and B , provided the new functions are weighted sums of the old:

$$R' = K_{11}R + K_{12}G + K_{13}B,$$

$$G' = K_{21}R + K_{22}G + K_{23}B,$$

$$B' = K_{31}R + K_{32}G + K_{33}B,$$

where K_{11} , K_{12} , . . . , K_{33} are constants that may be chosen arbitrarily provided the determinant differs from zero. It is legitimate to proceed therefore by trial and error to search for constants that will yield response curves, R' , G' , and B' , in accord with Young's hypothesis; that is, curves with no negative values anywhere, a green curve that is zero for as much as possible of the long-wave and short-wave portions of the spectrum, a red curve that is zero for the short-wave portion, and a violet curve that is zero for the long-wave portion. This choice of constants amounts to choice of red and violet primaries at the extremes of the spectrum.

This possibility was clearly explained by Helmholtz. He states, for example (p. 145): "The choice of the three fundamental colours is somewhat arbitrary. . . Young may have been guided by the consideration that the terminal colours of the spectrum seem to have special claims by virtue of their positions." The König-Dieterici

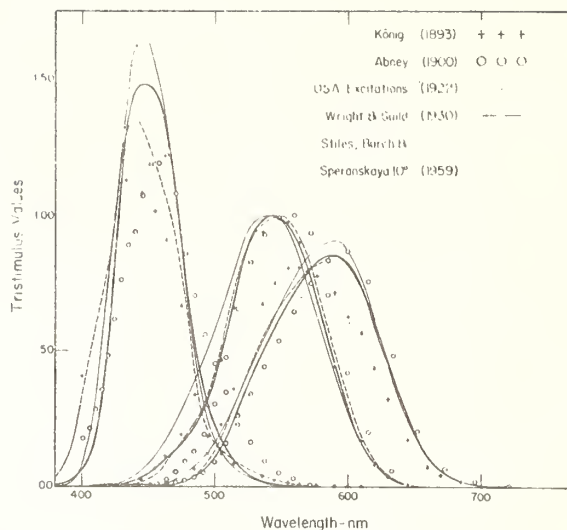


FIG. 2. Color-matching functions expressed relative to the Young primaries (spectrum red, spectrum violet, and green outside spectrum). By the Young theory these functions represent the spectral absorptances of the retinal photopigments multiplied by the spectral transmittances of the ocular media through which light passes to reach the retina. König (1893), crosses; Abney (1900), circles; OSA excitations (Troland, 1922), dashed lines; CIE standard observer based on Wright and Guild (1930), heavy lines; CIE supplementary observer for 10° viewing fields based on Stiles and Burch (1959), and Speranskaya (1959), light lines.

determinations of the color-matching functions were presented (1893) with Young's choice of primaries under the name "Elementar-Empfindungs-Curven," and Abney's were similarly presented (1900) under the name "sensation curves." This choice of primaries was used by Weaver, who averaged the König and Abney results to produce the "elementary excitation curves" recommended in 1922 by the Committee on Colorimetry, Optical Society of America, under Troland's chairmanship. Figure 2 compares these color-matching functions with those based on the work of Wright (1929-1930) and Guild (1931) recommended in 1931 by the International Commission on Illumination (CIE), and with those of Stiles and Burch, and Speranskaya (1959) for extramacular vision (field subtending 10°) recommended in 1964 by the CIE for large-field colorimetry.

The color-matching functions in Figure 2 all show the amounts of the Young primaries required to color-match unit radiant flux of each part of the spectrum. The scale values for the G' -functions are adjusted to make the maximum be unity; those for the R' - and B' -functions are adjusted to make the areas under the curves equal to that for the G' -function; that is, the so-called equal-energy source (source whose radiant flux per unit wavelength is constant) corresponds for each set of functions to equal amounts of the three primaries. To make the König-Dietrich determinations (shown by crosses) refer to unit amount of spectral radiant flux, it was assumed that König's sunlight was equivalent to sunlight at air mass 2 from the compilation by Parry Moon (1940). Similarly, the Abney data (shown by circles) were adjusted by assuming that the positive crater of the carbon arc used by him was equivalent to the blackbody at $3,800^\circ\text{K}$. It will be noted that Weaver's average of these two sets of color-matching functions (shown by dashed lines) falls

fairly satisfactorily between the crosses and circles; so he must have made a fairly similar adjustment. The Guild-Wright determination (shown by heavy solid lines) corroborates the Weaver average surprisingly well considering the rather large differences between the König and Abney data. The Stiles-Burch-Speranskaya determination (shown by light solid lines) departs from these chiefly in ways that correspond to the use of extramacular retinal regions instead of macular, and this indicates that the extramacular cones have spectral sensitivities not much different from those of foveal cones.

It will be noted that Young's opinion that the long-wave portion of the spectrum can excite the red response to the exclusion of the other two is not contradicted by any of the color-matching functions. These functions, however, do not permit the view that the short-wave extreme can excite the violet response alone; a small red response (about 1/200 of the maximum) has to be admitted.

There is one final remark on Figure 2. If the additional assumption be made that each type of cone contains its own characteristic photosensitive pigment, then these color-matching functions expressed relative to the Young primaries should correspond to the product of the spectral absorptance of the three retinal pigments by the spectral transmittance of the ocular media (principally lens and macula). When Rushton (1957) announced that by measuring the spectral reflectance of the retina both before and after bleaching by strong light, he had discovered the presence of two bleachable pigments, one with peak absorptance at 590 nm, the other at 540 nm, it was very tempting to point to these two color-matching functions peaking very closely at these two wavelengths, and to say that this is striking corroboration of the Young theory. When the spectral transmittances of the ocular media are divided out of these curves, however, the predicted wavelengths of maximum absorptance are found to be somewhat lower than those indicated by Rushton's very difficult technique.

Partial color blindness: Young's theory has a simple built-in explanation of the three forms of dichromatic vision. These forms (protanopia, deuteranopia, and tritanopia) are reduction forms of normal vision. Any dichromat finds any color match set up by an observer of normal color vision with the same ocular pigmentation to be an acceptable match for him also. The simplest explanation for dichromatic vision by the Young theory is to say that a protanopic observer is an otherwise normal observer who has lost the red response. Similarly, deuteranopia corresponds to loss of the green response, and tritanopia, to loss of the violet. Since in each case the remaining two responses are identical with those of normal vision, the dichromatic observer has no way to detect any error in a normal match. A corollary of this explanation, however, is that an observer who has lost the red response would be blind to the long-wave part of the spectrum where the other two mechanisms have zero sensitivity. Protanopes have reduced sensitivity to long-wave light, but not zero. Similarly, tritanopes have reduced sensitivity to short-wave light, but not zero.

Maxwell was among the first to recognize that dichromatism is a reduced form of normal trichromatism, and he described a method by which to determine precisely the normal color corresponding to the missing primary process. In a letter of June 4, 1855, to G. Wilson, Maxwell wrote: "The mathematical expression of the difference between colour-blind and ordinary vision is that colour to the former is a

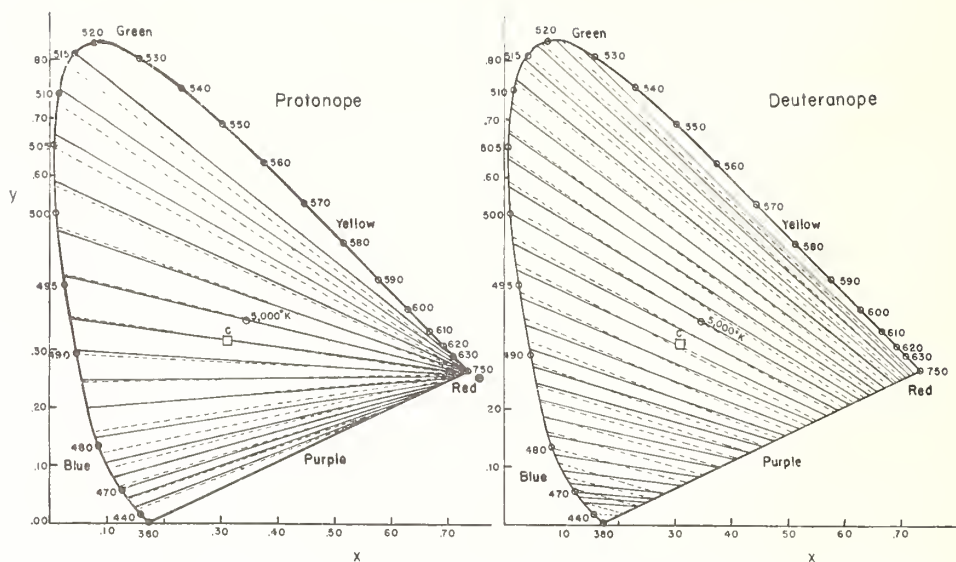


FIG. 3.—Chromaticity confusions of the protanope and deuteranope shown on the (x,y) -chromaticity diagram of the 1931 CIE colorimetric coordinate system from determinations by Pitt (1935). The points along each dotted line correspond to chromaticities found by Pitt to be indistinguishable in dichromatic vision. The solid lines intersect at a single point: protanope at $x = 0.753$, $y = 0.247$; deuteranope at $x = 1.000$, $y = 0.000$.

function of two independent variables, but to an ordinary eye, of three; and that the relation of the two kinds of vision is not arbitrary, but indicates the absence of a determinate sensation, depending perhaps upon some undiscovered structure or organic arrangement, which forms one-third of the apparatus by which we receive sensations of colour. . . . If we find two combinations of colours which appear identical to a colour-blind person, and mark their positions on the triangle of colours, then the straight line passing through these points will pass through all points corresponding to other colours, which, to such a person, appear identical with the first two. We may in the same way find other lines passing through the series of colours which appear alike to the colour-blind. All these lines either pass through one point or are parallel, according to the standard colours which we have assumed, and the other arbitrary assumptions we may have made. Knowing this law of colour-blind vision, we may predict any number of equations which will be true for eyes having this defect."

Figure 3 shows on the CIE (x,y) -chromaticity diagram by dotted lines the chromaticity confusions found by Pitt (1935) for six protanopes (on the left), and for six deuteranopes (on the right). The protanopic confusion lines are seen to intersect at very nearly one point as stated by Maxwell, and indicate the color for whose perception protanopes have no mechanism. The deuteranopic confusion lines are not as consistent, but the average intersection point is at about $x = 1.10$, $y = -0.10$. The solid lines intersect at $x = 1.00$, $y = 0.00$, which may be taken as an upper y -limit for Pitts' six deuteranopes. By basically the same method, though different in detail, König (1892) also evaluated the intersection points for confusion lines by two protanopes and one deuteranope. Similar evaluations for confusion lines were made by Farnsworth (1955) for one tritanope, and by Thomson and Wright (1953)

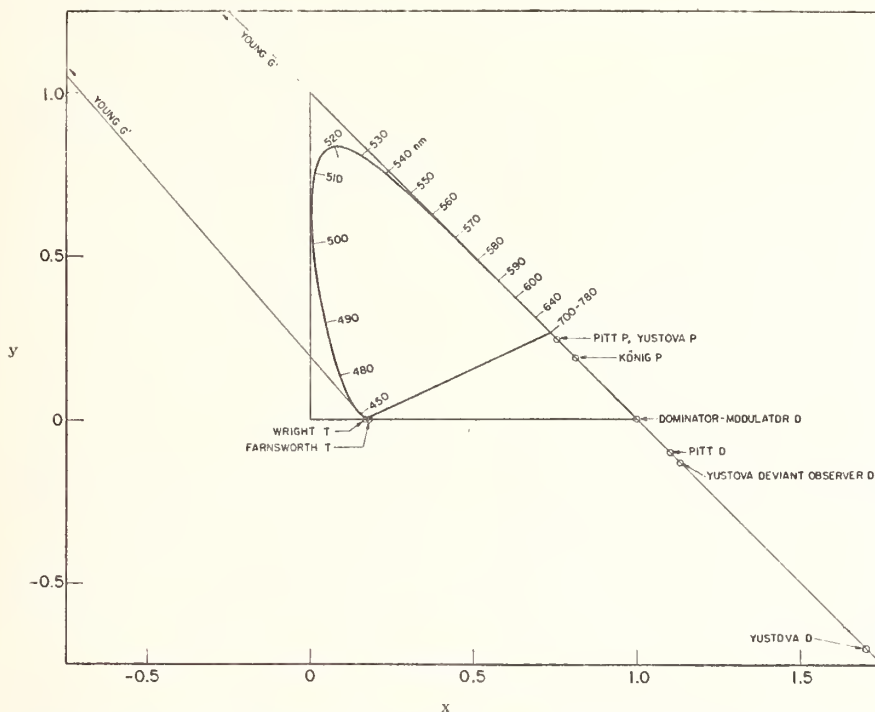


FIG. 4.—Intersection points of the chromaticity-confusion lines for the three types of dichromatic vision: protanopic, P ; deuteranopic, D ; and tritanopic, T , shown on the (x, y) -chromaticity diagram of the 1931 CIE colorimetric coordinate system. Note that the two determinations (Wright, 1952; Farnsworth, 1955) of the T -point fall close to $x = 0.17$, $y = 0.00$, and that the two recent determinations (Pitt, 1935; Yustova, 1953) of the P -point fall close to $x = 0.75$, $y = 0.25$. The determinations of the D -point (Pitt, 1935; Nuberg and Yustova, 1955), on the other hand, spread over a considerable range. The dominator-modulator theory implies a D -point at $x = 1.00$, $y = 0.00$. Young's theory in its original form implies that the deuteranope chromaticity confusion lines are nearly parallel on the (x, y) -chromaticity diagram (see the two straight lines labeled *Young G'*).

for seven tritanopes. Figure 4 shows the locations of the intersection points of the chromaticity-confusion lines for protanopes (P), deuteranopes (D), and tritanopes (T) from the studies mentioned above. König's deuteranope yields an intersection point off the graph.

Yustova (1949, 1953) carried out studies of dichromatic vision, and in 1955 Nuberg and Yustova published an extensive determination of intersection points of chromaticity-confusion lines for four protanopes and 12 deuteranopes. They used a long-overlooked method, also originated by Maxwell (1860), which yields the intersection point from a single pair of colors confused by the dichromat. The method is to obtain the tristimulus values set by the dichromatic observer to match any arbitrarily chosen color and to subtract them from the tristimulus values of the same color set by an observer having normal trichromatic vision. The differences specify the color for the perception of which no mechanism exists in the dichromatic observer (Judd, 1964). The Yustova results are also shown in Figure 4. It will be noted that the point found by Yustova for four protanopes agrees perfectly with that found by Pitt for six protanopes. Only one of the 12 deuter-

anopes, however, agrees with Pitt's *D*-point. The other 11 deuteranopes yield intersection points between Pitt's *D*-point and König's early determination for one deuteranope.

It must be pointed out that the location of the intersection points at various places along the line $z = 0$ in the CIE coordinate system correlates with the wavelength of the maximum of the long-wave response curve for the corresponding dichromat. This long-wave response curve is essentially the dichromatic luminosity function because the contribution of the short-wave response function to luminosity, if not precisely zero, differs negligibly from zero. For the intersection point at $x = 0.75$, $y = 0.25$, as for protanopes, the dichromatic luminosity function peaks at about 540 nm. For $x = 1.00$, $y = 0.00$, as in the dominator-modulator theory (to be discussed presently), the luminosity function is identical with that for normal vision, and peaks at 555 nm. For $x = 1.10$, $y = -0.10$, the peak is at about 560 nm, as for Pitt's six deuteranopes. But for Yustova's 11 deuteranopes yielding $x = 1.7$, $y = -0.7$, the peak is at 573 nm. The luminosity curves found by Hsia and Graham (1957) average to a curve with a peak at about 575 nm in good agreement with the Yustova determination. It should be recalled, also, that Willmer (1949) announced the discovery of two distinct types of deuteranopia, one with normal luminosity function, the other with luminosity function shifted toward the long-wave end of the spectrum compared to normal. If it is true, as announced by Willmer, that there are two types of deuteranope, then perhaps all of the *D*-points shown on Figure 4 are correct. By this view, all of Pitt's six deuteranopes, one of Hsia and Graham's six deuteranopes, and one of Yustova's 12 deuteranopes should be classed as having essentially normal luminosity functions; the remainder of the deuteranopes studied have luminosity functions shifted from the normal position to the long-wave end of the spectrum.

If the color-matching functions are expressed relative to the *P*-, *D*-, and *T*-points, they may be interpreted as the responses that three different kinds of cones would have to have to explain the three types of dichromatic vision. Figure 5 shows such response curves derived from the CIE 1931 color-matching functions in which the *D*-point is taken at $x = 1.00$, $y = 0.00$. By this choice, which is barely admissible from Pitt's determination, and inadmissible from Yustova's, deuteranopic luminosity is taken as identical to normal luminosity. The obvious difference between these response curves and those expressed relative to Young's primaries is that the long-wave curve peaks, not at 590 nm, but at 555 nm; and indeed this curve is precisely the normal luminosity function. The other important, though not obvious, difference is that the middle-wave curve does not drop all the way to zero at either end of the spectrum until the other curves do. The short-wave curve remains unchanged. The usual explanation of the differences between these two sets of response curves is that those referred to the Young primaries reflect the spectral absorptances of photosensitive retinal pigments, while those expressed relative to the intersection points of the chromaticity-confusion lines for the three types of dichromatic vision reflect the responses of three types of cones. The fact that the short-wave curves of the two sets are identical is taken to mean that the cones giving violet or blue signals contain only the short-wave-absorbing pigment. The fact that the middle- and long-wave curves expressing cone response are broader than those expressing absorptance of photosensitive pigments may be taken to mean that

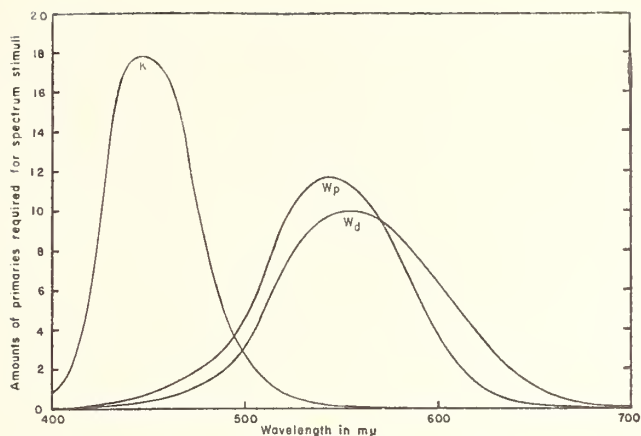


FIG. 5.—Color-matching functions for the 1931 CIE standard observer expressed relative to the primaries implied by the König (1897) tentative form of dominator-modulator theory. The red primary is placed at the protanopic intersection point ($x = 0.753, y = 0.247$); the violet primary is placed at the titranopic intersection point ($x = 0.18, y = 0.00$); and the green primary is placed at the deutanopic intersection point ($x = 1.00, y = 0.00$) implied by the dominator-modulator theory. By this theory the W_d curve represents for normal color vision both the luminous-efficiency function and the red response function; for deutanopic vision it is both the luminous efficiency function and the “warm” response function. Protanopic vision does not have this function. Similarly, the W_p curve represents for protanopic vision both the luminous-efficiency function and the “warm” response function. Finally, the K curve represents the response function for the violet modulator in normal vision and for the “cold” modulator for protanopic and deutanopic vision. Tritanopic vision does not have this function.

the cones giving green signals contain, in addition to middle-wave-absorbing pigment, slight amounts of the other two pigments, and the cones giving red signals contain about equal amounts of middle-wave- and long-wave-absorbing pigments, with perhaps a slight admixture of short-wave-absorbing pigment. This interpretation is based on the idea that the cause of dichromatic vision is a “loss” of one of the three components of normal vision. Another well-explored interpretation of the relation of dichromatic vision to normal trichromatic vision is that proposed by Leber (1869) and Fick (1879). They said that deuteranopia, for example, might be explained by assuming that both the cones signaling red and those signaling green alike contain the same admixture of middle-wave- and long-wave-absorbing pigments. This idea leads to the same cone-response curve as the “loss” hypothesis, but has the advantage that by this view the dichromat could never experience red separate from green, but would always see what unilateral dichromats testify they see: either blue or yellow (Judd, 1948).

The set of response functions in Figure 5 accords with the particular variety of three-components theory tentatively championed by König following his study of tritanopia in 1897. For example, v. Kries in a note specially prepared in 1924 for insertion in the English edition of Helmholtz' *Physiological Optics*, states (p. 412): “König long ago called attention incidentally to the curious fact that the luminosity values of the colours . . . turn out to be very nearly the same function of wave-length for deuteranopes and persons with normal vision. Since in the case of deuteranopes light of long wave-lengths has no effect except on the red component, we must suppose also that this action depends on the wave-length in the same way. But

then the further result is that the distribution of luminosity is not affected, or at least almost inappreciably affected, by the addition of the green component by which the deuteranopic visual organ is converted into a normal organ; in other words, that in the case of normal vision also the luminosity goes practically hand in hand with the action of the red component." König was, I believe, the first among three-components theorists to suggest the possibility that the production of the brightness sensation might be entrusted to a single one of the three receptor mechanisms, with the other two mechanisms having the function of modulating this response to produce the chromatic aspects of vision. After Granit's work on retinas of various animals with the microelectrode indicated (1943) that these retinas contained a preponderance of brightness sensors, such theories of color vision came to be known as dominator-modulator theories.

To summarize the 1960 status of three-components theories, there was one for the photosensitive pigments of the retina, and there were several cone-response theories, including dominator-modulator theories, depending on the relationship assumed between deuteranopic and normal vision. All of these theories assume that yellow is produced by the sum of red and green responses and that white is produced by the sum of equal amounts of red, green, and violet responses; but none of them so far discussed here provide any explanation for these assumptions. The Ladd-Franklin theory (1892) is a skillful attempt to formulate a three-components theory overcoming this defect. Its outstanding weaknesses are its failure to account for tritanopia and its implication that the blue process contributes to luminosity for protanopic vision, but subtracts from it for normal and deuteranopic vision.

Opponent-colors theory: This theory was proposed and explained in detail by Ewald Hering in 1878. It is based on an analysis of sensations of color rather than of the stimuli required to evoke them. It assumes that there are six independent unitary colors (red, yellow, green, blue, white, and black), no one of which partakes of any other; that is, for example, yellow is a basic color in its own right, not the product of combining red with green. The Hering theory tacitly assumes that light is absorbed in the receptors by photopigments, that this absorption starts activity in the rest of the visual system, and that this activity is directly responsible for the colors we see. This activity is not found in six separate systems, but in three opposing pairs of processes: black-white, yellow-blue, and red-green. Black and white blend to produce gray, but equal amounts of yellow and blue, and of red and green cancel to zero.

This formulation of human vision supplies a specific model for the colors perceived, which the three-components theory does not. It has been very fruitful in suggesting many researches in such aspects of vision as chromatic adaptation, the influence of the surround on the perceived color, Bezold-Brücke phenomenon, and dependence of color perception on luminance of test field and surround; and it has provided a framework within which the results of such studies can be clearly stated and analyzed.

Just as is true for the various three-components theories, so also do color-matching functions provide quantitative information regarding the dependence of the opponent-color processes on wavelength. It is necessary only to express the color-matching functions relative to the primaries implied by the special assumptions

of the particular form of opponent-colors theory under consideration. This quantitative information will be presented under stage theories of vision.

Stage theories of human vision: It has already been pointed out that there are three-components theories for two stages in the visual process: the retinal photopigment stage and the cone-response stage. It was suggested by v. Kries (1905) that the Young-Helmholtz three-components theory may be taken as valid at the receptor level but that the signals from the receptors are so processed that at some later stage the opponent-colors theory of Hering applies. Schrödinger (1925) derived the response curves for the two stages implied by the v. Kries proposal.

A somewhat similar two-stage theory has been used by Hurvich and Jameson (1951 on) in their comprehensive and detailed studies of color vision. The opponent-colors theory has probably received at their hands its most successful quantification and application.

There has been considerable uncertainty as to precisely at what stage of the visual process the signals from the receptors can be said to be organized in opponent colors, whether this be in the retina, the optic nerve, or the occipital lobe of the cortex. Adherents of the opponent-colors theory have been greatly encouraged by the work of Svaetichin and MacNichol (1953-1961) on shallow-water fish with the microelectrode technique. They obtained two types of response curves from the bipolar-cell layer of the fish retina. Both types of curves changed polarity depending on the wavelength of the light introduced into the retina. One corresponded in spectral location and shape to a yellow-blue process of an opponent-colors theory, the other to a red-green process. They suggested that the *Y-B* and *R-G* spectral response curves are expressions of signals delivered by *Y-B* and *R-G* cones, the chromoreceptors which form the basic mechanism for the color vision of this fish. This result for fish vision suggests that the signals in human vision might be organized according to opponent colors in the retinal stage immediately following the receptors themselves.

Müller three-stage theory: This first three-stage theory was developed by G. E. Müller between 1920 and 1930. The three stages are: photopigment stage, cone-response stage, and optic-nerve stage. The photopigment stage follows the three-components theory based on the Young primaries (see Fig. 6, upper-left quadrant). The cone-response stage follows an opponent-colors form with yellowish-red opposing bluish-green, and greenish-yellow opposing reddish-blue; see upper-right quadrant of Figure 6. These response curves (Judd, 1949) derived from the CIE color-matching functions are strikingly similar to those found by Svaetichin and MacNichol from the retinas of shallow-water fish. The optic-nerve stage is the opponent-colors formulation of Hering with red opposing green, and blue opposing yellow (see lower-right quadrant). This is close to the formulation used in the second stage by Schrödinger and by Hurvich and Jameson. The details of Müller's ingenious explanation of all three principal forms of dichromatic vision in terms of these three stages need not be explored here. Some hint of them, however, is shown in the lower-left quadrant. Müller assumed that the photopigment stage contributes directly to luminosity in such proportions as yields closely the protanopic luminosity function (see W_p); then for normal vision the *yR-bG* process makes another contribution of such size as to make up the difference between the normal and protanopic luminosity functions. Protanopia is explained by the loss of the

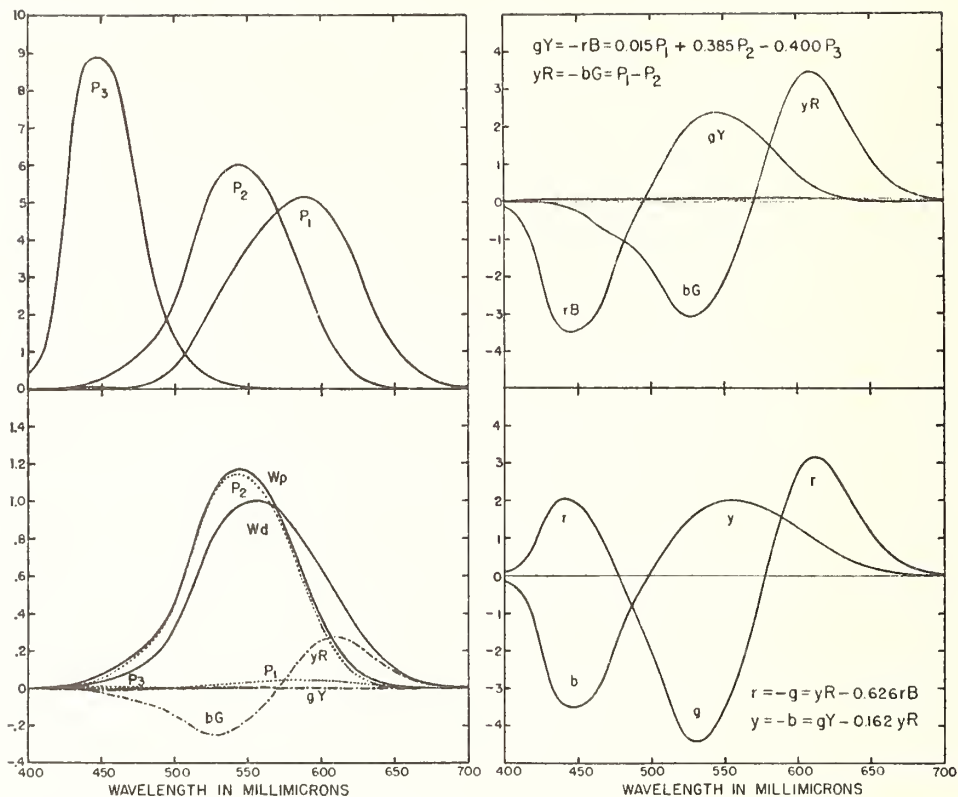


FIG. 6.—Response functions of the Müller stage theory evaluated by means of the 1931 CIE standard observer. *Upper-left* quadrant shows the response functions at the photopigment stage (same as Fig. 2, *heavy lines*). *Upper-right* quadrant shows the opponent-color response functions at the receptor stage. *Lower-right* quadrant shows the opponent-color response functions at the optic-nerve stage. These optic-nerve response functions also correspond to the Hering opponent-colors theory, and to the optic-nerve stage of the Adams theory. The *lower-left* quadrant shows how the Müller theory explains the luminous efficiency W_d function as the sum of contributions from the P_1 , P_2 , P_3 processes of the photopigment stage and from the yR and gY processes of the receptor stage. By this theory protanopia is a defect of the receptor stage whereby the yR ($= -bG$) process is lost. This loss explains both the inability of the protanopic observer to distinguish red from green, and the shortening of the luminous-efficiency function at the long-wave end whereby the W_d curve is supplanted by the W_p curve. Deuteranopia, on the other hand, is explained as a defect in the optic-nerve stage whereby the r ($= -g$) process is lost. This loss explains the inability of the deuteranopic observer to distinguish red from green, but it implies that the luminous-efficiency function of the deuteranopic observer must be identical to that for the normal observer.

yR - bG process, and it is clear that such a loss, in addition to accounting for inability of the protanope to make these red-green chromatic discriminations, also endows him with a luminosity function peaking correctly at about 540 nm.

Adams three-stage theory: Elliot Q. Adams proposed a three-stage theory of human color vision in 1923, and applied it in 1942 rather successfully to prediction of the perceptual size of color differences. The first and last stages correspond to the Young primaries and to the Hering opponent colors, respectively, just as in the Müller theory. The Adams second stage is a three-components formulation and, surprisingly enough, corresponds precisely to the primaries of the CIE system. Figure 7 shows the CIE color-matching functions in their official form. Adams

takes these without change to be the cone-response functions. The central function, which is the normal luminosity function, refers to brightness-perceiving cones, and a portion of their signals is supposed to be transmitted as nonlinear transforms (Munsell value function from Newhall, 1943) of luminance directly to the optic nerve as white signals. The similarly nonlinear signals from the red cones are partially inhibited by another portion of the white signal, and the difference is transmitted to the optic nerve in the form of the Hering red-green response. Similarly, the nonlinear signals from the blue cones are inhibited by the remainder of the signals from the white cones, and the difference is transmitted to the optic nerve in the form of the Hering blue-yellow response. The outstanding weakness of the Adams three-stage theory is that it provides no built-in explanation of two of the three forms of dichromatic vision. It is noteworthy, however, for two reasons. In the first place, the second and third stages are strictly dominator-modulator in type. The Adams theory antedates the coining of the phrase "dominator-modulator" by 20 years. In the second place, it is the first theory of color vision to take explicit account of the possibility that the frequency of firing of the nerve leading from the cones may not be linear with the rate at which radiant flux is absorbed within the cone, but is likely to be a diminishing function of it, such as the logarithm. This ends the summary of the principal theories of human color vision, and the demonstration that color-matching functions are basic to their quantification.

Chromatic Adaptation.—Another method of studying human color vision, less fundamental perhaps, than the information to be derived from setting of color matches, is to submit the eye to pre-exposure of various known kinds, and to study the differences in response so caused. It has been known since studies by v. Kries and König (about 1890) that such pre-exposures of whatever wavelength composition and over a wide range of retinal illuminances [up to 10,000 trolands according to Wright (1936)] have no influence on color matches set up for foveal vision. This "persistence of optical matches," as v. Kries phrased it, means that the color-matching functions whose determination has already been summarized apply to a wide range of chromatic adaptations. All of the theories mentioned above provide for this fact.

Helmholtz (1860) early indicated the connection conceived by him between chromatic adaptation and the three independent mechanisms of Young's theory by saying (p. 235): "... the nervous substance in question is less sensitive to new reacting light falling on it than the rest of the retina that was not previously stimulated. . . . Thus [p. 240] an eye which has been acted on by yellow light, say, is thereafter in a condition in which the blue components of white light affect it more than yellow does. Accordingly, the effect of fatiguing the retina is not uniformly extended to every kind of stimulation, but chiefly to stimulation similar to the primary stimulation. This fact is explained very simply by Young's assumption of three different kinds of sensory nerves for the different colours. For since coloured light does not stimulate these three kinds of nerves all to the same extent, different degrees of fatigue must also be the result of different degrees of stimulation. When the eye has been exposed to red, then the red-sensitive nerves are strongly stimulated and much fatigued; whereas the green-sensitive and violet-sensitive nerves are feebly stimulated and not much fatigued. If afterwards white light falls on the eye, the green-sensitive and violet-sensitive nerves will be relatively more af-

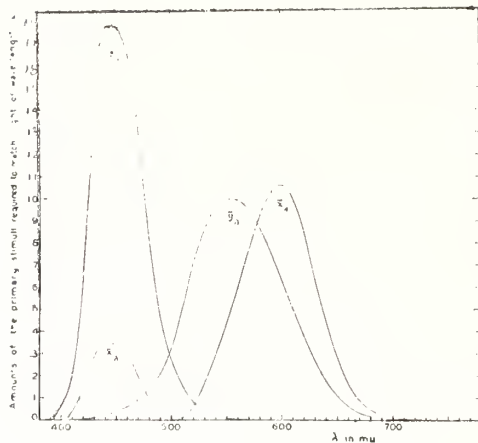


FIG. 7. —Color-matching functions for the 1931 CIE standard observer expressed in the CIE colorimetric coordinate system. The primaries are red (\bar{x}_λ), green (\bar{y}_λ), and blue (\bar{z}_λ), all outside the spectrum. The \bar{y}_λ -curve is identical to the photopic luminous-efficiency function for normal color vision, and this choice of \bar{y}_λ -curve ensures that the \bar{x}_λ - and \bar{z}_λ -curves are unassociated with luminosity. These color-matching functions apply to the receptor stage of the Adams theory (1923, 1942) which for this stage has the dominator-modulator form.

affected by it than the red-sensitive nerves; and hence the impression of blue-green, which is complementary to red, will predominate in the sensation." This explanation of the qualitative changes caused by local chromatic adaptation is quite successful, and equally good predictions are possible by the opponent-colors theory.

The predictions can, however, be subjected to a quantitative test. Suppose one retinal region (one eye, say) be maintained in a state of adaptation to daylight, and another retinal region (the other eye, say) be pre-exposed to green light. All stimuli presented to the green-adapted portion of the retina will appear more purplish than when presented to the daylight-adapted eye, and a measure of this adaptive color shift may be obtained by allowing the observer to alter the light stimulating the daylight-adapted region until he finds a stimulus yielding a color preception equal to that from the green-adapted region. Helmholtz' ideas on chromatic adaptation were quantitatively formulated by v. Kries, as follows: If R , G , V specify the color of any stimulus for neutral adaptation, then for any chromatic adaptation, the resulting color perception of this stimulus will be similarly specified by R' , G' , V' , where

$$R' = S_r R, \quad G' = S_g G, \quad V' = S_v V,$$

where S_r , S_g , S_v are coefficients specifying the state of chromatic adaptation of the eye following the pre-exposure. This formulation is known as the v. Kries coefficient law (v. Kries, 1905; p. 211).

The v. Kries coefficient law will work to some degree or other even if the coordinate system within which R , G , V are evaluated departs rather widely from that based on the true cone responses, but it should work perfectly only for the true cone responses. Studies of chromatic adaptation can therefore be used to check the accuracy of response curves derived by some other means, such as by reference to the protanopic, deutanopic, and tritanopic intersection points. Studies by Wright (1934), Walters (1942), MacAdam (1956), Brewer (1954), Wassef (1955), and Burnham, Evans, and Newhall (1957) have indicated that the v. Kries coefficient law is at least a good first approximation to the facts, but the primaries yielding the closest approximation to correct predictions do not agree with the dichromatic intersection points. The P - and T -points are found in some studies to

be reasonably successful, but the best third primary is extremely variable. For some studies (MacAdam, 1956) it falls within the range of *D*-points, but for others, far from that range. MacAdam (1958) has suggested a nonlinear extension of the *v*. Kries law for improved agreement.

A more rewarding line of attack is that by Stiles (1949, 1959) who determined for spectrum colors the least amount required to be added to the center of a large field of another spectrum color to be just detectable. The large field, of course, controls the chromatic adaptation of the eye of the observer, and may render two of the receptor mechanisms relatively insensitive, in which case the thresholds found should be inversely related to the spectral sensitivity of the third mechanism. If the wavelength of the large field is such as to reduce chiefly the sensitivity of but one of the receptor mechanisms, then the threshold for a given test stimulus will still be related chiefly to the sensitivity of the most sensitive of the remaining two. Only if the sensitivities of the two mechanisms are nearly equal does the interpretation of the increment-threshold data depend on an assumption regarding the combination of the responses of two mechanisms to produce an increment threshold.

The result of Stiles' researches by the two-color increment-threshold method is in one sense a very gratifying check on the cone-response curves derivable from the color-matching functions by taking the *D*-point in agreement with the Yustova values: $x = 1.7$, $y = -0.7$. One cone mechanism was found to peak at 440 nm, another at 540 nm, and a third at 575 nm. The shape of the "red" cone response curve found by this method does not, however, agree too well with that derived from color-matching functions. Its maximum is considerably more flat. In still another sense the results are puzzling. Not one but three "blue" mechanisms were found with response curves peaking at 440 nm. Furthermore, the response curves for the "green" and "red" cones changed significantly depending on whether the large field provided high or low retinal illuminances. The response for the "green" cone shifted at high illuminances toward somewhat shorter wavelengths, and that for the "red" cone toward longer, so that the maximum was pushed over to 587 nm, that is nearly in agreement with the "red" curve of the photopigment level. Perhaps this means that the photopigment that does not belong in the cone is less stable than the one that does. Evidence both from studies of the *v*. Kries coefficient law and from the two-color increment-threshold method suggests that chromatic adaptation may be a somewhat more complicated phenomenon than can be adequately described by any simple form of three-components theory.

Summary.—Between 1860 and 1960 several alternative theoretical possibilities were carefully worked out for human color vision but progress has been handicapped by lack of crucial evidence that might decide between these possibilities.

The color-matching functions determined since Maxwell's time (1860) have been reduced to the primaries previously (1807) proposed by Young, and the modern determinations are shown to be in good agreement. The principal theories of color vision are summarized under the headings: three-components (Young, Helmholtz, König), opponent-colors (Hering), two-stage theories (*v*. Kries-Schrödinger and Hurvich-Jameson), and three-stage theories (Müller, Adams). The results of quantifying these theories by appeal to modern determinations of color-matching functions is shown. For example, by the Young theory predictions of the spectral absorptance of three retinal photopigments are derived, and from the Helmholtz-

König theories, the responses of three types of retinal receptors responsible for color vision are derived. The various experimentally determined relations between the three principal forms of partial color-blindness and normal color vision are intercompared and checked against the various theories, and a brief review of the results of studies of chromatic adaptation (v. Kries coefficient law; Stiles two-color increment thresholds) and their theoretical implications is given.

Key words: Chromatic adaptation, color blindness, color-matching functions, color vision, deuteranopia, opponent-colors theory, protanopia, three-components theory of color vision, tritanopia.

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Physiological Optics at the National Bureau of Standards

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Published work in physiological optics at the National Bureau of Standards is summarized under the headings: evaluation of light-dark patterns, light measurements, color measurement, color differences, and color perception. The bearing of this work on standard methods of the American Society of Testing and Materials, of the Illuminating Engineering Society, and of the American Standards Association, and on recommendations of the International Commission on Illumination is indicated. The purposes and methods of current work are briefly described.

Introduction

In its development of instruments, methods, and standards for measurements made by the human eye, the National Bureau of Standards has obtained information on the response of that physiological mechanism to stimulation by radiant energy. Sometimes this information is specifically sought, but more often it is incidental to another purpose. This paper shows how the need for this information has arisen, recalls some of the most important studies of physiological optics carried out at the National Bureau of Standards, indicates the applications made of their results, and explains briefly the purposes of the current researches. It is convenient to discuss physiological optics under the headings: evaluation of light-dark patterns, light measurement, color measurement, color differences, and color perception.

Evaluation of Light-Dark Patterns

In 1938 Raymond Davis and Milo A. Durand sought a relationship between the legibility of a microcopy of typed material and the resolving power of the microcopying system. By experimenting with various styles of types they found that if the height of the lower case e on the film subtended three cycles of the highest spatial frequency resolved, the copy would be legible with difficulty. If the e subtended five cycles, the copy was legible without difficulty although serifs and fine details of type were not clearly defined. If the e subtended eight or more cycles, the details of the type were clearly defined. Davis included this correlation in the instructions for use of the National Bureau of Standards Microcopy Resolution Test Chart. If one defines a

quality index, these instructions may be restated as $m_r e = qR$, where m_r is the resolving power in cycles/mm on the film, e is the height of the lower case e on the original in millimeters, q is a quality index having values 3, 5, and greater than 8 as described above, and R is the reduction.¹ McCamy¹ recently succeeded in making a barely legible microcopy with a total reduction of 1200, enough reduction to permit the entire King James version of the Old and New Testaments to be recorded on an area considerably less than a thumbnail. The quality index q of this microcopy was found to be 2.34, in good agreement with Davis' legibility relation. By taking account of the fact that the human observer can average the density along a long band so as to perceive the band even though it may be somewhat broken, McCamy¹ showed that the information capacity of a photographic film in bits/mm² is equal to the square of the resolving power in cycles/mm: $C = m_r^2$. This formula was shown to yield results on four Kodak films in good agreement with information capacity computed by R. Clark Jones. It also indicates the information capacity of the film used ($m_r = 1800$ cycles/mm) to be 3.24×10^6 bits/mm². McCamy showed on the basis of an analysis of the information characteristics of literal test that the actual information recorded on this film with a reduction of 1200 was 79% of the information capacity computed from this formula.

These studies have served to evaluate the considerable ability of the human visual mechanism to abstract from a pattern afflicted with considerable noise the regularities (lines, letters) therein. These noisy patterns arise from the urge to put the information to be recorded into as small an area of microfilm as possible. A much more widespread use of photography, however, is to produce pictures of as high quality as possible. These pictures are intended to be viewed by the human eye, and the quality of them is appraised by the human eye by little-known processes. The picture containing

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the most information about a given scene is not necessarily judged to be the highest in quality. Judgments of quality are used to set up measures of the speed of photographic materials and to indicate optimum exposures. The National Bureau of Standards has studied methods of estimating exposures for daylight and moonlight photography without the use of an exposure meter. These studies have been carried out in cooperation with ASA Committee PH2.7, and the results have been embodied in the American Standard Photographic Exposure Guide (ASA PH2.7-1966, 7 April 1966). The optimum exposure is found from the speed of the film and the luminance value B_v of the scene which, in turn, is found as the illumination value V_s (derived from latitude, longitude, and apparent solar time) diminished by the scene value X_s which is determined by visual appraisal by the photographer of the scene (distant, nearby, close-up, shaded, unshaded, clear sky, hazy sky, front lighted, side lighted, and so forth). The method works. Is this an important achievement of physiological optics? The author thinks it is. We turn now to visual responses to a simple pattern: two fine dark lines viewed against a light background. The observer must move one line until it is coincident with the other. Washer² observed this pattern to determine the effect of magnification on the precision of indoor telescope pointing because it was needed in connection with the design of range finders. He found that the probable error of a single setting for coincidence expressed in seconds of arc (PE_s) was $PE_s = 4.962/M + 0.068$. The constant 4.96 sec is the Vernier acuity of the observer's eye, and this value has corroborated those cited by Walls.

In 1937, Hunter³ classified glossiness as having five types: (1) specular, (2) sheen, (3) contrast, (4) absence of bloom, and (5) distinctness of reflected image. He showed that suitable goniophotometric correlates of these five types are (1) specular gloss at 60°, (2) specular gloss at grazing incidence, (3) ratio of specular gloss at 45° to nonspecular reflectance factor, (4) ratio of somewhat off-specular reflectance factor to specular gloss, and (5) the same ratio for very slightly off-specular viewing. Hunter and Judd⁴ proposed a method for classifying paints according to specular gloss for adoption by ASTM. This proposal was accepted for tentative use, and the current ASTM standard D523-62T is essentially the same. Nimeroff⁵ in 1952 showed that the kurtosis index of the goniophotometric curve of polished and sand-blasted glass correlated better with subjective estimates of glossiness than the conventional correlate: 60° specular gloss; and in 1957⁶ he showed that judgments of distinctness-of-image glossiness could be predicted better from the ratio of the specular gloss measured with a small aperture to that measured with a large aperture than from any single-parameter gloss method. The two parameter method proposed by Nimeroff has been incorporated into ASTM method D1471-57T.

In 1938, Neeland *et al.*⁷ reported on the conspicuousness of airway beacons that are rotated to supply a series

of flashes visible to pilots. As a measure of this conspicuousness, they had the observer vary the intensity of a fixed light until it was judged just as conspicuous as the flash of light from the beacon. They showed that the ratio of the fixed-light equivalent intensity I_e of a rotating beacon to its maximum intensity I increases with increasing period of rotation in accord with the Blondel-Rey formula: $I_e/I = t/(a + t)$, where t is duration of an abrupt flash in seconds, and a is a constant equal to 0.2 sec for near-threshold values of I and decreases for flashes considerably above threshold. Neeland *et al.* evaluated I as the maximum intensity of the beacon, and took the duration as the time interval between the rise of intensity to 10% of maximum and the decline to 10% of maximum. They obtained the anomalous result that observation of the beacon at short range sometimes gave lower effective intensities than observation at long range where the maximum intensity was near threshold. The equivalent intensity I_e per unit luminous energy received by the eye from the flash was found to be a maximum for the shortest durations tried (0.001 sec); this suggests that the initial portion of a flash well above threshold desensitizes the retina so the later portions are less effective. In 1957, Douglas⁸ showed that an expression of the form originally proposed by Blondel and Rey for the effective intensity of a flashing light, whether or not the flashes are abrupt:

$$I_e = \int_{t_1}^{t_2} I dt / (a + t_2 - t_1)$$

has a maximum value when t_1 and t_2 are taken as the times at which I reaches the absolute threshold value. This definition of the duration of a nonabrupt flash had also been suggested by Blondel and Rey. For some intensity time patterns, an increase in I uniformly without changing the time pattern may result in a decrease in I_e , as found by Neeland *et al.* for rotating beacons, despite the lower values of a corresponding to intensities far above threshold.

In 1957, Projector⁹ pointed out that for t less than $a/100$, the Blondel-Rey formula implies, for a given value of effective intensity I_e , a reciprocal relation between I and t in a manner analogous to the equations used for determining photographic exposure. In 1959 Mullis and Projector¹⁰ proposed to define the effective efficiency of a flashing light as the product of a Blondel-Rey Merit Factor by the ratio of the luminous energy in the flash to the electrical energy producing the flash. The Blondel-Rey Merit Factor is defined as five times the ratio of the effective flux in the flash to the effective flux of a flash with the same total luminous energy compressed into an extremely short time interval. By this definition, the effective efficiency is the ratio of the effective flux—computed by the Douglas method with a set equal to 0.2 sec—to the electrical energy producing the flash. Effective intensity by the Douglas method is currently used for the appraisal of all flashing signal lights; see *IES Guide for Calculating the Effective Intensity of Flashing Signal Lights*. Effective ef-

iciency by the Projector method is often used in addition to effective intensity, particularly for appraisal of battery-powered blinking lights.

Light Measurement

Light is the aspect of radiant energy of which a human observer is aware through the stimulation of the retina of the eye. The measurement of light was accomplished by visual photometry at the National Bureau of Standards for many years without determining its precise relation to radiant energy. In 1906, Hyde¹¹ showed that Talbot's law served to predict within the experimental uncertainties the photometric effect of a rotary sector disk, both for incandescent lamp light and for such light viewed through red, green, and blue filters. Helmholtz stated Talbot's law as follows: "If any part of the retina is excited with intermittent light, recurring periodically and regularly in the same way, and if the period is sufficiently short, a continuous impression will result which is the same as that which would result if the total light received in each period were uniformly distributed throughout the whole period". By applying Talbot's law and the inverse square law, the determination of the intensity of an unknown source by visual comparison to a standard source of the same spectral quality was conveniently possible. Nevertheless, in 1908 Nutting¹² had already raised the question of the relation of light to radiant energy by determining roughly the maximum luminous efficacy of radiant energy. The value obtained (170 lumens/W) was of the right order of magnitude but still rather far from the presently accepted value (680 lumens/W). Nutting used luminous efficiency functions determined by König in Germany in the 1890's for both photopic and scotopic conditions. He pointed out that, because of the Purkinje effect, light cannot be proportional to radiant energy for all ranges, and stated that the logical primary standard of light is one in which the measured quantity is radiant energy measured by some form of radiometer. This suggestion has remained dormant for fifty years, but is now showing some signs of life.

If the test lamp had a spectral distribution different from that of the standard, it usually had a different chromaticity also, and settings for a brightness match not only became much less certain but also strongly dependent on what observer was doing it. The National Bureau of Standards found itself under pressure to supply light standards of a number of chromaticities so that the knotty problem of heterochromatic photometry need not be dealt with by every photometrist. These chromaticities, expressed in terms of color temperature, included 2080°K (carbon filament lamps), 2360°K (acetylene flame, vacuum tungsten lamps), and later 2788°K (gas-filled tungsten filament lamps). The unit of luminous intensity then used (international candle) was embodied in a group of carbon filament lamps, and to calibrate a vacuum tungsten lamp in terms of candles a visual setting for equality of brightness had to be made despite the chromaticity

difference. Three techniques were used to improve the reliability of the setting: (1) the step-by-step or cascade method in which the chromaticity difference was broken down into a number of small steps, each one which could be taken with reduced uncertainty; (2) the flicker method in which the lights to be compared were presented alternately at a few cycles per second and the observer adjusted the fields for a minimum of flicker; and (3) the filter method in which the chromaticity difference was reduced nearly to zero by the introduction of an appropriate filter whose luminous transmittance had to be separately determined. Thus we find Priest¹³ in 1915 proposing a method of heterochromatic photometry in which the filter is produced by a quartz plate between two polarizers, one fixed and the other adjustable, the luminous transmittance of the filter to be computed by means of a "visibility function based on the average of a large number of determinations of visibility, officially adopted as standard". And in 1916 Middlekauf and Skogland¹⁴ gave the results of an interlaboratory photometric comparison of glass screens and of tungsten lamps involving color difference, showing that each observer maintained a fairly definite relation to the mean of the observers in his laboratory so that significant and large individual observer differences cast doubt on the reliability of the average. Crittenden and Richtmyer¹⁵ measured by equality-of-brightness and flicker photometry the ratio of luminous transmittances of a potassium dichromatic solution and a copper sulfate solution (Y/B ratio) for 115 observers. The ratios ranged from 0.83 to 1.19. They proposed to rely for heterochromatic photometry on the flicker settings of a small number of observers of known Y/B ratio whose settings were to be corrected to give normal values, that is, values characteristic of $Y/B = 1$.

A frontal attack on the basic problem of the relation between light and radiant energy was launched in 1917 by Coblentz and Emerson¹⁶ who determined by flicker photometry the spectral luminous efficiencies of 130 observers, seven of whom were known to be color-blind, and checked the results by equality-of-brightness settings; these agreed for 588 nm but with a wide spread, and gave higher values (by 5% to 500%) in the extremes of the visible spectrum. They proposed a modification of the liquid filter designed by Ives and Kingsbury to duplicate better the average luminous efficiency function found by them, and reported the results of tests of a physical photometer produced by combining this filter with a thermopile. They also gave a formula closely approximating their values of spectral luminous efficiency as a function of wavelength, and in a separate publication¹⁷ used this formula to compute the luminous efficacy of the blackbody radiator as a function of temperature. The maximum efficacy was found at about 6500°K which gave about 85 lumens/W. They also used this formula in connection with determinations of the maximum luminous efficacy which they found to be about 620 lumens/W.

In 1923, Gibson and Tyndall¹⁸ published the results of what is presently the definitive study of the relation between light and radiant energy. They determined the spectral luminous efficiency functions for fifty-two observers by the cascade method. The spectrum was traversed in twenty-three steps, the field size was 3.5°, and the field luminance varied from 53 millilamberts at 580 nm to about one-twentieth of that luminance at 430 nm and 700 nm. In addition, they made a detailed comparison of the luminous efficiency functions of the eighteen observers studied by them who had previously served as observers for the Coblentz-Emerson study by the flicker method, and they analyzed the differences among the averages of seven separate determinations of spectral luminous efficiency (the two NBS determinations and five others). On the basis of this analysis, values of spectral luminous efficiency were derived to be recommended as representative of normal photopic human vision. They adjusted the recommended values to yield precisely the wavelength center of gravity for the blackbody at 2077°K found by Ives (581.6 nm) from a large number of observers in connection with the development of his physical photometer solution, but warned that the values recommended for the shortwave (violet) extreme are uncertain and must be considered as tentative. In 1924 the Commission Internationale de l'Eclairage (CIE) adopted the Gibson-Tyndall recommended average for tentative use.

In 1924, Tyndall and Gibson¹⁹ showed that this function could be closely approximated by a formula consisting of four exponential terms; and in 1931 Judd²⁰ extended the recommended values of the function to intervals of 1 nm by osculatory interpolation. In 1931, the CIE made this function one of the three defining the standard observer for colorimetry, and in 1939 the CIE made it the unique fundamental basis for all photometric measurements in the photopic range. It was subsequently adopted by the General Conference on Weights and Measures and has been embodied in the legal standard of light in many countries including the United States.

Luminous flux Φ_v in lumens of a source of spectral concentration of radiant flux $(\Phi_e)_\lambda$ in watts per unit wavelength is

$$\Phi_v = 680 \int_0^\infty (\Phi_e)_\lambda V(\lambda) d\lambda, \quad (1)$$

where $V(\lambda)$ is the spectral luminous efficiency function recommended in 1923 by Gibson and Tyndall.

In 1938, Gibson *et al.*²¹ constructed a combination glass-liquid filter whose spectral transmittances were closely proportional to the standard spectral luminous efficiencies; in 1939 they²² developed a more reproducible filter satisfying the same condition; and in 1941 Teele²³ constructed a physical photometer by combining this filter with a thermopile and specially designed potentiometer to measure the small values of electromotive force generated by the thermopile. This photometer is in current use at the National Bureau of Standards.

In 1945, Teele²⁴ constructed a visual photometer for luminescent materials. The luminances of these materials correspond to scotopic and mesopic vision and so cannot be evaluated properly by foveal vision, which, for most observers, is rod-free. The field used is elliptical divided along the major axis which subtends 20°; the minor axis subtends 15°. The scotopic or photopic luminance of the material is taken as the photopic luminance of the standard half-field of color temperature 2360°K found by the observer to appear as bright as the material viewed in the other half-field. Although chromatic differences are so small that they cause little trouble, the between-observer spread was found to vary from about 5% at 10 effective microlamberts to 25% at five thousandths of an effective microlambert.

In 1959, Barbrow²⁵ showed how to use the standard photopic luminous efficiency function combined with the maximum value (680 lumens/W) of luminous efficacy to derive, from measurements of color temperature and luminous intensity (lumens per steradian) of an incandescent lamp, the spectral concentration of radiant intensity of the lamp in microwatts per steradian in each band of 10 nm of the spectrum from 400 nm to 700 nm.

Color Measurement

Color is the aspect of light by means of which an observer may distinguish differences between two fields of view of the same size, shape, and structure. Color differences usually correspond to differences in spectral composition of the stimuli. Early in its history the National Bureau of Standards was called upon to recommend methods of measuring and specifying the colors of light sources and those of light reflected by or transmitted through objects by virtue of which the objects themselves may be said to have colors. Thus, we find Nutting²⁶ in 1912 describing "a new precision colorimeter" to measure color in terms of wavelength of dominant or complementary hue, purity, and luminance, and Priest²⁷ in 1915 proposing to measure the color of cottonseed oils by reference to the same filter (quartz plate between two polarizers) of adjustable spectral transmittance proposed by him for heterochromatic photometry. In 1917, Priest and Peters²⁸ showed that the saturation of various tints of yellow butter and oleomargarine could be specified by the spectral reflectance ratio for 436 nm and 578 nm.

In 1918, Priest²⁹ produced a close approximation to average noon sunlight according to Abbot's determinations by combining an incandescent lamp with his quartz rotatory dispersion filter. This artificial sunlight became known as Abbot-Priest average noon sunlight. And in 1923 Lofton³⁰ specified the color characteristics of white papers by giving the luminous reflectance measured on the Pfund multiple-reflection colorimeter at 625 nm, 550 nm, and 460 nm.

In 1920, Priest³¹ presented inductive evidence supporting the conclusion that if any two lights, however different in spectral composition, have equal colors,

they have equal spectral centroids of light; and he also showed³² that the frequencies of the spectrum colors yielding mixtures that match a standard white are related closely by the rectangular hyperbola: $(580 - f)(f_{\text{comp}} - 608) = 220$, where f and f_{comp} are expressed in cycles per trillionth of a second. In the same year Priest *et al.*³³ published a spectrophotometric examination of the Munsell color system by which object colors could be specified, and showed that one of the parameters of the Munsell system—Munsell value—is proportional to the square root of luminous reflectance.

None of the methods of specifying color dealt with in these early papers had general applicability. The Munsell system, for example, did not apply to light sources, and the charts in the Atlas of the Munsell Color system by no means covered the entire gamut of object colors. The quartz rotatory dispersion colorimeter had even greater limitations, and the Nutting colorimeter required standardization of the neutral stimulus used as a reference "white" in the determination of dominant wavelength and purity. The key to a color specification method of general applicability was given in 1922 by the report of the Optical Society of America Committee on Colorimetry written by Troland³⁴ but based on a preliminary unpublished report by Priest. This key is the set of three tristimulus functions of wavelength defining all color matches set up by normal color vision. The OSA report presented such a set under the name of "elementary color excitations". Troland³⁴ wrote (p. 579): "One of the main interests of the present Committee is to provide means by which color specifications in terms of different systems can be reduced to a common denominator and, so far as possible, be interconverted. Spectrophotometric data are potentially convertible into the data of any other system whatsoever, but no specifications which are based upon simple color-matching can be reduced to spectrophotometric terms, without additional information. However, a satisfactory common denominator for all systems is apparently provided by the *elementary color excitations*. Values of these excitations can be found which will specify completely the color characteristics of any stimulus, and each member or possible specification in every color system can be reduced to such excitation values, In this way the data of separate systems can be definitely intercompared, and can be interconverted in so far as the representations of the several systems overlap; with the obvious restriction that peculiarities of the stimulus—such as spectrophotometric details—which determine no characteristic excitation values, are necessarily lost." The OSA excitations were derived as an average of color-matching data by Abney (1891) in England, and König and Dieterici (1892) in Germany, obtained by matching each spectrum color in turn by appropriate mixtures of three lights chosen arbitrarily as primaries. The color-matching data were reduced to a common set of primaries (spectrum red, spectrum violet, and an extraspectral green, defined on the Maxwell triangle as the intersection of the tangents to the spectrum locus at these two extremes) by application of

Grassmann's additivity law which states that equally appearing stimuli added to equally appearing stimuli produce equally appearing sums. The report gives excitation values for the equal energy spectrum adjusted so the areas under the three curves of the products of the excitations by the spectral distribution of Abbot-Priest noon sunlight are equal. The report also gives excitation values for the blackbody as a function of temperature, for the parameters (dominant wavelength and purity) of the Nutting colorimeter, for a selection of colors obtained by the quartz rotatory dispersion colorimeter, and for certain of the colors in the Munsell Atlas.

In 1931, Judd³⁵ extended the OSA excitation curves to intervals of 1 nm by osculatory interpolation.

With great persistence and skill, Priest continued his studies of various methods of color specification. In 1921 he³⁶ showed by data of four observers of a 3.5° field at moderate brightness with dark surround that the color temperature of the light appearing neither yellowish nor bluish varies from 4600°K to 6500°K with an average of 5300°K. Use of such a stimulus in determinations of dominant wavelength and purity should yield optimum correlation with hue and saturation of the perceived color. In 1922, Priest³⁷ showed how to measure color temperature by the quartz rotatory dispersion colorimeter; and in 1923 he³⁸ showed how to measure luminance and spectral centroid of light sources by the same colorimeter. In 1924, he³⁹ assembled equipment to measure dominant wavelength, purity, and luminance, and embarked on a series of determinations of these quantities by a group of ten observers for spectrally known stimuli as a check on the applicability of the OSA excitations. Priest *et al.*⁴⁰ in 1924, Priest⁴¹ in 1926, and Judd⁴² in 1926 published formulas for the computation of purity from given tristimulus values. Gibson⁴³ in 1925 showed that the spectral centroid of the light transmitted by a daylight filter varies linearly with the thickness of the filter for a source of fixed color temperature. If the same filter is used for sources of different color temperature, the spectral centroids of incident and transmitted lights are linearly connected. Peters and Phelps⁴⁴ in 1927 found it possible to specify the color of sugar solutions in terms of the number of units of coloring matter in 1 g of saccharine dry substance computed as 206 times the logarithm of the reciprocal of the internal transmittance of the sample at the wavelength (560 nm) of the spectral centroid of the light transmitted by the sample illuminated by Abbot-Priest average noon sunlight. This method is still widely used.

Gibson and Harris⁴⁵ in 1927 published a spectrophotometric analysis of the glasses of the Lovibond color system. This system is based on three sets of glass filters (a red, a yellow, and a blue set), and each filter is assigned a number with the intention that, if light is passed through two glasses of the same set in turn, the color produced will equal that of a glass whose number is the sum of those identifying the two being combined, and further with the idea that glasses of equal numbers from each of the three sets will produce a neutral filter

(one whose chromaticity is the same as that of the daylight source used in the calibrations). The Lovibond color system has been widely used in commerce for the colors of transparent liquids and solids (paint vehicles, petroleum products, vegetable oils, resins, chemical solutions, and so forth) because of the convenience of the glasses and because the Lovibond notation indicates clearly the approximate color perceived to belong to the specimen. Greens are produced by combinations of yellow and blue Lovibond glasses, purples by red and blue, and orange by red and yellow. The Gibson-Harris spectrophotometric results permitted computation, by means of the OSA excitation curves, of specifications of the color of any combination of Lovibond glasses. The results of computations of this sort were published in 1947 by Haupt and Douglas⁴⁶ for about 60 Lovibond red, yellow, and blue glasses, singly and in pairs. Graphs of the computed chromaticity points permitted one to read approximately the Lovibond designation corresponding to any chromaticity within the Lovibond gamut. These graphs also revealed clearly some irregularities in the calibration of the Lovibond glasses in the possession of the National Bureau of Standards. The Gibson-Harris spectrophotometric results, supplemented by additional measurements made in England, also permitted the derivation of the Lovibond red, yellow, and blue units in terms of the spectral internal transmittances of the corresponding glasses. Judd *et al.*⁴⁷ carried out this derivation in 1962 and published the chromaticity coordinates of enough one- and two-part combinations of Lovibond glasses illuminated by incandescent lamplight to permit accurate readings of the Lovibond notation for any point on the chromaticity diagram within the Lovibond gamut.

In 1934 Wensel *et al.*⁴⁸ established a scale of color temperature by setting up visual color matches between incandescent lamps and the blackbody at three fixed temperatures, the freezing points of platinum (2042°K), rhodium (2233°K), and iridium (2716°K). This scale replaced the Nela scale of color temperature used up to that time and is still being used for calibration of incandescent lamps by the National Bureau of Standards.

Perhaps it is clarifying to point out that the central difficulty grappled with in these early papers on color measurement and color systems is the physiological fact that the human eye does not differentiate between the colors of groups of lights whose spectral distributions are widely different. Pairs of such lights make what is called a metameric match, and the lights are said to be metamers. Observers differ in setting up such matches. A metameric pair for one observer is a bad mismatch for another observer even though both have normal color vision. Varying degrees of lens pigment and macular pigment can cause this. The OSA excitations were an early solution to this problem. In his checks of these excitations by means of measurements of dominant wavelength, purity, and luminance, Priest found as expected that none of the ten observers set metamers in precise accord with the OSA excitations, but the metamers predicted by computation often did not even

fall among the experimental matches. Priest had not finished his attempts to revise the OSA excitations to correct these fatal discrepancies when the International Commission on Illumination (CIE) received from the British National Committee a proposal for a revised set of color-matching functions based on determinations of spectral metamers by Wright at the University of London, and by Guild at the National Physical Laboratory. The metamers defined by the British proposal were found to be in considerably improved agreement with Priest's experimental determinations, and Priest decided to support them.

The CIE recommendation of 1931 for a standard observer and coordinate system for colorimetry, and for three standard sources—source A representative of incandescent lamplight, source B representative of noon sunlight, and source C representative of average daylight—has served the immensely important practical purpose of defining standard metamers in satisfactory accord with those set by observers of normal color vision. These recommendations are still in force. The National Bureau of Standards contributed an important feature of the coordinate system, and supplied the liquid filters that, combined with the gas-filled incandescent lamp at a color temperature of 2854°K (source A), were used to produce sources B and C. In 1930, Judd⁴⁹ had showed that, if a weighted sum of the color-matching functions is equal to the standard luminous efficiency function, this function could be substituted for any of the original color-matching functions without any change in the metamers specified by them. This idea was incorporated into the 1931 CIE coordinate system for colorimetry. In 1931, Davis and Gibson⁵⁰ had developed an extensive series of liquid filters to be used with incandescent lamps for the reproduction of close approximations to sunlight and daylight, and had showed how to use these filters in the determination of color temperature. CIE sources B and C made use of two of these Davis-Gibson filters. Judd⁵¹ published in 1933 a description of the 1931 CIE standard observer and coordinate system for colorimetry, and gave tables facilitating the computation of dominant wavelength and purity from the CIE chromaticity coordinates (x, y).

The 1931 CIE coordinate system for colorimetry has since been applied at the National Bureau of Standards to solve other problems of interconversion of color specifications from one system to another. It was used by Haupt and Douglas⁴⁶ and by Judd *et al.*⁴⁷ to evaluate the colors of the Lovibond system as already mentioned. And it was used by Gibson and Nickerson⁵² in 1940 and Kelly *et al.*⁵³ to evaluate the colors in the original Atlas of the Munsell System and those in the Munsell Book of Color, respectively. Newhall *et al.*⁵⁴ also used it in the Final Report of the OSA Committee on the Spacing of the Munsell Colors as the system within which smoothed and adjusted Munsell notations, known as Munsell renotations, were defined. These Munsell renotations are used by the Munsell Color Company for their current production of color standards.

They have been recognized in ASTM Standard D 1535-58T, and they form one of the alternate methods in American Standard Methods of Measuring and Specifying Color (ASA Z58.7.3-1951). In 1960, Reinholdt and Menard⁵⁵ developed a program whereby the Munsell renotations corresponding to any set of tristimulus values in the CIE system can be found by automatic digital computation.

Judd and Kelly⁵⁶ in 1939 defined the first ISCC-NBS Method of Designating Colors. This cooperative work of the Inter-Society Color Council and the National Bureau of Standards defined, in terms of the Munsell Book of Color, ranges of colors for each of 319 designations by simple color names. Kelly⁵⁷ extended the ISCC-NBS color designations to self-luminous sources viewed with a dark surround. All parts of the CIE (x , y)-chromaticity diagram receive one or another of twenty-two color names (red, reddish orange, orange, yellowish orange, yellow, and so on) except a rather large central area for which no hue name is appropriate. The human observer adapts quickly to lights of these chromaticities viewed with a dark surround and sees them either as without hue or nearly so. The great importance of chromatic adaptation may be seen from the fact that this central area for which no hue name is appropriate covers a majority of the colors in the Munsell book, many of which, viewed with middle gray to white surround, are perceived to have colors far from gray. Kelly and Judd⁵⁸ in 1955 published the revised definition of the ISCC-NBS method of color designations in terms of the Munsell renotations, and thus indirectly in terms of the 1931 CIE coordinate system for colorimetry. They also included a dictionary defining about 7,500 color names including 267 groups of synonyms and near synonyms. In 1958, Kelly⁵⁹ computed the Munsell renotations for the centroids of the color ranges for the 267 ISCC-NBS designations, and in 1965 the National Bureau of Standards made available charts showing, in the form of glossy paint films on paper, these centroid colors. In the same year, Kelly⁶⁰ showed how the centroid charts could be used in three levels for surveys of customer acceptance of color in manufactured goods, and how the ISCC-NBS designations fit into a system of color specification with six levels of precision ranging from thirteen colors used in customer surveys to about five million specifications possible either by the CIE coordinate system or by the Munsell renotation system.

The 1931 CIE standard observer and coordinate system was also used for the statement of the color confusions made by observers with partially color-blind vision, that is, protanopes, deuteranopes, and tritanopes. Judd⁶¹ in 1943 showed how it would be possible for partially color-blind observers to detect camouflaged positions undetectable by observers with normal color vision, and proceeded⁶² to derive from the 1931 CIE standard observer a set of color-matching functions which, taking all three together, define normal metamers, but which, taken in pairs, also define the metamers for the three types of partially color-blind

vision. These dichromatic metamers were taken from the studies by König in Germany on tritanopia in 1897 and by Pitt in England on protanopia and deuteranopia in 1935. The color-matching functions conform to the three components theory of vision. In 1948, Judd⁶³ analyzed the published reports of people having one partially color-blind and one color-normal eye, and found that the yellow seen by protanopes and deuteranopes corresponds closely to the hue of the spectrum at 575 nm seen by people with normal color vision, and their blue corresponds to 470 nm. From this finding he developed protanopic and deuteranopic Munsell notations that correlate with what these partially color-blind people see in the same way that conventional Munsell notations correlate with what people with normal color vision see. In 1949, Judd⁶⁴ derived from the 1931 CIE standard observer the visual response functions corresponding to the photopigment stage, the receptor stage, and the optic-nerve stage of the Müller stage theory of color vision. The photopigment stage is explained by a three-components theory like that proposed by Young and Helmholtz; the other two stages take formulations of the opponent-colors type, the final stage being precisely the type originated by Hering. The Müller three-stage theory, quantified in this way, explains metamerism for observers of normal and partially color-blind vision, the final stage explains color perceptions by such observers, and the intermediate stage explains the Priest-Brickwedde data on minimum perceptible purity to be discussed presently in connection with color differences. In 1950, Judd *et al.*⁶⁵ showed that the color confusions and spectral luminous efficiencies of an atypical congenital tritanope corresponded in detail with the properties to be expected of a classical tritanope with five or six times normal ocular pigmentation.

One of the chief causes of individual differences in metamers for observers of normal color vision arises from the fact that normal eyes have a more or less dense spot of yellow pigment (probably xanthophyll) on the retina surrounding the fovea and extending from it by 2° to 4°. This pigment is called the macular pigment. Many metamers set up for a large field, for example 10°, fail to match in central vision, one side often appearing red, the other green. This red spot follows the fixation point around on the one side, and a green spot is seen on the other side. This spot, called the Maxwell spot after its discoverer, would seem to be an entoptic projection of the macular pigment, and was so explained by Maxwell. In 1953, Judd⁶⁶ showed that a particular form of three components theory, that used to explain partial color-blindness, yields predictions of the colors of the Maxwell spot produced by alternately viewing a uniform field of incandescent lamplight through a dark gray filter and a particular purple filter (Wratten No. 2389) used by Walls for this purpose. The predictions were found to be in nearly perfect agreement with those reported by observers of various types of vision studied by Walls (see Table I).

Table 1. Colors of the Maxwell Spot Obtained with Incandescent Lamplight Filtered through a Wratten No. 2389 Purple Filter

Type of color vision	Theory	Experiment (Walls)
Normal	Dark and reddish violet	Red or pink
Protanopia	Dark and blue	Dark or blue
Deutanopia	Dark	No spot and no color for 16 observers; faintly dark and bluish for 1 observer
Tritanopia	Dark and red	—

Why most deuteranopes would fail to see the Maxwell spot under these conditions is not known. In 1958, Kelly⁶⁷ carried out a study of ocular and macular pigmentation by means of a celebrated metameric pair known as the Granville grays. He found that the yellow pigmentation (probably melanin) of the lens was strongly correlated with the age of the observer. Macular pigmentation showed no correlation with age, but some observers' macular pigmentation was found to be greater than that of others by 120 microrreciprocal degrees (μ rd). A difference of 1 μ rd is about the least difference detectable with certainty.

We have seen that the color-matching functions known as the OSA excitation curves were used for about five years before definite evidence was collected that some of the metamers defined by them lay outside the range of metamers set up by observers with normal color vision. The 1931 CIE standard observer first came under fire fifteen years after being recommended. In 1949, Judd⁶⁸ corroborated a report by Jacobsen (National Lead Company) that it did not work for large field colorimetry of the rutile and anatase forms of titanium dioxide. Fields of angular subtense considerably greater than 2° (for which the standard observer was derived) are used widely in industry because of the higher precision of color matching achievable thereby. The CIE took up the challenge and developed (1951 to 1959) a supplementary observer for large field colorimetry. In 1964, Nimeroff⁶⁹ showed that large field settings on the Donaldson colorimeter by twenty-one observers (eleven from the National Bureau of Standards, ten from the National Research Council of Canada) agreed with the predictions of the proposed CIE supplementary observer for larger field colorimetry in the sense that, for eight of the eleven colors tested, the predicted chromaticity fell among the twenty-one observed chromaticities, three were borderline, and none was outside. Partly because of this support the CIE gave its official stamp of approval for the supplementary observer in that same year. Nimeroff⁷⁰ also showed that Grassmann's additivity law, on which the derivation of all color-matching functions must rely, significantly and systematically fails for 10° fields of less than 150 trolands. This failure is like that caused by rod intrusion, but might be owing to

interaction between the chromatic response components of the visual system. This result indicates that the 1964 CIE supplementary observer for large field colorimetry should be expected to agree only with experiments such as those carried out at luminances corresponding to considerably more than 170 trolands.

In 1957, Nimeroff⁷¹ showed how to apply the theory of error propagation to chromaticity coordinates computed from spectrophotometric data to predict the chromaticity regions within which observed chromaticity points will fall for a specified fraction of observers provided that the variances of the spectrophotometric data and the between-observer variances of the color-matching functions used for the comparison are both known. Availability of the latter variances permit discrepancies between computed and observed chromaticities to be placed in proper perspective. In 1961, Nimeroff *et al.*⁷² computed both the within-observer and between-observer variances of the 1964 CIE supplementary observer from data supplied by Stiles and Burch at the National Physical Laboratory in England and by Speranskaya at the State Optical Institute in Russia. In 1966, Nimeroff⁷³ showed that the uncertainty ellipses computed from these data agree well with those derived from the settings of color match by eleven actual observers.

The 1931 CIE standard observer has been used widely in recent years for indirect colorimetry by means of automatic spectrophotometers with digital readout, the computations of tristimulus values and chromaticity coordinates being done by high speed digital computers. Photoelectric tristimulus colorimeters, such as that developed at the National Bureau of Standards by Hunter⁷⁴ in 1942 and by Nimeroff and Wilson⁷⁵ in 1954, calibrated by color standards spectrophotometrically evaluated in the CIE coordinate system, are also used extensively for specimens forming a not too highly metameric match for an available standard. Nimeroff and Yurow⁷⁶ in 1965 developed an index for degree of metamerism and showed that it correlates well with the between-observer spread of chromaticity settings on the Donaldson colorimeter.

Color Differences

Interest of the National Bureau of Standards in the perceptual size of color differences became evident at an early date. In 1909, Nutting⁷⁷ proposed to form "a natural scale of pure color" by integration of the Steindler data on just noticeable difference in wavelength along the spectrum. In 1920, Karrer and Tyn-dall⁷⁸ determined the relations between visual angle, contrast, and background luminance required for a target to be visible. The data were used in connection with target detection by searchlights. In the same year, Priest *et al.*³³ showed that the value scale embodied in the 1915 Atlas of the Munsell Color System corresponds to Munsell value set proportional to the square root of the luminous directional reflectance factor. They said that, for equal visual spacing, the reflectance factor should form a geometric series in

conformity with the Weber-Fechner law, and reported that visual inspection of the Munsell gray scale indicated nonuniformities corresponding to failure to so conform. In 1930, Judd⁷⁹ showed that precision in settings of color temperature and the impression of size of the color temperature difference improves with angular subtense of the field and deteriorates with the prominence of any color separating the two fields being compared. Davis⁸⁰ in 1931 developed an empirical method for finding the temperature of the blackbody yielding the closest chromaticity match to any light source slightly off the blackbody locus from the chromaticity coordinates of that source. This temperature was, and still is, called the correlated color temperature. In 1932, Judd⁸¹ showed for several types of chromaticity difference (along the spectrum, along lines of constant dominant wavelength, along lines of constant purity, along the blackbody locus, and along the locus of variable Lovibond red combined with 35-yellow) that equal differences in the r coordinate or the g coordinate of the OSA chromaticity diagram, whichever difference is larger, correspond closely to differences found in previously published work to be of the same perceptual size. In 1933, Judd⁸² showed that chromaticity differences among lamps of different color temperature perceived to be equal to six observers corresponded closely to equal differences in either the r coordinate of the OSA (r, g)-chromaticity diagram, or in the spectral centroid, or in reciprocal color temperature. In the same year, Priest⁸³ proposed that the reciprocal of color temperature be used to specify the chromaticities of incandescent lamps and various phases of daylight. The unit was called the microrreciprocal degree (μrd or mired) when the quantity $10^6/T_c$ with T_c as color temperature in degrees Kelvin is used. A difference in reciprocal color temperature of 1 μrd is close to the smallest chromaticity difference perceptible with certainty. This scale is still widely used. In the same year, Tyndall⁸⁴ evaluated for his own eye the just noticeable difference in dominant wavelength as a function of purity at constant dominant wavelength for various dominant wavelengths; and Judd⁸⁵ evaluated for eight observers the intervals in purity perceived as equal for a dominant wavelength of 575 nm. In 1935, Judd⁸⁶ developed a projective transformation of the 1931 CIE (x, y)-chromaticity diagram on which equal distances correspond approximately to chromaticity differences perceived as of equal size. Thus, the "natural scale of pure color" proposed by Nutting in 1909 corresponds approximately to steps of equal length along the spectrum locus plotted on this projective transformation, and the Priest-Brickwedde⁸⁷ measures of minimum perceptible purity as a function of dominant wavelength correspond to approximately equal radii from the sun-light point, and so on. This transformation was called the "uniform-chromaticity-scale" (UCS) diagram. In 1935, McNicholas⁸⁸ used the UCS diagram to estimate the value N_R of Lovibond red that would yield the closest chromaticity match on the 35-yellow, N_R -red locus for an off-locus sample of vegetable oil. In 1936, Judd⁸⁹ applied the UCS diagram to the problem of finding the

correlated color temperature of any light source whose chromaticity point is slightly off the blackbody locus. The results agreed well with those found by the Davis⁸⁰ empirical method already mentioned. In the same year, McNicholas⁹⁰ used the UCS diagram to arrange signal colors to be tested on an approximately uniform perceptual scale. In 1939, Breckenridge and Schaub⁹¹ developed a rectangular form of the UCS diagram, known as the RUCS diagram. It has essentially the spacing of the UCS diagram, but the chromaticity point for the equal energy spectrum is placed at the origin of the coordinate system, and the longwave branch of the spectrum locus is set along the vertical. The RUCS diagram has been used to check the spacing of the chromaticity ranges within which signal colors must fall; see the United States Standard for the Colors of Signal Lights, by Breckenridge⁹², which appeared in 1964.

In 1960, the CIE recommended the use of a modification of the CIE diagram, with a simplified relation to the CIE (x, y)-chromaticity diagram, proposed by MacAdam in 1937. This modification was recommended for use "whenever a diagram yielding colour spacing perceptually more nearly uniform than the (x, y)-diagram is desired." In 1963, Kelly⁹³ applied the 1960 CIE-UCS diagram to derivation of the lines of constant correlated color temperature on the (x, y)-diagram; and in 1964 Nimeroff⁹⁴ computed the spectral tristimulus values implied by the 1960 CIE-UCS system.

In 1939, Judd⁸⁵ used the UCS diagram to derive a measure of the perceptual sizes of color differences in terms of what has now become known as the NBS unit of color difference. Distance on the UCS triangle was weighted by the square root of the reflectance factor to form a measure of the size of the chromaticity component, and the square root of reflectance formed the measure of the size of the lightness component. The size of the color difference is computed as the square root of the sum of the squares of the components weighted so the unit of color difference is about four or five times the size of the just detectable difference. Many commercial products are purchased with color tolerances of about one NBS unit.

In 1950, Judd *et al.*⁹⁶ used the NBS unit of color difference to improve the spacing of the Union Colorimeter scale used for lubricating oil and petrolatum. This respaced scale was incorporated into ASTM Method D 1500-58T which is currently in force.

In 1942, Hunter⁷⁴ modified the UCS projective transformation to make it agree better with the spacing of the Munsell colors and with spacing data published by Wright in England. Eickhoff and Hunter⁹⁷, in the same year, used the revised NBS unit of color difference to express the fading rate of paints exposed outdoors. The colors were measured by photoelectric tristimulus colorimetry.

In 1953, Nimeroff⁹⁸ showed that the uncertainties in chromaticity coordinates caused by uncertainty in spectral reflectance or transmittance uniform regardless of wavelength for a representative series of objects cor-

responded closely to the uncertainties of setting a chromaticity match visually found by MacAdam at Eastman Kodak.

In 1959, Richmond and Harrison⁹⁹ obtained estimates of the perceived character and size of about 180 small color differences between specimens of vitreous enamel from each of thirty-four observers. The geometric standard deviations of the size estimates was about 1.8 and the between-observer spread of the character estimates was also similarly large. Estimates for colors perceived as of middle lightness and low saturation were more reliable than those for other colors, but no significant difference in reliability was found between the nineteen observers classed as experienced and the fifteen observers classed as inexperienced.

The current work in physiological optics at the National Bureau of Standards is largely confined to further research on methods of predicting the perceived sizes of color differences from the CIE tristimulus values of the two colors involved. In 1955, a progress report by the Optical Society of America Committee on Uniform Color Scales, written by Judd¹⁰⁰, was published. It indicated that four basic methods of estimating the size of color differences were then in use by industry: one based on projective transformations of the CIE (x , y)-diagram developed at the National Bureau of Standards, a second based on the Munsell spacing, a third based on the Adams chromatic-value diagram, and a fourth based on the MacAdam ellipses. The Committee agreed to embark on the development of a 500-chip uniform tridimensional color scale to be derived by a regular rhombohedral sampling of the paint gamut such as described by Wyszecki¹⁰¹ in 1954. Each chip whose color fell in the interior of the gamut was to be surrounded by twelve nearest neighbors each differing from the color of the central chip by amounts perceptually as nearly equal as could possibly be achieved. In 1966, Judd¹⁰² published a second progress report detailing the steps taken to achieve this result. The first experiment (1956) determined a locus of chromaticity points on the CIE (x , y)-diagram for colors of Munsell value 6/perceived as having the same saturation (departure from gray). The second experiment (1957) determined forty chromaticity points on this locus equally spaced in hue. The third experiment (1961) determined chromaticity spacing for near grays of 3/, 6/, and 8/. The fourth experiment, still in progress, was based on the production of forty-three chips intended to give a uniform triangular sampling of the paint gamut at 6/. The uniformity of this spacing has been checked by eight observers, each making judgments of "larger" or "smaller" for one hundred and forty-two of the pairs of differences among the forty-three chips. The committee has spent more than a year analyzing these statistical data. These data support the previous finding of MacAdam that perceptual color space has a Gaussian curvature significantly different from zero, and this property prevents any tridimensional sampling from being strictly uniform in spacing. They indicate that the perceptual size of a

chromaticity difference is not linearly related to distance on any uniform-chromaticity-scale diagram, but varies with powers less than one (0.37 to 0.80) of these distances. And they support the previous finding of Nickerson that, for equal distances on any chromaticity diagram adjusted for approximately uniform spacing, saturation differences are perceived as smaller than hue differences. Appraisal of these differences by means of any of the four currently used methods does not correlate to any significant degree with the statistical measures of the perceived sizes; but similar measures obtained by fractionating the eighty observers into four groups of twenty observers each show good correlation with each other, and excellent correlation with the results from the entire group of eighty observers. The National Bureau of Standards is currently engaged in studies that are intended to find out what factor is operative in the statistical studies that is not taken into account in any of the current methods of estimating the perceptual size of color differences. Perhaps the presence of two colors defining the second difference in close juxtaposition with those defining the first exerts a significant influence on the perceived size of the first difference and vice versa. Modifications of each of the four currently used methods are being explored in an attempt to find a formula yielding reasonable correlation with these statistical measures of perceived size.

Color Perception

An observer may describe what he sees in his own terms independent of any measures of the stimuli acting on his retina. The terms of color perception are red, yellow, green, blue, and their intermediates for the perceived hues; gray, moderate, and vivid for the perceived saturations; black, dark, medium, light, and white for the lightnesses of colors perceived to belong to non-self-luminous bodies, and dim, bright, and dazzling for the colors perceived to belong to self-luminous bodies or light sources. Interest of the National Bureau of Standards in color perception first became evident in 1907 when Nutting¹⁰³ published an expression for the brightness perceived to belong to a spot of light viewed with a dark surround as a function of the luminance of the spot and the absolute threshold of luminance. In 1910, Ives¹⁰⁴ computed how much light from the Welsbach mantle, from the carbon filament lamp, and from the tungsten filament lamp had to be added to mercury arc light to produce the color of daylight, but remarked that the merit of the resulting artificial daylight should be judged by the degree to which the artificial daylight rendered, in accord with natural daylight, the colors of objects to which the eye is particularly sensitive, such as the colors of flesh, lips, and other commonly observed objects. In 1918, Priest¹⁰⁵ announced in discussion of a paper by Troland that the color temperature found by preliminary experiment for a bright spot viewed with dark surround to be perceived without hue is 4000°K; and in 1921 he¹⁰⁶ gave the final values of 4600°K to 6500°K with an average of 5300°K. In 1925, Priest¹⁰⁷ showed that skies perceived as gray may have a

higher luminance than objects in the same scene perceived as white. In 1926, he¹⁰⁸ showed that twenty-five observers found either Abbot–Priest average noon sunlight or an equal energy source to be perceived as without hue when viewed alone with a dark surround; but when viewed in juxtaposition neither appeared achromatic or neutral. In the same year, Priest¹⁰⁹ showed that a given element in a scene was perceived as blue, if it was seen as a body of water (Skinner's Lake), but white, if seen (as it really was) as a snow bank.

In 1929, Judd¹¹⁰ measured the least retinal illumination required in his own eye as a function of wavelength for the production of the "blue arcs of the retina", an entoptic projection of the fibers of the optic nerve, and showed that stimulation of the retinal rods initiated the nerve activity rendering the fibers visible.

In 1936, Judd¹¹¹ used the UCS diagram in developing a correlate for paper whiteness. The correlate was the luminous reflectance factor diminished by an appropriate fraction of the distance on the UCS diagram between the chromaticity point for the specimen and the chromaticity point of the standard accepted by the inspector as white. In 1941, Judd¹¹² proposed a revised formula for whiteness of any object which correlated perceived whiteness with the degree of approach of the object color to the color accepted as white measured in NBS units. With either of these formulas for whiteness it is quite possible to increase the whiteness of a sheet of yellowish paper by dyeing it with blue dye even though the luminous reflectance is thereby considerably decreased. In the same year, Judd¹¹³ objected to the phrases "black sensation" and "white sensation" and proposed definitions of black and white as names for colors perceived to belong to objects themselves perceived as non-self-luminous.

In 1940, Judd¹¹⁴ developed inductively, from judgments of six observers, empirical formulas for the hue, lightness, and saturation of the color found to belong to any flat object viewed with any surround as functions of the tristimulus values of the flat object and those of the surround. Saturation was expressed as proportional to distance on the UCS diagram between the achromatic point and the object point. Hue depended upon the direction of the line connecting these two points and lightness was expressed as an hyperbolic function, previously proposed by Adams, of the directional reflectance factor of the object and surround. The achromatic point was specified as an empirical function of the surround chromaticity, the object chromaticity and the object lightness. The correlation between formula and the judgments of the six observers was excellent for red surrounds, good for yellow and green, but only fair for blue surrounds. Evidence was found for a significant influence of light scattered by eye media of the observers viewing dark objects with light blue surrounds.

In 1950, Judd *et al.*¹¹⁵ used the Munsell color system to develop an index of color fidelity that can be used to appraise systems of color television. In 1952, Helson *et al.*¹¹⁶ showed that the color changes perceived for

objects on passing from daylight to incandescent lamp illumination are in approximate conformity to the v. Kries coefficient law applied to the primaries used for the response functions for the three types of dichromatic vision. The v. Kries coefficient law states that, if the colors perceived to belong to objects by an eye adapted to daylight are correlated with the tristimulus values R , G , B , of those objects, then for an eye adapted to a source of non daylight chromaticity the perceived colors will similarly correlate with tristimulus values R' , G' , B' found by applying coefficients α , β , γ characterizing the sensitivities of the components of vision as changed by adaptation to the non daylight source; thus $R' = \alpha R$, $G' = \beta G$, and $B' = \gamma B$. In 1956, Helson *et al.*¹¹⁷ showed for white, coolwhite, and daylight fluorescent lamps that the v. Kries coefficient law gave predictions within the between-observer spread for 85% of the 390 hue judgments, for 86% of the 390 saturation judgments, and about 97% of the 390 lightness judgments.

In 1954, Wyszecki¹¹⁸ showed how to determine graphically on the (x, y) -chromaticity diagram the v. Kries transformation based on any choice of fixed points (primaries).

In 1942, Hunter⁷⁴ proposed indices of hue, lightness, and saturation of colors perceived to belong to a specimen measured by photoelectric tristimulus colorimetry. These indices apply to specimens illuminated by daylight and viewed with a gray to white surround by an observer with normal color vision. Indices for whiteness and yellowness of near white objects were also proposed.

In 1959, Judd¹¹⁹ described a number of demonstrations of perceived colors including the blue arcs of the retina, the Maxwell spot, the derivation of the variables' hue, lightness, and saturation (Desert Island experiment), color perceptions of red-green confusers, metamorphism, chromatic adaptation, and object-color rendition.

In 1960, Judd¹²⁰ analyzed the Land demonstrations of color perceptions arising from two-primary color projections and showed that the formulas published¹¹⁴ in 1940 predicted the perception of colors of all hues in accord with Land's report for a red primary combined with an incandescent lamp primary.

In 1961 there was published¹²¹ a five-attribute system of describing the nonshape, nontextural, and non-temporal aspects of visual appearance worked out by Judd and Gibson some thirty years previously. The attributes are hue, lightness, saturation, transparency, and glossiness for objects perceived as non-self-luminous and hue, brightness, saturation, and transparency for those perceived as self-luminous.

In 1966, Takasaki¹²² studied the influence of a light-dark difference between target and surround on the lightness perception of the target by presenting one gray surround with a fixed gray target side by side with a different gray surround with variable gray targets and asking the observer to select the one making the two targets appear equally light. He found that this influence was particularly marked if one of the targets differed only slightly from the surround, and he referred

to this property of the human eye as "crispening". He showed that the Munsell value of the matching gray could be predicted by setting the perceived lightness L_i of the target equal to the following function of the Munsell value V_i of the target, the Munsell value V_{bi} of the surround and the average Munsell value \bar{V}_b of two surrounds,

$$L_i = V_i - C_1 V_{bi} + C_2 \bar{V}_b [(V_i - V_{bi})/C_3] e^{-|V_i - V_{bi}|/C_3},$$

$$i = 1, 2,$$

and finding the Munsell value V_2 for which $L_1 = L_2$. The constants C_1 , C_2 , and C_3 , which have to be adjusted for each observer, refer to induction, amount of crispening, and reciprocal sharpness of crispening, respectively. Unpublished work shows that formulas of the same type apply to chromaticity variations in the red-green and violet-green-yellow senses at constant luminance. The crispening phenomenon implies that a significant correction to the Munsell value function near the value of the surround should be applied, and it may explain some of the failures of the v. Kries coefficient law to predict the influence of chromatic adaptation on the perceived color.

Current work at the National Bureau of Standards on color perception stems from a study by McNicholas⁹⁰ who showed in 1936 that point sources appearing orange or yellow to thirty-eight observers at low intensity appeared shifted in hue toward red or orange, respectively, at ten or fifteen times this low intensity. Similarly, point sources appearing bluish green at low intensities shifted in hue at higher intensities toward green. Both of these shifts are opposite to the hue shifts (Bezold-Brücke phenomenon discovered in the 1870's) found to apply to extended fields of relatively high luminance. The Bezold-Brücke hue shifts correspond to a lower stability of the red-green sense relative to the yellow-blue sense at high luminances. The McNicholas hue shifts correspond to a lower stability of the red-green sense at low values of luminous flux entering the eye (low product of luminance by solid angular size of source). In view of the fact discovered by König in 1894 in Germany and corroborated by Willmer and Wright in 1945 and Hartridge in 1947, both in England, that the normal human eye makes tritanopic mistakes (violet-green-yellow confusion) if the field size is smaller than about 0.1° , the McNicholas hue shifts are difficult to explain. The National Bureau of Standards has undertaken a series of measurements to determine the variation of chromaticity sensibility as a function of field luminance and angular subtense, and to discover to what extent an increase in luminance can compensate for a decrease in solid angular subtense, and vice versa. If it is true that at low luminance and/or angular subtense the normal human eye tends toward tritanopia, these measurements should reveal it, and also show how the transition from trichromatic to dichromatic vision takes place. It is expected that these results will yield a basis not only for explaining the McNicholas hue shifts but also for an improved selection of colors for signalling.

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Colorimetry in Parafoveal Fields.* I. Color-Matching Functions

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Minor defects in the 1931 CIE Standard Observer System, derived from 2° foveal viewing conditions, have increased interest in large parafoveal viewing fields. As the parafoveal region of the retina gives rod as well as cone responses, the CIE is considering a new observer system (recommended for adoption at the June 1963 CIE meetings in Vienna, Austria, by the Experts of Committee E 1.3.1) in which contributions of the rods to over-all vision have been removed so that the system reflects only the cone response. The author, a normal trichromat in 2° foveal vision, has measured his 10°-field color-matching functions on the Wright visual tristimulus colorimeter. These color-matching functions, reported here, are compared with other such data, and with data taken for foveal, parafoveal, and extrafoveal fields. The author's 10°-field data confirm, with the retinal illuminance achieved in the Wright visual tristimulus colorimeter, that rod responses are an important part of the total visual response. It is concluded that rod responses should not be neglected in color-matching with retinal illumination less than 1000 trolands.

1. INTRODUCTION

THE standard observer system for colorimetry recommended by the Commission Internationale de l'Eclairage (International Commission on Illumination), was adopted in two stages. The spectral luminous efficiency function V_λ was provisionally recommended in 1924¹ while the spectral tristimulus-value functions were made final in 1931.² These functions were based on data obtained with an approximately 2° foveal field of view.

This system has proved to be eminently successful for color measurement during the 30 years since its inception. It has been used in the color evaluation of many different kinds of materials used in commerce and industry, where "absolute" color measurement is required. The system has served as a common basis for intercomparing results. It has also formed the basis of the design of photoelectric colorimeters for relative color measurements.

The most outstanding defect found with the system is its inability to predict metameric matches in fields of large angular subtense, particularly, the Granville Grays³, the Simon textiles⁴, and the anatase and rutile forms of titanium dioxide. This last difficulty with the standard observer was first reported by Jacobsen in 1948⁵ for TiO_2 and has led to the consideration of the possible revision of the 1931 standard observer.

The question of revising the 1931 standard observer was raised by Judd⁶ in 1949. Judd⁷ suggested in 1951 that the luminous-efficiency function should be re-

investigated, particularly at the short-wave region of the visible spectrum, and that the spectral tristimulus values be adjusted accordingly. This suggestion was based first on the evidence that the luminous-efficiency function is low in the short-wave region of the visible spectrum. Such evidence is found in the work of Gibson and Tyndall⁸ in 1923, and in the subsequent work of Thompson,⁹ Ishak,¹⁰ and others. The second basis for Judd's suggestion was that, when he derived tristimulus-value functions with Gibson-Tyndall luminous-efficiency data instead of the recommended 1924 V_λ data, the difficulties encountered in the colorimetry of TiO_2 are resolved.

As early as 1941, however, a suggestion had been made by Perry¹¹ that a standard observer should be based on color-matching data obtained with large-field observing conditions. In 1948, at the National Physical Laboratory, Stiles began his investigations of the 10°-field color-matching functions. Because Stiles' investigation was well along in 1951, the CIE committee on colorimetry decided to await the outcome of his research before recommending either a revised standard observer system or a new system. The CIE committee decided further, that, before taking any final action of its own, the utility of any proposal was to be subjected to field trials, and that the outcome of these trials was to be scrutinized thoroughly. These field trials were to deal with problems of both metamerism and additivity in large-field vision. There is evidence that although failures of the additivity law are negligibly small in foveal vision, such failures are not negligible in vision that involves parafoveal responses.

During the time of the Stiles investigations, work was begun in Russia by Speranskaya to supply additional data for consideration in the new CIE system. At the present time, the total number of observers studied by Stiles and by Speranskaya is 80. The investigation

* The work reported here was done partially in fulfillment of the requirements of the Diploma of Membership of the Imperial College, University of London, England.

¹ Proc. Intern. Comm. Illumination 6, 67 (1924).

² Proc. Intern. Comm. Illumination 8, 19 (1931).

³ W. C. Granville and D. B. Judd, J. Opt. Soc. Am. 39, 632A (1949).

⁴ A. Cruz and J. Juan, Detailed Report of Progress, CIE Working Committee W-1.3.1 (September 1957–April 1959), Appendix B.

⁵ A. E. Jacobsen, J. Opt. Soc. Am. 38, 661 (1948).

⁶ D. B. Judd, J. Res. Natl. Bur. Std. 43, 227 (1949); J. Opt. Soc. Am. 39, 945 (1949); errata 40, 52 (1950).

⁷ D. B. Judd, Proc. Intern. Comm. Illumination 12, 1.7.1 (1951).

⁸ K. S. Gibson and E. P. T. Tyndall, Sci. Papers Bur. Std. 19, 131 (1923–1924); Transactions IES 19, 176 (1924).

⁹ L. C. Thompson, Proc. Phys. Soc. (London) B62, 787 (1949).

¹⁰ Proc. Intern. Comm. Illumination 13, 1.4.1–9 (1955–I).

¹¹ J. W. Perry, Nature 148, 691 (1941).

described here may be considered to bear on the field trials of the color-matching functions developed from data obtained by Stiles and by Speranskaya.

In this paper the color-matching functions of a number of different areas, foveal, parafoveal, and extrafoveal, are discussed. By the foveal area is meant the fovea and the area that surrounds it, up to 2 deg from the fovea centralis. By the parafoveal area is meant any area centered on the fovea that includes but is larger than the fovea. By the extrafoveal area is meant any retinal area that does not include the foveal area.

2. FOVEAL SMALL-FIELD COLOR-MATCHING FUNCTIONS

The tristimulus-value functions defining the CIE standard observer system for colorimetry are based on the color-matching data of Guild¹² and Wright.¹³ The Guild data represent the average of seven observers and the Wright data the average of 10 observers. The Wright data were augmented by "white-point" data for 26 observers. The observations of Guild and Wright were made on apparatus for which the angular subtense of the visual field was about 2° centered on the fovea of the retina. This small field of the retina was chosen because of its known stability of response, because the range of illuminance to which this region responds covers the usual viewing conditions, and because it is well within the macular-pigment region and the rod-free region of the retina. Beyond 2° the number of rods increases relative to the number of cones.

Figure 1 shows the color-matching data of Guild and of Wright, and the ranges of these data among the individual observers. The essential reason for the difference between the Guild data and Wright data lies in the choice of their respective instrumental primaries. These primaries were chosen for convenience on the respective instruments and are filter bands with dominant wavelengths about 460, 545, and 620 nm for the Guild data and monochromator bands centered at 460, 530, and 650 nm for the Wright data, respectively, for the blue, green, and red primaries. Differences were resolved when these data were transformed to a common reference primary system, 435.8, 546.1, and 700.0 nm by using Grassmann's laws of additivity.¹⁴

3. STILES-BURCH PARAFOVEAL LARGE-FIELD INVESTIGATIONS

The large-field investigations of Stiles and Burch, started in 1950, exposed a 10° field surrounding the foveal region. The red, green, and blue instrumental primaries employed in this investigation were 645.2, 526.3, and 445.4 nm for the "green" run; and 645.2, 588.2, and 470.6 nm, respectively, for the "yellow" run. The data obtained in these two runs have been trans-

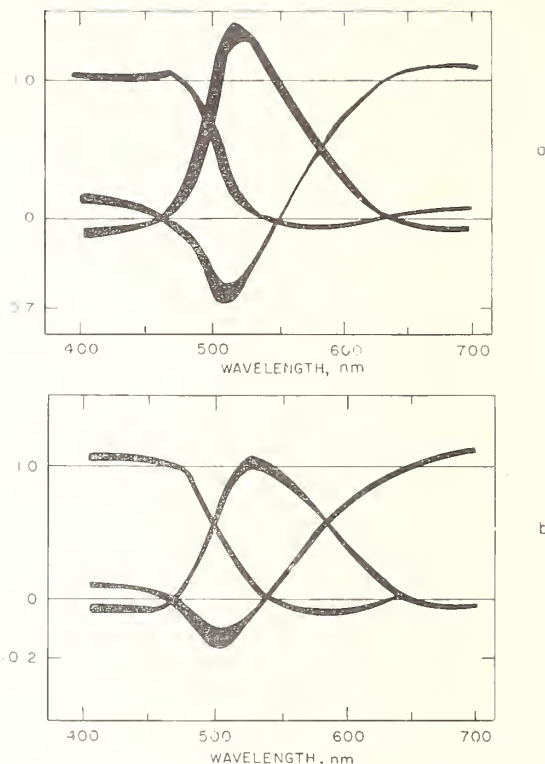


FIG. 1. Ranges of color-matching functions: (a) Guild data for 7 observers, (b) Wright data for 10 observers after W. D. Wright, *The Measurement of Color* (The Macmillan Company, New York, 1958).

formed to a system with primaries at wavelengths 645.2, 526.3, and 444.4 nm, respectively.

In the Stiles-Burch investigations, the visual technique used in making matches in the bipartite viewing field was that of disregarding the Maxwell spot and paying attention only to the surrounding area within the 10° field presented to the observer. The reason given for taking these precautions is that when a large area of the retina is used in making matches, and these matches are metameric, a central area is seen differently from the surrounding annulus. This central area is called the Maxwell Spot¹⁵ and corresponds at least chiefly to entoptic perception of the macula lutea (yellow spot). The retinal illuminance in the Stiles-Burch investigation was reported to be nominally 500 trolands, but ranged from approximately 20 to 3000 trolands.

The pilot results for 10 observers of the Stiles-Burch investigations were reported in 1955¹⁶ and the final results were reported in 1959.¹⁷

4. SPERANSKAYA PARAFOVEAL LARGE-FIELD INVESTIGATIONS

During the time of the Stiles-Burch investigations, work on large-field (10°) color-matching was under-

¹² J. Guild, *Phil. Trans. Roy. Soc. (London)* **A230**, 149 (1931).

¹³ W. D. Wright, *Trans. Opt. Soc.* **30**, 141 (1928-1929).

¹⁴ H. Grassmann, *Poggendorff's Ann.* **89** (1860).

¹⁵ J. C. Maxwell, *Proc. Roy. Soc. (London)* **10**, 404 (1860).

¹⁶ W. S. Stiles, *Opt. Acta* **2**, 168 (December 1955); with appendix by W. S. Stiles and J. M. Burch, p. 176.

¹⁷ W. S. Stiles and J. M. Burch, *Opt. Acta* **6**, 1 (1959).

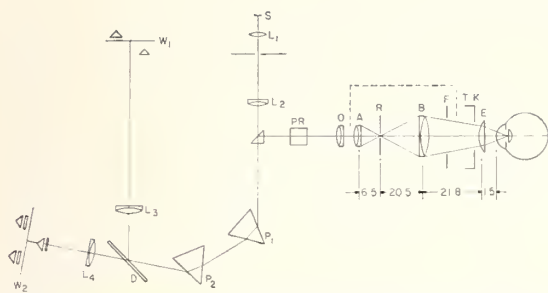


FIG. 2. Schematic plan view of Wright visual tristimulus colorimeter with 10° -field adaptor. The spacing of the optical parts in the adaptor is shown in cm.

taken also by Speranskaya. The red, green, and blue instrumental primaries employed in the Speranskaya investigations were filters, which, together with the spectral distribution of radiant flux from the source, had dominant wavelength of about 640, 545, and 465 nm, respectively. The data obtained have been transformed to a system with primaries at wavelengths 645.2, 526.3, and 444.4 nm, respectively.

In the Speranskaya investigations a visual technique, different from that of Stiles, was used in most of the observations to remove the effect of the Maxwell spot. As the presence of the Maxwell spot may be confusing to the inexperienced or unpractised observer, Speranskaya preferred to block out the central 2° region of the 10° bipartite field of view, and force the observers in this investigation to make matches with a 10° annulus. To aid the observer in locating his fovea, a bright fixation point was provided in the center of the blocked-out area. The retinal illuminance was reported to be nominally 5 trolands, but ranged from approximately 1 to 40 trolands.

Results of this work were reported in several stages, first for four observers, then for 18 observers, and finally for 27 observers. Most of the Speranskaya work was reported to CIE working committee 1.3.1 in advance of publication.¹⁸

5. NIMEROFF PARAFOVEAL LARGE-FIELD COLOR-MATCHING FUNCTIONS

a. Instrument and Calibration

As an additional source of color-matching data for 10° -field viewing conditions, it was suggested that the author determine his own color-matching functions. The instrument used in this determination was the Wright visual tristimulus colorimeter¹⁹ which is located at the Imperial College in London. To obtain such data for 10° -field viewing conditions, the Wright colorimeter had to be adapted to fields larger than the $1^\circ 20'$ for

¹⁸ N. I. Speranskaya, CIE Committee Report NPL Symposium 8, 317 (1958); Opt. Spectry 7, 424 (1959) [Opt. i Spektroskopiya, 7 (1959)].

¹⁹ W. D. Wright, *Researches on Normal and Defective Colour Vision*, (Henry Kimpton, London, 1946, distributed in U. S. by C. V. Mosby Company, St. Louis, Missouri, 1947).

which it had been designed. Such an adapting device had been designed by Clarke²⁰ for his work in large extrafoveal-field color vision. The device is shown schematically in Fig. 2 as it is installed in the Wright colorimeter.

Collimated lights from the two sets of spectral wedges, W_1 (test), and W_2 (comparison primaries) are brought into juxtaposition at the photometer rhomb PR. The lights are focused at the plane of the exit pupil by the objective lens O. The attachment for adapting the instrument to fields larger than $1^\circ 20'$ is placed between the objective and the exit-pupil plane. The optical paths of the attachment, as shown in Fig. 2, are explained as follows: Rays from Objective O are brought to focus in the plane of Diaphragm R by Lens A placed just beyond O. Light from R is collected by Lens B and focused through Field Aperture F to form an image of the source at the focal point of Lens E. Lens E is provided for accommodation of the observer and is mounted just outside the artificial pupil K which is at the end of the stray-light tube T. The field aperture F is selected to give a field size of any angular subtense up to 10° as desired.

The primaries used in the investigation on the Wright colorimeter were at 650, 530, and 460 nm. Although slightly different from those of the Stiles-Burch investigation, the result should not be influenced by the choice of primaries, provided Grassmann's laws hold as indicated earlier, and transformation from one system of primaries to another system can be easily accomplished by use of those laws.

The required calibrations of the Wright colorimeter are: (1) the wavelength scales of the primaries, of the test colors, and the desaturation colors, all of which are taken from the spectrum; (2) the relative spectral emittance of source; and (3) the relative densities of the neutral wedges of the primaries. The first was done by use of mercury and cadmium lamps whose spectral-line wavelengths are known. The second calibration was done by use of a photoelectric photometer for which the relative spectral sensitivity of the photodetector was known. The third calibration was achieved by the use of sectors whose transmittances were known by measurement of their angular apertures.

As expected, the calibrations found for 10° -fields did not differ substantially from those previously found for the Wright colorimeter for 2° -fields.

b. Observation Conditions and Results

The visual technique employed in this investigation involved making the best over-all matches within the macular spot as well as its surround. This technique is quite different from that employed in the Stiles-Burch investigations, and from that employed in the Speranskaya investigations, but is believed to be more repre-

²⁰ F. J. J. Clarke, Ph.D. thesis, Imperial College, University of London, 1959.

sentative of normal matching practices. Normally, an individual does not consciously reject, or neglect, a part of the response of the retina, nor does he see with a part of his field obscured. However, he does permit his entire retina to be exposed, in so far as the sizes of the color samples or test field permit, and then allows the natural processes to convey the retinal responses to the brain for interpretation. The technique employed by the author results in data that may permit its use to determine whether wide-field vision is predominantly a foveal or extrafoveal response or is a compromise (weighted average) response.

The luminous-efficiency data for the 10°-field were obtained by the author using a flicker technique in the spectral region between 700 and 460 nm, and by direct match using a step-down technique in the region between 460 and 400 nm. The results are shown in Fig. 3 and indicate a somewhat higher response in the blue portion of the spectrum and a somewhat lower response in the red portion than in the results obtained by Stiles' observers. This difference is probably caused by the retinal illuminance, which is about 100 trolands (ranging from 5 to 200 trolands) in the Wright colorimeter equipped with the 10°-field adaptor having a reduced exit pupil, while in the colorimeter used by Stiles the illuminance is approximately 500 trolands (ranging from 20 to 3200 trolands). With low levels of retinal illuminance, rods are more likely to contribute to the over-all response of the retina than at higher levels. Aguilar and Stiles²¹ have found that at a retinal illumi-

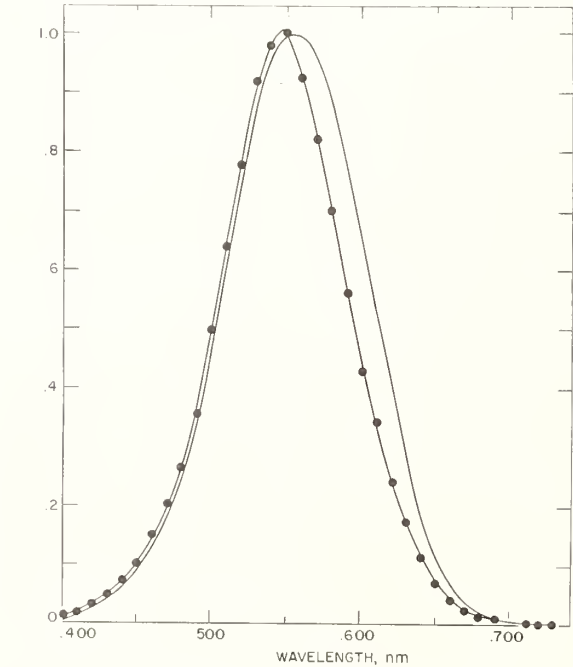


FIG. 3. Relative spectral luminous efficiency with large fields: Stiles —, Nimeroff ●.

²¹ M. Aguilar and W. S. Stiles, Opt. Acta 1, 59 (1954).

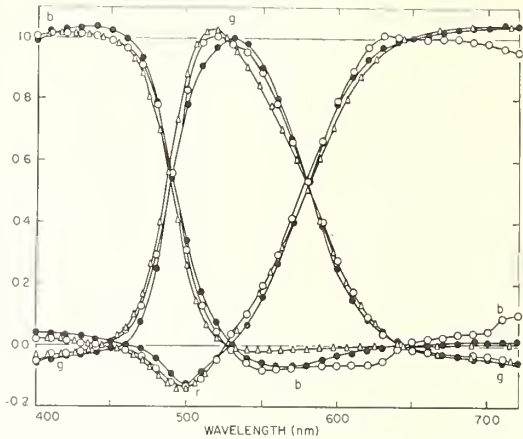


FIG. 4. Color-matching functions for 10°-fields: Stiles Δ, Speranskaya ○, Nimeroff ●.

nance of about 1000 trolands all the rods respond maximally and hence make no differential contribution, which would disturb matches.

The luminous-efficiency data V_λ , for Stiles' observers were deduced from a linear combination of the relative luminances of the primaries, L_R, L_G, L_B , as evaluated by flicker photometry and the color-matching functions $\bar{r}_\lambda, \bar{g}_\lambda, \bar{b}_\lambda$. The equation for this linear combination is:

$$V_\lambda = L_R \bar{r}_\lambda + L_G \bar{g}_\lambda + L_B \bar{b}_\lambda. \tag{1}$$

TABLE I. Equalization wavelengths (crossover) and reference-primary wavelengths of the Stiles-Burch, Speranskaya, and Nimeroff investigations.

Data source	Wavelengths, nm				
	Red-green cross over	Blue-green cross over	Reference primaries		
			Red	Green	Blue
Stiles-Burch	579.7	488.2	645.2	526.3	444.4
Speranskaya	580	490	645.2	526.3	444.4
Nimeroff	580	490	650	530	460

The data obtained by the author on *rgb* color-matching in a WDW system (units of primaries adjusted to make $r_{\lambda_1} = g_{\lambda_1}$, where λ_1 is a wavelength near 580 nm, and to make $g_{\lambda_2} = b_{\lambda_2}$, where λ_2 is near 490 nm) are shown in Fig. 4 and the *r, g* chromaticity diagram is shown as Fig. 5. For comparison, the color-matching data of Stiles and of Speranskaya are also shown in Fig. 4. To facilitate the comparison, the Speranskaya data and the author's data were adjusted for equalization of the primaries at wavelengths similar to those at which the Stiles-Burch data were equal. Table I lists the equalization wavelengths (crossover) as well as wavelengths of instrumental and reference primaries of the three investigations.

The similarity among these data probably would be increased if the primaries and the retinal illuminances were the same. The differences between the two sets of

TABLE II. Summary of the 10°-field data for the author's chromaticity coordinates, spectral luminous efficiency, and tristimulus values.

λ nm	r_λ	g_λ	b_λ	V_λ	$L_R \bar{r}_\lambda$	$L_G \bar{g}_\lambda$	$L_B \bar{b}_\lambda$
400	0.028	-0.023	0.995	0.012	0.0050	-0.0053	0.012
10	0.026	-0.037	1.011	0.019	0.011	-0.019	0.027
20	0.024	-0.049	1.025	0.030	0.023	-0.061	0.067
30	0.020	-0.050	1.030	0.047	0.036	-0.115	0.122
40	0.015	-0.044	1.029	0.072	0.039	-0.142	0.173
450	0.006	-0.027	1.021	0.100	0.015	-0.084	0.169
60	0.000	0.000	1.000	0.150	0.000	0.000	0.150
70	-0.007	0.058	0.949	0.202	-0.011	0.117	0.099
80	-0.027	0.200	0.827	0.265	-0.026	0.239	0.052
90	-0.069	0.465	0.604	0.355	-0.044	0.374	0.026
500	-0.100	0.702	0.398	0.500	-0.063	0.547	0.016
10	-0.070	0.855	0.215	0.642	-0.045	0.678	0.009
20	-0.036	0.950	0.096	0.780	-0.025	0.800	0.004
30	0.000	1.000	0.000	0.920	0.000	0.920	0.000
40	0.057	1.000	-0.057	0.980	0.044	0.940	-0.003
550	0.145	0.930	-0.075	1.000	0.112	0.891	-0.004
60	0.250	0.845	-0.095	0.925	0.180	0.750	-0.004
70	0.381	0.717	-0.098	0.820	0.248	0.575	-0.004
80	0.506	0.574	-0.087	0.700	0.293	0.411	-0.003
90	0.642	0.427	-0.069	0.560	0.309	0.254	-0.002
600	0.769	0.280	-0.049	0.430	0.297	0.133	-0.001
10	0.867	0.167	-0.034	0.344	0.278	0.066	-0.001
20	0.923	0.100	-0.023	0.240	0.213	0.028	...
30	0.962	0.049	-0.011	0.172	0.162	0.010	...
40	0.988	0.017	-0.005	0.113	0.111	0.002	...
650	1.000	0.000	0.000	0.000	0.070	0.000	0.000
60	1.009	-0.014	0.005	0.005	0.041	-0.0007	...
70	1.017	-0.027	0.010	0.024	0.027	-0.0008	...
80	1.025	-0.037	0.012	0.014	0.015	-0.0007	...
90	1.031	-0.045	0.014	0.0081	0.0089	-0.0005	...
700	1.037	-0.052	0.015	0.0049	0.0053	-0.0003	...
10	1.040	-0.056	0.016	0.0027	0.0029	-0.0003	...
20	1.042	-0.060	0.018	0.0018	0.0019	-0.0001	...

data reflect mostly the need for more negative blue in the region between 530 and 650 nm than was required by the observers in the Stiles-Burch investigation.

The author required some small amount of positive blue in making matches in the longwave region, beyond 650 nm, as in Wright's 1°20' field data. The Stiles-Burch observers also required some small amounts of

blue in this region although Stiles argues against inclusion of all this positive blue response because a portion is attributed to "rod intrusion." The merits of applying corrections for this rod intrusion, or more correctly called rod response, in deriving the proposed CIE 10°-field system will be discussed in Part II, Additivity Failure.

The relative luminances of the primaries (matching stimuli) were found for the author by direct heterochromatic comparison to be $L_R=80.9$, $L_G=100.0$, and $L_B=5.3$.

The spectral tristimulus values obtained by the author for the 10°-field are shown in Fig. 6. These have been derived by using the equations that relate chromaticity coordinates with tristimulus values, assuming that luminance is additive. Ishak²² has shown, in a simple and direct way, the relationship between the chromaticity coordinates r_λ , g_λ , b_λ , and the tristimulus values, $\bar{r}_\lambda L_R$, $\bar{g}_\lambda L_G$, $\bar{b}_\lambda L_B$, through the relative spectral luminous efficiency function V_λ , and the relative luminances of the primaries, L_R , L_G , L_B , thus,

$$\begin{aligned}\bar{r}_\lambda L_R &= V_\lambda r_\lambda L_R / L_{C\lambda}, \\ \bar{g}_\lambda L_G &= V_\lambda g_\lambda L_G / L_{C\lambda}, \\ \bar{b}_\lambda L_B &= V_\lambda b_\lambda L_B / L_{C\lambda},\end{aligned}\quad (2)$$

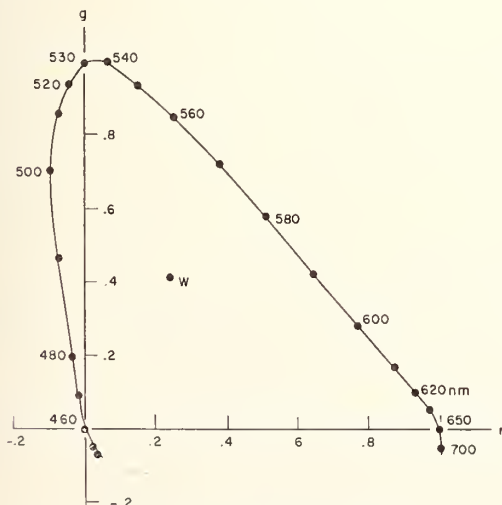


FIG. 5. Nimeroff chromaticity diagram.

²² I. G. H. Ishak, J. Opt. Soc. Am. 42, 844 (1952).

where $L_{CA} = r_{\lambda}L_R + g_{\lambda}L_G + b_{\lambda}L_B$. The color-matching data of the author are listed in Table II.

6. EXTRAFOVEAL OBSERVATIONS

In studying the response of the retina, a number of different areas have been exposed. Figure 7 shows the regions of the retina that had been exposed during the investigations mentioned in this paper. The term "eccentricity" is used to describe the angle, apex at the eye lens, between the fovea and the center of the exposed retinal area.

a. Wright's Extrafoveal Observations

Wright¹⁹ conducted some preliminary studies of extrafoveal vision in the region of the retina which was

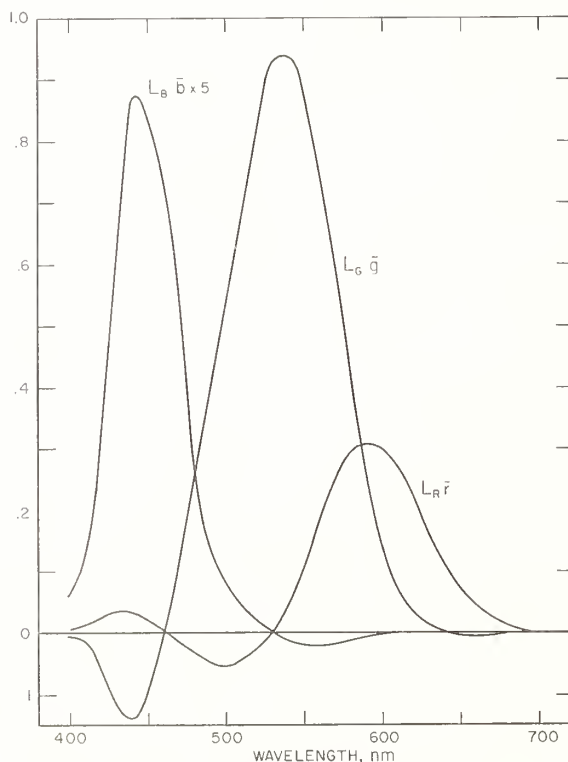


FIG. 6. Nimeroff tristimulus-value diagram.

of 4° eccentricity. He used a 2°-field of retinal exposure to make his observations. The results of this investigation are in accord to some extent with Nimeroff's observations in the 10° parafoveal field. The Wright extrafoveal data, Fig. 8, show that there is a decrease in the amount of negative red in the region of 460 to 530 nm compared with his foveal 1°20' color-matching functions shown in Fig. 1. Also, there is an increase in the amount of negative blue in the region of 530 to 650 nm, compared with the foveal 1°20' color-matching functions on which the present CIE standard-observer system is based.

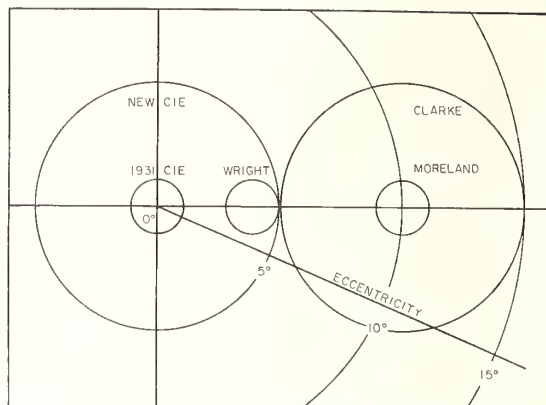


FIG. 7. Areas of retina exposed during experiments described in the text.

b. Moreland's and Clarke's Extrafoveal Observations

Moreland²³ performed similar experiments with a small (2°) extrafoveal field of 10° eccentricity. His data confirm those of Wright in that there is a decrease in the amounts of negative red and an increase in the amount of negative blue required in the color-matching functions. These functions also are shown in Fig. 8.

Clarke²⁰ also performed similar experiments. These were conducted with a large (10°) field of 10° eccentricity. Clarke, however, did not do a complete set of color-matching functions, but in his thesis there are sufficient data from which to evaluate his color-matching functions approximately, by interpolation. The interpolated functions also are shown in Fig. 8. Here also there is confirmation of the data of Wright.

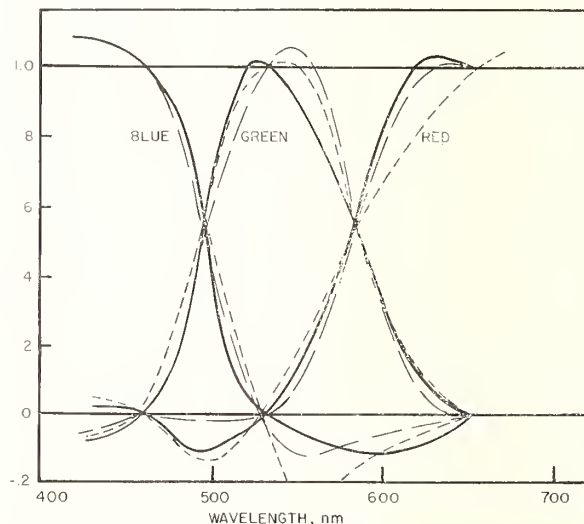


FIG. 8. Nonparafoveal color-matching functions: Wright's small extrafoveal field —, Moreland's small extrafoveal field — — —, Clarke's large extrafoveal field — · — · —.

²³ J. D. Moreland, Ph.D. thesis, Imperial College, University of London, 1955.

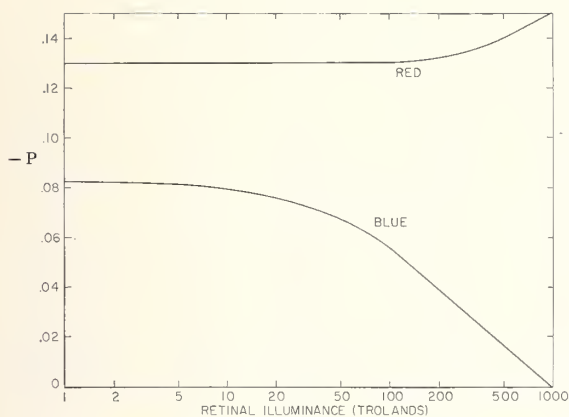


FIG. 9. Relation between retinal illuminance and the maximum, $-P$, of the negative lobes of the red and blue color-matching data for 10° -fields. Red maximum at 500 nm and blue maximum at 550 nm.

7. DISCUSSION

Because there are appreciable numbers of rods in the annulus about the central region of the retina that contribute to the over-all response to light stimuli in wide-field vision, and because the CIE committee was charged with developing a standard observer for wide-field vision which is free from rod response, the data of Stiles and Speranskaya were corrected for "rod intrusion" before transforming these data in an rgb system with real primaries to an xyz system with imaginary primaries. It is to be supposed that in removing "rod intrusion" any instability resulting from the combination of cone and rod response might be eliminated.

This proposed system, free from rod-response contributions, presumes that all color-matching will be performed with sufficient retinal illuminance (at least 1000 trolands) to remove any effects of rod contribution. This, of course, precludes the use of such a system for color matching in cases where the retinal illuminance is considerably less than 1000 trolands.

For such cases some method would need to be devised by which corrections could be applied to color-matching data obtained with low levels of retinal illuminance, to bring them into accord with the proposed rod-corrected system. Alternatively, corrections could be applied to the system to make the system applicable to the observing conditions.

As a first step in devising such a correction method, the color-matching data that have already been obtained for foveal, parafoveal, and extrafoveal vision may be examined. Table III lists the maxima of the negative blue and red lobes at 550 and 500 nm, respectively, obtained by a number of investigators. The close agreement of the data of Wright, König, and Abney,¹⁹ for small foveal fields, shows that the response of the foveal region is very stable. It should be noted that the equalization wavelengths for the data given by Wright were at 582.5 nm for red-green and at 492.9 nm for blue-

TABLE III. Comparison of viewing conditions and maximum of negative lobes of the colormatching functions obtained by several investigators.

Data source	Viewing field		Maximum of negative lobes	
	Diameter deg	Eccentricity deg	blue at 550 nm	red at 500 nm
Wright	2	0	0.02	0.19
König	2	0	0.03	0.19
Abney	2	0	0.04	0.19
Stiles	10	0	0.016	0.14
Speranskaya	10	0	0.081	0.13
Nimeroff	10	0	0.056	0.13
Wright	2	4	0.04	0.13
Moreland	2	10	0.07	0.03
Clarke	10	10	0.19	0.19

green. The values in Table III are on the basis of equalization at 580 nm and 490 nm, respectively.

There is general agreement among the parafoveal data of Stiles, of Speranskaya, and of Nimeroff, for the negative red lobe. There are, however, small differences among those obtained for the foveal region by Wright, König, and Abney. For the negative blue lobes, only the Stiles value is quite close to that obtained by Wright for a 2° field; the other parafoveal data for the negative

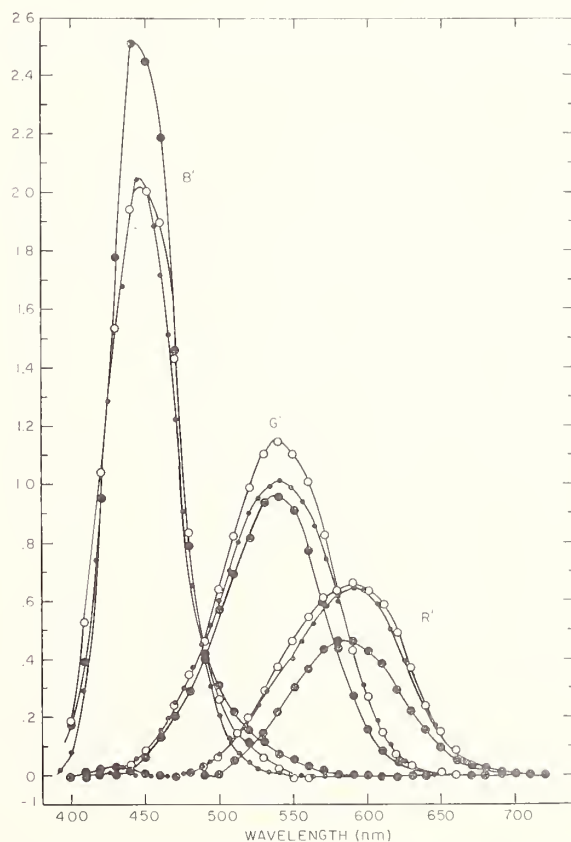


FIG. 10. Fundamental response curves derived for the 10° -field data of Stiles. Speranskaya \circ , and Nimeroff, \bullet .

TABLE IV. Fundamental response curve data.

λ nm	Stiles-Burch ^a			Speranskaya			Nimeroff		
	R'	G'	B'	R'	G'	B'	R'	G'	B'
400	0.0011	0.00013	0.082	0.0011	-0.006	0.191	0.0063	0.0047	0.176
10	0.0040	0.00000	0.397	0.0056	-0.001	0.530	0.012	0.0023	0.392
20	0.0068	0.00074	0.955	0.0080	-0.002	1.045	0.022	-0.0003	0.954
30	0.0070	0.015	1.530	0.0070	0.012	1.534	0.030	0.0016	1.783
40	0.0043	0.044	2.000	0.0047	0.040	1.943	0.029	0.015	2.511
450	0.0016	0.088	1.965	0.0019	0.087	2.000	0.0070	0.068	2.450
60	0.00093	0.156	1.710	0.0014	0.149	1.849	0.0003	0.135	2.182
70	0.0047	0.250	1.249	0.0016	0.246	1.432	0.0042	0.207	1.458
80	0.017	0.348	0.728	0.012	0.351	0.836	0.0051	0.289	0.791
90	0.038	0.461	0.392	0.032	0.473	0.460	0.0030	0.401	0.422
500	0.074	0.605	0.203	0.068	0.641	0.260	0.0040	0.569	0.309
10	0.126	0.762	0.099	0.123	0.823	0.152	0.060	0.696	0.219
20	0.194	0.910	0.046	0.199	0.994	0.108	0.098	0.818	0.165
30	0.268	0.980	0.020	0.296	1.101	0.064	0.152	0.938	0.118
40	0.350	1.018	0.011	0.374	1.145	0.026	0.214	0.960	0.078
550	0.423	0.980	0.0070	0.462	1.105	-0.0044	0.301	0.913	0.059
60	0.502	0.893	0.0053	0.550	1.009	-0.0021	0.368	0.774	0.034
70	0.570	0.770	0.0040	0.609	0.825	-0.0002	0.430	0.599	0.016
80	0.617	0.595	0.0020	0.646	0.629	0.0036	0.465	0.437	0.0068
90	0.648	0.430	0.00020	0.657	0.431	0.0016	0.460	0.279	0.0019
600	0.630	0.276	-0.00080	0.647	0.271	-0.0017	0.425	0.156	0.0008
10	0.565	0.152	-0.0013	0.584	0.152	-0.0003	0.389	0.086	-0.0009
20	0.455	0.077	-0.0013	0.490	0.074	0.0016	0.294	0.044	-0.0001
30	0.340	0.032	-0.0011	0.368	0.031	0.0045	0.221	0.021	0.0005
40	0.214	0.012	-0.00046	0.241	0.012	0.014	0.151	0.010	0.00045
650	0.140	0.0042	-0.00018	0.154	0.0040	0.012	0.096	0.0050	0.00043
60	0.080	0.0011	-0.00007	0.088	0.0017	0.0079	0.056	0.0022	0.00035
70	0.041	0.00002	0.00000	0.047	0.0004	0.0043	0.037	0.0011	0.00033
80	0.021	0.00000	0.00009	0.024	0.00005	0.0025	0.020	0.00034	0.00017
90	0.010	0.00000	0.00001	0.012	0.00006	0.0013	0.012	0.00014	0.00010
700	0.0050	0.00000	0.00000	0.0058	0.00000	0.00068	0.0071	0.00005	0.00006
10	0.0023	0.00000	0.00000	0.0027	-0.00003	0.00045	0.0040	0.00001	0.00004
20	0.0011	0.00000	0.00000	0.0012	-0.00001	0.00022	0.0026	0.00000	0.00002

^a Graphical interpolation.

blue lobe are greater than any of the foveal data, and differ from each other as well as from that of Stiles. It should be noted here that these data were obtained with different retinal illuminances: Stiles 500 trolands (range 20–3200 trolands), Speranskaya 5 trolands (range 1–40 trolands), Nimeroff 100 trolands (range 5–200 trolands). Figure 9 shows the relationship that might be inferred from the differences in values of the functions and retinal illuminances.

It should be noted from Fig. 4 that the positive blue lobes at the extreme red end, $\lambda > 645$ nm, are consistent with the concept that in parafoveal vision the rods become saturated and do not contribute to the over-all response.

Because these data of Stiles, Speranskaya, and Nimeroff, were obtained with somewhat different primaries and with different equalization wavelengths, objections may be raised to the making of direct comparisons of their respective chromaticity-coordinate data. To put the data on a comparable basis the tristimulus values, R , G , and B , were transformed to fundamental response curves by a procedure devised by König²⁴ and described in detail by Szekeres.²⁵ Table IV

and Fig. 10 show the fundamental response curves R' , G' , and B' , that were derived by the transformation equations:

$$\begin{aligned} R_{SB}' &= 0.78187R_{SB} + 0.24235G_{SB} + 0.00147B_{SB}, \\ G_{SB}' &= 0.03257R_{SB} + 0.96289G_{SB} + 0.03023B_{SB}, \\ B_{SB}' &= -0.00106R_{SB} + 0.02675G_{SB} + 1.00000B_{SB}; \end{aligned} \quad (3)$$

$$\begin{aligned} R_S' &= 0.87441R_S + 0.25590G_S + 0.00186B_S, \\ G_S' &= 0.03479R_S + 1.06828G_S + 0.02910B_S, \\ B_S' &= 0.05801R_S + 0.07416G_S + 1.00000B_S; \end{aligned} \quad (4)$$

$$\begin{aligned} R_N' &= 0.96673R_N + 0.16538G_N + 0.00012B_N, \\ G_N' &= 0.05039R_N + 1.01982G_N + 0.06202B_N, \\ B_N' &= 0.00438R_N + 0.12785G_N + 1.00000B_N; \end{aligned} \quad (5)$$

where the subscripts, SB , S , and N , indicate the Stiles-Burch, Speranskaya, and Nimeroff data, respectively.

From these fundamental response curves we may note (see Table IV) that the short-wave portion of the R' curves are highest for the Nimeroff data, intermediate for the Speranskaya data, and lowest for the Stiles data. If we neglect the Speranskaya data because of the very low retinal illuminance, this variation between the

²⁴ A. König, *Abhandlungen zur Physiologischen Optik* (Johann Ambrosius Barth, Leipzig, 1903).

²⁵ G. Szekeres, *J. Opt. Soc. Am.* **38**, 350 (1948).

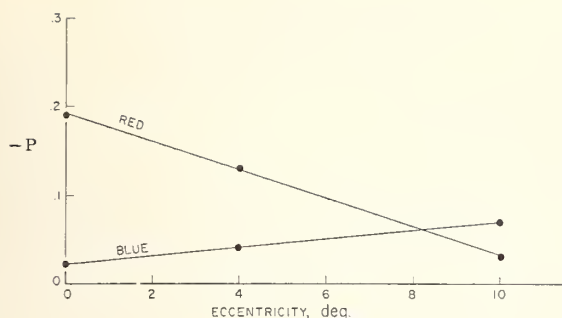


FIG. 11. Relation between eccentricity and maximum, $-P$, of the negative lobes of the red and blue color matching data for 2° -fields. Red maximum at 500 nm and blue maximum at 550 nm.

Stiles data and the Nimeroff data is consistent with the idea that retinal fluorescence is of greater importance for the relatively lower illuminances and of less importance for the higher illuminances.

The G' curves (see Fig. 10) are broader for the Stiles data and Speranskaya data, and narrower for the Nimeroff data. This may have resulted from the fact that the Stiles data and the Speranskaya data represent the averages of a number of observers (53 and 27, respectively), while the Nimeroff data represent only one observer. The averaging process tends to result in a broader curve than that found for any individual observer, if the maxima are not all located at the same place. It should be noted here that the Nimeroff maxima of each of these curves are located at shorter wavelengths than are the Stiles and the Speranskaya maxima. This may have resulted from the presence of less selective absorption in Nimeroff's ocular media than in that of the average of the Stiles and the Speranskaya observers.

In the long-wave portion of the B' curves (see Table IV) the Speranskaya data are higher at 690 nm where B' is 13×10^{-4} ; the Nimeroff data, intermediate at 690 nm where B' is 10×10^{-5} ; and the Stiles data, lowest at 690 nm where B' is 1×10^{-5} . This gradation with retinal illuminance is consistent with that deduced from the chromaticity-coordinate data shown in Fig. 4.

The B' fundamental response curves for the intermediate wavelengths, in the vicinity of 530 nm, do not show a gradation that is consistent with retinal illuminance. This lack of consistency may result from the

presence in the Nimeroff data, of what Stiles calls the π_2 component mechanism.²⁶ It may be either that such a component mechanism is very active in the Nimeroff retina, or that its contribution shows a reversal of trend for the retinal illuminance used in the Nimeroff investigation.

In the extrafoveal data of Wright and the extrafoveal data of Clarke, the negative red lobe maxima are in general agreement with those of the foveal data, but in the extrafoveal data of Moreland there is great disagreement. The negative blue lobe data for these researchers differ among themselves and from the foveal and parafoveal data. There is, however, a decided trend among the small field (2°) data as a function of the eccentricity, which is shown by the plot of these data in Fig. 11.

Admittedly many of these data are based on only one observer, for each condition, although 10 observers were used in Wright's foveal data, 53 observers in Stiles' parafoveal data, and 27 observers in Speranskaya's parafoveal data. Clearly, if the indicated relationships between maxima and retinal illuminance on the one hand and eccentricity on the other are to be established, further data are required. On the basis of further evidence, a method may be devised by which color-matching observations made under low levels of retinal illuminance may be made useful. Such a method may be applied either to the data, to transform them to the system, or to the system to transform it to the framework in which the data were obtained.

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²⁶ W. S. Stiles, Proc. Natl. Acad. Sci. **45**, 100 (1959).

Colorimetry in Parafoveal Fields.* II. Additivity Failure

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Field trials of the proposed 10°-field observer system (recommended for adoption at the June 1963 CIE meetings in Vienna, Austria, by the Experts of Committee E-1.3.1), which was derived by removing rod response from the colormatching data, indicate that many visual color matches made on existing additive colorimeters require correction for rod response. But even if such corrections are made, additivity failure still may persist. Such additivity failures may result from rod-cone interactions, either inhibitive or facilitative. Additivity checks made by the author are in general agreement with results obtained in the extrafoveal investigations of Wright, of Moreland, and of Clarke; and with the results of the 10°-parafoveal field trials of Wright and Wyszecki, and of Stiles and Wyszecki.

1. INTRODUCTION

ONE of the basic assumptions of the 1931 Standard Observer System for Colorimetry is that Grassmann's law of additivity¹ holds for all viewing conditions. Trezona² and Blottiau³ have shown that within statistical uncertainties this law does hold for 2° viewing fields. In field trials of the proposed 10°-field observer system, developed by Judd⁴ from the data of Stiles and Burch⁵ and of Speranskaya,⁶ discrepancies have been found between the predictions of matches made by the proposed observer system and the observed matches. Stiles and Wyszecki⁷ have speculated that these differences may be ascribed to failure of the additivity law. Direct checks of the law of additivity were undertaken to determine the magnitude and direction of such failures.

2. ADDITIVITY

a. Additivity of Color Equations

If c_1 units of Color C_1 match quantities of a set of three lights, r_1R , g_1G , and b_1B , and c_2 units of Color C_2 match different quantities of the same set of lights, r_2R , g_2G , and b_2B , then Grassmann's law of additivity requires that if the two colors are combined, the resulting color can be represented by the sums of the tristimulus values of the other two, thus: $cC \equiv c_1C_1 + c_2C_2 \equiv (r_1 + r_2)R + (g_1 + g_2)G + (b_1 + b_2)B$, where $c_1C_1 \equiv r_1R + g_1G + b_1B$ and $c_2C_2 \equiv r_2R + g_2G + b_2B$.

The Trezona² technique for testing additivity consists of setting up a combination of three primary lights to match a given amount of, say, a red monochromatic test color, $c_1C_1 \equiv t_1(\lambda_{t1})$, to which has been added a given amount of blue desaturating primary, $d_1(\lambda_{d1})$, and then

measuring the amount of d_1 . Thus: $c_1C_1 + d_1(\lambda_{d1}) \equiv t_1(\lambda_{t1}) + d_1(\lambda_{d1}) \equiv r_1R + g_1G + b_1B$, and $d_1(\lambda_{d1}) \equiv b_{d1}B$. Therefore $c_1C_1 \equiv r_1R + g_1G + (b_1 - b_{d1})B$.

A match is then also made with another, say, blue monochromatic test color, $c_2C_2 \equiv t_2(\lambda_{t2})$, and its red desaturating primary, $d_2(\lambda_{d2})$, which can be expressed as $c_2C_2 + d_2(\lambda_{d2}) \equiv t_2(\lambda_{t2}) + d_2(\lambda_{d2}) \equiv r_2R + g_2G + b_2B$, and $d_2(\lambda_{d2}) \equiv r_{d2}R$. Therefore $c_2C_2 \equiv (r_2 - r_{d2})R + g_2G + b_2B$.

Finally, a match is made of the combination of the two test colors, cC , where $cC \equiv t_1(\lambda_{t1}) + t_2(\lambda_{t2}) \equiv rR + gG + bB$.

If the additivity law holds, then the following equations are correct: $cC \equiv c_1C_1 + c_2C_2$, or $rR + gG + bB \equiv (r_1 + r_2 - r_{d2})R + (g_1 + g_2)G + (b_1 + b_2 - b_{d1})B$. The readings for the amounts of the R G B primaries were corrected to chromaticity coordinates so that predicted as well as observed chromaticities could be computed conveniently to check on the additivity law of Grassmann. The conversion to chromaticity coordinates was done in this investigation as well, it is supposed, as in the Trezona investigation by means of the Wright technique. This technique consists of determining the correction factor $k_i\bar{r}$ required to make the tristimulus values $k_1\bar{r} = \bar{g}_1$ and $k_2\bar{b} = \bar{g}_2$ at wavelengths 582.5 and 494.0 nm, respectively. These factors are then applied to all subsequent measurements of tristimulus values to obtain their chromaticity coordinates in the rgb system. Chromaticity coordinates are computed in the usual manner, as $r = \bar{r}/(\bar{r} + \bar{g} + \bar{b})$.

b. Small Foveal-Field Tests of Additivity

Trezona² has shown that for the small-field condition this law holds within experimental differences among observers. The work of Trezona confirms the additivity data of Blottiau³ for small field viewing. To acquire additional data for a limited number of combinations of monochromatic lights, the results shown in Table I were obtained for Observer I.N. for three combinations with the 1°20' field. Although the last one of these three additivity checks appears to show a somewhat large difference compared with the other two, the difference is still within the uncertainty of measurement, estimated about 0.01.

* The work reported here was done partly in fulfillment of the requirements of the Diploma of Membership of the Imperial College, University of London, England.

¹ H. Grassmann, Poggendorff's Ann. 89, 69 (1853).

² P. W. Trezona, Ph.D. thesis, Imperial College, University of London, 1951.

³ F. Blottiau, Rev. Opt. 23, 136 (1947).

⁴ D. B. Judd, CIE Proceedings A, 19 (1951).

⁵ W. S. Stiles and J. M. Burch, Opt. Acta 6, 1 (1959).

⁶ N. I. Speranskaya, CIE Committee Report, NPL Symposium 8, 317 (1958); Opt. Spectry [Opt. i Spektroskopiya 7, 429 (1959)].

⁷ W. S. Stiles and G. Wyszecki, J. Opt. Soc. Am. 52, 58 (1962).

TABLE I. Chromaticities obtained with combinations of 490 and 650 nm.

Observed			Predicted		
r	cC_g	b	r	$c_1C_1+c_2C_2_g$	b
0.254	0.293	0.454	0.253	0.294	0.454
0.529	0.194	0.277	0.534	0.192	0.275
0.606	0.155	0.240	0.617	0.150	0.233

c. Extrafoveal-Field Tests of Additivity

Moreland,⁸ in his experiments with a 2°-field of view at an eccentricity of 10°, studied the peripheral matching of Sources *A*, *P*, and *C*. These studies could serve well as tests of additivity in peripheral vision. The results he obtained under the above conditions by direct matches indicated a shift of the match chromaticities toward the blue primary from the chromaticities predicted from the spectral distributions of the sources and his color-matching functions for these conditions. Figure 1 shows his chromaticity diagram with these shifts.

Clarke's⁹ checks of the additivity law were made with a 10°-field of 10° eccentricity. The chromaticity results he obtained are shown in Fig. 2. His additivity checks were made by adding various amounts of 490 nm to various amounts of 650 nm and various amounts of 430 nm to various amounts of 550 nm. The former show a shift of the matches from the predicted chromaticities

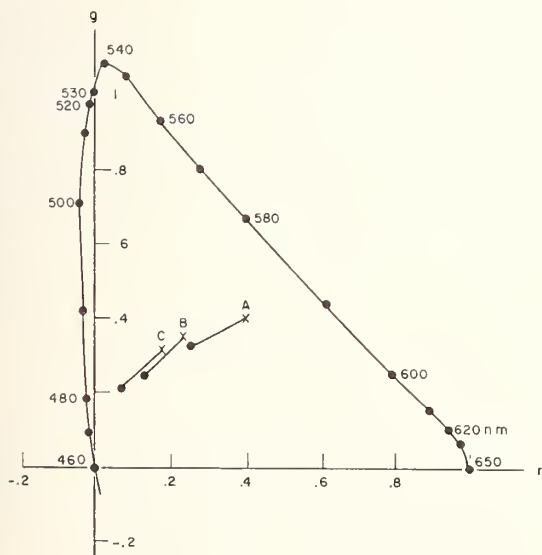


FIG. 1. Moreland's chromaticity diagram for extrafoveal vision, showing white matches for Sources *A*, *B*, and *C*; predicted, *X*; observed, ●.

⁸ J. D. Moreland, Ph.D. thesis, Imperial College, University of London, 1955.

⁹ F. J. J. Clarke, Ph.D. thesis, Imperial College, University of London, 1959.

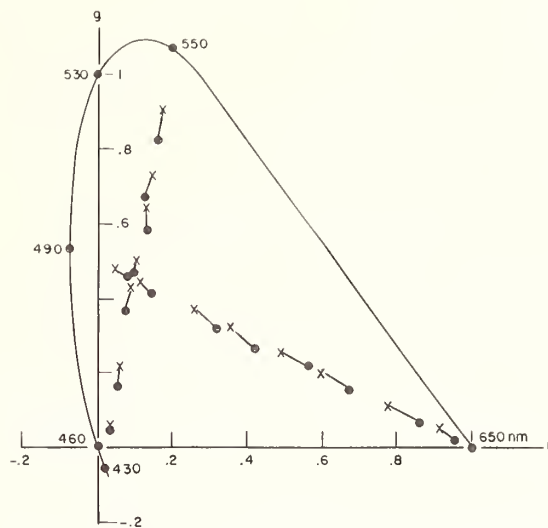


FIG. 2. Clarke's chromaticity diagram for extrafoveal vision, showing additivity failures for combinations of 430 with 550 nm, and 490 with 650 nm; predicted, *X*; observed, ●.

toward the red primary, although there is some tendency of a shift toward the blue primary as well. In the latter additivity checks there is a shift toward the blue primary. These data, showing a shift toward blue, are in general agreement with the departures from additivity found by Moreland.

d. Parafoveal-Field Tests of Additivity

The author's checks of additivity in the 10°-parafoveal field were made in the manner of Trezona² as described above. Various amounts of monochromatic light of wavelength 490 nm were added to various amounts of monochromatic light of wavelength 590 nm in one series of experiments and, similarly, of 480 added to 580 nm in another series. The chromaticity results obtained are shown in Fig. 3 and indicate a departure

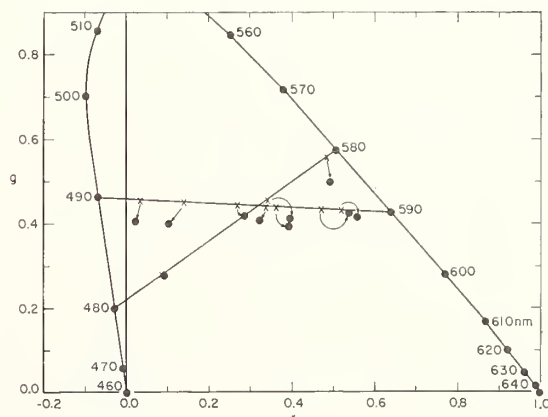


FIG. 3. Author's chromaticity diagram for parafoveal vision, showing additivity failures for combinations of 480 with 580 nm, and 490 with 590 nm; predicted, *X*; observed, ●.

from the predicted chromaticity toward the blue and red primaries. These data are not in conflict with the data of Wright,¹⁰ of Moreland, or of Clarke. The predicted and observed chromaticities in the *rgb* system are listed in Table II.

3. PHOTOMETRIC DETERMINATIONS OF RETINAL ILLUMINANCE

Absolute photometric calibration of the light from comparison-primary wedges of the Wright visual colorimeter may be accomplished by means of the geometric arrangement shown in Fig. 4. A standard lamp of known candlepower I is used to illuminate a "white"-coated sector of uniform reflectance ρ from a distance r and at an incidence angle θ . The light from any wedge may then be compared with that from the standard source by means of the rotating sector by a flicker technique. The illuminance E from the source onto the sector is then $E = I \cos\theta/r^2$, and the luminance B of the

TABLE II. Chromaticities obtained with combinations of 490 plus 590 nm and of 480 plus 580 nm.

Combinations (nm)	cC (observed)			$c_1C_1 + c_2C_2$ (predicted)		
	r	g	b	r	g	b
490+590	0.022	0.406	0.572	0.034	0.458	0.508
	0.107	0.401	0.492	0.142	0.451	0.408
	0.286	0.419	0.294	0.263	0.450	0.287
	0.320	0.405	0.276	0.331	0.440	0.230
	0.394	0.395	0.212	0.369	0.439	0.191
	0.538	0.425	0.038	0.469	0.429	0.102
	0.559	0.416	0.025	0.518	0.416	0.066
480+580	0.089	0.282	0.630	0.083	0.284	0.633
	0.395	0.416	0.189	0.343	0.453	0.203
	0.492	0.501	0.006	0.487	0.545	-0.032

sector is $B = \rho I \cos\theta/\pi r^2$. If the area of the exit pupil is a then the retinal illuminance RI is $RI = a\rho I \cos\theta/\pi r^2$. If a is a circular aperture of diameter t then $a = \pi t^2/4$ and the retinal illuminance may be computed from

$$RI = t^2 \rho I \cos\theta/4r^2. \quad (1)$$

The data for the components of the colorimeter, r , θ , I , ρ , and t are: $r = 0.86$ m, $\theta = 45^\circ$, $I = 1370$ cds, $\rho = 0.80$, $t = 0.606$ mm, and give a retinal illuminance of 97 trolands. The number of trolands was found by multiplying the luminance in candelas per square meter of the viewed surface by the pupil area, in square millimeters.

To complete the calibration, two additional factors need to be taken into account. The first is that, as shown in Fig. 4, the exit pupil of area a' is related to the area of the entrance pupil a by the ratio $(v/u)^2$. But as u (entrance pupil-to-lens distance) is approximately equal

TABLE III. Retinal illuminances, in trolands, for 10° parafoveal color checks of additivity.

$t_1 + d_1$	d_1	t_1	$t_2 + d_2$	d_2	t_2	$t_1 + t_2$	Pre- dicted	Ob- served
590+460	460	590	490+650	650	490			
22	1	21	61	14	47	68		61
36	1	35	35	7	28	63		61
61	1	60	28	14	14	74		71
60	0	60	20	9	11	71		74
74	1	73	39	27	12	85		85
117	0	117	22	13	9	126		121
132	0	132	15	8	7	139		110
						Pre- dicted		Ob- served
580+460	460	580	480+650	650	480			
35	2	33	67	42	25	58		60
93	1	92	23	13	10	102		116
141	1	140	55	53	2	142		139

to v (lens-to-exit pupil distance), this correction is negligible in determining only approximate retinal-illuminance values. The second factor comes about because there are 10 air-glass interfaces in the lens system for which reflection losses are to be accounted, as can be seen in Fig. 2 of Part I,¹¹ two at each of the following optical components: photometer rhomb, objective lens, two field magnifying lenses, and accommodation lens. Losses of flux on reflection are therefore approximately $1 - (0.96)^{10}$, or about $1/3$. Consequently, the retinal-illuminance calibrations computed from Eq. (1) need to be multiplied by a factor of $2/3$.

4. RETINAL ILLUMINANCE FOR ADDITIVITY MATCHES

In the additivity checks for 10° -parafoveal field described above, an attempt was made to keep the retinal illuminance as constant as possible (within an order of magnitude) with the filter combinations that were available. Weaver¹² has shown that luminance is accurately additive if retinal illuminance and adaptation

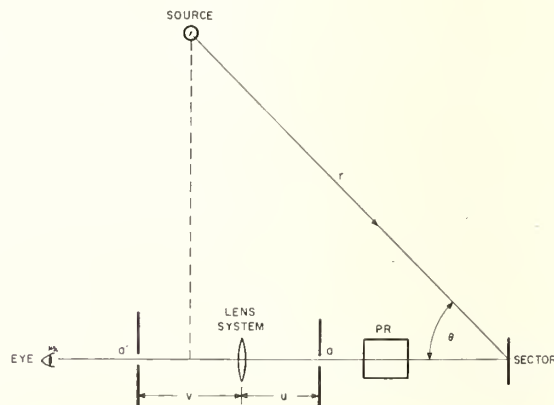


FIG. 4. Optical arrangement for measurement of retinal illuminance.

¹⁰ W. D. Wright, *Researches on Normal and Defective Colour Vision* (Henry Kimpton, London, 1946, distributed in U. S. by C. V. Mosby Company, St. Louis, Missouri, 1947).

¹¹ I. Nimeroff, *J. Opt. Soc. Am.* **54**, 824 (1964).

¹² K. S. Weaver, *J. Opt. Soc. Am.* **39**, 278 (1949).

are kept constant. The attempt at constancy was made for each part of the test and throughout all the tests. Table III gives the retinal illuminances in trolands for each of the matches involved in the check tests. The data in the table are in terms of quantities of test colors t_1 and t_2 , desaturating primaries d_1 and d_2 , and the additions t_1+t_2 (predicted and observed). The retinal illuminances for t_i+d_i ranged from about 15 to 130 trolands, for combinations of 590 and 490 nm, while the mixtures ranged from about 60 to 140 trolands. The retinal illuminances for t_i+d_i ranged from about 20 to 140 trolands for combinations of 580 and 480 nm, while the mixtures ranged from about 60 to 140 trolands.

5. FIELD TRIALS ON VISUAL COLORIMETERS

a. MacAdam Visual Colorimeter

The CIE committee on colorimetry has conducted several field tests of the proposed 10° -field standard-observer system for colorimetry. This proposal was derived by Judd⁴ from the rod-corrected color-matching functions of Stiles and Burch⁵ and of Speranskaya.⁶ One group of tests of the system was conducted at the National Research Council of Canada by Wright and Wyszecki.¹³ Their data were taken on a three-stimulus MacAdam visual colorimeter¹⁴ by five observers who made color matches of a number of filter combinations. Differences between the observed and predicted chromaticities of these filters show a shift predominantly toward the blue primary but partially toward the red primary. Because of the similarity between these differences and those found by the author in the additivity checks described above, the differences found by Wright and Wyszecki may be at least partly explained on the basis of failure of the additivity laws.

b. Donaldson Visual Colorimeter

Another field trial was conducted also at the NRC by Stiles and Wyszecki.⁷ This work was done on a Donaldson colorimeter¹⁵ with 10 observers and several filter combinations in addition to those in the Wright-Wyszecki trials. Several of these filter combinations have chromaticities similar to those obtained by the author in his additivity checks, described above. These are shown in Fig. 5 on the chromaticity diagram from the proposal derived by Judd. The departures of the observed chromaticities from the predicted chromaticities are in general accord with the Wright-Wyszecki departures, in that for both sets of data chromaticities less red than the chromaticity of the intersection of the lines that join 480 with 580 nm and 490 with 590 nm (two pairs of near complementaries) tend to shift toward the chromaticity of the blue primary, while

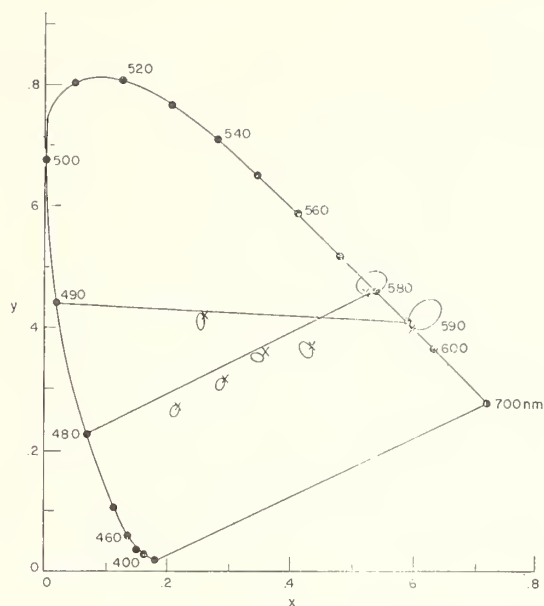


FIG. 5. Proposed CIE chromaticity diagram for 10° -field observer system, showing ranges (ellipses) of visual matches by the Stiles-Wyszecki observers and predictions by the system (X).

those less blue tend to shift away from the chromaticity of the blue primary. Consequently the departures may be explained partly on the basis of failure of the laws of additivity in large parafoveal fields.

An interesting part of the Stiles-Wyszecki data is that the Donaldson colorimeter matches of two of the filter combinations fall outside of the spectrum locus, in the proposed CIE System, which predicts that they may be seen as supersaturated colors, that is, colors that are more strongly saturated than monochromatic colors. Such colors do not exist, hence we must seek some technique for adjusting the data. To bring these data within the region of real colors, Stiles and Wyszecki perform additional rod corrections to the visual data of their observers with only partial success. It appears, however, that the proposed system fails partly because it is based on data that underestimates the negative blue of the color-matching functions in this region of the spectrum.

A comparison was made between the chromaticity predictions of the author's 10° color-matching functions and the field tests reported by Stiles and Wyszecki for two of the filters, Nos. 5 and 12. First the chromaticity coordinates for Source A flux distribution, E_A , were obtained for the two trial filters and the R, G, and B primary filters for the author's color-matching functions, thus $r_F = R_F/S = \sum_{\lambda} E_A \tau_F \bar{r}_{\lambda}/S$, $g_F = G_F/S = \sum_{\lambda} E_A \tau_F \bar{g}_{\lambda}/S$, $b_F = B_F/S = \sum_{\lambda} E_A \tau_F \bar{b}_{\lambda}/S$, where τ_F = spectral transmittance of each filter; F , \bar{r}_{λ} , \bar{g}_{λ} , \bar{b}_{λ} = author's spectral tristimulus values, and $S = R + G + B$. The tristimulus values, R, G, B, and the chromaticity coordinates, r , g , b , thus computed for the trial filters and the primary filters, are listed in Table IV.

¹³ H. Wright and G. Wyszecki, J. Opt. Soc. Am. **50**, 647 (1960).

¹⁴ D. L. MacAdam, J. Opt. Soc. Am. **40**, 589 (1950).

¹⁵ R. Donaldson, Proc. Phys. Soc. (London) **59**, 554 (1947).

From these data the mixtures of the Donaldson primaries, (*R*), (*G*), and (*B*), that on the author's system have the same chromaticities as filters 5 and 12, were found in trichromatic units, or *t* units. The match may be expressed in terms of the components (*R*), (*G*), (*B*), as

$$\begin{aligned}l(R) &= lr_R(R) + lg_R(G) + lb_R(B), \\m(G) &= mr_G(R) + mg_G(G) + mb_G(B), \\n(B) &= nr_B(R) + ng_B(G) + nb_B(B),\end{aligned}$$

where the subscripts, *R*, *G*, *B*, refer to the Donaldson primaries and *l*, *m*, *n*, are the relative amounts of these primaries, respectively, for the match. To solve for the matching mixture, the amounts of chromaticities, *r*, *g*, and *b*, in each of the primary filters are equated to the chromaticities *r_f*, *g_f*, and *b_f* of the test filters. Thus *r_Rl* + *r_Gm* + *r_Bn* = *r_f*, *g_Rl* + *g_Gm* + *g_Bn* = *g_f*, *b_Rl* + *b_Gm* + *b_Bn* = *b_f*.

Next the numbers of *t* units in each of the radiations from the source (distribution of Source *A*) that passed through the primary filters (*R*), (*G*), and (*B*) were found by dividing the relative amounts of the primaries used in a match, *l*, *m*, and *n*, by the sum of their respective tristimulus values, *R*, *G*, and *B*, thus:

$$\begin{aligned}l/(R_R + G_R + B_R) &= T_R l, \quad m/(R_G + G_G + B_G) = T_G m, \\n/(R_B + G_B + B_B) &= T_B n, \quad \text{where } T_R = 1/(R_R + G_R + B_R)\end{aligned}$$

and so on. Then the spectral energy distribution for the match, *E_{λ(RGB)}*, was computed as

$$E_{\lambda(RGB)} = E_A(\tau_R T_R l + \tau_G T_G m + \tau_B T_B n).$$

From this energy distribution the tristimulus values and the chromaticity coordinates of the *RGB* mixture required to match the two test filters were computed by using the proposed CIE 10°-field spectral tristimulus values.

$$\begin{aligned}X &= \Sigma_{\lambda} E_{\lambda(RGB)} \bar{x}_{\lambda}, \quad Y = \Sigma_{\lambda} E_{\lambda(RGB)} \bar{y}_{\lambda}, \quad Z = \Sigma_{\lambda} E_{\lambda(RGB)} \bar{z}_{\lambda}; \\x &= X/(X + Y + Z), \quad y = Y/(X + Y + Z), \\z &= Z/(X + Y + Z).\end{aligned}$$

In Table V, the chromaticity coordinates *x*, *y*, and *z*, computed in the manner described above, Column A, are compared with the averaged results reported in the field tests by Stiles and Wyszecki, S-W, and with those

TABLE V. Comparison of the chromaticity coordinates of field trial filters reported by Stiles and Wyszecki, S-W, and predicted by the proposed 10°-field observer P and the author A.

Chromaticity coordinates	Filter No. 5			Filter No. 12		
	P	S-W	A	P	S-W	A
<i>x</i>	0.294	0.288	0.294	0.597	0.626	0.616
<i>y</i>	0.316	0.306	0.312	0.403	0.432	0.392
<i>z</i>	0.390	0.406	0.395	0.000	-0.058	-0.008

predicted by the proposed 10° field observer, P. The results predicted by the color-matching functions of the author lie between those of the proposed system and the Stiles-Wyszecki field trials for these two filters.

6. DISCUSSION

The investigations described above evoke two items for discussion. One is the problem of what to do about the failure of additivity laws in parafoveal and extrafoveal viewing conditions. The second is the problem of the rod response that is found to occur at intermediate levels of illuminance for parafoveal conditions of view.

The evidence of the failure of the additivity law for color measurement, in which regions of the retina beyond the fovea are stimulated, should be considered seriously. This failure is shown in the work of Moreland, of Clarke, of Wright and Wyszecki, of Stiles and Wyszecki, and of the author. Further investigations should be made to determine the extent and direction of the additivity failure so that, perhaps, suitable corrections or allowances may be recommended in any newly formulated standard-observer system.

Additivity failures may be explained, for instance, by postulating the existence of interactions between different chromatic-response components of the visual system. These interactions may be inhibitive or facilitative, as has been found by Clarke.¹⁶ More knowledge about the magnitude of these interactions may reveal what corrections are to be made to compensate for additivity failure.

Additivity and color-matching experiments in parafoveal and extrafoveal vision have indicated that large-field vision is more like extrafoveal vision than like foveal vision. This conclusion is correct at least for the illuminance levels at which these experiments were conducted.

It is possible that rod response may be the cause of additivity failure. This response may also be the cause of apparent supersaturation of strong colors. The additivity experiments described above may be a case in point. For, if rod response makes 490-nm radiation appear relatively more desaturated than 460 nm, then matching 490 nm alone may require less than 460 nm without rod response. As 590-nm radiation is added to 490-nm radiation, the rod response becomes relatively

TABLE IV. Source *A* color characteristics of the Donaldson primary filters and the trial filters in the new system for the author's color-matching functions.

Filters	Tristimulus values			Chromaticity coordinates		
	<i>R</i>	<i>G</i>	<i>B</i>	<i>r</i>	<i>g</i>	<i>b</i>
(<i>R</i>)	0.8178	-0.0133	0.0046	1.0107	-0.0164	0.0057
(<i>G</i>)	0.3662	4.7331	0.0587	0.0710	0.9176	0.0114
(<i>B</i>)	0.1199	-0.1394	16.3626	0.0073	-0.0085	1.0012
(5)	2.6515	8.2600	22.8538	0.0785	0.2446	0.6768
(12)	30.7205	22.6934	-2.9959	0.6093	0.4501	-0.0594

¹⁶ F. J. Clarke, Opt. Acta 7, 355 (1960).

reduced, then desaturation of 460 nm increases relative to 490 nm and more of 460 nm may be required to achieve a color match. Hence, a shift of the observed chromaticity toward the blue primary from the predicted chromaticity point results.

The results of these investigations with 10° parafoveal viewing conditions tend to agree with the results obtained in the investigations of Wright, of Moreland, and of Clarke, which were performed under extrafoveal conditions of viewing. We are led to conclude from this agreement that, at the levels of retinal illuminance used in the investigations of the author, parafoveal vision may well be the result of the combined response of cones and rods. Under such conditions of illumination, the rods are not fully saturated and hence are able to make their contribution to the over-all response.

If, however, a standard observer system is formulated wherein rod response contributions are eliminated, then observations of color matches that are made under intermediate levels of illuminance would have rod-response contributions and hence would require the elimination of this rod contribution. Unless the rod contributions are removed, some colors may appear to lie outside the region of real colors, as in the Stiles-Wyszecki tests described above.

On the other hand, where large-field matching observations are made under higher levels of illuminance, the rods become saturated (always respond maximally) and have no influence on color matching data, as has been found by Aguilar and Stiles.¹⁷ Results derived from data obtained under these conditions depend entirely upon cones and hence would be similar to foveal vision.

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¹⁷ M. Aguilar and W. S. Stiles, *Opt. Acta* **1**, 59 (1954).

Field Trial of the 1959 CIE Supplementary Standard Observer Proposal*

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The field trials of the 10°-field supplementary standard observer proposal performed by W. S. Stiles and G. Wyszecki on the National Research Council three-primary Donaldson colorimeter have been repeated on the National Bureau of Standards Donaldson colorimeter. Only the least selective 11 of the 18 NRC filters were used in the NBS repetition, which employed 11 observers each making two sets of matches. The NBS Donaldson colorimeter with a 10°-field of view was calibrated both in terms of the 1931 CIE standard observer system and in terms of the 1959 supplementary proposal. As expected, visual matches with a 10° field compared quite unsatisfactorily with predictions made by the 1931 standard observer. The results are in general agreement with the Stiles-Wyszecki field-trial data. The agreement with the predictions by the 1959 proposal is improved when the results of the two field trials are combined, because the resulting arrays of individual-observer data encompass the predicted chromaticities of all of the filters but the red one. It is concluded that, although the proposal is not completely satisfactory, it does yield a significant improvement over the agreements achieved between predictions by the 1931 observer and observations.

1. INTRODUCTION

FIELD trials of the supplementary standard-observer system proposed in 1959 for large-field (10°) color matching are still (in December 1963) being conducted. The proposed system was derived by Judd¹ from the 10° color-matching data obtained by Stiles and Burch² and by Speranskaya.³ The Stiles-Wyszecki⁴ field trials of the proposal consisted of color matches for 18 filters obtained on a three-primary Donaldson colorimeter by ten observers. As a part of the field trials the work of the National Research Council was repeated on the Donaldson colorimeter located at the National Bureau of Standards. Color matches of only 11 of the 18 NRC filters yielding colors predicted by the supplementary observer to fall within the three-primary gamut were made by eleven observers in the NBS repetition of the field trial. This paper describes the results of the NBS field trial when the colorimeter is calibrated in terms of the 1931 2° standard-observer system and in terms of the 1959 10° proposed supplementary-observer system. All observations, however, were made with an approximately 10° field.

2. CALIBRATION OF THE COLORIMETER

a. Description

The Donaldson colorimeter,⁵ schematically shown as Fig. 1, is a visual colorimeter in which the observer judges his achievement of a match that appears in the double-trapezoid field of a Lummer-Brodhun photometer cube L . The visual-field trapezoid on one side and the surround on the other side are illuminated by light

that is emitted from the test source S_T reflected by a vitreous diffusing plate V and transmitted by the trial filter F_T . The other parts of the visual field are illuminated by light that is emitted from the "primary source," transmitted by six primary filters F_P (red, orange, yellow-green, green, blue-green, and blue) focused by a condenser lens L_P into an MgO-lined integrating sphere S and reflected by a 45° prism. The light on the test side is varied by changing the distance or by inserting a sector between S_T and V ; the light on the primary side is varied by changing the shutter openings before the primary filters or by moving the rotating star wheel through which light passes from the sphere to the eye of the observer. During the field trials only the shutters located before the red, green, and blue primary filters were opened, thereby constituting a three-primary colorimeter. The operating voltage on each of the sources was set so that their output had a spectral energy distribution consistent with the color temperature of CIE Source A .

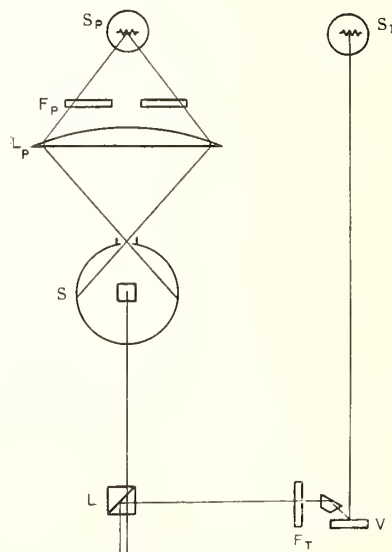


FIG. 1. Schematic diagram of Donaldson colorimeter.

* Recommended for adoption at the June 1963 meetings of the Commission Internationale de l'Eclairage, held in Vienna, Austria, by the Experts of Committee E 1.3.1.

¹ D. B. Judd, Proc. CIE (Brussels) Vol. A⁷(1959).

² W. S. Stiles and J. M. Burch, Opt. Acta 6, 1 (1959).

³ N. I. Speranskaya, CIE Comm. Repts., NPL Symp. 8 1, 313 (1958); Opt. Spectry. 7, 429 (1959) [Opt. i Spektroskopiya 7 (1959)].

⁴ W. S. Stiles and G. Wyszecki, J. Opt. Soc. Am. 52, 58 (1962).

⁵ R. Donaldson, Proc. Phys. Soc. (London) 59, 554 (1947).

Six auxiliary filters, not shown in Fig. 1, intended to be identical to the six primary filters, are also provided with the colorimeter. These filters have two purposes: (1) in calibration of the instrument and (2) in the use of the instrument for color matching. In calibration of the colorimeter the auxiliary filters are placed, in turn, at the trial-filter position so that the observer can determine the primary shutter settings for a luminance match between the primary and auxiliary filters. In the use of the colorimeter for color matching these filters are placed, in turn, between the photometric cube and the observer's eye to enable him to equate radiant flux from the trial beam to that from the primary filter. In this way the beam synthesized from the primaries is adjusted to a near energy match for the trial beam throughout the visible spectrum.

b. Calibration Theory

Tristimulus values, X_m , Y_m , Z_m , of any mixture m of the n primaries are, by Grassmann's additivity law, simply the sums of the tristimulus values of the components contributed by the working primaries. If X_n , Y_n , Z_n , are the calibration coefficients, that is the tristimulus values of one unit ($R_{nm}=1$) of the n th primary, then X_m , Y_m , Z_m may be written as:

$$\begin{aligned} X_m &= \sum_n X_n R_{nm}, \\ Y_m &= \sum_n Y_n R_{nm}, \\ Z_m &= \sum_n Z_n R_{nm}. \end{aligned} \quad (1)$$

The amounts R_{nm} of the primaries fail to be precisely equal to the scale readings r_{nm} of the shutter openings. This failure results chiefly because the filter irradiance varies from one end of the filter to the other. The primary energy R_{nm} transmitted by the filter-shutter combination was therefore photometrically evaluated at the exit pupil as a function of the shutter setting r_{nm} . This function was expressed as a polynomial in r_{nm} of the form

$$R_{nm} = q(r_{nm}) + r(r_{nm})^2 + s(r_{nm})^3, \quad (1a)$$

where q , r , and s are constants found by least-squares fitting.

Source- A chromaticity coordinates, x_n , y_n , z_n , of the light from the n primary filters were computed from the spectral characteristics of the colorimeter components. Luminous transmittance, Y_{na} , of the auxiliary filters that have closely similar Source- A chromaticities were also computed. Computations were made for the 1931 standard and the 1959 supplementary observer systems. The measured spectral characteristics of these components are listed in Table I.

Tristimulus values of the n primaries are related to their chromaticity coordinates by

$$\begin{aligned} X_n &= x_n(X_n + Y_n + Z_n), \\ Y_n &= y_n(X_n + Y_n + Z_n), \\ Z_n &= z_n(X_n + Y_n + Z_n). \end{aligned} \quad (2)$$

Calibration was effected as follows. The illuminance of the auxiliary filters, inserted in turn at the trial-filter position, was set to an arbitrary value E . Several observers adjusted the shutter opening in front of the corresponding primary filters to a value r_{nm} required to produce a brightness match between the light from the two filters. Average values of r_{na} were then converted to values of R_{na} by Eq. (1a). As the auxiliary filter had nearly the same chromaticity as the corresponding primary filter the settings produced nearly perfect color matches. For a brightness match between the light transmitted by the auxiliary filter and that transmitted by the corresponding primary filter we have

$$EY_{na} = Y_n R_{na}, \quad \text{or} \quad Y_n = EY_{na}/R_{na}.$$

But as

$$Y_n = y_n(X_n + Y_n + Z_n),$$

then

$$X_n + Y_n + Z_n = EY_{na}/R_{na}y_n.$$

Therefore, the required calibration coefficients are

$$\begin{aligned} X_n &= (EY_{na}/R_{na}y_n)x_n, \\ Y_n &= (EY_{na}/R_{na}y_n)y_n, \\ Z_n &= (EY_{na}/R_{na}y_n)z_n. \end{aligned} \quad (3)$$

One of the complicating features of the Donaldson colorimeter is that the two optical paths, test and primary, do not exactly balance each other. In the test path light is diffusely reflected by the vitreous plate, while in the primary path light is transmitted through the condensing lens and the integrating sphere. The chromaticity coordinates of the test filters and the primary filters and the luminous transmittance of the auxiliary filters used in the calibrations include the spectral characteristics of the corresponding optical components. The spectral transmittances of red, green, and blue primary and auxiliary filters, T_{RP} , T_{GP} , T_{BP} , T_{RA} , T_{GA} , T_{BA} , respectively, and the condenser lens, t_L , and the spectral reflectance of the vitreous diffusing plate, R_V , were measured on the NBS Cary 14 spectrophotometer. The spectral transmittance of the integrating sphere, t_s , was measured on the NBS ratio-recording spectroradiometer. The spectral characteristics of the combination of the integrating sphere and the condenser lens, T_L , was obtained from the product $t_L t_s$. These spectral characteristics are listed in Table I.

c. Calibration Coefficients

Calibration coefficients, X_n , Y_n , Z_n , for the three primaries $n=R, G, B$ (red, green, blue, respectively), were derived for the 1931 2° standard observer system and the 1959 10° proposed supplementary observer system. These calibration coefficients, of course, have the same relative proportions as the chromaticity coordinates of the primaries for the observer system; see Eq. (3). The results thus obtained are listed in Table II for the two

TABLE I. Spectral characteristics of Donaldson colorimeter components.

λ (nm)	E_A	T_{RP}	T_{RA}	T_{GP}	T_{GA}	T_{BP}	T_{BA}	T_L	R_V
380	0.0979					0.1811	0.1698	0.3710	0.8950
390	0.1209					0.2490	0.2372	0.3840	0.8850
400	0.1471					0.3004	0.2909	0.398	0.8750
410	0.1768					0.3317	0.3238	0.410	0.865
420	0.2100					0.3466	0.3403	0.420	0.859
430	0.2467					0.3381	0.3338	0.431	0.857
440	0.2870				0.0000	0.3166	0.3137	0.445	0.856
450	0.3309			0.0000	0.0001	0.2764	0.2742	0.453	0.863
460	0.3782			0.0002	0.0002	0.2155	0.2133	0.462	0.871
470	0.4287			0.0008	0.0009	0.1249	0.1237	0.472	0.874
480	0.4825			0.0020	0.0024	0.0480	0.0483	0.4800	0.876
490	0.5391			0.0107	0.0176	0.0122	0.0123	0.4889	0.878
500	0.5986			0.0680	0.0891	0.0031	0.0034	0.4939	0.879
510	0.6606			0.1432	0.1556	0.0005	0.0005	0.4997	0.882
520	0.7250			0.1541	0.1565	0.0001	0.0001	0.5029	0.884
530	0.7913			0.1202	0.1199	0.0000	0.0000	0.5065	0.885
540	0.8595	0.0000	0.0000	0.0793	0.0787			0.5082	0.886
550	0.9291	0.0001	0.0001	0.0453	0.0449			0.5099	0.886
560	1.0000	0.0002	0.0002	0.0229	0.0228			0.5087	0.886
570	1.0718	0.0003	0.0003	0.0104	0.0103			0.5048	0.885
580	1.1444	0.0006	0.0006	0.0043	0.0043			0.5020	0.883
590	1.2173	0.0009	0.0009	0.0016	0.0016			0.4974	0.880
600	1.2904	0.0011	0.0011	0.0006	0.0006			0.4867	0.878
610	1.3634	0.0014	0.0014	0.0002	0.0002			0.4810	0.875
620	1.4362	0.0024	0.0025	0.0001	0.0001			0.475	0.872
630	1.5083	0.0080	0.0084	0.0000	0.0000			0.467	0.869
640	1.5798	0.0362	0.0375					0.461	0.867
650	1.6503	0.1050	0.1086					0.454	0.865
660	1.7196	0.1822	0.1849					0.448	0.863
670	1.7877	0.2480	0.2203					0.441	0.862
680	1.8543	0.2828	0.2835					0.435	0.861
690	1.9193	0.2979	0.2992					0.428	0.860
700	1.9826	0.3010	0.3020					0.421	0.859
710	2.0441	0.2965	0.2965					0.415	0.857
720	2.1036	0.2866	0.2875					0.408	0.856
730	2.1612	0.2748	0.2754					0.402	0.855
740	2.2166	0.2612	0.2612					0.395	0.853
750	2.2700	0.2480	0.2483					0.389	0.852
760	2.3211	0.2351	0.2348					0.382	0.851
770	2.3701	0.2206	0.2214					0.377	0.850

observer systems. Figure 2 shows the chromaticity coordinates of the primaries for the two observer systems.

3. FIELD TRIAL

a. Filters

The Stiles–Wyszecki field trial⁴ of the 1959 supplementary observer was conducted on 18 filters. The chromaticity coordinates of seven of these fell outside the gamut of the three primaries of the Donaldson colorimeter. Although the NBS Donaldson colorimeter is equipped with a desaturating beam, this part of the colorimeter had not been calibrated at the time the trials were conducted. Consequently, only the 11 filters, yielding characteristics that lie within the three-primary

gamut were used in the NBS repetition of the field trial. The chromaticity coordinates predicted by the 1931 observer system and the 1959 supplementary system are listed in Table III and shown in Fig. 2.

These predicted chromaticities do not agree precisely with the chromaticities predicted from the NRC spectral transmittance measurements. The predicted NBS chromaticities are closer to Source A than those predicted by NRC. Such differences in spectrophotometry between two laboratories may result from either overcorrection by one laboratory or undercorrection by the other laboratory for photometric scale errors, particularly zero error. In general the disagreement between the results obtained by NRC and NBS is of the order of that found in other comparisons.

TABLE II. Calibration coefficients and chromaticity coordinates of the primaries of the Donaldson colorimeter.

Primaries	1931 observer system						1959 observer system					
	X_n	Y_n	Z_n	x_n	y_n	z_n	X_n	Y_n	Z_n	x_n	y_n	z_n
<i>R</i>	1.8429	0.7046	0.0000	0.723	0.277	0.000	1.4213	0.5807	0.0000	0.710	0.290	0.000
<i>G</i>	0.8756	3.5129	0.4188	0.182	0.731	0.087	0.9795	3.1562	0.2692	0.222	0.716	0.061
<i>B</i>	1.1003	0.1840	5.9054	0.153	0.026	0.821	1.1062	0.3651	5.9734	0.149	0.049	0.802

TABLE III. Chromaticity coordinates of field-trial filters predicted by the 1931 standard observer and the 1959 supplementary observer.

F. T. filters	1931 observer system			1959 observer system		
	x	y	z	x	y	z
3	0.211	0.259	0.530	0.214	0.274	0.512
5	0.295	0.315	0.390	0.296	0.317	0.387
6	0.248	0.417	0.334	0.262	0.422	0.317
7	0.348	0.415	0.237	0.356	0.413	0.230
8	0.423	0.420	0.157	0.430	0.417	0.153
9	0.287	0.606	0.107	0.317	0.595	0.088
14	0.645	0.314	0.042	0.635	0.321	0.044
15	0.561	0.340	0.099	0.554	0.345	0.101
16	0.511	0.266	0.223	0.495	0.271	0.234
17	0.380	0.263	0.357	0.370	0.268	0.362
18	0.354	0.196	0.450	0.337	0.210	0.453

b. Observers

The observers were selected from among the available persons as those who had normal color vision, and had relatively little difficulty in making reasonably duplicable matches. Some attention was given to the individual's age and complexion so that we would obtain a group of observers to represent all ages and complexions. How well we were able to make a representative selection may be seen in Table IV.

c. Procedure

During the field trials performed at the NBS, the test source was placed at convenient distances from the trial filters. Each observer was asked to adjust the primary-shutter openings and the movable rotating star wheel to obtain a color (chromaticity and luminance)

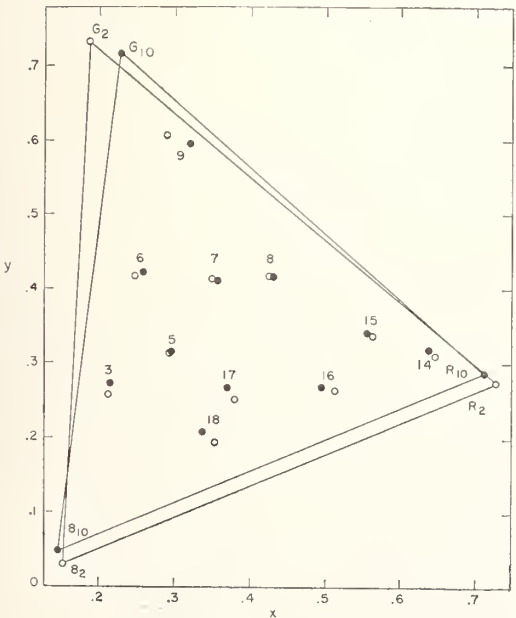


FIG. 2. Chromaticity coordinates of the field-trial filters in the 1931 CIE standard observer system \circ and in the 1959 proposed observer system, \bullet .

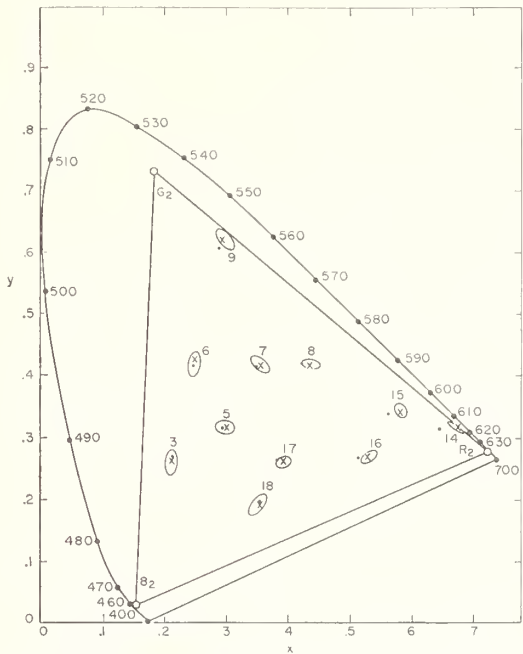


FIG. 3. NBS observations of chromaticity coordinates of the field-trial filters in the 1931 CIE standard observer system compared with predictions by that system. Predictions \bullet , observation average \times , observation boundary, ellipse.

match between the two viewing fields. The positions of the shutter for the red, green, and blue primaries r_{Rc} , r_{Gc} , r_{Bc} , were then noted. Each of the 11 observers made two matches on the 11 filters. The two matches were made on separate occasions to minimize memory effects. From these data the chromaticity coordinates of the mixture of the primaries could be obtained in the two systems, the 1931 standard-observer, and the 1959 proposed supplementary-observer system.

Thus, if $X_R \cdots Z_B$ are calibration coefficients and $R_{Rc} \cdots R_{Bc}$ are primary energies, the tristimulus values X , Y , and Z of the trial filters are determined by the following equations:

$$\begin{aligned} X &= X_R R_{Rc} + X_G R_{Gc} + X_B R_{Bc}, \\ Y &= Y_R R_{Rc} + Y_G R_{Gc} + Y_B R_{Bc}, \\ Z &= Z_R R_{Rc} + Z_G R_{Gc} + Z_B R_{Bc}. \end{aligned}$$

The chromaticity coordinates, x , y , and z , can be

TABLE IV. Ages and complexions of the observers employed in the trials.

Age range (years)	Number of observers of complexion		Totals
	Fair	Dark	
20-30	0	2	2
30-40	2	1	3
40-50	2	0	2
50-60	1	1	2
60-70	1	1	2
Totals	6	5	11

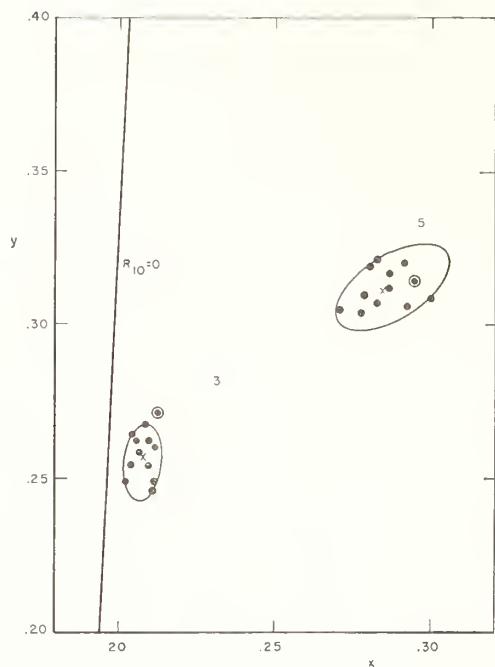


FIG. 4. Individual observations for field-trial filters No. 3 and No. 5. Individual observations ●, observation average ×, predictions ○.

computed from

$$\begin{aligned}x &= X/(X+Y+Z), \\y &= Y/(X+Y+Z), \\z &= Z/(X+Y+Z).\end{aligned}$$

4. RESULTS

Figure 3 shows the results obtained with the colorimeter calibrated to make the trial-filter chromaticity predictions in the 2° observer system while observations are made with a 10° viewing field. In Figs. 3 through 10 the observation boundaries are ellipses drawn to include all or most of the observed chromaticities. Consequently, these ellipses may not be centered at exactly

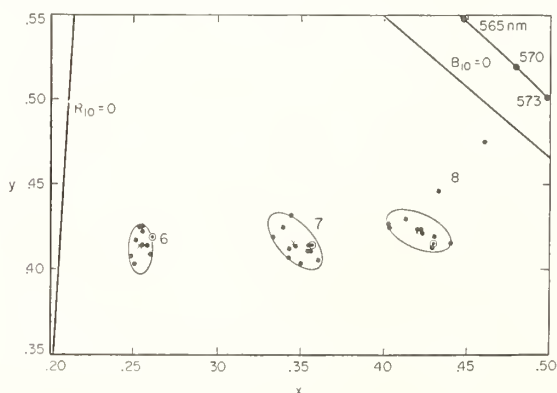


FIG. 5. Individual observations for field-trial filters Nos. 6, 7, and 8. Individual observations ●, observation average ×, predictions ○.

TABLE V. Ages and complexions of the outlying observers.

F. T. filters	Complexion	Age ranges (years)
8	Fair	60-70
	Dark	60-70
15, 16	Dark	20-30
	Fair	60-70
	Dark	60-70
17	Dark	30-40
	Dark	50-60
	Fair	60-70
18	Dark	50-60

the average chromaticity point. The average point, however, does lie well within the elliptical boundary. The data of Fig. 3 show conditions of agreement between observations and predictions in the blue-purple to blue-

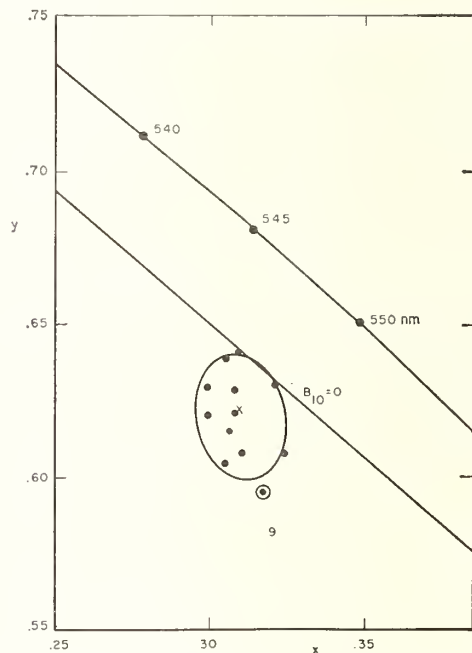


FIG. 6. Individual observations for field-trial filter No. 9. Individual observations ●, observation average ×, prediction ○.

green regions, borderline agreement in the near-neutral region, and disagreement in the green to red regions. The term agreement means that the predicted chromaticity point lies among the chromaticity points determined from the settings made by the observers.

Figures 4 through 8 show the average chromaticity coordinates obtained by the individual observers for the 11 field-trial filters when the colorimeter is calibrated so that predictions are made for the 10° proposed-observer system while observations are made with a 10° viewing field. Also shown in Figs. 4 through 8 are the boundaries of these observations. The boundaries for the field-trial filters 8, 15, 16, 17, and 18 do not include the data of all of the observers. These outlying observers were dark complexioned, over age 60, or both as shown

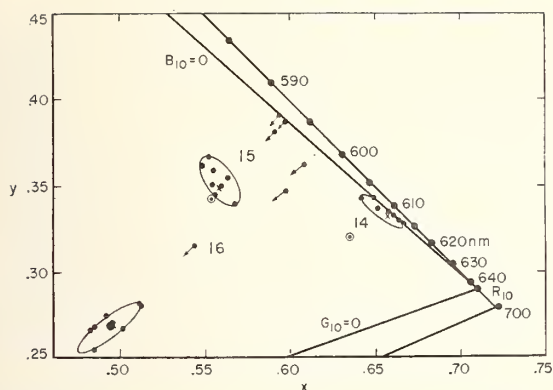


FIG. 7. Individual observations for field-trial filters Nos. 14, 15, and 16. Individual observations \bullet , observation average \times , prediction: Arrows point toward the average observed chromaticity of the filter to which the outlying observations belong.

in Table V. The data for these observers were neglected in determining the boundaries because, it is supposed, the heavy ocular pigmentation of the observers may have produced observations that differ considerably from those expected for observers aged 30. The same data are shown in Fig. 9 to reveal the conditions of agreement between observations and predictions in the blue-green to yellow regions, borderline in the blue-purple region, and disagreement in the red and green regions.

Because of unavoidable spectrophotometric errors the chromaticity-coordinate predictions made by the 10° observer system, as well as those made by the 2° observer system, are uncertain. On the basis of analysis of errors of spectrophotometric measurements⁶ of filters

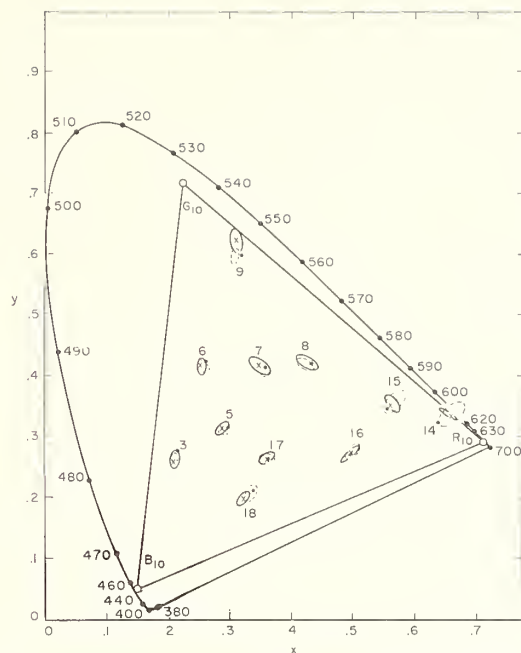


FIG. 9. NBS observations of chromaticity coordinates of the field-trial filters in the proposed observer system compared with NRC observations and with predictions by that system. Prediction \bullet , NBS observation average \times ; NBS observation boundary, solid ellipse; NRC observation boundary, dashed ellipse.

similar to those of this field trial, the uncertainties in the predicted chromaticity coordinates of the field-trial filters are estimated to be about one-tenth of the maximum difference among the observers.

Also shown in Fig. 9 are the boundaries of the chroma-

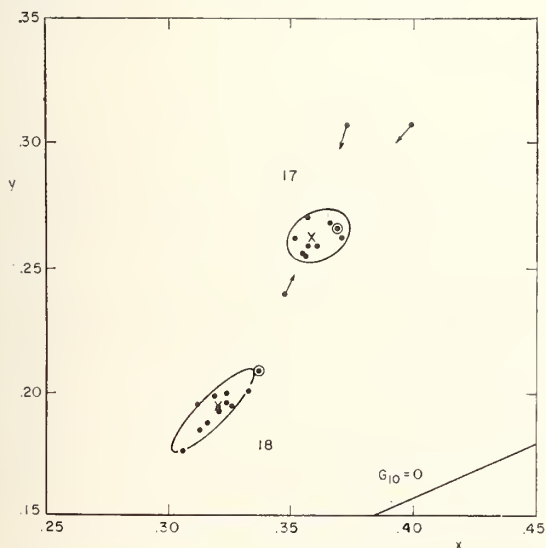


FIG. 8. Individual observations for field-trial filters Nos. 17 and 18. Individual observations \bullet , observation average \times , prediction: Arrows point toward the average observed chromaticity of the filter to which the outlying observations belong.

⁶ H. J. Keegan, J. C. Schleiter, and D. B. Judd, J. Res. Natl. Bur. Std. 66A, 203 (1962).

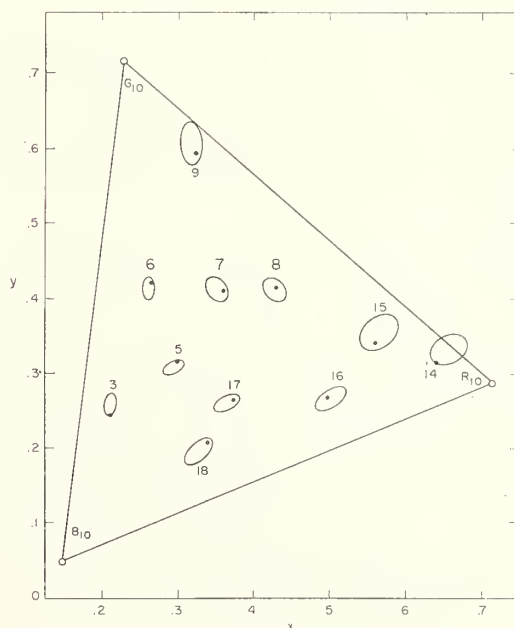


FIG. 10. Combination of NRC and NBS observations compared with predictions by the 1959 proposed observer system. Prediction \bullet , NRC-NBS observation boundary ellipse.

ticity coordinates determined from the observers in the field trials conducted by Stiles and Wyszecki at the National Research Council. The NRC observer data show, in general, the same degree of disagreement with predictions by the proposed observer system as do the data obtained by the NBS observers. In some instances the NRC observer data are in closer agreement with predictions by the proposed system while in others the NBS data are in closer agreement.

In Table VI are listed the comparisons of the large-viewing-field results obtained in the two field trials, NBS and NRC, with the predictions made by the 2° and 10° observer systems. It may be noted that the data obtained in each of these field trials of the proposed system for large field viewing do not agree quite satisfactorily with the predictions made by the 10° system. Both sets of data, however, show some improvement over the agreements of the predictions made by the 2° standard observer system for observations made with

TABLE VI. Comparison of observation agreement with predictions by the 2° and 10° observer systems. (A=agreement, B=boderline, D=disagreement).

F. T. filters	2°			10°						Combined		
	A	B	D	A	B	D	A	B	D	A	B	D
3	X				X				X			X
5	X			X				X		X		
6	X			X				X		X		
7		X		X				X		X		
8		X		X			X			X		
9			X			X			X			X
14			X			X		X		X		
15			X	X			X			X		
16			X	X			X			X		
17		X		X			X			X		
18	X			X				X				X
Total	4	3	4	5	4	2	4	5	2	8	3	0

wide-field viewing. The improvement is indicated by the totals shown in Table VI.

It can be seen from Fig. 9 that the location and direction of the NBS boundary ellipses for all filters but 14 and 15 are in substantial agreement with those of NRC. If the outlying observations for filter 15 had been included, the boundary ellipse for the NBS observers would have agreed well with that for the NRC observers for this filter also. In the matching of Filter 14, contrary to the prediction by the supplementary observer, two observers indicated that they would like to have had the use of a desaturating beam. One of these observers, over age 60, gave outlying results for Filters 15, 16, and 17, while the other observer, also over age 60, gave outlying results for Filters 15 and 16. Consequently, it may be stated that the difference between NBS and NRC boundary ellipses for Filters 14 and 15 arises from our practice of neglecting outlying results ascribable to abnormally heavy ocular pigmentation.

It may be argued that if in each of these two field trials, NRC and NBS, of the proposed supplementary-observer system an insufficient number of observers were employed to make matches, then the two sets of data should be combined. It may be suggested that because these two field trials were conducted on two different instruments, combination of their results may not be rigorously justified. This objection may be raised despite the fact that the proposed 10° field observer system itself is derived from essentially three different sets of color matching data, Stiles' two sets, and Speranskaya's one set. Nimeroff has discussed the difference between the two sets of Stiles data.⁷

There is this to be said in behalf of the combination of the data of these two field trials: The matching data were obtained with the very same filters in both trials, the instruments were of the same type and were calibrated in essentially the same manner, and in each trial the same basic observer system was used. Notwithstanding any proof or disproof of the justification, an attempt can be made to combine the results to determine whether employment of an increased number of observers in either or both trials would have given better agreement. Figure 10 shows the boundaries of the chromaticity coordinates determined from the combined observers settings in both trials.

If the combined results do represent the condition for an increased number of observers in either of the field trials, a tabulation of the combined results of comparisons between predictions with a 10° observer system and the large-field observations obtained in the NRC and the NBS field trials may indicate the success or failure of the proposed system. The list of the comparisons is given in Table VI and indicates a decided improvement of agreement over the two individual field trials, NRC and NBS.

5. ENERGY MATCHES

In each of the Donaldson-colorimeter trials of the 1959 proposed supplement to the standard-observer system, all of the matches achieved by the observers have been metameric. This metamerism resulted from the fact that the colorimeter was a three-primary device. In making the matches of light that passed through the field-trial filters the observers saw light emitted by the test source, passed through the filter, and reflected from the vitreous diffusing plate. This light was matched by the light that was emitted by the primary source and transmitted through the primary filters, the primary shutters, the lens and the sphere. This match may be represented by the equation

$$\int_0^\infty (E_A T_f R_V) d\lambda \equiv K_y \sum_n \frac{Y_{nc} R_{nc}}{Y_n} \int_0^\infty (E_A T_N T_L) d\lambda, \quad (4)$$

⁷ I. Nimeroff, J. R. Rosenblatt, and M. C. Dannemiller, J. Opt. Soc. Am. **52**, 685 (1962).

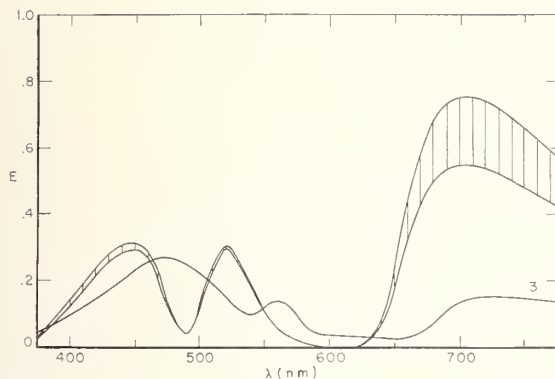


FIG. 11. Range of spectral energy distribution of match compared with transmitted energy for field-trial filter No. 3.

where E_A is irradiance, in arbitrary units, from Source A per unit wavelength interval; T_f is spectral transmittance of the field-trial filters; R_V is spectral reflectance of the vitreous diffusing plate; M is observed color match; T_n is spectral transmittance of the n primary filters, $n=R, G, B$; T_L is spectral transmittance of the sphere and lens; R_{nc} is transmittance of the primary shutters; Y_{nc} is luminance coefficients (calibration) of the primaries; and Y_n is luminous transmittance of the primaries. Because the matches were made at equal luminance as well as at equal chromaticity, a proportionality factor, K_y , is required which is defined as

$$K_y = \int_0^\infty (E_A T_f R_V \bar{y}) d\lambda / \sum_n \frac{Y_{nc} R_{nc}}{Y_n} \int_0^\infty (E_A T_n T_L \bar{y}) d\lambda,$$

where \bar{y} is the luminosity (relative luminous efficiency) function.

Figures 11, 12, and 13 show the ranges of energy distributions for the matches set by the observers in the NBS trial for Field-Trial Filters 3, 9, and 17. The energy distributions transmitted by the trial filters are also shown in Figs. 11, 12, and 13, thereby indicating the metamerism of the matches. It is this metamerism that renders the trial filters good tests of the proposed supplement to the standard observer system. The ranges of the energy distributions for the matches set by the observers in the NBS trial for the remaining eight filters are similar to the ranges of these filters. The energy distributions transmitted by the trial filters, together with the matching distributions of the NRC Donaldson colorimeter as predicted by the 10° supplementary observer system, already have been shown by Stiles and Wyszecki.⁴

6. ADDITIVITY FAILURE

Additivity failure has been suggested⁴ as a possible cause of the difference between predicted and observed chromaticity coordinates. Figure 14 shows the author's⁸ results of a direct check on the effect of additivity failure under the extremely metameric conditions of matching

⁸ I. Nimeroff, DIC thesis, Imperial College, University of London.

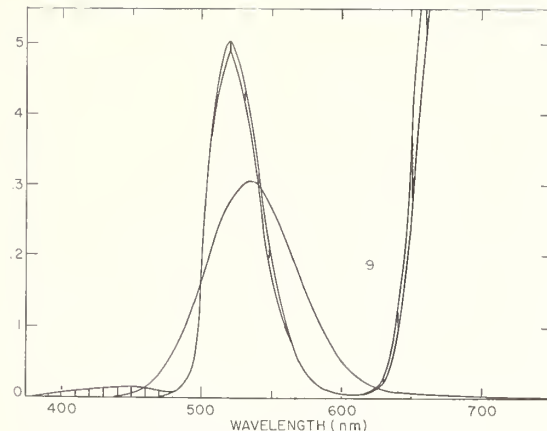


FIG. 12. Range of spectral energy distribution of match compared with transmitted energy for field-trial filter No. 9.

combinations of 480 with 580 nm and 490 with 590 nm matched by the three primaries of the Wright visual tristimulus colorimeter, 460, 530, and 650 nm. From these data we might expect additivity failure to cause blue to green colors to be matched bluer and yellow to red colors to be matched redder than predicted by Grassmann's law⁹ of additivity.

As the Donaldson colorimeter yields matches that are metameric, as seen on Figs. 11, 12, and 13, we attempted to determine whether the difference found between predictions by the proposed supplementary 10°-observer system and observations were in accord with differences indicated by the additivity-failure experiments. From the energy distribution for the matches obtained by two of the observers and their individual color-matching functions, chromaticity coordinates were computed in the system from which their color-matching functions were derived. The color-matching functions for one observer (IN) were obtained in the Wright (WDW) system of red, green, and blue primaries, 650, 530, and 460 nm, respectively, while those of the other observer (DBJ) were obtained in the Stiles

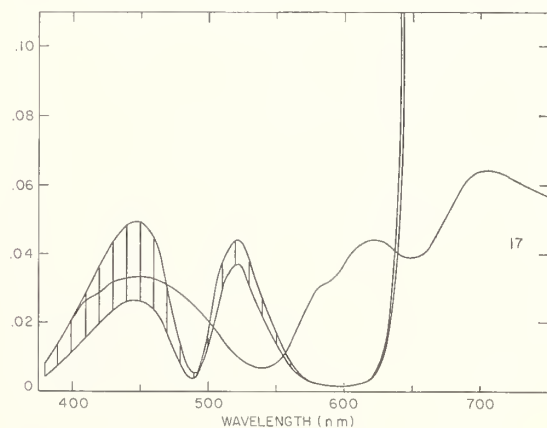


FIG. 13. Range of spectral energy distribution of match compared with transmitted energy for field-trial filter No. 17.

⁹ H. Grassmann, Poggendorf's Ann. 89, 69 (1853).

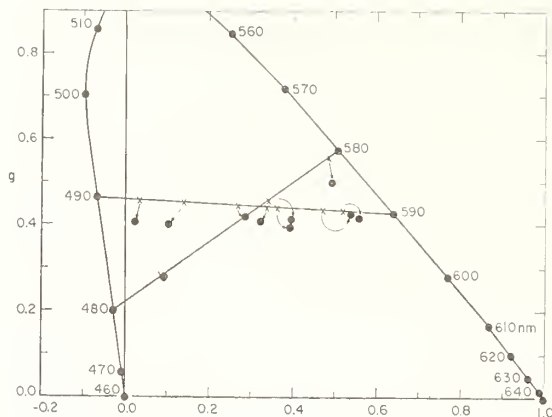


FIG. 14. Direct additivity check by author obtained on the Wright visual tristimulus colorimeter.

(WSS) system of red, green, and blue primaries, 645.2, 536.3, and 444.4 nm, respectively.

The chromaticity coordinates in the individual system derived from the system of filters can be compared with the chromaticities predicted from the matches. Figure 15 shows the chromaticity coordinate data for one observer. The directions of the differences are generally in accord with the additivity-failure predictions shown in Fig. 14 but are smaller in magnitude. The blue-purple-to-blue-green field-trial filters (18, 3, 5, 6, and 9) show shift toward stronger blue, the purple-to-neutral field-trial filters (17, 7, and 8) show little shift, while the red-to-magenta field trial filters (14, 15, and 16) show rather large shifts toward orange. The same trend of general consistency with additivity-failure predictions is indicated in Fig. 16.

It is reasonable to conclude from the data of these two observers that the lower-magnitude departures result from the decrease of metamerism in the Donaldson field trials compared with the metamerism in the direct additivity investigation where color matches were made with monochromatic lights.

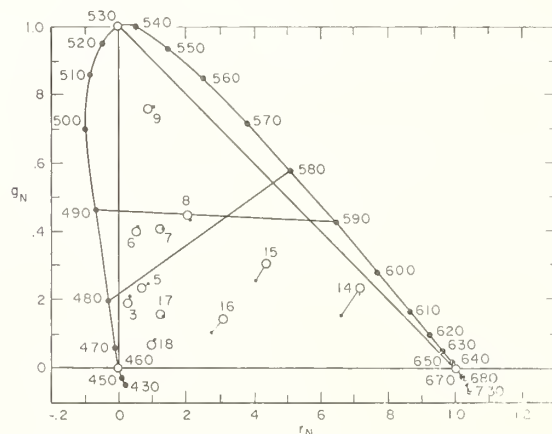


FIG. 15. Chromaticity coordinates of the field-trial filters in Nimeroff's 10° observer system obtained with the WDW primaries. Prediction ●, match ○.

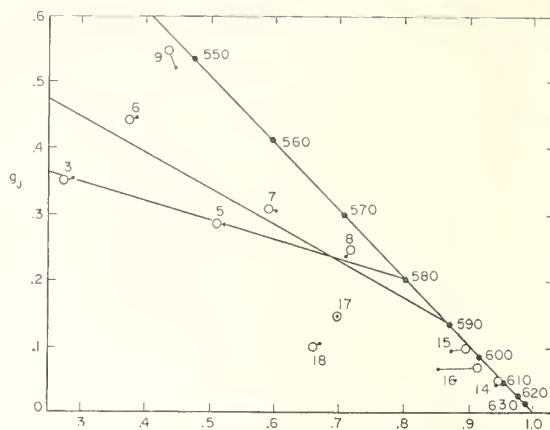


FIG. 16. Chromaticity coordinates of the field-trial filters in Judd's 10° observer system obtained with the WSS primaries. Prediction ●, match ○.

Aguilar and Stiles¹⁰ have found that when large-field color-matching observations are made under relatively high levels of retinal illuminance the rods become saturated and the response is entirely the contribution of the cones. In intermediate and low levels of illuminance, however, the combined system of rod and cone response leads to additivity failure if metamerism is present. On the other hand, if there is very little or no metamerism, additivity failure is negligible or nonexistent, regardless of the level of illuminance. To establish the joint effect of the degree of metamerism and level of illuminance on the magnitude and direction of additivity failures would require further study.

7. CONCLUSION

We may conclude from these Donaldson-colorimeter field trials that the proposed supplementary observer makes better predictions of chromaticity for large-field viewing conditions than does the present standard-observer system. These field trials, conducted independently at NRC and at NBS each do not yield completely satisfactory agreement with predicted chromaticities. It is suggested, however, that in both of these trials an insufficient number of observers was employed. The combining of these two independent field trials, wherein about 20 observers are employed, results in a decided improvement of agreement between predicted and observed chromaticity coordinates.

ACKNOWLEDGMENTS

The author is pleased to acknowledge the encouragement, advice, and assistance afforded him by the following persons: D. B. Judd for guidance and helpful criticism, G Wyszecki for the loan of the field-trial filters, and J. C. Schleter for programming the extensive calculations for automatic computation. Thanks are given also to the persons who served as observers.

¹⁰ M. Aguilar and W. S. Stiles, *Opt. Acta* 1, 57 (1954).

Lightness Change of Grays Induced by Change in Reflectance of Gray Background*

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A relatively small gray sample looks lighter (darker) when it is placed on darker (lighter) background. This phenomenon is an example of the well-known contrast effect. The purpose of this paper is to make a quantitative determination of the effect, and to derive a formula for it.

For any given gray sample and gray surround on the left side, the observer was instructed to choose, for the different gray surround on the right, a different gray sample appearing as light as that on the left. It was found that sample lightness changes rapidly with reflectance when sample reflectance is close to that of the background. This effect was named the "crispening effect."

Several models (von Kries coefficient law, Hurvich-Jameson induction) were tried, but none of them reproduced the experimentally discovered crispening effect. A fairly successful empirical formula was developed by adding a term for the crispening effect to the formula for the induction theory.

INDEX HEADINGS: Vision; Reflectance.

1. INTRODUCTION

THE lightness perceived for a gray sample depends on the reflectance of the background. A gray sample on a black background looks much lighter than the same gray sample on a white background.

This is an example of the contrast effect discussed in detail by Helmholtz.¹ There are many qualitative explanations of contrast but very few quantitative studies except for the work of Jameson and Hurvich.²

The purpose of this paper is to make a quantitative determination of the effect, and to derive a formula for it. That is, given the reflectances Y_{b1} and Y_{b2} of two backgrounds, and the reflectance R_1 of a sample on the first background, find a method of determining the reflectance Y_2 needed for a sample on the second background to make it have the same lightness as the first sample.

2. EXPERIMENTAL PROCEDURE

Two series of gray samples are mounted on plywood wheels in order. One is the special 60-step series produced by the Munsell Company.³ These gray samples were mounted on a 14-in. wheel and placed in front of the observer to his right. The other series (eleven samples) is composed of every sixth sample extracted from the 60-step series. The extracted series is mounted on a 6-in. wheel and placed at the left front of the observer.

Two 5- by 8-in. gray backgrounds with $\frac{1}{2}$ -in. sq windows at the centers are placed side by side along the 8-in. side, and also on top of the wheels so that the

gray samples on the two wheels appear in the windows. The left-hand background and samples are referred to as standard background and standard samples; and those at right, are referred to as second background and matching samples.

The task of the observer is to adjust the large wheel so that the standard and matching samples look equally light. To make the uncertainty of the measurement clear, a limit method was used; that is, two matching samples, one which looks a little lighter and one a little darker than a given standard, are chosen by the observer. The observer is allowed to look from standard sample to matching sample, and vice versa, as he does when he compares two objects in daily life.

The background pair was interchanged left to right to eliminate the effect of possible nonuniform illumination. The background pairs are N1/-N9/, N3/-N7/, N4/-N6/, N1/-N5/, and N5/-N9/.

3. PRELIMINARY OBSERVATIONS

Figure 1 shows some results of observer T. The range of reflectance of the matching sample, which looks as light as the standard sample on the standard background, is plotted against the reflectance of the standard sample on a log-log diagram.

When the two backgrounds are the same there is no lightness change. The straight 45° line shows this situation. When the second background is lighter (darker) than the standard background, a sample with higher (lower) reflectance looks as light as the standard sample on the standard background. The observation is made for the background pair of neutral 5 and 7, 3 and 7, and 1 and 9.

An unexpected hump in the lightness-change curve was observed, besides the anticipated general shift.

Some preliminary observations were made to explore the reason for the hump.

First, the illuminance was changed from 100 lx to 600 lx. No significant change was observed.

Second, the width of the test area was reduced by

* Presented at Spring 1965 Meeting of Optical Society of America, J. Opt. Soc. Am. 55, 609A (1965).

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¹ H. V. Helmholtz, *Treatise on Physiological Optics*. Translated from the 3rd German edition, J. P. C. Southall, Ed. (The Optical Society of America, 1924; reprinted by Dover Publ., Inc., New York, 1964), Vol. 2, p. 231.

² D. Jameson and L. M. Hurvich, J. Opt. Soc. Am. 51, 46 (1961).

³ W. C. Granville, D. Nickerson, and C. E. Foss, J. Opt. Soc. Am. 33, 376 (1943).

factors of $\frac{1}{2}$ down to $\frac{1}{8}$ of the original width. The hump increased its magnitude with decreasing test-area width but not by much. The $\frac{1}{2}$ -in.-sq test area shows almost the full extent of the hump.

However, by changing the average Munsell value of the backgrounds, but keeping the value difference equal to 4, both the position and magnitude of the hump changed as shown in Fig. 2. The hump must be caused by the relation between the reflectance of the samples and backgrounds. The curve may be said to be composed of a general shift plus a hump.

So far the left sample and background were considered to be standard; but by interchanging backgrounds left to right, a second curve is obtained. If interchanging backgrounds had no influence on the observation, this second curve would be symmetrical with the first about the 45° line.

This means that the curves shown below the 45° line may be symmetrically reflected into the upper area; these reflected curves are comparable with the corresponding curves originally plotted above the 45° line.

In practice there is considerable difference, which can be attributed to possible nonuniformities of illumination and visual response. These effects may be eliminated by plotting the two series of observations above the 45° line.

4. COMPARISON OF SEVERAL ASSUMPTIONS

Jameson and Hurvich applied the induction theory to their work on color contrast.² A model analogous to

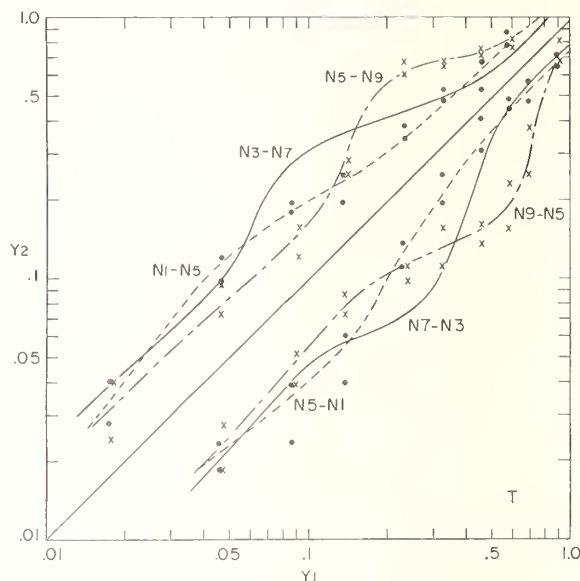


FIG. 2. The effect of average Munsell value of background pair on reflectance of the matching sample. Background pairs are chosen to have the same Munsell value difference 4 but of progressively higher average Munsell value. Curves are fitted to the plotted points by eye. Data points for N3-N7 and N7-N3 confirm those shown in Fig. 1, and are omitted for clarity.

their theory says that a certain part of the excitation from the background is inversely induced on the test area and reduces its lightness. Applying this model directly to the reflectances of the test and background areas yields lightness estimation for test areas 1 and 2:

$$L_1 = f(Y_1 - CY_{b1}) \quad (1)$$

$$L_2 = f(Y_2 - CY_{b2}), \quad (2)$$

where L_1 and L_2 are lightness estimations, Y_1 and Y_2 are the reflectances of the test area, Y_{b1} and Y_{b2} are the reflectances of the backgrounds, and the f function is a suitable monotonic nonlinear scaling function such as the logarithmic, the square root, the cube root, or the Munsell value function. When the lightness match is obtained.

L_1 equals L_2 ; thus Eq. (3) holds and Y_2 is solved as shown in Eq. (4):

$$Y_1 - CY_{b1} = Y_2 - CY_{b2}, \quad (3)$$

$$Y_2 = Y_1 + C(Y_{b2} - Y_{b1}). \quad (4)$$

Y_2 is plotted against Y_1 for several amounts of induction in Fig. 3.

If the reflectance range of the test area is limited, it seems possible to explain the general lightness shift by adjusting the induction coefficient. However, if a wide range of applicability is wanted, the effect of induction is too small in the high- Y region and too large in the low- Y region.

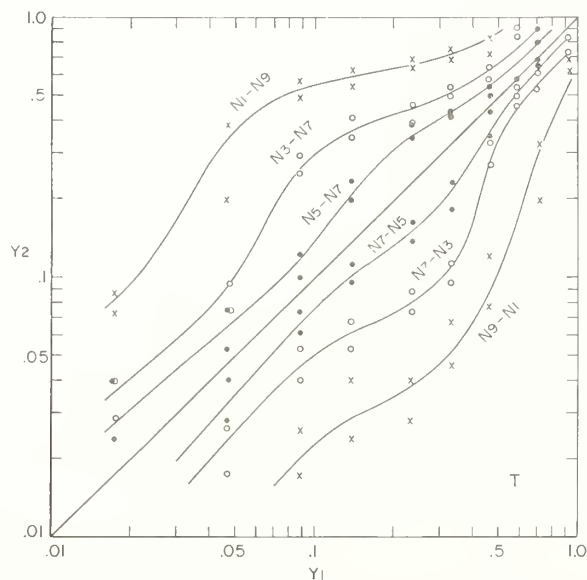


FIG. 1. Reflectance Y_2 of matching samples found by observer T to appear as light as the standard sample of reflectance Y_1 . Symbols beside each curve stand for Munsell values of each background pair. The order of the two Munsell-value notations coincide with the position of standard and second backgrounds. The uncertainty of the observations is indicated by the distances between the points of each pair (xx, oo, ●●), one of which refers to a matching sample slightly lighter than the standard and the other, slightly darker.

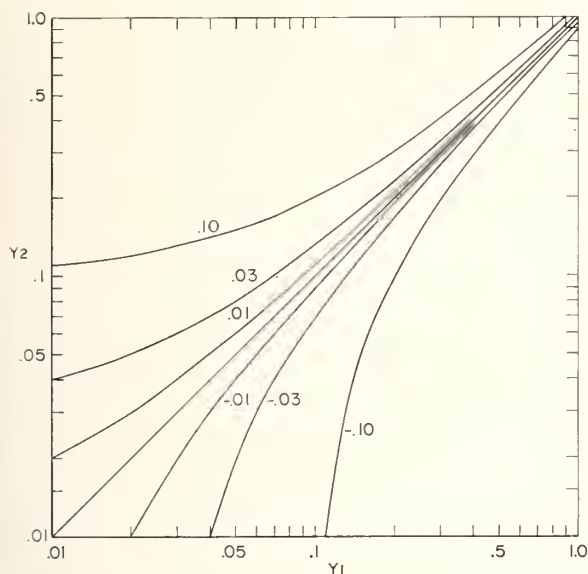


FIG. 3. A graphical representation of reflectance of matching sample (Y_2) predicted by the induction theory applied to reflectance of samples and background. The total amount of induction $C(Y_{b2} - Y_{b1})$ is shown beside each curve.

The von Kries coefficient theory⁵ was applied next. This theory says that the sensitivity of each independent mechanism of the eye varies in inverse proportion to the total excitation determining the adaptive state of the mechanism. We assumed that the adaptation is defined by the reflectances of the two backgrounds. When test area 1 is observed, the effect of background 1 may be

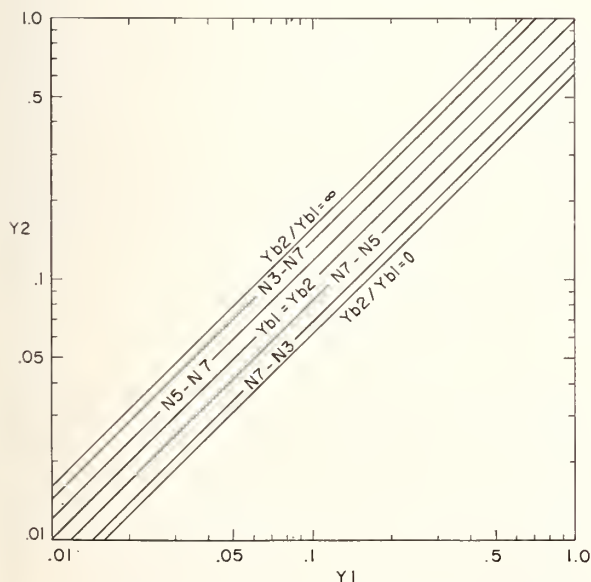


FIG. 4. A graphical representation of reflectance of matching sample (Y_2) predicted by the von Kries coefficient law with a weight factor $n = 1.6$.

⁵ J. von Kries, "Die Gesichtsempfindungen," in *Handbuch der Physiologie des Menschen*, W. A. Nagel, Ed. (Vieweg & Sohn, Braunschweig, 1905), Vol. 3, pp. 109-282.

larger than that of background 2, because the former surrounds the test area and the latter is relatively far away from the test area. For test area 2, the situation interchanges. Thus, for each test area, the lightness-estimation is written as Eqs. (5) and (6):

$$L_1 = f[Y_1/(nY_{b1} + Y_{b2})], \quad (5)$$

$$L_2 = f[Y_2/(nY_{b2} + Y_{b1})], \quad (6)$$

where n is the weight factor for the effect of immediate surround, and f function is again a monotonic nonlinear scaling function.

From the equality of lightness ($L_1 = L_2$), Y_2 is found as:

$$Y_2 = Y_1(nY_{b2} + Y_{b1})/(nY_{b1} + Y_{b2}). \quad (7)$$

Y_2 is calculated for background pairs of Munsell neutral 5 and 7, 3 and 7, and 0 (black) and 10 (white) with the weight factor $n = 1.6$, because it has been shown by preliminary calculation that $n = 1.6$ gives pretty good agreement with observation when the background contrast is low or moderate.

The result is, as shown in Fig. 4, a group of straight lines with inclination of 45° on the log-log diagram. This is not too different from the observed general shift. The amount of shift is reasonable when the background contrast is moderate.

This theory fails seriously when the background contrast is high. The calculated lightness change approaches n with increasing background contrast but the observed lightness change increases with increasing background contrast. If n is changed to fit the observation, for high background contrast, the fit becomes poor for low and medium background contrast.

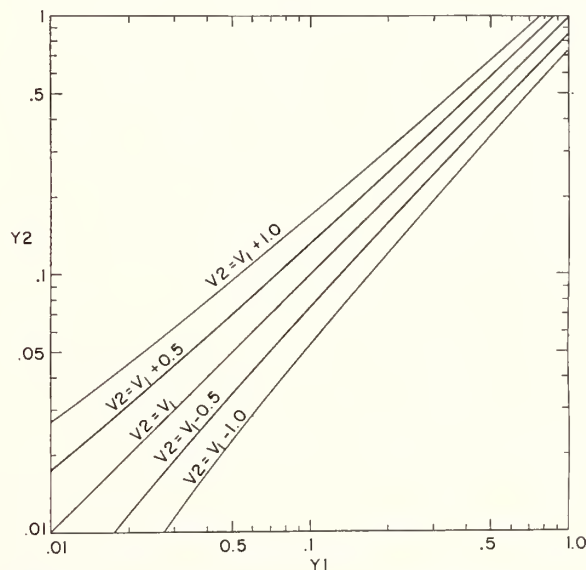


FIG. 5. A graphical representation of reflectance of matching sample (Y_2) predicted by the induction theory applied to the Munsell-value function. The total amount of induction $C(V_{t2} - V_{b1})$ is changed by steps of 0.5.

Because Eq. (7) is in the form of a product, taking logarithms yields an equation in an additive form, like the formula obtained from the induction theory. But V is changed to $\log V$.

This means that the coefficient theory applied to reflectance is equivalent to the induction theory applied to the logarithm of reflectance. It seems worth trying to apply the induction theory to another nonlinear function of reflectance.

The induction theory was applied to the Munsell-value function, which yields lightness estimation formulas (8) and (9),

$$L_1 = V_1 - CV_{b1} \quad (8)$$

$$L_2 = V_2 - CV_{b2}, \quad (9)$$

where V is Munsell value of the sample or background.

From the equality of lightness,

$$V_2 = V_1 + C(V_{b2} - V_{b1}) \quad (10)$$

The results are shown in Fig. 5 still using the log-log diagram. It shows less effect in the high- V region and larger effect in the low- V region. In addition the amount of shift increases with increasing background contrast. Because these properties agree well with the observed general shift, the induction theory applied to Munsell value was adopted for the rest of the study.

5. OBSERVATIONS

Further observations were made by five observers (Gerald Howett, D. B. Judd, Miyo Kasuya, H. Takasaki, and G. Yonemura) and all except those for

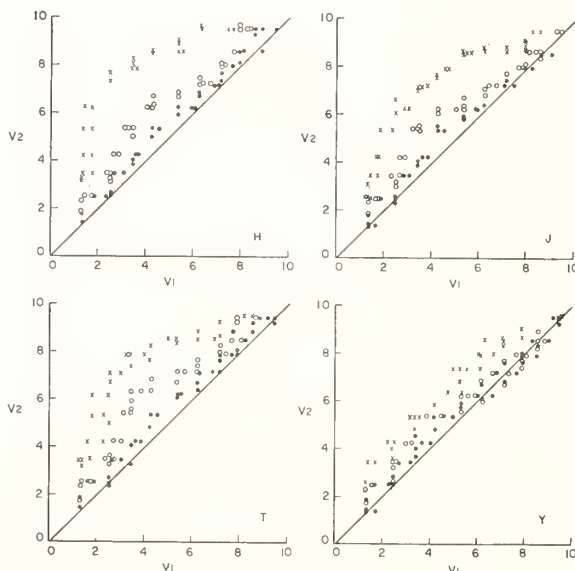


FIG. 6. Observations for four observers (H,J,T,Y) for background pairs of same average Munsell value. Observed points formerly plotted under the 45° line are plotted in upper area from the line, thus \times is for N1-N9 and N9-N5, \circ is for N3-N7 and N7-N3, and \bullet is for N4-N6 and N6-N4.

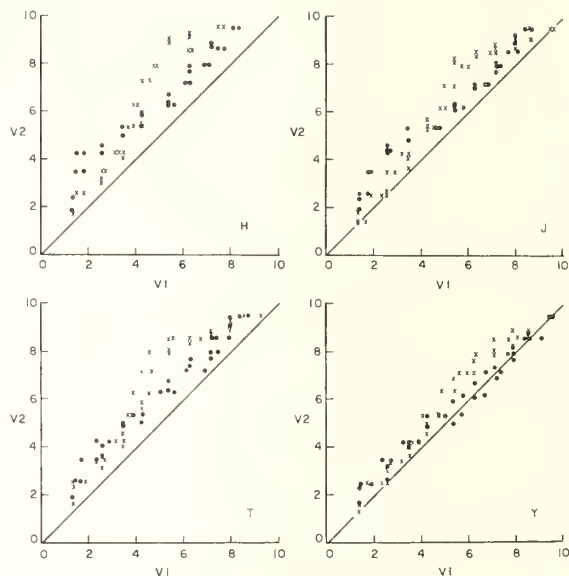


FIG. 7. Observations for four observers (H,J,T,Y) for background pairs of same background value difference but different average Munsell value. \times is for N5-N9 and N9-N5 and \bullet is for N1-N5 and N5-N1.

Kasuya are shown in Figs. 6 and 7. The results are now shown by using the V_1-V_2 diagram, in which V_1 is taken as the Munsell value of the test sample placed on the darker of the two backgrounds. By this convention the plotted points ought all to appear above the 45° line.

As is seen, the individual difference is very large. Observers H and T show large lightness change. Observer Y shows small changes and, in one case, a two-peaked lightness change. Observers J and K (K not shown in the figure), fall between the two extreme cases.

During the course of the observations, all observers noticed that the lightness of the test area changes rapidly with change in sample reflectance when the reflectance of the sample is close to that of the surround. This phenomenon is named the crispening effect. The prediction formula should explain this crispening effect.

6. EMPIRICAL SIMULATION OF OBSERVATIONS

A term which represents the crispening effect was introduced into the lightness estimation formula (8), which yields the following formula:

$$L = V - CV_b + f(V, V_b, \bar{V}_b). \quad (11)$$

The function shown in Eq. (12) was assumed as the crispening function.

$$f(V, V_b, \bar{V}_b) = C_2 \bar{V}_b [(V - V_b)/C_3] e^{-|V - V_b|/C_3}, \quad (12)$$

where \bar{V}_b is the average Munsell value of the two backgrounds. The estimated lightnesses L_1 and L_2 for the

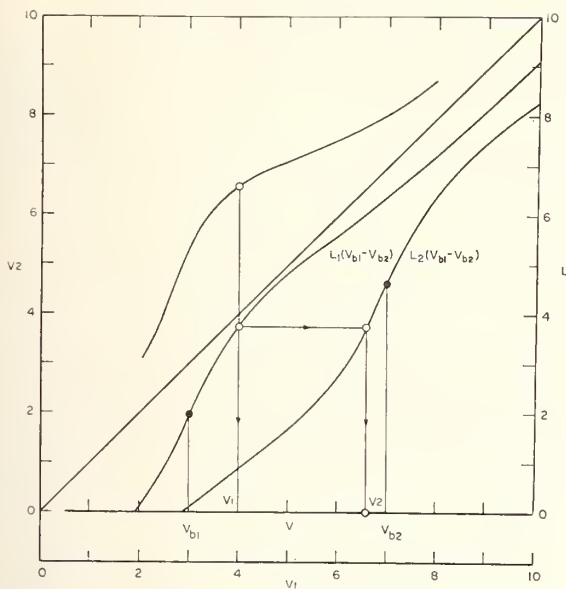


FIG. 8. Graphical method of obtaining the Munsell value of a sample which looks as light as a given background pair. The labels for the curves $L_1(V_{b1}-V_{b2})$ and $L_2(V_{b1}-V_{b2})$ mean that the curves represent lightness L_1 or L_2 of each sample for the given pair of backgrounds with Munsell values V_{b1} and V_{b2} .

two test areas are written as follows:

$$L_1 = V_1 - C_1 V_{b1} + C_2 \bar{V}_b [(V_1 - V_{b1})/C_3] e^{-|V_1 - V_{b1}|/C_3} \quad (13)$$

$$L_2 = V_2 - C_1 V_{b2} + C_2 \bar{V}_b [(V_2 - V_{b2})/C_3] e^{-|V_2 - V_{b2}|/C_3}; \quad (14)$$

C_3 adjusts the reciprocal sharpness of the crispening, C_2 is the amount of crispening, and C_1 the amount of induction.

By putting $L_1 = L_2$, V_2 may be found. An explicit analytic expression for V_2 has not been found, but V_2 has been determined by a graphical method.

To find V_2 for any value of V_1 , plot L_1 against V_1 on the $L-V$ diagram, and plot L_2 against V_2 as shown in Fig. 8; find L_1 for given V_1 ; then, from the L_2-V_2 curve, read V_2 corresponding to $L_2 = L_1$. Figure 8 also shows V_2 found in this way plotted against V_1 .

By adjusting the three constants, the observations of each observer can be simulated as shown in Figs. 9 to 12 according to Eqs. (13) and (14). Observations characterized by large and round-peak lightness change are simulated by large but gradual crispening and large induction (Figs. 9 and 10). Observations showing small and two-peaked lightness changes suggest small but sharp crispening and small induction (Fig. 12). In this case, however, simulation for the background pair of N1-N5 is poor in the high V region.

The values of the constants found by trial and error to simulate the observations of the five observers are given in Table I.

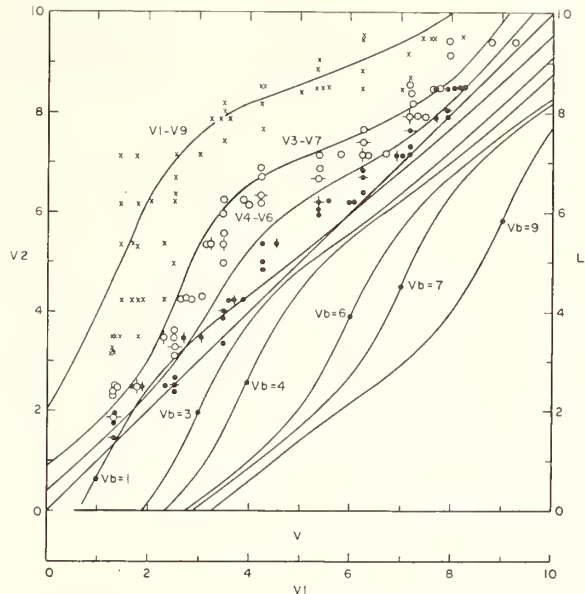


FIG. 9. Empirical simulation of the pooled observations of observers H and T, for the background pairs of same average Munsell value. The lightness-prediction formula is:

$$L_i = V_i - 0.35 V_{bi} + 0.5 \bar{V}_b [(V_i - V_{bi})/1.5] e^{-|V_i - V_{bi}|/1.5} \\ i = 1, 2 \quad \bar{V}_b = (V_{b1} + V_{b2})/2.$$

Lightness L is plotted against Munsell value V under the 45° line on the figure. The curves above the 45° line are a plot of V_2 against V_1 which have same L value. A pair of coincident data points is indicated by a vertical or horizontal line through the dot and a trio of coincident points by an additional cross on the dot.

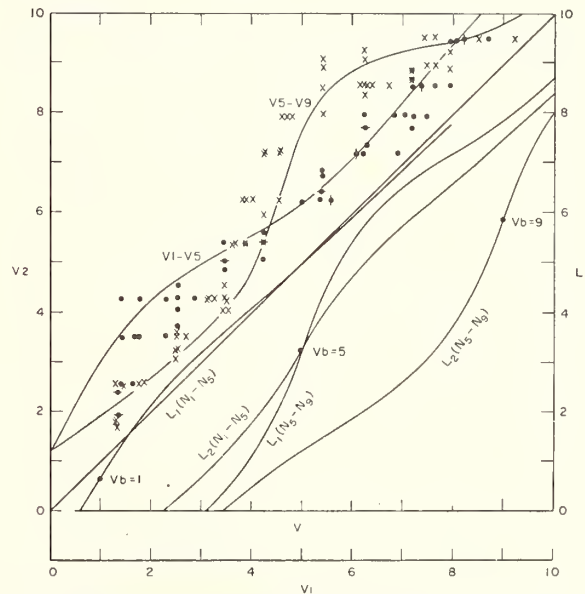


FIG. 10. Empirical simulation of the pooled observations of observers H and T, for background pairs of same Munsell-value difference with different average value. The lightness prediction formula is the same as in Fig. 9. Note that the average background value \bar{V}_b is different for the background pair of N1-N5 and N5-N9; accordingly, formula (13) or (14) requires two different $L-V$ curves for the N5 background, one applying for the background pair N1-N5 and the other for N5-N9.

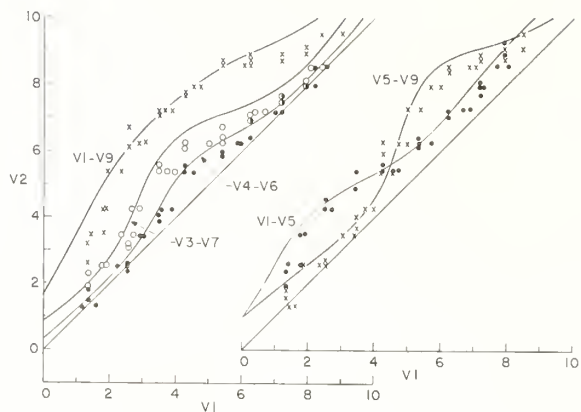


FIG. 11. Empirical simulation of the observations of observer J. The lightness-prediction formula is:

$$L_i = V_i - 0.30V_{bi} + 0.4\bar{V}_b[(V_i - V_{bi})/1.4]e^{-|V_i - V_{bi}|/1.4}.$$

7. DISCUSSION

The prediction formula is seen from Figs. 9 to 12 to agree fairly well with the observations for a wide range of experimental conditions; that is, the general shape of the experimental curves is reproduced and the predicted values fall almost within the observed dispersion of the observations. The formula can be adjusted to the observations of any particular observer by using the appropriate set of constants from Table I.

It is desirable that such a formula be based on some assumption on the mechanism of perception. Our formula is still on the basis of an empirical approach. Kaneko has recently published a beautiful modification of the Adams-Cobb lightness formula.⁵ His lightness formula was derived from Cobb's photochemical model.

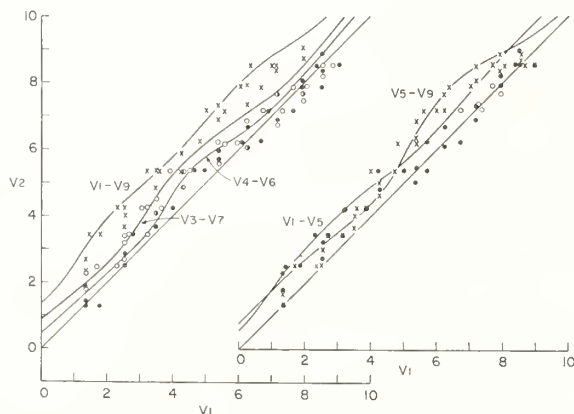


FIG. 12. Empirical simulation of the observations of observer Y. The lightness-prediction formula is:

$$L_i = V_i - 0.20V_{bi} + 0.15[(V_i - V_{bi})/0.8]e^{-|V_i - V_{bi}|/0.8}.$$

⁵ T. Kaneko, *Acta Chromatica* 1, 103 (1964).

TABLE I. Constants required for simulation of the observations of the five observers.

Observer	Induction C_1	Amount of crispening C_2	Reciprocal sharpness of crispening C_3
H	0.35	0.5	1.5
J	0.30	0.4	1.4
K	0.23	0.25	1.5
T	0.35	0.5	1.5
Y	0.20	0.15	0.8

There is however a nonnegligible discrepancy between his formula and observations; on the log \bar{Y} -lightness diagram, the fitted curves from his formula show decreasing slope particularly for the white background in the high- \bar{Y} region, but the observations indicate increasing slope. Our average empirical formula gives a crispening that is too sharp to agree with Kaneko's observations. Perhaps the effect of background is larger for the experimental condition of one test sample on each background than for his condition of eleven test samples viewed simultaneously on one background. For our conditions the observer adaptation was controlled largely by the background; for his, the test samples themselves may have played an important role.

A conceivable weakness of our formula is that the adaptive state of the eye is assumed to be decided solely by the two backgrounds. So far the condition of observation secured a large background effect and the formula worked fairly well, but the assumption may not be true in general.

Some modification in evaluation of adaptive state would be necessary to generalize the formula. In addition, the lightness-change estimation of the present formula might be improved by modifying the nonlinear function of reflectance used in the induction part of the formula.

8. CONCLUSION

A formula to estimate the contrast effect was developed by applying the induction theory to Munsell value and adding a term for the crispening effect. The formula involves three constants. An appropriate set of constants yields fairly good prediction of the lightness change for any particular observer for a wide range of experimental conditions.

ACKNOWLEDGMENT

The author wishes to express his sincere thanks to the staff of the National Bureau of Standards, especially Dr. D. B. Judd for his guidance and kind arrangements for the study.

Chromatic Changes Induced by Changes in Chromaticity of Background of Constant Lightness

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A relatively small gray sample looks reddish when it is placed on a green background, and yellowish when it is placed on a blue background. This phenomenon is an example of the well-known contrast effect. The purpose of this paper is to make a quantitative determination of the effect, and to check whether the empirical formula previously derived for lightness contrast applies also to chromatic contrast. Two series of color samples were prepared so that, in the dominator-modulator system proposed by Judd, only one of the three fundamental responses varied. For a given member of one of these two series viewed on a surround of any member of the same series on the left side, the observer was instructed to choose, for a surround of a different member of the series on the right, the member of the series producing most nearly the same color appearance. The crispening effect which was found in lightness was also found in each of the chromatic responses, green and violet; and the empirical formula derived for lightness contrast was found to apply with fair success to the observations of each of the five observers, each with a different set of constants.

INDEX HEADINGS: Color; Color vision; Vision.

THIS study has been designed as a basic part of a complete study of chromaticness change induced into a relatively small color sample from a chromatic background. Preliminary observations of this sort were made and a prediction formula based on the von Kries law¹ was tested. When the background color contrast was moderate, the prediction worked fairly well. But when highly contrasting backgrounds were used the predictions failed to work regardless of the choice of primaries such as those of Brewer,² MacAdam,³ and Judd.⁴ It was supposed that there must be a basic discrepancy between what is going on in the visual mechanism and the simple model based on the von Kries law.

The study was planned to start from the very beginning by using a more systematic choice of background and target colors. The basic idea is the three-components theory of color vision. The Judd primaries⁴ have been used because of their simple relation to the CIE coordinate system. The former study⁵ by the author on lightness-contrast effect of grays is the first part of the basic study.

The present study is the second part of the basic study and takes the form of quantitative observations of the contrast effect on the G and V Judd primaries in order to see if the same form of empirical formula obtained in the former study on lightness is applicable to the G and V primary responses.

The Judd primaries are used mainly because of simplicity of transformation. In the case of the Judd primaries the amount of the R primary is equal to the luminous reflectance Y . Because the lightness is de-

termined by the luminous reflectance, the result of the experiment on lightness contrast must be closely related to the amounts of the R primary.

EXPERIMENTAL PROCEDURE

Two series of color samples, which were expected to change only one of the Judd primary responses by small steps while keeping the other two constant, were prepared by the Munsell Color Company. One is a red-to-green series which changes only the G -primary response and the other is a reddish-blue to greenish-yellow series which varies only the V primary response. Figure 1 shows the chromaticity coordinates (x, y) of the color

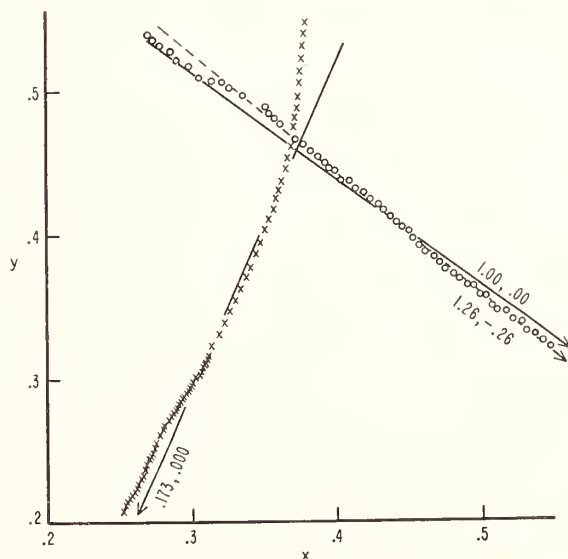


FIG. 1. Chromaticity plot of two series of color samples. One is a red-green series designed to vary only the green response of the Judd primaries and another is a reddish-blue-greenish-yellow series which is designed to vary only the violet response of the Judd primaries.

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¹ J. von Kries, *Handbuch der Physiologie des Menschen*, W. Nagel, Ed. (Vieweg & Sohn, Braunschweig, 1905), Vol. 3, p. 109-282.

² W. Lyle Brewer, *J. Opt. Soc. Am.* 44, 207 (1954).

³ D. L. MacAdam, *J. Opt. Soc. Am.* 46, 500 (1956).

⁴ H. Helson, D. B. Judd, and M. H. Warren, *Illum. Eng.* 47, 221 (1952).

⁵ H. Takasaki, *J. Opt. Soc. Am.* 56, 504 (1966).

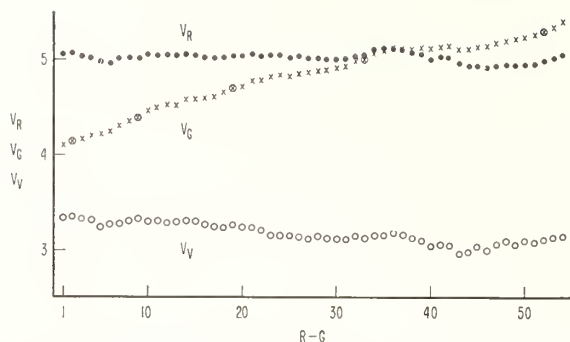


FIG. 2. The Munsell-value transformation of the three fundamental (Judd) responses of the red-green series of samples. The abscissa shows the serial numbers of the samples.

samples. The Munsell-value transforms of the fundamental responses of the samples are shown in Fig. 2 and Fig. 3.

Visual inspection of the red-to-green series shows good continuity in chromaticness. The chromaticity plot of the series closely approaches a straight line. But the straight line points approximately toward $x=1.26$, $y=-0.26$, $z=0$ instead of toward the Judd primary point $x=1.00$, $y=z=0$, as intended. Furthermore, the change of fundamental response G is not everywhere greater than the changes of the other responses and the change is somewhat irregular around serial number 40. Because there might be some different effect within this region owing to the irregularity, the observed values for samples 36 to 44 were treated with less weight.

The chromaticity plot of the reddish-blue to greenish-yellow series deviates considerably from a straight line; the samples are too greenish at the greenish-yellow end of the series. On the other hand, change of the V fundamental response is greater than the residual variations of the R and G responses in the series.

All samples of a series are pasted on a 35-cm wheel to form the matching color wheel. Selected samples chosen to subdivide the series into nearly equal steps of Munsell-value transform are pasted on a 15-cm wheel. The 35-cm wheel is placed at the right, in front of an observer and the 15-cm wheel is placed at his left; the wheels are covered with two 20- by 13-cm chromatic backgrounds placed side by side along their 20-cm sides. The backgrounds, which are also chosen from the same series, have 1.3-cm square holes at their centers through which are exposed the samples on the wheels. Thus the selected samples are seen in the left window and the matching sample in the right window. Vertical illumination of about 180 lux from a xenon arc lamp was used.

The observer was instructed to define the range of match of the matching samples to the sample color by rotating the big wheel. The sample series on the large wheel sometimes fails to give a perfect match. This failure made the judgment difficult. In such cases, the observer was allowed to look at the selected samples

pasted in order on neutral 6/paper to get an idea of the color change exhibited by the series. He was then asked to make a judgment based on the major attribute of change. For instance, for a sample color of unsaturated reddish blue, the range of matching was indicated by judging whether the matching color was a more saturated reddish blue than the sample or less saturated.

RESULTS

The observed data are shown in Figs. 4 and 5, where $V_{v1}(V_{g1})$ on the abscissa is the Munsell-value transform of the violet or green response corresponding to the sample on the background which was the lower value compared with the other background $V_{vb2}(V_{gb2})$. Thus, in the case where the V (or G) response of the background was less for the selected sample, the matching range is represented by short vertical lines, and when the background of the selected sample had a higher V (or G) response compared with the background of the matching sample, the matching range is represented by horizontal short lines.

The spread of observations and the individual differences, as expected, are very large. But a hump in the curve which suggests a chromatic crispening effect, analogous to the lightness crispening previously noted,⁵ was clearly observed for all observers and for both V and G responses.

The two kinds of observations which are represented by vertical or horizontal short lines should result in the same curve. In practice sometimes a clear discrepancy was observed, as is seen in Fig. 4 (observers H and T). This is believed to be due to possible non-

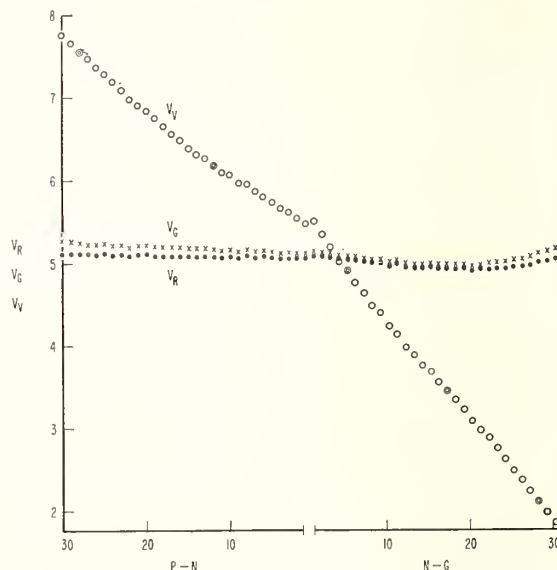


FIG. 3. The Munsell-value transformation of the three fundamental (Judd) responses of the reddish-blue-greenish-yellow series of samples. The abscissa shows the serial numbers of the samples.

⁵ N. T. Fedorow, Proc. Acad. Sci. USSR 67, 445 (1949).

uniformity of illumination or of visual sensitivity or of both, from one side of the visual field to the other. This problem, however, is put aside and both types of observations are treated as samplings from the same lot.

EMPIRICAL REPRESENTATION OF OBSERVATIONS

The empirical formula

$$L = V - C_1 V_b + C_2 \bar{V}_b [(V - V_b)/C_3] \times \exp(-|V - V_b|/C_3) \cdots (1)$$

which was obtained from the former study, was used to represent the lightness observations. In that formula

L = estimated lightness

V = Munsell value of the sample

V_b = Munsell value of the immediate background

\bar{V}_b = mean V_b of the two backgrounds

C_1 = constant for induction

C_2 = constant for amount of crispening

C_3 = constant for reciprocal sharpness of crispening.

This formula was found to be fairly successful in predicting lightness matches among grays viewed with any gray surround. In the Judd system, the correlate for lightness is the red response; so we have a satisfactory formula for equality of red response for backgrounds of any red response provided the target and surround colors all correspond to the same ratios of red, green, and violet responses.

In the present study, the colors of the targets and surrounds were taken from series in which two of the primary responses were held constant. It seems likely that the same type of formula developed for grays might apply to these observations of chromatic colors. Accordingly an attempt was made to fit curves based on this type of formula to the observations by trial-and-

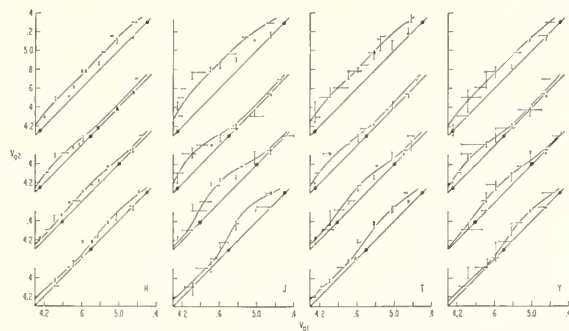


FIG. 5. Observation and prediction of the contrast effect analogous to those explained in Fig. 4, but on the Munsell-value transform of the Judd green response (V_{g2}).

error adjustments of the constants for four sets of observations of different background contrast for each of the observers. The fit was made as follows. The basic formula is of the same form as Eq. (1) but instead of perception of the red response, which is called lightness L in Eq. (1), substitute either the perception of the green response P_g , or the perception of the violet response P_v . Also, instead of the Munsell-value transform of the red response V , V_b , and \bar{V}_b , for the sample, the background, and the average of the two backgrounds, substitute either the Munsell-value transform of the green response, V_g , V_{gb} , and \bar{V}_{gb} , respectively, or the Munsell-value transform of the violet response, V_v , V_{vb} , and \bar{V}_{vb} ; V_{g2} or V_{v2} of the sample on the second background which matches for the green or violet perception with the sample on the first background ($P_{g2} = P_{g1}$; $P_{v2} = P_{v1}$) is found by the graphical method detailed in the earlier paper.⁵

The fitted curves are shown on Figs. 4 and 5. The sets of constants for each observer and each fundamental response are shown in Table I together with the constants for lightness which were obtained in the former study.

DISCUSSION

The present study is at the stage of making quantitative observations of the contrast effect and developing an appropriate formula to represent the obser-

TABLE I. Constants required for simulation of the observation of the five observers for the lightness and green and violet fundamental responses.

Observer	Induction			Amount of crispening			Reciprocal sharpness of crispening		
	C_1			C_2			C_3		
	V	V_V	V_G	V	V_V	V_G	V	V_V	V_G
H	0.35	0.10	0.10	0.50	0.10	0.016	1.5	2.0	0.12
J	0.30	0.20	0.15	0.40	0.10	0.042	1.4	0.8	0.20
K	0.23	0.15	0.15	0.25	0.10	0.042	1.5	0.6	0.20
T	0.35	0.20	0.15	0.50	0.10	0.021	1.5	0.6	0.16
Y	0.20	0.10	0.10	0.15	0.16	0.021	0.8	2.0	0.16

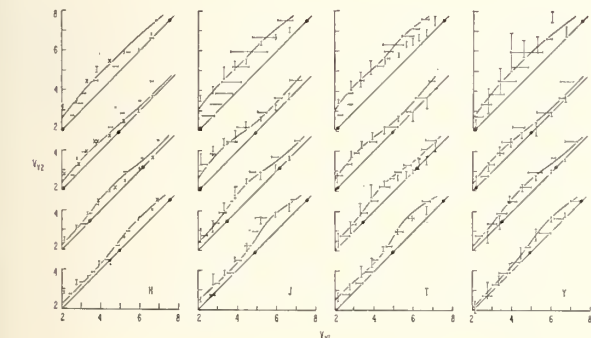


FIG. 4. Munsell-value transformation V_{v2} of the Judd violet response of the second color sample which matches the chromaticness of the first color sample that has the Munsell-value transformation of the Judd violet response, V_{v1} . The first and the second color samples are placed on backgrounds with different Judd violet response. The backgrounds are shown by a pair of solid circles on the 45° lines. The background which has the lower value among the two is taken as the first background. Predicted V_{v1} vs V_{v2} curves are also shown in the figure. The results for four observers out of five are shown.

variations. As is seen, the errors in observations of this kind are rather large. Under such conditions, we may be satisfied if the empirical formula agrees fairly well for a variety of experimental conditions without serious failure.

The fitted curves agree in general with the observations; the fit is usually not perfect. The curves show the following features:

- (1) A hump between the points (solid circles) corresponding to the backgrounds.
- (2) A shift of hump with shift of the background pair.
- (3) A larger hump for higher contrast of the backgrounds.

Two series of data (Fig. 4 observer *T* top curve and Fig. 5 observer *J* top curve) out of forty show what appears to be a significant deviation from the fitted curve. The fitted curve is symmetrical about the -45° straight line but the points are skewed toward low V_{g1} or V_{v1} .

Except for this discrepancy, Eq. (1) obtained for the lightness contrast seems to represent quite successfully the contrast effect in the green and violet fundamental responses.

Table I summarizes the characteristics of each fundamental response. Compared with the contrast effect in lightness, the constant for general chromatic induction is a little less for the green and violet fundamental responses. The contrast for the amount of crispening differs drastically for each primary; about one third for violet responses and about one tenth for green responses relative to the constant for lightness. On the other hand crispening for the green response is about ten times as sharp as for the lightness response.

The indicated difference of contrast effects for each fundamental response seems possible, but it introduces a basic problem. Fedorov⁶ made a qualitative analysis of the contrast effect based on an induction theory like that of Jameson and Hurvich⁷ but he applied it to three fundamental responses rather than to opponent responses.⁶ In the case of contrast of neutral grays no

chromatic effect is observed. This means that the 1:1:1 ratio of fundamental responses to gray is not disturbed by the contrast effect with grays of different reflectances. The amount of the contrast effect, hence, should be equal for the three fundamentals.

No chromatic effect due to contrast of grays has ever been observed except for cases that can be attributed to the dynamic response of the retina or to chromatic aberration of the eye. However, such big differences in the contrast effect as we found between the three fundamental responses, would result in predictions of significant chromatic changes induced by grays with different reflectances. This wrong prediction results because our experimental findings are interpreted by the three components theory.

If these findings were expressed in terms of the opponent-colors theory, no such wrong prediction would result. In the case of contrast composed of grays, contrast exists only for the black-and-white response. Contrast cannot occur for the two chromatic responses because no gray yields any chromatic response.⁷

The Müller second-stage transformation⁸ was calculated for the color series used. For the reddish blue-yellowish green series, the change of the $gY-rB$ response is dominant as is the $yR-bG$ response for red-green series. The amounts of these changes are comparable and correspond to the visual sensations better than does the very small R -response change of the red-green series and relatively great change of V response for the reddish-blue to greenish-yellow series, based on the Judd primaries. Some problems such as how to apply a nonlinear transformation to the chromatic responses of the opponent colors theory are expected, but data analysis based on the opponent colors theory seems valuable.

ACKNOWLEDGMENT

The author wishes to express his sincere appreciation to Dr. D. B. Judd of the National Bureau of Standards for his guidance during the study, and for his helpful discussion of the results.

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⁷ D. Jameson and L. M. Hurvich, J. Opt. Soc. Am. 51, 46 (1961).

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RELATION BETWEEN NORMAL TRICHROMATIC VISION AND DICHROMATIC VISION REPRESENTING A REDUCED FORM OF NORMAL VISION

DEANE B. JUDD*

In 1860 Maxwell subtracted the tristimulus values of a color set by a dichromatic observer from the tristimulus values of the same color set by an observer with normal color vision, and stated that the three differences specified "that color, the lack of sensation of which forms a defect of the dichromatic eye." Nuberg and Yustova, in 1955, gave a vectorial proof of this relation, and in the present paper an analytical proof is given.

INTRODUCTION

In an important paper, investigation of color vision of dichromats, Nuberg and Yustova [1] drew attention to the method used by Maxwell [2] to evaluate "that color, the lack of sensation of which forms a defect of the dichromatic eye." Maxwell's method was to obtain the tristimulus values set by the dichromatic observer to match any color and to subtract them from the tristimulus values of the same color set by an observer having normal trichromatic vision. The differences specify the color for the perception of which no mechanism exists in the dichromatic observer.

Nuberg and Yustova [1] further pointed out that prior to their own work no further use of Maxwell's method had been made. Instead, the chromaticity coordinates of the missing primary for various types of dichromatic vision have been determined by the more complicated and less precise method of finding two pairs of chromaticities, C_1 , C_2 , and C_3 , C_4 (See Fig. 1) such that the two chromaticities of each pair are indistinguishable to the dichromat but as different as possible to the observer of normal color vision. The two chromaticity points of each pair define a straight line on the chromaticity diagram on which the chromaticity point for the missing primary must be located; so

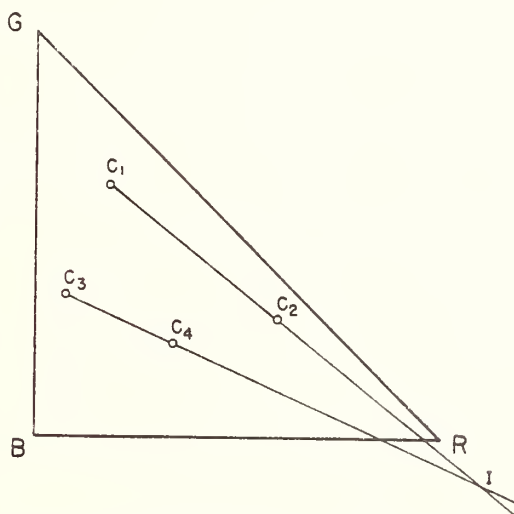


Fig. 1. Chromaticity diagram for any system of working primaries, R, G, B. If the dichromatic observer finds chromaticities C_1 and C_2 indistinguishable, and also C_3 indistinguishable from C_4 , the chromaticity point for the missing primary is given by the intersection point I.

this chromaticity point is determined as the intersection I of the two lines defined by the two pairs of chromaticities.

Nuberg and Yustova [1] stated the condition that must be satisfied for either method to be valid, and gave a vectorial proof of Maxwell's method. The condition is that all matches set by the trichromatic observer must hold for the dichromatic observer (that is, the vision of the dichromatic observer must be what is

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called a reduction form of that of the trichromatic observer). It is the purpose of the present paper to give an analytical proof of Maxwell's method.

Given: An observer with trichromatic vision defined by color-matching functions, $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$, expressed in any arbitrarily chosen tristimulus system, and an observer with dichromatic vision for whom all metameric matches set by the trichromatic observer hold. Let R_1, G_1, B_1 be the tristimulus values of any arbitrarily chosen color set by the trichromatic observer and let R_2, G_2, B_2 be the tristimulus values of the same color set by the dichromatic observer such that $R_1 \neq R_2$, $G_1 \neq G_2$, or $B_1 \neq B_2$, that is, set so that the trichromatic observer sees a definite mismatch between the arbitrarily chosen color and the match set by the dichromatic observer.

REQUIRED TO PROVE

That the intersection point on the chromaticity diagram of the arbitrarily chosen tristimulus system for the straight lines each of which is a locus of chromaticities confused by the dichromat has coordinates r_I, g_I , equal to the ratios of the differences, $R_1 - R_2, G_1 - G_2, B_1 - B_2$, to their sum; that is, to prove that:

$$\left. \begin{aligned} r_I &= (R_1 - R_2) / [(R_1 - R_2) \\ &\quad + (G_1 - G_2) + (B_1 - B_2)], \\ g_I &= (G_1 - G_2) / [(R_1 - R_2) \\ &\quad + (G_1 - G_2) + (B_1 - B_2)]. \end{aligned} \right\} \quad (1)$$

PROOF

Let $L_{\lambda,1}$ and $L_{\lambda,2}$ be the spectral radiances of any two patches of light. The conditions that must be satisfied if these two patches are to appear identical to the trichromatic observer whose color-matching functions are $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$, are

$$\left. \begin{aligned} \int_0^\infty (L_{\lambda,1} - L_{\lambda,2}) \bar{r}(\lambda) d\lambda &= 0, \\ \int_0^\infty (L_{\lambda,1} - L_{\lambda,2}) \bar{g}(\lambda) d\lambda &= 0, \\ \int_0^\infty (L_{\lambda,1} - L_{\lambda,2}) \bar{b}(\lambda) d\lambda &= 0. \end{aligned} \right\} \quad (2)$$

If the two patches of light are spectrally identical, that is, if $L_{\lambda,1} - L_{\lambda,2} = 0$ throughout the visible spectrum, Eq. (2) is satisfied regardless of what form the color-matching functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ may take, and the match is what is known as a nonmetameric, or isomeric, match. On the other hand, if $L_{\lambda,1} - L_{\lambda,2}$ is different from zero somewhere within the visible spectrum, the match depends on the color-matching functions of the trichromatic observer, and is known as a metameric match. A dichromatic observer requires but two color-matching functions, $\bar{w}(\lambda)$ and $\bar{k}(\lambda)$, and if this dichromatic observer finds all metameric matches set by the trichromatic observer also to be matches for him, then the two color-matching functions of the dichromat must be weighted sums of the three color-matching functions of the trichromat; that is, $\bar{w}(\lambda)$ and $\bar{k}(\lambda)$ must have the form:

$$\left. \begin{aligned} \bar{w}(\lambda) &= c_{11} \bar{r}(\lambda) + c_{12} \bar{g}(\lambda) + c_{13} \bar{b}(\lambda), \\ \bar{k}(\lambda) &= c_{21} \bar{r}(\lambda) + c_{22} \bar{g}(\lambda) + c_{23} \bar{b}(\lambda), \end{aligned} \right\} \quad (3)$$

in order to ensure that

$$\begin{aligned} \int_0^\infty (L_{\lambda,1} - L_{\lambda,2}) \bar{w}(\lambda) d\lambda \\ = \int_0^\infty (L_{\lambda,1} - L_{\lambda,2}) \bar{k}(\lambda) d\lambda = 0. \end{aligned}$$

The tristimulus values of the first light patch are defined in the usual way as:

$$\begin{aligned} R_1 &= \int_0^\infty L_{\lambda,1} \bar{r}(\lambda) d\lambda, & G_1 &= \int_0^\infty L_{\lambda,1} \bar{g}(\lambda) d\lambda, \\ B_1 &= \int_0^\infty L_{\lambda,1} \bar{b}(\lambda) d\lambda, \end{aligned}$$

and $R_2, G_2, B_2, W_1, K_1, W_2, K_2$ are defined similarly.

Chromaticity coordinates (r, g, b) for the trichromatic observer and those (w, k) for the dichromatic observer are defined in the usual way as:

$$\begin{aligned} r &= R/(R+G+B), & g &= G/(R+G+B), \\ b &= B/(R+G+B), \\ w &= W/(W+K), & k &= K/(W+K), \end{aligned}$$

so that

$$r+g+b=1, \text{ and } w+k=1.$$

CHROMATICITY COORDINATES OF THE INTERSECTION POINT

The chromaticity coordinates, r_I, g_I , of the intersection point of the dichromatic chromaticity-confusion lines depend on the weights (c_{11}, c_{12} , and so on from Eq. (3)) by which the dichromatic color-matching functions are related to the trichromatic. For each chromaticity confusion line, the dichromatic chromaticity coordinates, w, k , become fixed at, say, w_0, k_0 , and this condition may be expressed as:

$$w_0 = W/(W+K),$$

or, solving for W/K :

$$W/K = w_0/(1-w_0),$$

and from Eq. (3) this condition may be written as:

$$\frac{c_{11}R + c_{12}G + c_{13}B}{c_{21}R + c_{22}G + c_{23}B} = \frac{w_0}{1-w_0}.$$

By dividing numerator and denominator by $R+G+B$, we obtain:

$$\frac{c_{11}r + c_{12}g + c_{13}b}{c_{21}r + c_{22}g + c_{23}b} = \frac{w_0}{1-w_0}.$$

By substituting for b its equivalent, $1-r-g$, we obtain:

$$\frac{(c_{11}-c_{13})r + (c_{12}-c_{13})g + c_{13}}{(c_{21}-c_{23})r + (c_{22}-c_{23})g + c_{23}} = \frac{w_0}{1-w_0},$$

which may take the form of:

$$(c_1r + c_2g + c_3) - \frac{w_0}{1-w_0}(c_4r + c_5g + c_6) = 0, \quad (4)$$

where $c_{11}-c_{13}$ is abbreviated as c_1 and $c_{12}-c_{13}$ as c_2 ; and so forth. Since Eq. (4) is a linear equation in r and g , it defines a straight line on the (r, g) -chromaticity diagram for any chosen fixed value of w_0 ; and if w_0 be given a series of fixed values, Eq. (4) defines a family of straight lines intersecting at a common point. To find the intersection point, solve the equations:

$$\left. \begin{aligned} c_1r + c_2g + c_3 &= 0, \\ c_4r + c_5g + c_6 &= 0, \end{aligned} \right\} \quad (5)$$

simultaneously, and substitute the resulting values, r_I and g_I , into Eq. (4). It is seen that these particular values (r_I, g_I) of r and g satisfy Eq. (4) regardless of what fixed value may be chosen for w_0 . The values of the chromaticity coordinates of the intersection point are:

$$\left. \begin{aligned} r_I &= \frac{c_2c_6 - c_3c_5}{c_1c_5 - c_2c_4} \\ &= (c_{12}c_{23} - c_{13}c_{22}) / (c_{12}c_{23} - c_{13}c_{22} \\ &\quad + c_{13}c_{21} - c_{11}c_{23} + c_{11}c_{22} - c_{12}c_{21}), \\ g_I &= \frac{c_3c_4 - c_1c_6}{c_1c_5 - c_2c_4} \\ &= (c_{13}c_{21} - c_{11}c_{23}) / (c_{12}c_{23} - c_{13}c_{22} \\ &\quad + c_{13}c_{21} - c_{11}c_{23} + c_{11}c_{22} - c_{12}c_{21}). \end{aligned} \right\} \quad (6)$$

SPECIFICATION OF A COLOR PAIR INDISTINGUISHABLE IN DICHROMATIC VISION BUT DIFFERENT IN TRICHROMATIC VISION

If the tristimulus values of any color be taken as R_1, G_1, B_1 , the tristimulus values, R_2, G_2, B_2 , of any other color indistinguishable in dichromatic vision must be such that:

$$W_1 = W_2 \text{ and } K_1 = K_2,$$

and these conditions by virtue of Eq. (3) can be written:

$$\left. \begin{aligned} c_{11}R_1 + c_{12}G_1 + c_{13}B_1 \\ &= c_{11}R_2 + c_{12}G_2 + c_{13}B_2, \\ c_{21}R_1 + c_{22}G_1 + c_{23}B_1 \\ &= c_{21}R_2 + c_{22}G_2 + c_{23}B_2. \end{aligned} \right\} \quad (7)$$

With no loss of generality we can ensure that the color R_2, G_2, B_2 is different in trichromatic vision from color R_1, G_1, B_1 simply by taking R_2 arbitrarily at any value different from R_1 . Let G_2 and B_2 be expressed as unknown fractions, f_g and f_b , of G_1 and B_1 , respectively; that is:

$$G_2 = f_g G_1, \text{ and } B_2 = f_b B_1.$$

By making these substitutions in Eq. (7) we obtain two equations with the two unknowns, f_g and f_b . Solution of these two

equations simultaneously results in:

$$\left. \begin{aligned} f_g &= 1 + (c_{11}c_{23} - c_{13}c_{21})(R_1 - R_2) / \\ &\quad (c_{12}c_{23} - c_{13}c_{22})G_1, \\ f_b &= 1 + (c_{12}c_{21} - c_{11}c_{22})(R_1 - R_2) / \\ &\quad (c_{12}c_{23} - c_{13}c_{22})B_1. \end{aligned} \right\} \quad (8)$$

The tristimulus values of the second color different in trichromatic vision but identical in dichromatic vision are thus:

$$R_2, f_g G_1, f_b B_1,$$

for f_g and f_b as given in Eq. (8). The differences whose ratios to their sums we must prove (see Eq. (1)) to be equal to the coordinates of the intersection point r_I, g_I, b_I are thus:

$$\begin{aligned} R_1 - R_2 &= R_1 - R_2, \\ G_1 - G_2 &= (R_1 - R_2)(c_{13}c_{21} - c_{11}c_{23}) / \\ &\quad (c_{12}c_{23} - c_{13}c_{22}), \\ B_1 - B_2 &= (R_1 - R_2)(c_{11}c_{22} - c_{12}c_{21}) / \\ &\quad (c_{12}c_{23} - c_{13}c_{22}). \end{aligned}$$

The ratios of these numbers to their sum are:

$$\left. \begin{aligned} (R_1 - R_2) / [(R_1 - R_2) + (G_1 - G_2) + (B_1 - B_2)] \\ &= (c_{12}c_{23} - c_{13}c_{22}) / (c_{12}c_{23} - c_{13}c_{22} + c_{13}c_{21} - c_{11}c_{23} + c_{11}c_{22} - c_{12}c_{21}), \\ (G_1 - G_2) / [(R_1 - R_2) + (G_1 - G_2) + (B_1 - B_2)] \\ &= (c_{13}c_{21} - c_{11}c_{23}) / (c_{12}c_{23} - c_{13}c_{22} + c_{13}c_{21} - c_{11}c_{23} + c_{11}c_{22} - c_{12}c_{21}). \end{aligned} \right\} \quad (9)$$

Comparison with Eq. (6) shows these ratios also to be equal to the chromaticity coordinates of the intersection point, r_I, g_I , which was to be proved (Eq. (1)).

SUMMARY

If, in an arbitrarily chosen tristimulus system, color R_2, G_2, B_2 appears in dichromatic vision to be like color R_1, G_1, B_1 but in trichromatic vision to be different, and if every color match set in accord with this trichromatic vision also appears to be a match for this kind of dichromatic vision, then the differences $R_1 - R_2, G_1 - G_2, B_1 - B_2$ and the chromaticity coordinates, r_I, g_I, b_I of the intersection point of the dichromatic confusion lines are alike proportional to:

$$c_{12}c_{23} - c_{13}c_{22}, \quad c_{13}c_{21} - c_{11}c_{23}, \quad c_{11}c_{22} - c_{12}c_{21},$$

where the color-matching functions $\bar{v}(\lambda), \bar{k}(\lambda)$ defining this kind of dichromatic vision in the same arbitrarily chosen tristimulus system are related to the color-matching functions $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ defining this kind of trichromatic vision in accord with:

$$\begin{aligned} \bar{v}(\lambda) &= c_{11}\bar{r}(\lambda) + c_{12}\bar{g}(\lambda) + c_{13}\bar{b}(\lambda), \\ \bar{k}(\lambda) &= c_{21}\bar{r}(\lambda) + c_{22}\bar{g}(\lambda) + c_{23}\bar{b}(\lambda). \end{aligned}$$

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Manuscript received 11 November, 1964.

Twenty-Two Colors of Maximum Contrast

by Kenneth L. Kelly

Requests for sets of colors which would be maximally different for use in color coding, providing maximum contrast for those with deficient color vision, have resulted in the selection of 22 shades from the ISCC-NBS Centroid Colors.

A number of requests have been made to the Inter-Society Color Council for sets of colors, 2 up to 22 colors per set, which would be maximally different for use in color coding. The author was requested to determine such sets from the ISCC-NBS centroid colors since these colors will be widely distributed and always available in chart form as well as in 9-by 12-inch sheets, both at very reasonable prices. The list of centroid colors in Table 1 is given as the solution to this request.

The development and designation of the ISCC-NBS centroid colors have been covered in great detail in the literature over the past 30 years. The three documents that give information basic to this problem are the set of ISCC-NBS Centroid Color Charts (1), the color names dictionary (5) and a Universal Color Language (4).

The selection of the ISCC-NBS centroid colors from which to choose colors for coding was made for the following reasons: the centroid colors were developed by the Inter-Society Color Council and the National Bureau of Standards (ISCC-NBS), they will be widely available, the price is

extremely reasonable, they may be obtained in one-by-one-inch size in the centroid chart form and in 9-by-12-inch sheets from Munsell Color Company, Inc., Baltimore, Maryland, and repaints will color match the originally selected colors within very stringent tolerances.

It was immediately obvious that to list these colors in sets of 2, 3, 4, 5, 6, 7 and so on up to 22, would involve duplication of the colors of one set in those of the next higher numbered set. Therefore, it was decided to present these centroid colors in a list in such an order that the desired number would be obtained by taking that number from the top of the list. That is, if one desires 3, 7, or 19 colors for a job, one would simply use the first 3, 7 or 19 colors from this list. If the colors for a job are selected from this list but not in numerical order as described above, the desired effectiveness with respect to color blindness and visual contrast will be reduced.

It was also obvious that the second question by the user would concern the effect of color blindness on the selection and thus the use of these colors. Earlier work by Judd (2) (3)

led to the selection of nine colors which would be maximally different not only to the color-normal observer but to most red-green deficient observers, and would, at the same time, be readily distinguished by color name. It was therefore decided to use the nine centroid colors which would effectively duplicate the nine colors in Judd's selection as the first nine colors in this list. These colors have been arranged in such an order that the second color contrasts maximally with the first, the third color contrasts satisfactorily with the first pair, the fourth color contrasts maximally with the third, and pairs of maximum contrast are added up through the eighth color. The ninth color is medium gray. This is as far as any claim can be made with respect to red-green confusers, and this division is indicated by a line drawn across the list between colors 9 and 10.

The succeeding 13 colors (10 through 22) in this list are claimed to contrast maximally only for the color-normal observer. Each successive color has been selected so that it will contrast maximally with the color just preceding it and satisfactorily with the

earlier colors in the list. That is, the color following Green is Purplish Pink which differs considerably from Green in lightness and is almost complementary to it in hue. The next color, Blue, differs also from Purplish Pink considerably in lightness and is almost complementary to it in hue. This method was followed as far as possible in the selection of the centroid colors in the rest of the list.

Application

Some color codes such as diagrams, maps and illustrations, apply to detailed patterns shown by black lines on white paper and require that the first two colors, black and white, of this list be omitted from the code. Thus, for this purpose, the required number of colors is taken from the top of the list starting with color number 3. For other color codes where white and black are acceptable, such as to indicate contents of containers (for drugs, chemicals or any industrial material) or to indicate numerical values of some property of a manufactured article or component, the desired number of colors is taken from the top of the list starting with color number 1.

List of Colors

Table I lists the 22 colors in order, giving for each its serial and therefore its selection number, the general color name, the ISCC-NBS centroid or color-name block number, the ISCC-NBS color name and the Munsell renotation taken from the table in the front of the set of ISCC-NBS Centroid Color Charts.

To summarize: the ISCC-NBS centroid colors listed in Table I have been carefully selected to provide maximum contrast in color coding. The first nine colors have been selected so as to yield satisfactory contrast for red-green-deficient as well as color-normal observers, while the following 13 colors have been selected, of necessity, to be useful only for color-normal observers. This list of colors is intended primarily to be of use in establishing new color codes and secondarily as a guide in the revision of existing color codes. Such revision might consist of shifting the color for a designated meaning so that it will be more effectively seen by a greater percentage of the population

(normal plus red-green-deficient observers). This list of colors can also serve as a guide when new colors have to be added to an existing color code.

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The Author

KENNETH L. KELLY, a physicist in the Photometry and Colorimetry Section of the National Bureau of Standards, Washington, D. C., has been involved in the color names project for nearly 30 years. In 1936 he commenced work at the National Bureau of Standards to assign boundaries to color ranges for designations derived on the basis laid down by I. H. Godlove, then chairman of the Inter-Society Color Council Committee on Measurement and Specification.

Three years later, Mr. Kelly and Dr. Deane B. Judd prepared research paper RP1239, Method of Designating Colors, which was approved by the ISCC for use in pharmaceutical literature. In 1955, again with Dr. Judd, he prepared the ISCC-NBS Method of Designating Colors and a Dictionary of Color Names. Three years later he authored the research paper, Central Notations for the Revised ISCC-NBS Color-Name Blocks.

Mr. Kelly, long active in many phases of color, is chairman of the ISCC Subcommittee on Color Names and a member of the subcommittee on Expression of Historical Color Usage.

Color Serial or selection number	General color name	ISCC-NBS centroid number	ISCC-NBS color- name (abbreviation)	Munsell renotation of ISCC-NBS Centroid Color
1	white	263	white	2.5PB 9.5/0.2
2	black	267	black	N 0.8/
3	yellow	82	v.Y	3.3Y 8.0/14.3
4	purple	218	s.P	6.5P 4.3/9.2
5	orange	48	v.O	4.1YR 6.5/15.0
6	light blue	180	v.l.B	2.7PB 7.9/6.0
7	red	11	v.R	5.0R 3.9/15.4
8	buff	90	gy.Y	4.4Y 7.2/3.8
9	gray	265	med.Gy	3.3GY 5.4/0.1
<hr/>				
10	green	139	v.G	3.2G 4.9/11.1
11	purplish pink	247	s.pPk	5.6RP 6.8/9.0
12	blue	178	s.B	2.9PB 4.1/10.4
13	yellowish pink	26	s.yPk	8.4R 7.0/9.5
14	violet	207	s.V	0.2P 3.7/10.1
15	orange yellow	66	v.OY	8.6YR 7.3/15.2
16	purplish red	255	s.pR	7.3RP 4.4/11.4
17	greenish yellow	97	v.gY	9.1Y 8.2/12.0
18	reddish brown	40	s.rBr	0.3YR 3.1/9.9
19	yellow green	115	v.YG	5.4GY 6.8/11.2
20	yellowish brown	75	deep yBr	8.8YR 3.1/5.0
21	reddish orange	34	v.rO	9.8R 5.4/14.5
22	olive green	126	d.OIG	8.0GY 2.2/3.6

3. Color Specification and Order Systems

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Foreword

An important function of colorimetry is the development of systems in which colors can be described quantitatively and qualitatively with reasonable precision. This section deals with color specifications and color systems that have been developed to achieve reasonable and adequate descriptions.

The first document in this section is "Colorimetry", NBS Monograph 104. This is an updated version, prepared by Nimeroff, of the NBS Circular 478 originally written by Judd in 1950. Aside from bringing current some of the parts of the earlier work and deleting methods no longer in use, this monograph contains a new section on the limitations and uncertainties of several methods used in colorimetry.

The second paper in this section treats the network of colors that can be achieved with ideal Lovibond glasses. The relationship between ideal and actual Lovibond scales that result from the red, yellow, and blue glasses is discussed. These glasses in various combinations and densities are widely used in color grading of various items of commerce (vegetable oils, petroleum, naval stores, paint vehicles, etc.).

Kelly's paper, "A Universal Color Language," is the next document included in this section. It describes a method of color designation to provide a universal means of color communication among people of varying technical backgrounds. This language consists of a hierarchical arrangement of increasingly complex color orders, starting with a level that has 13 generic hue names and culminating in a level in which about 5 million colors can be specified. It has the advantage that it can be used to specify colors at a desirable level of complexity.

NBS Monograph 75, completed by Breckenridge after his retirement from the National Bureau of Standards, is the last document in this section. This document is intended to serve as a reference work in the selection, specification, and use of signal-light colors. It is particularly useful for all applications where signal-light systems are required to regulate safety in traffic of various sorts: on the land, in the air, and at sea.

Colorimetry

I. Nimeroff

Institute for Basic Standards
National Bureau of Standards
Washington, D.C. 20234



National Bureau of Standards Monograph 104
(Supersedes NBS Circular 478)

Issued January 1968

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Foreword

NBS Circular 478, Colorimetry, was issued 1 March 1950 and has been reprinted several times in response to a continuing demand. Such have been the advances in colorimetry that in 1965, instead of authorizing another reprinting of a work obsolete in many details, I asked Mr. I. Nimeroff to up-date it. I am grateful to him for undertaking this tedious but important task, and I am much pleased with the result.

Deane B. Judd
Assistant Division Chief,
Metrology Division

II

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Colorimetry*

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The definition of color, as a characteristic of light, and the basic principles of its measurement are given. The reduction of spectrophotometric data to three chromaticity coordinates by means of the three-function CIE standard observer system for colorimetry is described. Various methods of direct colorimetry, and visual and photoelectric methods of colorimetry by differences from material standards are treated. The most useful collections of material color standards are described and the most widely used one-dimensional color scales are explained. The limitations of several colorimetric methods: spectrophotometric, photoelectric, and visual, are discussed.

Key Words: Color dictionaries, color vision, colorimeters, photoelectric tristimulus, spectrophotometric colorimetry, visual colorimetry.

1. General Considerations

1.1. Introduction

It is common practice to regard color as a property of objects, and in a limited sense this view is justified. We have color comparators for solutions in which the color is taken as an index of the composition of the solution; and in applying a suitably prepared set of color standards in a color comparator, the color of the unknown behaves as if it were a property of the solution itself, just as the concentrations of the constituents which it indicates. However, this document deals with color for its own sake; and, for this purpose, a broader view is useful. Lights have colors as well as objects. The flame of a Bunsen burner can be changed from bluish purple to orange by the introduction of sodium. And since even objects lose their colors and become invisible unless they reflect, scatter, or transmit radiant energy, or form a part of an illuminated scene, this broader view is that color is a property of light, and of light alone.

As it is possible to measure with a spectrophotometer the spectral energy distribution of any light beam, and as the color of a light correlates closely with its spectral composition, some of the more physically minded people have contended that color is a physical property of radiant energy; but this is not the most useful view. The color change of the Bunsen flame from bluish purple to orange can be shown by a purely physical measurement to be caused by a change in the spectral composition of the emitted energy, but it takes more than physics to decide whether this flame has the same

color as the light reflected from the peel of a given citrus fruit. Application of the spectrophotometer to the orange peel will show that the spectral composition of the light reflected from it under daylight illumination is radically different from that emitted by the sodium flame. It has a continuous spectrum relatively strong in the long-wave (550 to 770 nm)¹ portion of the visible spectrum (380 to 770 nm). The visible energy of the sodium flame is nearly all confined to two narrow bands (589.0 and 589.6 nm). Physically, therefore, the two lights are different, but they have closely the same color. The two lights must therefore be identical in some other respect. This identity consists in some aspect of the response made by a normal observer to the sodium flame being the same as the corresponding aspect of the response to the peel of the citrus fruit. The broader view of color must, therefore, include not only the spectral composition of the radiant energy reaching the eye of the observer, but the properties of the observer as well. These properties have been evaluated by finding equivalent stimuli, like the energy of the sodium flame and that reflected from an orange peel, which have different spectral compositions but still manage to stimulate the same color response to the normal observer. Such equivalent stimuli are called *metamers*. In this chapter there will be presented the standard method for

* This monograph is an updating of NBS Circular C-478, Colorimetry, by D. B. Judd (1950). With the permission of Judd, much of the original text has been kept intact where applicable.
¹ The unit nanometers (nm), 10⁻⁹ meters, was formerly called millimicrons (mμ).

finding by computation whether or not any two lights form equivalent stimuli. The fundamental method of color specification based upon equivalent stimuli plus spectrophotometry will be described in detail together with other methods of obtaining the same numbers. And finally some discussion will be given on the use of the Munsell color system and color dictionaries such as the ISCC-NBS (Inter-Society Color Council-National Bureau of Standards) Method of Designating Color and a Dictionary of Color Names, National Bureau of Standards Circular 553, 1956 [79].²

1.2. Definition of Color

The most widely accepted technical definition of color is that given by the Committee on Colorimetry of the Optical Society of America [15]: "Color consists of the characteristics of light other than spatial and temporal inhomogeneities; light being that aspect of radiant energy of which a human observer is aware through the visual sensations which arise from the stimulation of the retina of the eye." It will be noted that this definition relates color and light to radiant energy only in so far as the energy produces a visual effect within an observer. On this account color and light are said to be psychological entities that can be evaluated by means of psychophysical quantities, and in their evaluation it is ordinarily not necessary to pay attention to energy of wavelength shorter than 380 nm, nor longer than 770 nm because the eye is relatively insensitive to such energy.

1.3. Tridimensionality of Color

If an observer with normal color vision attempts to adjust one element of his visual field whose color is under his control so that it matches a neighboring element, he will ultimately discover that three independent adjustments have to be made. If he is using the red, yellow, and blue paints frequently found in primary grade schools, only by chance will he obtain a match from a mixture of two of them. Even a brown color requires blue in addition to red and yellow. Within the color gamut of the three paints, an exact match for a given paint color is easily possible, but three is the irreducible minimum. Similarly, if he is trying to color-match one spot of light by shining several spotlights of different colors onto the same neighboring spot of a screen, he finds, in general, that either three lights of fixed spectral composition are required, or, if only two lights be added together, not only the amounts of both but also the spectral composition of at least one has to be adjustable. The same rule applies to rotary mixture on a sector disk; four sectors, giving three independent adjustments, are necessary and sufficient.

As the color vision of a normal observer is at least tridimensional, it follows that there must be at least three independent excitations in the optic nerve fibers corresponding to each patch of the

visual field. Theories of color vision have been derived mostly from speculation as to the character of these excitations. It also follows that a color specification is expressible by three numbers. For normal observers three numbers are necessary; for partially color-blind observers only two numbers are necessary; and for totally color-blind observers only one is necessary.

In the examples given (paints, spotlights, sector disks), the observer by adjustment of three variables obtains a color match, that is, he has to set up a second stimulus equivalent to the first. Except by accident, however, the ternary or binary mixture does not match the unknown in spectral composition. In the usual case the mixture is equivalent to the unknown in color but not in spectral composition, and the unknown and the mixture are said to form a *metameric pair*. There are, however, degrees of difference in spectral composition. If one painted panel be matched by a mixture of red, yellow, and blue paints, the degree of metamerism is likely to be only moderate; but if the paint panel illuminated by daylight be matched by shining on a white card three spotlights each of which contains energy restricted to a narrow wavelength band (such as spectrum red, green, and blue), the degree of metamerism will ordinarily be large.

1.4. Metamerism and the Macular Pigment

Studies of extremely metameric pairs in which mixtures of two parts of the spectrum are set up to color-match other two-part spectrum mixtures have yielded our most valuable knowledge regarding the properties of the average normal eye [1, 44, 84, 99, 158]. An outstanding fact derived from observation of such metamers is that the center of the retina (fovea centralis) has somewhat different properties from that part of the retina immediately surrounding it; that is, a color match set up for the central 2 or 3 deg of the retina becomes an easily detectable mismatch if the eye be turned so as to allow the stimuli forming the metameric pair to affect a portion of the retina, say, 6 deg from the fovea. Furthermore, if the metamers are compared in large patches so as to subtend 6 deg or more at the eye of the observer, this mismatch causes a central spot, known as the Maxwell spot [98], to appear temporarily on either field even though the field is physically uniform. After the spot has faded away, change of fixation to the other field will renew the spot. This dependence of metamerism on the portion of the retina used arises chiefly from the existence of a spot of brownish or yellowish pigment irregularly covering and interpenetrating the central 3 or 4 deg of the normal retina; it is called the *macula lutea* or sometimes the *yellow spot* (see fig. 1). Figure 1 shows a horizontal cross section of the eye. Light enters the tear-film ff, passes through the cornea aa, the aqueous humor B, the pupil bb, the crystalline lens A, the vitreous humor C, and the macula p before reaching the retina i. The macular pigment acts as

² Figures in brackets indicate the literature references at the end of this paper.

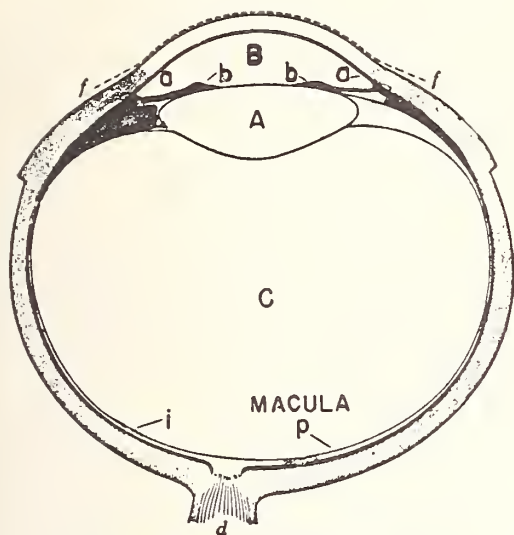


FIGURE 1. Horizontal cross-section of the normal human eye.

a selective filter interposed between the vitreous humor C and the retina i. Metamers set up for one normal observer usually fail to hold strictly for anyone else. This failure is ascribable to variations in amount of pigmentation of the eye media (cornea, lens, humors, macula), the macular pigment being one of the chief variables. The properties of the normal eye derived from a small-field study of these extreme metamers therefore refer only to the central 2 deg of the retina, and they refer to an hypothetical average eye. Nobody has been found whose eye differs so little from this average eye that the differences could not be detected. Practically speaking, therefore, nobody has an eye that is colorimetrically normal.

1.5. Grassman's Law

From a knowledge of spectral metamers, it has been possible to summarize concisely the properties of the average normal eye. This summary is made in accord with the principle known as Grassman's law [42] foreshadowed by Newton's laws of color mixture. If a light composed of known amounts of three components (called primaries) is equivalent in color to an unknown light, the three known amounts may be used as a color specification for this light. These amounts are called the *tristimulus values* of the color. Grassman's law states that, when equivalent lights are added to equivalent lights, the sums are equivalent. Thus, if an unknown spot of color were matched by shining on the same spot of a white screen two component spotlights of tristimulus values, X_1 , Y_1 , Z_1 , and X_2 , Y_2 , Z_2 , respectively, by Grassman's law, the tristimulus values, X , Y , Z , of the unknown spot of color would be simply:

$$\begin{aligned} X &= X_1 + X_2 \\ Y &= Y_1 + Y_2 \\ Z &= Z_1 + Z_2 \end{aligned} \quad (1)$$

Any beam of light, whether it originates from a

self-luminous body or comes, by transmission, scattering, or reflection, from a nonself-luminous object, may be considered as made up of a large number of portions of the spectrum. The amounts of these various portions may be determined by spectrophotometry. The spectral values, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, of each of these portions have been determined for a number of normal observers, and average values are given in table 1 in arbitrary units for a spectrum of unit spectral irradiance.

The principle expressed in Grassman's law has been established by repeated experiment over a wide middle range of retinal illuminances. It breaks down for very high retinal illuminances [159] that begin to approach those sufficient to do the eye permanent harm, and it breaks down if the illumination of the whole retina continues for several minutes to be so slight that vision by the retinal rods (twilight vision) intrudes significantly [84]. Between these two extremes, however, Grassman's law holds independently of the adaptive state of the eye. Thus, if two stimuli of different wavelength distributions of energy be found that are once responded to alike by the eye, they will be seen alike even after exposure of the eye to another stimulus sufficient to change considerably the appearance of the two equivalent stimuli. For example, if a portion of the spectrum near 640 nm (red) be superposed on a portion near 550 nm (yellowish green), it will be found possible to obtain the color of this combination from an intermediate portion of the spectrum, say, 590 nm (orange). If the retina of the eye be highly illuminated by light of wavelength near 640 nm, and its sensitivity to radiant flux of this wavelength region considerably reduced in this way, it is found that, although neither of the equivalent stimuli any longer appears orange, they still give identical colors; for example, they may yield identical yellows or identical greenish yellows. The eye thus cannot be trusted to yield the same color perception from a given stimulus; simultaneous and successive contrast affect it profoundly. But the eye is still a satisfactory null instrument and obeys Grassman's law.

By Grassman's law it is possible to test whether any two beams of light of differing spectral composition form a metameric pair. The condition for metamerism of two beams of light of spectral irradiance, E_1 and E_2 , is that simultaneously:

$$\begin{aligned} \sum_{\lambda}^{\infty} (E_1)_{\lambda} \bar{x}(\lambda) \Delta\lambda &= \sum_{\lambda}^{\infty} (E_2)_{\lambda} \bar{x}(\lambda) \Delta\lambda \\ \sum_{\lambda}^{\infty} (E_1)_{\lambda} \bar{y}(\lambda) \Delta\lambda &= \sum_{\lambda}^{\infty} (E_2)_{\lambda} \bar{y}(\lambda) \Delta\lambda \\ \sum_{\lambda}^{\infty} (E_1)_{\lambda} \bar{z}(\lambda) \Delta\lambda &= \sum_{\lambda}^{\infty} (E_2)_{\lambda} \bar{z}(\lambda) \Delta\lambda \end{aligned} \quad (2)$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ characterize the observers' spectral responses. (These equations are written in accord with the CIE notation adopted in 1963. Symbols with subscripted λ , as E_{λ} , indicate spectral concentration, while symbols with parenthetical λ , as $\bar{x}(\lambda)$, indicate other spectral relationships not critically dependent on choice of wavelength interval.) The wavelength interval,

$\Delta\lambda$, to be used in these summations depends upon the irregularity of the curve of spectral irradiance with wavelength; intervals of 20 nm are sometimes sufficient to yield a significant result; intervals of 10 nm often do; and intervals of 5 nm are usually sufficient except for discontinuous spectral distributions such as those characterising gaseous discharge lamps. De Kerf [27] has reported instances where 1-nm intervals have been required.

The summations of eq (2) form the tristimulus values of the color and are customarily given the symbols X, Y, Z , respectively, so that the condition for a color match would ordinarily be written:

$$\begin{aligned} X_1 &= X_2 \\ Y_1 &= Y_2 \\ Z_1 &= Z_2 \end{aligned} \quad (2a)$$

and would mean that the two colors are identical since their tristimulus values are identical. Thus, to match color one requires X_1 parts of the X primary, Y_1 parts of the Y primary, and Z_1 parts of the Z primary; and because to match color two requires the same amounts of the same primaries as does color one, the two colors are the same.

Any three lights may be used as primaries in a system of tristimulus color specifications, provided only that no one of them is equivalent to a combi-

nation of the other two. Tristimulus specifications X, Y, Z , expressed relative to one set of primaries, may be transformed into specifications R, G, B , relative to any other set, by transformation equations of the form:

$$\begin{aligned} R &= K_1X + K_2Y + K_3Z \\ G &= K_4X + K_5Y + K_6Z \\ B &= K_7X + K_8Y + K_9Z \end{aligned} \quad (3)$$

The constants K_1 to K_9 may take on any arbitrary values, positive, negative, or zero, provided they are not such as to make one of the new primaries identical to a combination of the other two; that is, provided that:

$$\begin{vmatrix} K_1 & K_2 & K_3 \\ K_4 & K_5 & K_6 \\ K_7 & K_8 & K_9 \end{vmatrix} \neq 0 \quad (3a)$$

As the exceptions that cause the determinant of the system to vanish are trivial, the choice of coordinate system is very wide. The primaries do not even have to correspond to physically realizable lights. Imaginary lights defined by spectral compositions having negative values for some parts of the spectrum are admissible, and indeed are preferred for routine colorimetry because by their use the computation of tristimulus values from spectrophotometric data is somewhat simplified.

2. The 1931 CIE Standard Colorimetric System and 1964 Supplement

2.1. General Principles

In 1931, the Commission Internationale de l'Eclairage (CIE)³ recommended that all subsequent color data be expressed in terms of the same tristimulus system so that the results would be immediately comparable. The standard observer and coordinate system recommended [49, 65, 135, 143] is defined by the tristimulus values of the spectrum colors given in table 1a and plotted in figure 2. The supplementary observer for large fields, adopted by the CIE in 1964 [59] is given in table 1b. It will be noted that the primaries chosen are such that none of these tristimulus values is less than zero. It is further true that the green primary chosen, whose amounts are designated by Y , is such as to carry all of the luminosity, the other two primaries (red, blue) whose amounts are designated by X and Z , respectively, being unassociated with luminosity. Therefore, the values of \bar{y} for the spectrum correspond to the standard luminosity function, and it is convenient and customary to express the Y value of a luminous area as its *luminance* (photometric brightness) in terms of some recognized unit (such as candles per square meter, millilambert, or foot-lambert). The Y value of an opaque specimen may be conveniently expressed as its *luminous reflectance* (ratio of reflected to incident luminous flux); and the Y value of a transmitting specimen is customarily put in terms of

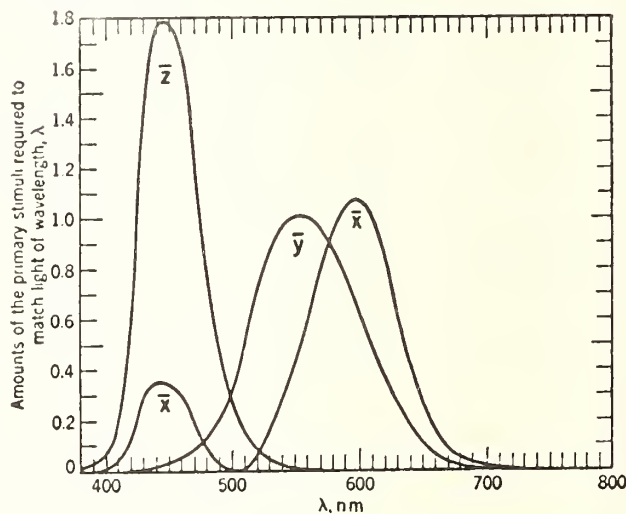


FIGURE 2. Spectral tristimulus values according to the 1931 CIE standard observer.

luminous transmittance (ratio of transmitted to incident luminous flux).

If, as is usual, light combinations are not the chief interest, it is convenient to substitute for the tristimulus values, X, Y, Z , the two ratios, $X/(X + Y + Z)$ and $Y/(X + Y + Z)$, combined with the luminous value, Y . The two ratios are known as *chromaticity coordinates*, x, y , because they serve to

³ This was formerly called ICI for the initial letters of the English name of this commission.

TABLE 1(a). The 1931 CIE standard observer (2°)

Wave-length, nm	Spectral tristimulus values of equal-energy spectrum			Wave-length, nm	Spectral tristimulus values of equal-energy spectrum		
	\bar{x}_2	\bar{y}_2	\bar{z}_2		\bar{x}_2	\bar{y}_2	\bar{z}_2
380	0.0014	0.0000	0.0065	580	0.9163	0.8700	0.0017
385	.0022	.0001	.0105	585	.9786	.8163	.0014
390	.0042	.0001	.0201	590	1.0263	.7570	.0011
395	.0076	.0002	.0362	595	1.0567	.6949	.0010
400	.0143	.0004	.0679	600	1.0622	.6310	.0008
405	.0232	.0006	.1102	605	1.0456	.5668	.0006
410	.0435	.0012	.2074	610	1.0026	.5030	.0003
415	.0776	.0022	.3713	615	0.9384	.4412	.0002
420	.1344	.0040	.6456	620	.8544	.3810	.0002
425	.2148	.0073	1.0391	625	.7514	.3210	.0001
430	.2839	.0116	1.3856	630	.6424	.2650	.0000
435	.3285	.0168	1.6230	635	.5419	.2170	.0000
440	.3483	.0230	1.7471	640	.4479	.1750	.0000
445	.3481	.0298	1.7826	645	.3608	.1382	.0000
450	.3362	.0380	1.7721	650	.2835	.1070	.0000
455	.3187	.0480	1.7441	655	.2187	.0816	.0000
460	.2908	.0600	1.6692	660	.1649	.0610	.0000
465	.2511	.0739	1.5281	665	.1212	.0446	.0000
470	.1954	.0910	1.2876	670	.0874	.0320	.0000
475	.1421	.1126	1.0419	675	.0636	.0232	.0000
480	.0956	.1390	0.8130	680	.0468	.0170	.0000
485	.0580	.1693	.6162	685	.0329	.0119	.0000
490	.0320	.2080	.4652	690	.0227	.0082	.0000
495	.0147	.2586	.3533	695	.0158	.0057	.0000
500	.0049	.3230	.2720	700	.0114	.0041	.0000
505	.0024	.4073	.2123	705	.0081	.0029	.0000
510	.0093	.5030	.1582	710	.0058	.0021	.0000
515	.0291	.6082	.1117	715	.0041	.0015	.0000
520	.0633	.7100	.0782	720	.0029	.0010	.0000
525	.1096	.7932	.0573	725	.0020	.0007	.0000
530	.1655	.8620	.0422	730	.0014	.0005	.0000
535	.2257	.9149	.0298	735	.0010	.0004	.0000
540	.2904	.9540	.0203	740	.0007	.0003	.0000
545	.3597	.9803	.0134	745	.0005	.0002	.0000
550	.4334	.9950	.0087	750	.0003	.0001	.0000
555	.5121	1.0002	.0057	755	.0002	.0001	.0000
560	.5945	0.9950	.0039	760	.0002	.0001	.0000
565	.6784	.9786	.0027	765	.0001	.0000	.0000
570	.7621	.9520	.0021	770	.0001	.0000	.0000
575	.8425	.9154	.0018	775	.0000	.0000	.0000
580	.9163	.8700	.0017	780	.0000	.0000	.0000
Totals -----					21.3713	21.3714	21.3715

TABLE 1(b). *The 1964 CIE supplementary observer (10°)*

Wave-length, nm	Spectral tristimulus values of equal-energy spectrum			Wave-length, nm	Spectral tristimulus values of equal-energy spectrum		
	\bar{x}_{10}	\bar{y}_{10}	\bar{z}_{10}		\bar{x}_{10}	\bar{y}_{10}	\bar{z}_{10}
380	0.0002	0.0000	0.0007	580	1.0142	0.8689	0.0000
385	.0007	.0001	.0029	585	1.0743	.8256	.0000
390	.0024	.0003	.0105	590	1.1185	.7774	.0000
395	.0072	.0008	.0323	595	1.1343	.7204	.0000
400	.0191	.0020	.0860	600	1.1240	.6583	.0000
405	.0434	.0045	.1971	605	1.0891	.5939	.0000
410	.0847	.0088	.3894	610	1.0305	.5280	.0000
415	.1406	.0145	.6563	615	0.9507	.4618	.0000
420	.2045	.0214	.9725	620	.8563	.3981	.0000
425	.2647	.0295	1.2825	625	.7549	.3396	.0000
430	.3147	.0387	1.5535	630	.6475	.2835	.0000
435	.3577	.0496	1.7985	635	.5351	.2283	.0000
440	.3837	.0621	1.9673	640	.4316	.1798	.0000
445	.3867	.0747	2.0273	645	.3437	.1402	.0000
450	.3707	.0895	1.9948	650	.2683	.1076	.0000
455	.3430	.1063	1.9007	655	.2043	.0812	.0000
460	.3023	.1282	1.7454	660	.1526	.0603	.0000
465	.2541	.1528	1.5549	665	.1122	.0441	.0000
470	.1956	.1852	1.3176	670	.0813	.0318	.0000
475	.1323	.2199	1.0302	675	.0579	.0226	.0000
480	.0805	.2536	0.7721	680	.0409	.0159	.0000
485	.0411	.2977	.5701	685	.0286	.0111	.0000
490	.0162	.3391	.4153	690	.0199	.0077	.0000
495	.0051	.3954	.3024	695	.0138	.0054	.0000
500	.0038	.4608	.2185	700	.0096	.0037	.0000
505	.0154	.5314	.1592	705	.0066	.0026	.0000
510	.0375	.6067	.1120	710	.0046	.0018	.0000
515	.0714	.6857	.0822	715	.0031	.0012	.0000
520	.1177	.7618	.0607	720	.0022	.0008	.0000
525	.1730	.8233	.0431	725	.0015	.0006	.0000
530	.2365	.8752	.0305	730	.0010	.0004	.0000
535	.3042	.9238	.0206	735	.0007	.0003	.0000
540	.3768	.9620	.0137	740	.0005	.0002	.0000
545	.4516	.9822	.0079	745	.0004	.0001	.0000
550	.5298	.9918	.0040	750	.0003	.0001	.0000
555	.6161	.9991	.0011	755	.0002	.0001	.0000
560	.7052	.9973	.0000	760	.0001	.0000	.0000
565	.7938	.9824	.0000	765	.0001	.0000	.0000
570	.8787	.9555	.0000	770	.0001	.0000	.0000
575	.9512	.9152	.0000	775	.0000	.0000	.0000
580	1.0142	.8689	.0000	780	.0000	.0000	.0000
Totals -----					23.3294	23.3323	23.3343

TABLE 2(a). Chromaticity coordinates of the spectrum colors (2°)

Wave-length, nm	Chromaticity coordinates			Wave-length, nm	Chromaticity coordinates		
	x_2	y_2	z_2		x_2	y_2	z_2
380	0.1741	0.0050	0.8209	580	0.5125	0.4866	0.0009
385	.1740	.0050	.8210	585	.5448	.4544	.0008
390	.1738	.0049	.8213	590	.5752	.4242	.0006
395	.1736	.0049	.8215	595	.6029	.3965	.0006
400	.1733	.0048	.8219	600	.6270	.3725	.0005
405	.1730	.0048	.8222	605	.6482	.3514	.0004
410	.1726	.0048	.8226	610	.6658	.3340	.0002
415	.1721	.0048	.8231	615	.6801	.3197	.0002
420	.1714	.0051	.8235	620	.6915	.3083	.0002
425	.1703	.0058	.8239	625	.7006	.2993	.0001
430	.1689	.0069	.8242	630	.7079	.2920	.0001
435	.1669	.0086	.8245	635	.7140	.2859	.0001
440	.1644	.0109	.8247	640	.7190	.2809	.0001
445	.1611	.0138	.8251	645	.7230	.2770	.0000
450	.1566	.0177	.8257	650	.7260	.2740	.0000
455	.1510	.0227	.8263	655	.7283	.2717	.0000
460	.1440	.0297	.8263	660	.7300	.2700	.0000
465	.1355	.0399	.8246	665	.7311	.2689	.0000
470	.1241	.0578	.8181	670	.7320	.2680	.0000
475	.1096	.0868	.8036	675	.7327	.2673	.0000
480	.0913	.1327	.7760	680	.7334	.2666	.0000
485	.0687	.2007	.7306	685	.7340	.2660	.0000
490	.0454	.2950	.6596	690	.7344	.2656	.0000
495	.0235	.4127	.5638	695	.7346	.2654	.0000
500	.0082	.5384	.4534	700	.7347	.2653	.0000
505	.0039	.6548	.3413	705	.7347	.2653	.0000
510	.0139	.7502	.2359	710	.7347	.2653	.0000
515	.0389	.8120	.1491	715	.7347	.2653	.0000
520	.0743	.8338	.0919	720	.7347	.2653	.0000
525	.1142	.8262	.0596	725	.7347	.2653	.0000
530	.1547	.8059	.0394	730	.7347	.2653	.0000
535	.1929	.7816	.0255	735	.7347	.2653	.0000
540	.2296	.7543	.0161	740	.7347	.2653	.0000
545	.2658	.7243	.0099	745	.7347	.2653	.0000
550	.3016	.6923	.0061	750	.7347	.2653	.0000
555	.3373	.6589	.0038	755	.7347	.2653	.0000
560	.3731	.6245	.0024	760	.7347	.2653	.0000
565	.4087	.5896	.0017	765	.7347	.2653	.0000
570	.4441	.5547	.0012	770	.7347	.2653	.0000
575	.4788	.5202	.0010	775	.7347	.2653	.0000
580	.5125	.4866	.0009	780	.7347	.2653	.0000

TABLE 2(b). Chromaticity coordinates of the spectrum colors (10°)

Wave-length, nm	Chromaticity coordinates			Wave-length, nm	Chromaticity coordinates		
	x_{10}	y_{10}	z_{10}		x_{10}	y_{10}	z_{10}
0.380	0.1813	0.0197	0.7990	0.580	0.5386	0.4614	0.0000
.385	.1809	.0195	.7996	.585	.5654	.4346	.0000
.390	.1803	.0194	.8003	.590	.5900	.4100	.0000
.395	.1795	.0190	.8015	.595	.6116	.3884	.0000
.400	.1784	.0187	.8029	.600	.6306	.3694	.0000
.405	.1771	.0184	.8045	.605	.6471	.3529	.0000
.410	.1755	.0181	.8064	.610	.6612	.3388	.0000
.415	.1732	.0178	.8090	.615	.6731	.3269	.0000
.420	.1706	.0179	.8115	.620	.6827	.3173	.0000
.425	.1679	.0187	.8134	.625	.6898	.3102	.0000
.430	.1650	.0203	.8147	.630	.6955	.3045	.0000
.435	.1622	.0225	.8153	.635	.7010	.2990	.0000
.440	.1590	.0257	.8153	.640	.7059	.2941	.0000
.445	.1554	.0300	.8146	.645	.7103	.2898	.0000
.450	.1510	.0364	.8126	.650	.7137	.2863	.0000
.455	.1459	.0452	.8088	.655	.7156	.2844	.0000
.460	.1389	.0589	.8022	.660	.7168	.2832	.0000
.465	.1295	.0779	.7926	.665	.7179	.2821	.0000
.470	.1152	.1090	.7758	.670	.7187	.2813	.0000
.475	.0957	.1591	.7452	.675	.7193	.2807	.0000
.480	.0728	.2292	.6980	.680	.7198	.2802	.0000
.485	.0452	.3275	.6273	.685	.7200	.2800	.0000
.490	.0210	.4401	.5389	.690	.7202	.2798	.0000
.495	.0073	.5625	.4302	.695	.7203	.2797	.0000
.500	.0056	.6745	.3199	.700	.7204	.2796	.0000
.505	.0219	.7526	.2256	.705	.7203	.2797	.0000
.510	.0495	.8023	.1482	.710	.7202	.2798	.0000
.515	.0850	.8170	.0980	.715	.7201	.2799	.0000
.520	.1252	.8102	.0646	.720	.7199	.2801	.0000
.525	.1664	.7922	.0414	.725	.7197	.2803	.0000
.530	.2071	.7663	.0267	.730	.7195	.2806	.0000
.535	.2436	.7399	.0165	.735	.7192	.2808	.0000
.540	.2786	.7113	.0101	.740	.7189	.2811	.0000
.545	.3132	.6813	.0055	.745	.7186	.2814	.0000
.550	.3473	.6501	.0026	.750	.7183	.2817	.0000
.555	.3812	.6182	.0007	.755	.7180	.2820	.0000
.560	.4142	.5858	.0000	.760	.7176	.2824	.0000
.565	.4469	.5531	.0000	.765	.7172	.2828	.0000
.570	.4790	.5210	.0000	.770	.7169	.2831	.0000
.575	.5096	.4904	.0000	.775	.7165	.2835	.0000
.580	.5386	.4614	.0000	.780	.7161	.2839	.0000

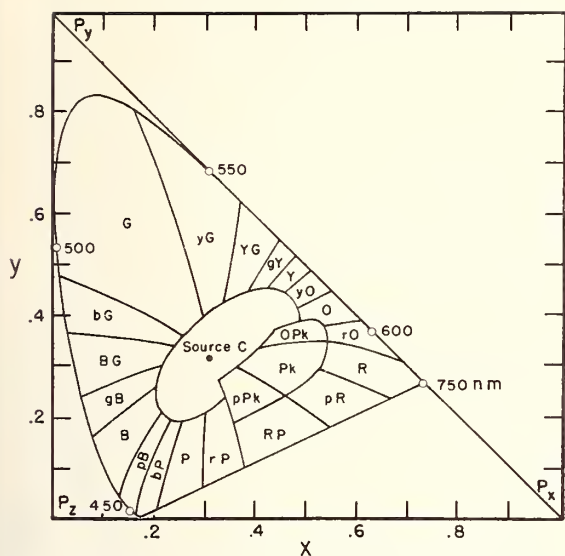


FIGURE 3. The (x,y) -chromaticity diagram, showing the spectrum locus and the purple boundary.

Wavelength is indicated in millimicrons. The hue names are those proposed by Kelly (77).

specify the chromatic aspect of the light. The analogous ratio, $Z/(X + Y + Z)$, is also known as a chromaticity coordinate, z , but only two of the three coordinates, x, y, z , give independent information since by definition the sum of all three is unity. Table 2a gives the chromaticity coordinates, x, y, z , of the spectrum colors for 2° and table 2b for 10° fields [59]. Figure 3 shows the points representing the spectrum colors in the (x, y) -chromaticity diagram. This diagram is also known as a *Maxwell triangle* because of Maxwell's first use of such a diagram [98]. Furthermore, it has aptly been called a *mixture diagram* because it indicates in a very simple way the chromaticity of the color resulting from the additive combination of any two lights. The point representing this chromaticity is found on the straight line connecting the points representing two lights. The primary lights are represented by points at the corners of a triangle, and every point within the triangle represents the chromaticity of a mixture of the primary lights whose proportions are indicated by the chromaticity coordinates, x, y, z . The spectrum colors are shown by a smooth curve known as the spectrum locus. A few points on this locus are identified by wavelength in nanometers. It will be noted from figure 3 that the spectrum locus is substantially straight from 540 nm to the long-wave extreme. This means that the standard observer would find binary mixtures of, say, 540 nm with 640 nm, closely equivalent to some intermediate portion of the spectrum. But the spectrum locus from 540 nm to the short-wave extreme is curved outward. This means that for the standard observer a binary mixture of 540 nm with, say, 440 nm would differ importantly in chromaticity from the

intermediate parts of the spectrum. By drawing straight lines through any central point (such as $x = y = 1/3$, representing the so-called equal-energy stimulus) and extending them until they cut the spectrum locus, we may find the spectral complementaries relative to a stimulus represented by that point; that is, we may find the two parts of the spectrum that, when combined in proper proportions, will for the standard observer be equivalent to the central stimulus.

The straight line in figure 3 joining the extremes of the spectrum locus represents the chromaticities of the mixtures of the two extremes of the visible spectrum. The area bounded by the closed curve made up of the spectrum locus and this straight line is the locus of all physically realizable chromaticities. Note that the points representing the primaries of the CIE coordinate system, the apices of the triangle ($x = 1, y = z = 0$; $y = 1, x = z = 0$; $z = 1, x = y = 0$), all fall outside this area; that is, the primaries are imaginary. Note also that both the X and Z primaries fall on the line $y = 0$, which is unassociated with luminosity and is known as the *alychne* or *lightless line*. The short-wave extreme of the spectrum locus comes close to this line; this means that, although it has the power to elicit in the standard observer a considerable X and Z response, resulting in a vivid bluish purple color, radiant flux of wavelength 380 to 420 nm is almost unassociated with luminosity. The areas in figure 3 corresponding to common color designations for lights are those proposed by Kelly [77] and will be discussed later.

2.2. Standard Sources

At the time of setting up the standard observer and coordinate system, the International Commission on Illumination [135], Commission Internationale de l'Eclairage (CIE), recommended use of three standard sources for colorimetry; source A, representative of gas-filled incandescent lamps; source B, representative of noon sunlight; and source C, representative of average daylight such as that from a completely overcast sky. Source A is an incandescent lamp operated at a color temperature of 2854°K , on the international temperature scale ($c_2 = 14,380$). Source B is obtained by using this same lamp in combination with a two-cell Davis-Gibson liquid filter designed to give a color temperature of about 5000°K . Source C is obtained similarly and results in a source of correlated color temperature about 6800°K . These sources are recommended for general use, or whenever there is no special reason for using some other source. Table 3 gives the relative spectral irradiance of Sources A, B, C, D_{5500} , D_{6500} , and D_{7500} . Sources D_{5500} , D_{6500} , D_{7500} represent several phases of daylight, closely represented by the subscripted correlated color temperatures. Tables 4a and 4b give computation forms for evaluation of the colors of non-self-luminous specimens that transmit, scatter, or reflect incident light for the 2° standard ob-

TABLE 3. Relative spectral irradiance E of the six standard sources, A , B , C , D_{3500} , D_{6500} , and D_{7500}

Wave-length nm	E_A	E_B	E_C	E_{5500}	E_{6500}	E_{7500}	Wave-length nm	E_A	E_B	E_C	E_{5500}	E_{6500}	E_{7500}
380	9.79	22.40	33.00	32.60	50.00	66.70	580	114.44	101.00	97.80	97.70	95.80	94.20
385	10.90	26.85	39.92	33.35	52.30	68.35	585	118.08	100.07	95.43	94.55	92.25	90.60
390	12.09	31.30	47.40	38.10	54.60	70.00	590	121.73	99.20	93.20	91.40	88.70	87.00
395	13.36	36.18	55.17	49.50	68.70	85.95	595	125.39	98.44	91.22	92.90	89.35	87.10
400	14.71	41.30	63.30	60.90	82.80	101.90	600	129.04	98.00	89.70	94.40	90.00	87.20
405	16.15	46.62	71.81	64.75	87.15	106.90	605	132.70	98.08	88.83	94.75	89.80	86.65
410	17.68	52.10	80.60	68.60	91.50	111.90	610	136.34	98.50	88.40	95.10	89.60	86.10
415	19.29	57.70	89.53	70.10	92.45	112.35	615	139.99	99.06	88.19	94.65	88.65	84.85
420	21.00	63.20	98.10	71.60	93.40	112.80	620	143.62	99.70	88.10	94.20	87.70	83.60
425	22.79	68.37	105.80	69.75	90.05	107.95	625	147.23	100.36	88.06	92.30	85.50	81.15
430	24.67	73.10	112.40	67.50	86.70	103.10	630	150.83	100.00	88.00	90.40	83.30	78.70
435	26.64	77.31	117.75	76.75	95.80	112.15	635	154.42	101.56	87.86	91.35	83.50	78.55
440	28.70	80.80	121.50	85.60	104.90	121.50	640	157.98	102.20	87.80	91.30	83.70	78.40
445	30.85	83.44	123.45	91.80	110.95	127.25	645	161.51	103.05	87.99	90.60	81.85	76.60
450	33.09	85.40	124.00	98.00	117.00	133.30	650	165.03	103.90	88.30	88.90	80.00	74.80
455	35.41	86.88	123.60	99.25	114.40	132.85	655	168.51	104.59	88.20	89.60	80.10	74.55
460	37.82	88.30	123.10	100.50	117.80	132.40	660	171.96	105.00	87.90	90.30	80.20	74.30
465	40.30	90.08	123.30	100.20	116.35	129.85	665	175.38	105.08	87.22	92.10	81.25	74.85
470	42.87	92.00	123.80	99.90	114.90	127.30	670	178.77	104.90	86.30	93.90	82.30	75.40
475	45.52	93.75	124.09	101.30	115.40	127.05	675	182.12	104.55	85.30	91.95	80.30	73.50
480	48.25	95.20	123.90	102.70	115.90	126.80	680	184.43	103.90	84.00	90.00	78.30	71.60
485	51.04	96.23	122.92	100.40	112.35	122.30	685	188.70	102.84	82.21	84.85	74.00	69.75
490	53.91	96.50	120.70	98.10	108.80	117.80	690	191.93	101.60	80.20	79.70	69.70	68.90
495	56.85	95.71	116.90	99.40	109.10	117.20	695	195.12	100.38	78.24	81.25	70.65	64.50
500	59.86	94.20	112.10	100.70	109.40	116.60	700	198.26	99.10	76.30	82.80	71.60	65.10
505	62.93	92.37	106.98	100.70	108.60	115.15	705	201.36	97.70	74.36	83.80	72.95	66.60
510	66.06	90.70	102.30	100.70	107.80	113.70	710	204.41	96.20	72.40	84.80	74.30	68.10
515	69.25	89.65	98.81	100.35	106.30	111.20	715	207.41	94.60	70.40	77.50	67.95	62.25
520	72.50	89.50	96.90	100.00	104.80	108.70	720	210.36	92.90	68.30	70.20	61.60	56.40
525	75.79	90.43	96.78	102.10	106.25	109.55	725	213.26	91.10	66.30	74.75	65.75	60.30
530	79.13	92.20	98.00	104.20	107.70	110.40	730	216.12	89.40	64.40	79.30	69.90	64.20
535	82.52	94.46	99.94	103.15	106.05	108.35	735	218.92	88.00	62.80	82.15	72.50	66.70
540	85.95	96.90	102.10	102.10	104.40	106.30	740	221.66	86.90	61.50	85.00	75.10	69.20
545	89.41	99.16	103.95	102.55	104.20	105.60	745	224.36	85.90	60.20	78.45	69.35	63.90
550	92.91	101.00	105.20	103.00	104.00	104.90	750	227.00	85.20	59.20	71.90	63.60	58.60
555	96.44	102.20	105.67	101.50	102.00	102.45	755	229.58	84.80	58.50	62.35	55.00	50.60
560	100.00	102.80	105.30	100.00	100.00	100.00	760	232.11	84.70	58.10	52.80	46.40	42.60
565	103.58	102.92	104.11	98.60	98.15	97.80	765	234.59	84.90	58.00	64.35	56.60	52.00
570	107.18	102.60	102.30	97.20	96.30	95.60	770	237.01	85.40	58.20	75.90	66.80	61.40
575	110.80	101.90	100.15	97.45	96.05	94.90	775	239.37	86.10	58.50	73.85	65.10	59.85
580	114.44	101.00	97.80	97.70	95.80	94.20	780	241.67	87.00	59.10	71.80	63.40	58.30

TABLE 4a. *Computation form*

CIE coordinates (2°)

Source A (2854 °K.)

Sample----- Source of Trans. Data-----

Submitted by-----

λ (nm)	$\overline{E\alpha x_2}$	$\overline{E\alpha y_2}$	$\overline{E\alpha z_2}$	τ	$\tau \overline{E\alpha x_2}$	$\tau \overline{E\alpha y_2}$	$\tau \overline{E\alpha z_2}$
380	1		6	0.			
90	5		23	.			
400	19	1	93	.			
10	71	2	340	.			
20	262	8	1256	.			
30	649	27	3167	.			
40	926	61	4647	.			
450	1031	117	5435	.			
60	1019	210	5851	.			
70	776	362	5116	.			
80	428	622	3636	.			
90	160	1039	2324	.			
500	27	1792	1509	.			
10	57	3080	969	.			
20	425	4771	525	.			
30	1214	6322	309	.			
40	2313	7600	162	.			
550	3732	8568	75	.			
60	5510	9222	36	.			
70	7571	9457	21	.			
80	9719	9228	18	.			
90	11579	8540	12	.			
600	12704	7547	10	.			
10	12669	6356	4	.			
20	11373	5071	8	.			
30	8980	3704		.			
40	6558	2562		.			
650	4336	1637		.			
60	2628	972		.			
70	1448	530		.			
80	804	292		.			
90	404	146		.			
700	209	75		.			
10	110	40		.			
20	57	19		.			
30	28	10		.			
40	14	6		.			
750	6	2		.			
60	4	2		.			
70	2			.			
Sums x_w, y_w, z_w	109828 0.4476	100000 0.4075	35547 0.1449	Sums $S = X+Y+Z$	X =	Y =	Z =

x, y , and z
 $(x = X/S, y = Y/S, z = Z/S)$

Planck 2854 °K., $c_2 = 14,380$
 Planck 2848 °K., $c_2 = 14,350$

Computed by----- Checked by-----

TABLE 4b. *Computation form*

CIE coordinates

Source C (Davis-Gibson, 2854 to 6800 °K.)

Sample----- Source of Trans. Data-----

Submitted by-----

λ (nm)	$E_c \bar{x}_2$	$E_c \bar{y}_2$	$E_c \bar{z}_2$	τ	$\tau E_c \bar{x}_2$	$\tau E_c \bar{y}_2$	$\tau E_c \bar{z}_2$
380	4		20	0.			
90	19		89	.			
400	85	2	404	.			
10	325	9	1570	.			
20	1238	37	5949	.			
30	2997	122	14628	.			
40	3975	262	19938	.			
450	3915	443	20638	.			
60	3362	694	19239	.			
70	2272	1058	14972	.			
80	1112	1618	9461	.			
90	363	2358	5274	.			
500	52	3401	2864	.			
10	89	4833	1520	.			
20	578	6462	712	.			
30	1523	7934	388	.			
40	2785	9149	195	.			
550	4282	9832	86	.			
60	5880	9841	89	.			
70	7322	9147	20	.			
80	8417	7992	16	.			
90	8984	6627	10	.			
600	8949	5316	7	.			
10	8325	4176	2	.			
20	7070	3153	2	.			
30	5309	2190	.	.			
40	3693	1443	.	.			
650	2349	886	.	.			
60	1361	504	.	.			
70	708	259	.	.			
80	369	134	.	.			
90	171	62	.	.			
700	82	29	.	.			
10	39	14	.	.			
20	19	6	.	.			
30	8	3	.	.			
40	4	2	.	.			
750	2	1	.	.			
60	1	1	.	.			
70	1		.	.			
Sums	98041	100000	118103	Sums	$X =$	$Y =$	$Z =$
x_w, y_w, z_w	0.3101	0.3163	0.3736	$s = X+Y+Z$			

$x, y,$ and z
 $(x = X/S, y = Y/S, z = Z/S)$

Planck 2854 °K., $c_2 = 14,380$
 Planck 2848 °K., $c_2 = 14,350$

Computed by----- Checked by-----

TABLE 4c. 10° Distribution coefficients
for sources S_A and S_C

λ (nm)	For S_A			For S_C		
	$E\alpha\bar{x}_{10}$	$E\alpha\bar{y}_{10}$	$E\alpha\bar{z}_{10}$	$E\bar{c}_{x10}$	$E\bar{c}_{y10}$	$E\bar{c}_{z10}$
380			1	1		2
90	3		11	9	1	43
400	25	3	111	103	11	463
10	132	14	605	581	60	2672
20	377	40	1795	1708	179	8122
30	682	83	3368	3011	370	14865
40	968	156	4962	3969	343	20349
450	1078	260	5802	3914	945	21058
60	1005	426	5802	3168	1343	18292
70	737	698	4965	2062	1952	13887
80	341	1076	3274	840	2675	8144
90	76	1607	1968	167	3484	4268
500	20	2424	1150	37	4398	2085
10	218	3523	650	327	5284	976
20	750	4854	387	971	6285	501
30	1644	6086	212	1973	7302	255
40	2847	7267	104	3275	8362	119
550	4326	8099	33	4744	8882	36
60	6198	8766		6322	8941	
70	8277	9002		7653	8322	
80	10201	8740		8444	7235	
90	11967	8317		8874	6168	
600	12748	7466		8583	5027	
10	12349	6327		7756	3974	
20	10809	5026		6422	2986	
30	8583	3758		4851	2124	
40	5992	2496		3226	1344	
650	3892	1561		2014	808	
60	2306	911		1142	451	
70	1277	499		598	233	
80	666	259		293	114	
90	336	130		136	53	
700	167	64		62	24	
10	83	33		28	11	
20	40	15		13	4	
30	19	8		5	3	
40	10	4		3	1	
750	6	2		2	1	
60	2			1		
70	2			1		
	111,159	100,000	32,200	97,298	100,000	116,137

server. Table 4a refers to Source A; table 4b, to Source C. Source B is relatively little used except in Great Britain. Table 4c gives corresponding values for the 10° observer.

2.3. Spectrophotometric Colorimetry

The fundamental nature of the tristimulus specification of color permits it to be used as a common denominator by means of which colorimeters involving color standards of glass, plastic, or solutions, or systems of material color standards, transparent and opaque, may be inter-compared. In order to demonstrate how the CIE standard observer and coordinate system may be used for this purpose, four printing inks, red purple, greenish yellow, greenish blue, and blue, have been evaluated, and the steps are reproduced here in detail. Figure 4 shows spectral reflectances of these four printing inks obtained on a recording spectrophotometer. Table 5a gives the spectral reflectances read from the originals of the curves of figure 4. These reflectances apply to the specimens measured except for small wavelength-scale and photometric-scale corrections

which have not been applied. Table 5b gives for the greenish yellow specimen the products indicated on the form for computation of luminous reflectance, Y , and chromaticity coordinates, x, y, z , under standard source C; see table 4b. The sums of these products are the tristimulus values, X, Y, Z . The luminous reflectance is found as $Y/100,000$; and the chromaticity coordinates, x, y, z , are found by dividing X, Y , and Z , respectively, by the corresponding sum, $X + Y + Z$. Table 5c lists these results for all four printing-ink specimens. Figure 5 is the (x, y) -chromaticity diagram on which have been plotted large dots to represent these chromaticity coordinates, x, y .

Comparison of figure 5 with figure 3 shows that the chromaticity points of the four printing-ink specimens correspond to the hue designations red purple, greenish yellow, and blue. This accords well with the color designations found by visual inspection of the specimens. Furthermore, it will be noted that one of the blues is greener than the other. The position of the chromaticity point for the greener-ink color is in accord with the greener hue of this ink. Note also that the greenish yellow is much lighter than the red purple or either of the blues; this accords with the luminous reflectance determinations (compare 0.74 with 0.221, 0.242, and 0.246 in table 5c).

2.4. Short-Cuts and Automatic Computation

The labor of computing X, Y, Z , or Y, x, y , corresponding to pairs of spectrophotometric curves to see how the colors of the corresponding specimens compare is considerably great. Often the degree of metamerism exhibited by the pair is sufficiently small that the comparison can be made directly from the curves themselves, and much product-control work can be handled in this way. There is still frequent need, in the establishment of color standards and tests for conformity to those standards, to compute the tristimulus values, X, Y, Z , by a short-cut method, the selected ordinate method, to reduce spectrophotometric data.

In this method the ordinates of the spectrophotometric curve are read at a series of selected wavelengths different for each source. Instead of multiplying by the tristimulus values of the spectrum of the source, $(E\bar{x})_\lambda$, $(E\bar{y})_\lambda$, $(E\bar{z})_\lambda$, the selected ordinates are spaced proportionately closer in the wavelength regions where the tristimulus values are higher, and the corresponding readings of spectral reflectance are simply added. Tables 6 gives [21, 49] selected ordinates for source A (incandescent lamp light) and source C (average daylight). Table 7 gives the spectral reflectances of the greenish yellow printing-ink specimen read from figure 4b for the selected ordinates for source C together with the sums of these spectral reflectances, both for ten ordinates and for thirty. It will be noted that, after applying the multiplying factors listed in table 6, the tristimulus values, X, Y, Z , for the greenish yellow print-

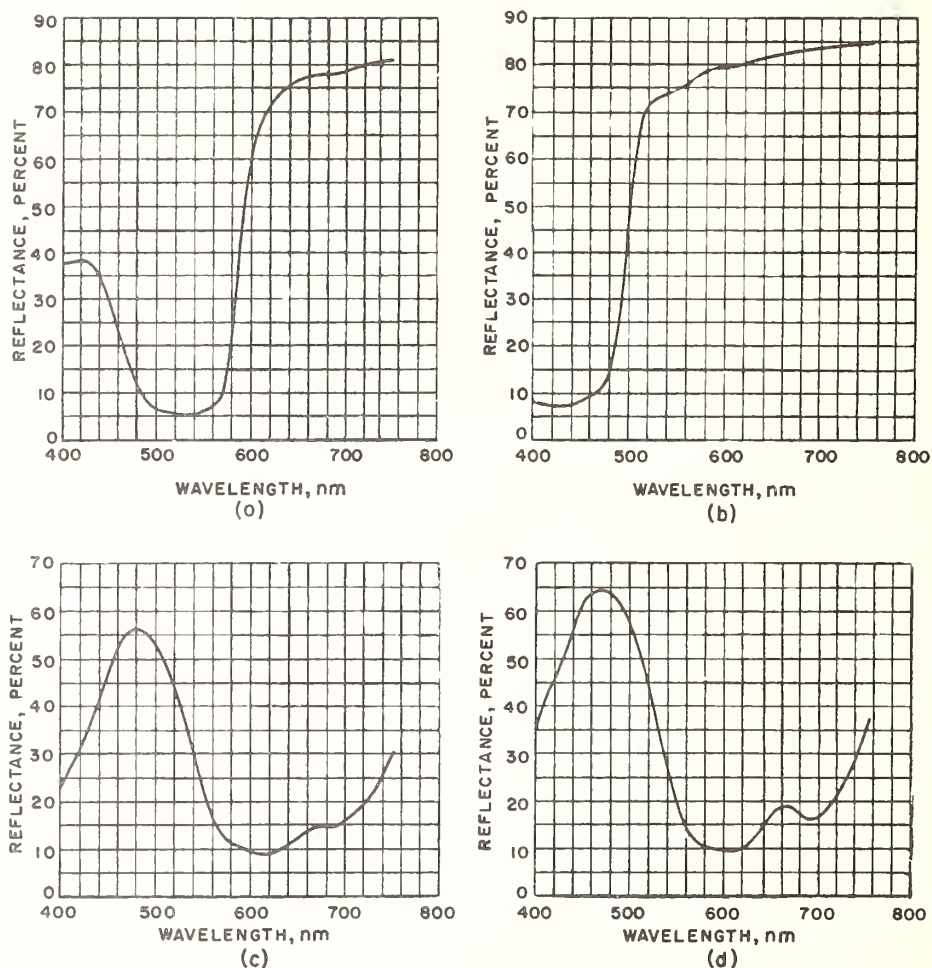


FIGURE 4. Spectral reflectance of printing-ink specimens: (a) red-purple; (b) greenish yellow; (c) greenish blue; (d) blue.

ing-ink specimen are found again to a close approximation (compare 0.630, 0.704, and 0.145 from table 7 with 0.631, 0.704, and 0.145, respectively, from table 5c).

Ten selected ordinates sometimes give significant information (see table 7); thirty selected ordinates are often sufficient (as above); and one hundred selected ordinates are sufficient for all but a few very irregular spectral distributions (such as produced by gaseous discharge tubes). These wavelengths are available for many sources in other publications [13, 21, 49]. Nickerson [114] and De Kerf [27] have published studies of the reliability of the selected-ordinate method of computation.

Analog and digital techniques have been developed for use with automatic computing devices to abbreviate the labor of computation required by spectrophotometric colorimetry. In the application of the analog technique [25] data are sent directly from the spectrophotometer to the computer without wavelength-scale and photometric-scale corrections. In one application of the digital

technique [12, 95] spectral data are punched directly on cards and fed into the computer, while in another [75] the spectral data are corrected and then punched on cards and fed into the computer.

2.5. Visual Tristimulus Colorimeters

Tristimulus values, X, Y, Z , may be obtained by direct comparison of the unknown light with an optical mixture of three primary lights in a divided photometric field. Since the primaries of the CIE standard colorimetric coordinate system are imaginary, such a tristimulus colorimeter cannot be made to read directly. It must be calibrated by measurements of four known stimuli, and then may yield tristimulus values, X, Y, Z , by a transformation the reverse of that indicated in eq. (3). Since the color matches set up in a tristimulus colorimeter designed to cover any substantial part of all possible colors with a single set of primaries exhibit serious metamerism, the field has to be relatively small, subtending about 2 deg at the observer's eye. This restriction to a small

TABLE 5a. Spectral reflectances of four printing-ink specimens

Wave-length, nm	Spectral reflectance relative to magnesium oxide			
	Red purple	Greenish yellow	Greenish blue	Blue
380	0.375*	0.091*	0.150*	0.230*
390	.375*	.089*	.187*	.293*
400	.376	.085	.228	.354
410	.379	.079	.269	.415
420	.381	.077	.306	.458
430	.373	.076	.353	.505
440	.345	.077	.407	.563
450	.295	.086	.467	.616
460	.235	.095	.520	.639
470	.174	.108	.552	.645
480	.120	.145	.560	.635
490	.083	.250	.548	.608
500	.066	.445	.523	.568
510	.061	.635	.483	.508
520	.057	.708	.432	.438
530	.054	.725	.363	.353
540	.055	.733	.292	.272
550	.062	.743	.220	.198
560	.071	.752	.162	.145
570	.095	.768	.128	.117
580	.220	.782	.113	.106
590	.440	.787	.102	.102
600	.597	.790	.093	.098
610	.676	.793	.088	.097
620	.715	.798	.088	.103
630	.739	.803	.098	.122
640	.756	.809	.110	.147
650	.768	.814	.124	.172
660	.776	.818	.136	.187
670	.780	.822	.145	.186
680	.782	.824	.147	.172
690	.783	.827	.149	.162
700	.788	.829	.160	.169
710	.794	.832	.177	.192
720	.799	.833	.196	.221
730	.805	.835	.218	.256
740	.809	.836	.258	.304
750	.812	.837	.298	.362
760	.815*	.838*	.338*	.422*
770	.817*	.839*	.375*	.484*

*Extrapolated.

angular size of field severely limits the precision of setting compared to what is possible by direct comparison of large specimens in daylight. Furthermore, the metamerism also prevents one normal observer from getting the same reading as another except by accident. If a reasonable approximation to the standard values of X, Y , and Z is to be assured, either the readings of a group of five or ten observers must be averaged, or a color standard yielding a spectral composition similar to that of the unknown specimen must be used. Because of industrial interest in large-field color matching Stiles and Burch [146] and Speranskaya [144] determined the color-matching functions for 10° -field viewing. In this determination either the observers were instructed to ignore the Maxwell spot [73, 98, 103, 104, 156] or it was masked. The color-matching functions thus found are significantly different from the 2° -field functions of the 1931 CIE Standard Observer. The difference is chiefly that expected from the removal of an intervening yellow filter, the macular pigment, from the field of view.

It will be seen that tristimulus colorimeters give

TABLE 5b. Computation of tristimulus values, X, Y, Z , and chromaticity coordinates, x, y , for the greenish-yellow printing-ink specimen under source C

(The computation form given as table 4b has been used.)

Wave-length, nm	Reflectance (ρ)	$\rho E_c \bar{x}$	$\rho E_c \bar{y}$	$\rho E_c \bar{z}$
380	0.091	0		2
390	.089	2		8
400	.085	7	0	34
410	.079	26	1	124
420	.077	95	3	458
430	.076	228	9	1112
440	.077	306	20	1535
450	.086	337	38	1775
460	.095	319	66	1833
470	.108	245	114	1617
480	.145	161	235	1372
490	.250	91	690	1319
500	.445	23	1513	1274
510	.635	57	3069	965
520	.708	408	4575	504
530	.725	1104	5752	281
540	.733	2041	6706	143
550	.743	3182	7305	64
560	.752	4422	7400	29
570	.768	5623	7025	15
580	.782	6582	6250	13
590	.787	7070	5215	8
600	.790	7070	4200	6
610	.793	6602	3312	2
620	.798	5642	2516	2
630	.803	4263	1759	0
640	.809	2988	1167	
650	.814	1912	721	
660	.818	1113	412	
670	.822	582	213	
680	.824	304	110	
690	.827	141	51	
700	.829	68	24	
710	.832	32	12	
720	.833	16	5	
730	.835	7	3	
740	.836	3	2	
750	.837	2	1	
760	.838	1	1	
770	.839	1	0	
Tristimulus values	X, Y, Z	63,076	70,395	14,495
Chromaticity coordinates	x, y, z	0.4263	0.4758	0.0980

TABLE 5c. Tristimulus values, X, Y, Z , under source C, luminous reflectance relative to magnesium oxide, Y/Y_0 , and chromaticity coordinates, x, y , computed from the spectral reflectance of four printing-ink specimens as in table 5b

Hue designation of specimen	Tristimulus values, arbitrary units			Luminous reflectance, Y/Y_0	Chromaticity coordinates	
	X	Y	Z		x	y
Red purple	39788	22124	30570	0.221	0.430	0.239
Greenish yellow	63076	70395	14495	.704	.426	.476
Greenish blue	19003	24245	54529	.242	.194	.248
Blue	21948	24633	69010	.246	.190	.213

only poor information regarding the unknown specimen. Their application to product-control problems is negligible, but because of the ease of calibration and simplicity of the theory they are very useful research tools. Tristimulus colorime-

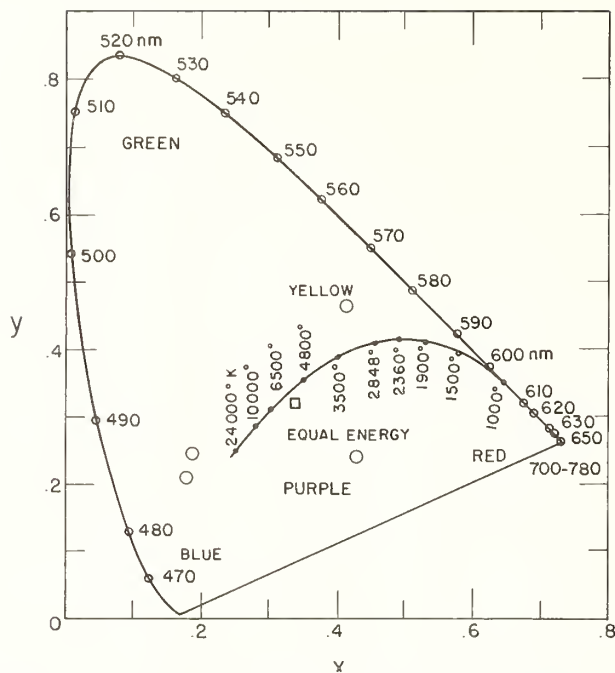


FIGURE 5. Points representing the colors of four printing-ink specimens whose spectral reflectances are shown in figure 9.4.

The colors of the ideal closed-cavity radiator are also shown, the temperatures of the radiators being indicated in degrees Kelvin. The smooth curve connecting these points is often called the Planckian locus.

ters have been built and described by Allen [3], Donaldson [28], Guild [43], McAdam [96], Newhall [110], Stiles [145], Verbeek [154], and Wright [157]. The Wright instrument has spectrum primaries; the other four have primaries formed by combining a light source with glass filters. To the Guild and Wright instruments we owe our accurate information regarding the properties of the normal visual system which have been expressed in terms of the standard observer.

The foregoing instruments make up the comparison-field mixture by optical combination of light beams from different sources so that a sum of the separate effects is obtained. A similar optical effect is obtained if the beams are caused to fall upon the same portion of the retina in such rapid succession that a nonflickering spot of color is seen. The effect is that of a time-weighted average of the separate beams. A very simple and widely used tristimulus colorimeter is obtained by taking four disks that have been cut along a radius, interlocking them so as to expose a sector of each, and causing them to rotate on the spindle of a motor so rapidly that neither the separate sectors nor even flicker is perceived. Such an arrangement for combining colors by rotary mixture is called a Maxwell disk. The four disks provide the necessary three degrees of freedom in the adjustment for a match, and if the tristimulus values of the component disks be known (X_1, Y_1, Z_1 ;

X_2, Y_2, Z_2 ; X_3, Y_3, Z_3 ; X_4, Y_4, Z_4), the tristimulus values of the mixture color can be computed from the fractions of the total area occupied by the respective sectors, f_1, f_2, f_3, f_4 :

$$\begin{aligned} X &= f_1 X_1 + f_2 X_2 + f_3 X_3 + f_4 X_4 \\ Y &= f_1 Y_1 + f_2 Y_2 + f_3 Y_3 + f_4 Y_4 \\ Z &= f_1 Z_1 + f_2 Z_2 + f_3 Z_3 + f_4 Z_4. \end{aligned} \quad (4)$$

If the disks are chosen anew for each kind of unknown color to be measured so as to be all fairly similar in color to the unknown, the spectral composition of the mixture color is usually sufficiently nonmetameric that no restriction to the central 2 deg of the retina is required. Furthermore any two normal observers with some experience at making the adjustment can check each other closely. The chief drawback of this simple arrangement for product-control work in color is the time lost in adjustment of the sector disk areas. The motor must be turned off, brought to a stop, the disks loosened and readjusted, the motor turned on and allowed to resume speed several times to obtain a final setting of reasonably good precision.

Nickerson has described a disk colorimeter [105] that avoids the difficulties of the elementary Maxwell disk. Light reflected from the unknown specimen fills one-half of a photometric field, and that from a stationary sector disk fills the other. By having the observer look at the sector disk through a rapidly rotating glass wedge, each sector is presented to view in sufficiently quick succession that no flicker is produced; and at the same time the sectors, since they are stationary, may be continuously adjusted until a color match is obtained. A further advantage is obtained by extending the rotary scanning to the unknown specimen. In this way the average color of a notably nonuniform specimen such as that made up of coarse salt crystals may be obtained. The disk colorimeter has been extensively used by the United States Department of Agriculture for the color-grading of food products and is well adapted to product-control colorimetry of many kinds.

2.6. Dominant Wavelength and Purity

Another way to identify a combination of lights to specify a color, alternate to the tristimulus method, is to determine the luminance (photometric brightness) of one spot of light of fixed spectral composition (such as average daylight) and the luminance of a spot of light of continuously variable spectral composition separately identified (as by wavelength in the spectrum). In this way the requisite three degrees of freedom in adjustment to a color match are supplied. This form of identification leads naturally into a specification in which the luminance [20] of the unknown is given and the chromaticity is specified by two variables in polar coordinates. One of these variables is an angle, the other a radius, and both

TABLE 6. *Selected Ordinates (In Nanometers) for Computing Tristimulus Values, X, Y, Z, for Specimens Under Standard Sources A and C [20]*

Number	Source A			Source C		
	X	Y	Z	X	Y	Z
1	444.0	487.8	416.4	424.4	465.9	414.1
2*	516.9*	507.7*	424.9*	435.5*	489.4*	422.2*
3	544.0	517.3	429.4	443.9	500.4	426.3
4	554.2	524.1	432.9	452.1	508.7	429.4
5*	561.4*	529.8*	436.0*	461.2*	515.1*	432.0*
6	567.1	534.8	438.7	474.0	520.6	434.3
7	572.0	539.4	441.3	531.2	525.4	436.5
8*	576.3*	543.7*	443.7*	544.3*	529.8*	438.6*
9	580.2	547.8	446.0	552.4	533.9	440.6
10	583.9	551.7	448.3	558.7	537.7	442.5
11*	587.2*	555.4*	450.5*	564.1*	541.4*	444.4*
12	590.5	559.1	452.6	568.9	544.9	446.3
13	593.5	562.7	454.7	573.2	548.4	448.2
14*	596.5*	566.3*	456.8*	577.3*	551.8*	450.1*
15	599.4	569.8	458.8	581.3	555.1	452.1
16	602.3	573.3	460.8	585.0	558.5	454.0
17*	605.2*	576.9*	462.9*	588.7*	561.9*	455.9*
18	608.0	580.5	464.9	592.4	565.3	457.9
19	610.9	584.1	467.0	596.0	568.9	459.9
20*	613.8*	587.9*	469.2*	599.6*	572.5*	462.0*
21	616.9	591.8	471.6	603.3	576.4	464.1
22	620.0	595.9	474.1	607.0	580.5	466.3
23*	623.3*	600.1*	476.8*	610.9*	584.8*	468.7*
24	626.9	604.7	479.9	615.0	589.6	471.4
25	630.8	609.7	483.4	619.4	594.8	474.3
26*	635.3*	615.2*	487.5*	624.2*	600.8*	477.7*
27	640.5	621.5	492.7	629.8	607.7	481.8
28	646.9	629.2	499.3	636.6	616.1	487.2
29*	655.9*	639.7*	508.4*	645.9*	627.3*	495.2*
30	673.5	659.0	526.7	663.0	647.4	511.2
Multiplying factors						
30 ordinates -	0.03661	0.03333	0.01185	0.03268	0.03333	0.03938
10 ordinates -	.10984	.10000	.03555	.09804	.10000	.11812

*Values for calculation with 10 selected ordinates.

can be computed from the chromaticity coordinates of the fixed spot of light, the variable spot of light, and the unknown, these coordinates serving to locate the respective positions in a chromaticity diagram. If the fixed light is nearly achromatic, the angle often correlates well with the hue of the color perception, and the radius fairly well with its saturation. The most fundamental way to specify the direction on a chromaticity diagram from the point representing the fixed light to the point representing the unknown light is by wavelength of the part of the spectrum required to make the match. If the unknown color can be matched by adding some part of the spectrum to the fixed light, it is said to have a spectral color, and the required wavelength is called *dominant wavelength*. But if a color match is produced for the fixed light by adding some part of the spectrum to the unknown color, the unknown is said to be nonspectral, and the required wavelength is called the *complementary wavelength*. Either dominant wavelength or complementary wavelength may be obtained for the standard observer by drawing on a chromaticity diagram a straight line through the point representing the fixed light

and that representing the unknown color, and then by reading the wavelength corresponding to the point at which this line extended intersects the locus of spectrum colors. If the unknown color is plotted between the fixed light and the spectrum, the intersection gives the dominant wavelength; but if the fixed light is represented by a point intermediate to the unknown and the intersection of the straight line with the spectrum locus, the intersection indicates the complementary wavelength.

The degree of approach of the unknown color to the spectrum color is commonly indicated by the ratio of the amount of the spectrum color to the total amount of the two-part combination; this ratio is called *purity*, and if the amounts are specified in luminance units, the ratio is called *luminance* (formerly colorimetric) *purity*. By far the most common convention, however, is to express the amounts in units of the excitation sum $X + Y + Z$; the resulting ratio is called *excitation purity* and corresponds simply to distance ratios on the chromaticity diagram of a colorimetric coordinate system [49, 64, 133]. Formulas have been derived by Hardy [49] and MacAdam [91] to

TABLE 7. *Spectral Reflectances of Greenish Yellow Printing-Ink Specimen (see fig. 4b) Read for the Selected Ordinates for Source C (see table 6)*

Number	X		Y		Z	
1	0.076		0.102		0.078	
2*	.076	0.076	.240		.076	0.076
3	.079		.460		.076	
4	.088		.615		.075	
5*	.096	.096	.683		.075	.075
6	.117		.711		.076	
7	.727		.720		.076	
8*	.739	.739	.725		.077	0.77
9	.745		.728		.077	
10	.750		.731		.078	
11*	.758	.758	.735		.080	.080
12	.765		.738		.082	
13	.772		.742		.084	
14*	.779	.779	.745		.086	.086
15	.783		.747		.088	
16	.785		.750		.089	
17*	.787	.787	.755		.091	.091
18	.788		.760		.093	
19	.789		.765		.095	
20*	.790	.790	.772		.097	.097
21	.791		.778		.100	
22	.792		.782		.103	
23*	.793	.793	.785		.106	.106
24	.795		.787		.110	
25	.797		.788		.119	
26*	.800	.800	.790		.133	.133
27	.803		.792		.155	
28	.807		.796		.210	
29*	.812	.812	.802		.342	.342
30	.819		.812		.650	
Totals ---	19.298	6.430	21.136	7.032	3.677	1.163
Times factor--	0.630	0.630	0.704	0.703	0.145	0.137

*Values for calculation with 10 selected ordinates.

convert from luminance purity to excitation purity, and the reverse.

Figure 6 indicates how dominant wavelength and excitation purity of the four printing-ink specimens would be found from their chromaticity coordinates x, y , relative to source C taken as the fixed light. Table 8 gives the dominant and complementary (C) wavelengths found as in figure 10 by the intersections of the straight lines with the spectrum locus. Table 8 also gives the excitation purities found by dividing the distance from the fixed point (source C) to the specimen point by the total distance from the fixed point to the boundary (spectrum locus plus straight line connecting its extremes). Large-scale charts for reading dominant wavelength and purity relative to source C are provided in the *Hardy Handbook of Colorimetry* [49].

Apparatus for the direct measurement of dominant wavelength and luminance purity has been designated by Nutting [125] and by Priest [132]. The degree of metamerism ordinarily obtained with such apparatus leaves it open to the same objections as have prevented tristimulus colorimeters with single sets of primaries from being use-

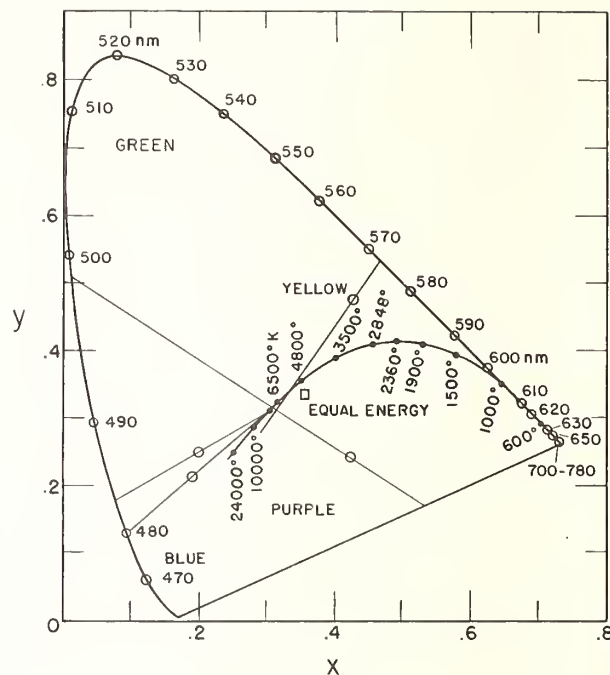


FIGURE 6. *Dominant wavelength and purity evaluated from the (x,y)-chromaticity diagram.*

The four points indicated by circles represent the colors of the four printing-ink specimens shown in figures 9.4 and 9.5.

TABLE 8. *Dominant wavelengths and excitation purities of four printing-ink specimens (see fig. 4)*

Hue designation of specimen	Chromaticity coordinates (from table 5c)		Dominant wavelength, nm	Excitation purity, percent
	x	y		
Red purple-----	0.430	0.239	498.9C*	53.7
Greenish yellow-----	.426	.476	573.2	74.0
Greenish blue-----	.194	.248	483.6	49.6
Blue-----	.190	.213	479.7	55.1

*C denotes complementary wavelength.

ful for production control. There is a further disadvantage in the direct measurement of luminance purity in that the luminance of the spectrum component has to be determined relative to the luminance of the mixture by separate photometry. Since there is usually a large chromatic difference between these two fields, simple equality-of-brightness settings are not reliable, and an auxiliary flicker photometers, as in Priest's apparatus [132], must be used. This method has been found to exaggerate individual-observer differences; oftentimes observers will differ only slightly in the mixtures of spectrum light and fixed light that they find to be equivalent to an unknown color, but they will disagree importantly in their photometry of the components.

3. Colorimetry by Difference

3.1. General Principles

In the fundamental colorimetry of lights and objects a single standard is used for each class of specimen. Opaque surfaces are referred to the ideal perfect diffuser, or to physically realizable near-perfect diffusers, such as a sufficiently thick layer of magnesium oxide deposited from the smoke of magnesium turnings or ribbon [108] or layers of barium sulfate [141, 6]. Transparent objects, such as gelatin films and crystal or glass plates, are referred to the equivalent thickness of air; transparent solutions, to the equivalent thickness of distilled water or solvent. Self-luminous objects, such as fluorescent lamps, cathode-ray tubes, and incandescent lamps, are measured relative to one of the standard sources, usually source A [19]. The colors of specimens closely resembling the respective standards can be evaluated quite precisely and accurately; those differing radically in spectral composition, only with relative uncertainty. That is, near-white specimens, nearly clear glass plates, and incandescent lamps nearly equivalent to source A present the simplest colorimetric problem; highly selective absorbers and emitters, like the rare-earth glasses and gaseous discharge tubes, present difficult measurement problems. In general, the greater the deviation in spectral composition between the unknown specimen and the standard, the greater the uncertainty of the result obtained by a visual or a photoelectric colorimeter.

Automatic spectrophotometry has greatly extended the application of both visual and photoelectric colorimetry. It has supplied a practical way to calibrate working standards of color. If a fairly large group of specimens is at hand to be measured, say twenty or more, all of similar spectral composition, the most satisfactory way to measure them in the present state of colorimetric science is to evaluate one or two of them carefully by means of the spectrophotometer to serve as working standards, then obtain the color specifications of the remainder by visual or photoelectric determination of the difference between specimen and standard.

3.2. Chromaticity Spacing, Perceptibility

In the interpretation of the importance of chromaticity differences based upon separation of the points representing the two chromaticities in the (x,y) -diagram a warning is necessary. This diagram is considerably expanded in the green portion relative to the other portions, much as the Mercator projection of the earth's surface is expanded near the poles. Thus, two points separated by a given distance in the green portion of this diagram correspond to chromaticities that are

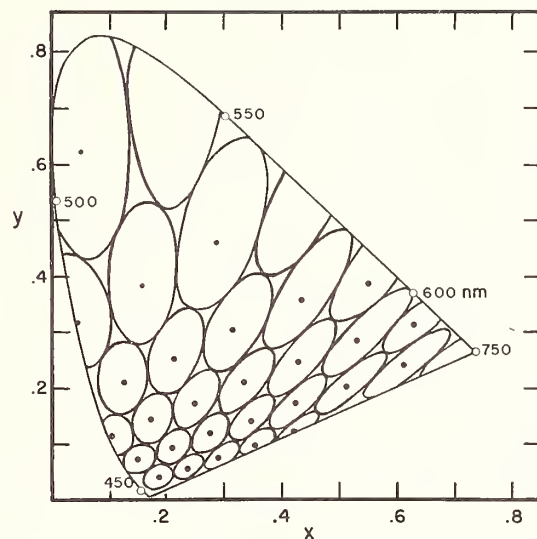


FIGURE 7. *Chromaticity spacing in the (x,y) -diagram.* Between any center and any point on the corresponding ellipse there are approximately 100 just noticeable chromaticity steps [67].

much harder to distinguish under ordinary viewing conditions than two chromaticities separated by the same amount in other portions of the diagram. Furthermore, the bluish purple portion of the diagram is correspondingly compressed. The system of ellipses shown on figure 7 serves to indicate approximately the metric properties of the (x,y) -diagram. Under moderately good observing conditions, the distances from the central point of each ellipse to any point on its boundary correspond approximately to one hundred times the chromaticity difference just perceptible with certainty. These ellipses were drawn from a review of the literature in 1936 [66, 67], and subsequent extensive work published by Wright [160, 161] and by MacAdam [92, 94] has corroborated the essential correctness of the indicated chromaticity spacing. Figure 7 not only indicates the extent to which the green portion of the diagram is expanded, and the bluish purple compressed, but also indicates that, in general, the chromatic importance of a distance on the (x,y) -diagram is a function both of the position of the central point and the direction of the deviation from it.

When sets of primaries other than those of the CIE standard observer system are expressed by transformations of the form of eq (3), the chromaticity spacings in the resulting Maxwell triangle may be made to vary widely. There have been several attempts to select primary sets that yield uniform chromaticity scales in which the chromaticity spacing corresponds to perceptibility [14, 55, 66, 90, 144]. The transformation equations for the chromaticity coordinates, r, g , of the uniform-

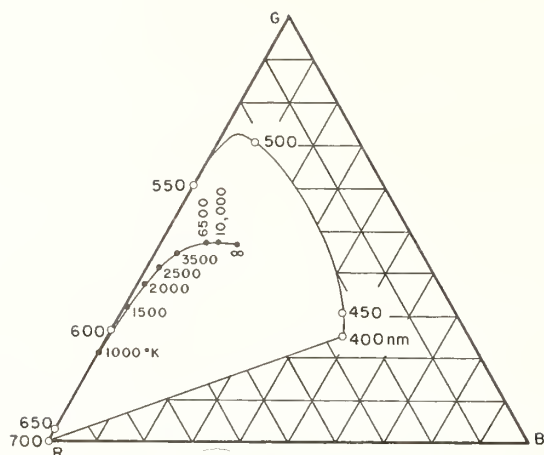


FIGURE 8. Uniform-chromaticity-scale triangle according to Judd [66].

The length of the straight line connecting the points representing any two chromaticities on this triangle is approximately proportional to the perceptibility of the chromaticity difference.

chromaticity-scale (UCS) System [66], from the CIE chromaticity coordinates, x, y , are

$$r = \frac{2.7760x + 2.1543y - 0.1129}{-1.0000x + 6.3553y + 1.5405} \quad (5)$$

$$g = \frac{-2.9446x + 5.0323y + 0.8283}{-1.0000x + 6.3553y + 1.5405}$$

The triangle resulting from this transformation is shown in figure 8.

In 1960 the CIE adopted a provisional recommendation that the transformation suggested by MacAdam in 1937 [89] be used whenever a spacing perceptually more uniform than that of the (x, y) -diagram is desired. This transformation is:

$$u = \frac{4x}{12y - 2x + 3} \quad (6)$$

$$v = \frac{6y}{12y - 2x + 3}$$

Although the transformation coefficients are simple, the (u, v) -diagram (fig. 9) has a spacing that closely resembles the UCS diagram.

3.3. Visual Devices

a. Martens Photometer

One of the most useful visual devices for determining relative luminance is the Martens photometer. Figure 10 shows the Martens photometer combined with a diffuse illuminator to form the Priest-Lange reflectometer [134]. This reflectometer is intended for the measurement of luminous reflectance of opaque specimens relative to reflecting standards of similar spectral reflectance. The Priest-Lange instrument is also adaptable to the measurement of luminous transmittance of transparent plates relative to transmitting standards

similar in spectral transmittance to the unknown. Finally, the Martens photometer, removed from the mounting, may be used for the determination of the luminance of an unknown self-luminous surface relative to a spectrally similar standard of known luminance. The superior usefulness of the Martens photometer arises from the convenience of the adjustment for equality of brightness between the two halves of the photometer field and from the fact that the dividing line between the half-fields is exceptionally narrow so that it is often invisible when a brightness match has been set.

b. Chromaticity-Difference Colorimeter

The determination of chromaticity coordinates, x, y , by comparison of the unknown specimen with a working standard of similar spectral reflectance can be carried out visually with high precision by means of a colorimeter described by Judd [64]. The adjustment of the chromaticity of the comparison field to match the standard field is by two double wedges, one of greenish and the other of yellowish glass. Since the light from the comparison field must pass through both the yellow and the green wedge, some of the radiant energy being subtracted by each, it is sometimes called a subtractive colorimeter; see figure 11 which gives a schematic diagram. The standard and comparison fields are brought into juxtaposition by means of a Lummer-Brodhun cube having a double-trapezoid pattern subtending $9 \times 13^\circ$ at the observer's eye. The adjustment to near equality of brightness to facilitate detection of chromaticity differences is by movement of the projection lamp that illuminates both standard and comparison surfaces.

A substitution method is usually employed with this colorimeter, a match first being set up between the standard and comparison surfaces by adjustment of the wedges. Then the unknown specimen is substituted for the standard, and the wedges readjusted to restore the match. The differences in wedge settings can be calibrated in terms of differences in the chromaticity coordinates, x, y , of the CIE standard coordinate system, provided the spectral reflectances of the comparison surface are known approximately, from the known spectral transmittances of the wedges. This calibration has been carried out for about 100 widely differing comparison surfaces. It has been found that the calibration is chiefly a function of the chromaticity coordinates, x, y , of the comparison surface; so calibrations for comparison surfaces intermediate in chromaticity to those already calibrated may usually be found satisfactorily by interpolation.

Because of the large patterned field of high luminance and the convenience of the brightness adjustment this instrument takes full advantage of the ability of the observer to detect small chromaticity difference. If specimen and comparison surfaces are similar in spectral composition, the settings for match may be repeated generally within 0.001 in chromaticity coordinates, x, y . The chromaticity-difference colorimeter has the disadvantage, how-

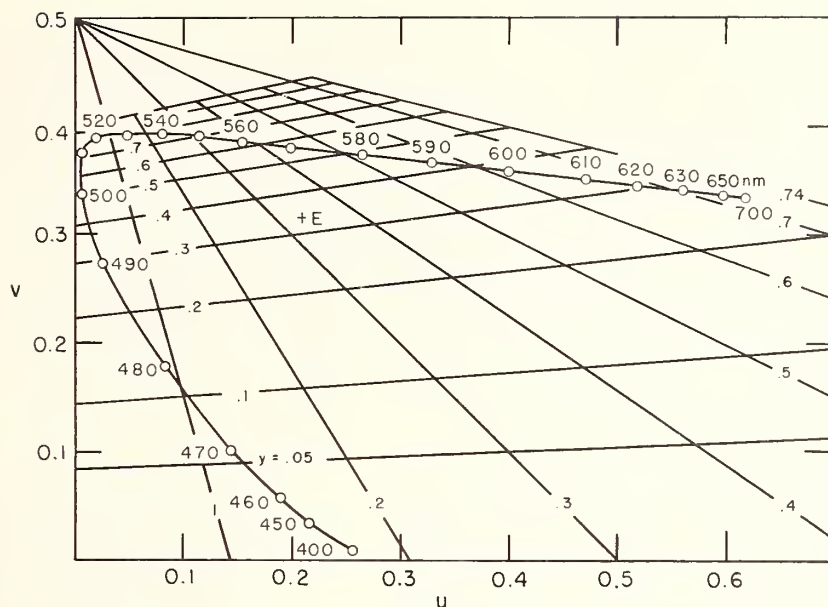


FIGURE 9. The (u,v) -diagram developed by MacAdam [90].

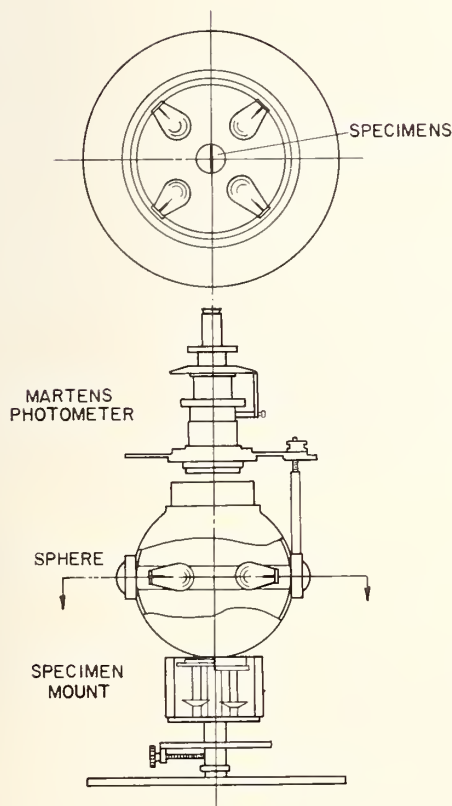


FIGURE 10. Vertical cross-section of the Priest-Lange [134] reflectometer, showing Martens photometer, diffuse illuminator, and specimen holder.

Horizontal cross section of the illuminator is above.

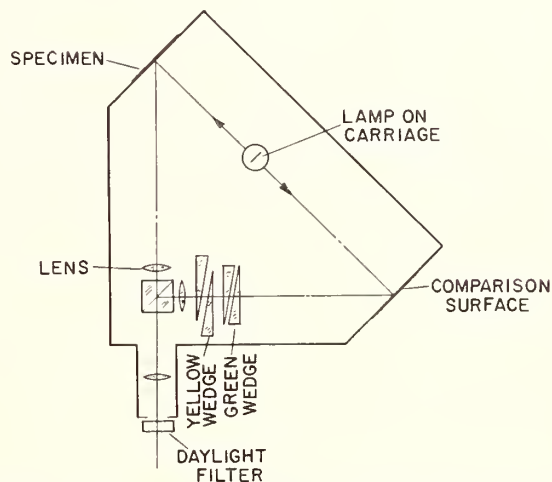


FIGURE 11. Schematic diagram of chromaticity-difference colorimeter [68].

ever, of requiring a skilled operator. Furthermore the calibrations, carried out in accord with table 4b, are time-consuming; and the instrument is not applicable to some highly selective samples because nearly homogeneous energy is too little changed in spectral composition by passage through the wedges. This colorimeter has been used in setting up a color standard for ruby mica [70]; for inspection of working standards, transparent and opaque, for conformity to a master standard; and for general colorimetry by difference, for both fluorescent and nonfluorescent specimens [137].

3.4. Photoelectric Devices

If three photocells could be adjusted, as by glass filters, so that their responses were proportional throughout the visible spectrum to some linear combination [as in eq. (3) of the standard CIE distribution curves (see fig. 2)], then they could be used to test whether any two light beams have the same color according to eq. (2) and could be made to yield direct measurements of tristimulus values, X, Y, Z [34, 45]. The X -function filter is the most difficult to design because it has two lobes, one short-wave lobe and one long-wave lobe. Two separate filters to cover a portion of one photocell have been designed for this bilobal function by Barnes [10] and by Nimeroff and Wilson [121]. Figure 12 shows the X -function match achieved in the Nimeroff-Wilson colorimeter. This function has also been fitted by a filter-photocell combination to approximate the long-wave lobe to which a portion of the Z -function is added, either electrically, as in the Color Difference Meter of Hunter [56, 57] and arithmetically, as in the Multipurpose reflectometer of Hunter [55], the Colormaster of Glasser and Troy [38] and the Color Eye of Bentley [11]. In these instruments the transformation equations to obtain X, Y, Z for source C may be represented as:

$$\begin{aligned} X &= 0.80R + 0.18B \\ Y &= 1.00G \\ Z &= 1.18B \end{aligned} \quad (7)$$

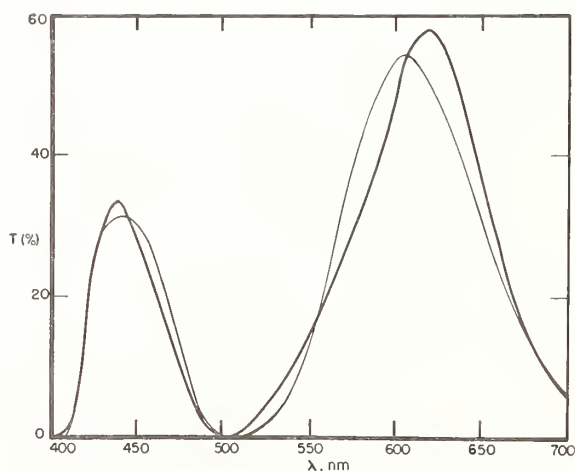


FIGURE 12. Spectral transmittance of the required (light line) and the approximate filter (heavy line) for CIE function X in the Nimeroff-Wilson colorimeter.

4. Color Specification by Material Standards

4.1. General Principle

Because of the convenience of material standards of color, they are often used in commerce in preference to specification according to the more funda-

where R, G , and B are settings with red, green, and blue filters, respectively. In some of these instruments the filter for the long-wave lobe of the X -function may be designated A. Van den Akker [152] has discussed the incompleteness of success of these colorimeters to duplicate the CIE standard observer system. Recent versions of Color Difference meters and the Color Eye are shown in figure 13.

Figure 14 shows the degree of success achieved by the filters designed by Hunter [55] to duplicate the CIE standard observer and simultaneously to adjust the spectral distribution of a projection lamp to that for CIE source C. Figure 15 shows the discrepancies that the filters of his multipurpose reflectometer introduce. These discrepancies are roughly proportional to the distance from the point representing the magnesium-oxide standard, and are frequently larger than 0.02 in x or y ; that is, more than 10 times a reasonable chromaticity tolerance for most colorimetric work. However, for the comparison of near-white surfaces this degree of duplication is sufficient. Figure 16 refers to the small rectangle near the center of figure 15 and indicates that the discrepancies are less than 0.001 in x or y for comparison of near-white surfaces with magnesium oxide. In general, a similar agreement can be expected in using this photoelectric tristimulus colorimeter for the determination of small chromaticity differences between nonmetameric pairs. And even for measurement of fairly sizable nonmetameric chromaticity differences, such as analyzed spectrophotometrically in the upper portion of figure 17 (BPB 8/2 versus MgO, BG 7/4 versus BG 6/4), and small chromaticity differences with a moderate metameric component, such as shown in the lower left portion of figure 17 (Y_1 versus Y_2), the discrepancy is in the neighborhood of 0.002 in x or y , which is negligible for many purposes. However, for highly metameric pairs, such as shown in the lower right portion of figure 17, the discrepancy may be expected to be in the neighborhood of 0.02, just as it is for large chromaticity differences.

If the limitations of photoelectric tristimulus colorimetry are appreciated, the method is most useful in product-control colorimetry of non-fluorescent specimens by difference from a working standard. The precision of the method is comparable, though perhaps not quite equal, to the best that can be done by eye. No unusual qualifications or extended special training is required by the operator; and, compared to visual colorimetry or to indirect colorimetry by the spectrophotometer, the results are obtained very rapidly.

mental CIE system. Material standards may be carried from place to place, and, if the colors are sufficiently closely spaced in the neighborhood of the unknown color, the nearest match may be found by visual comparison. The color specification con-

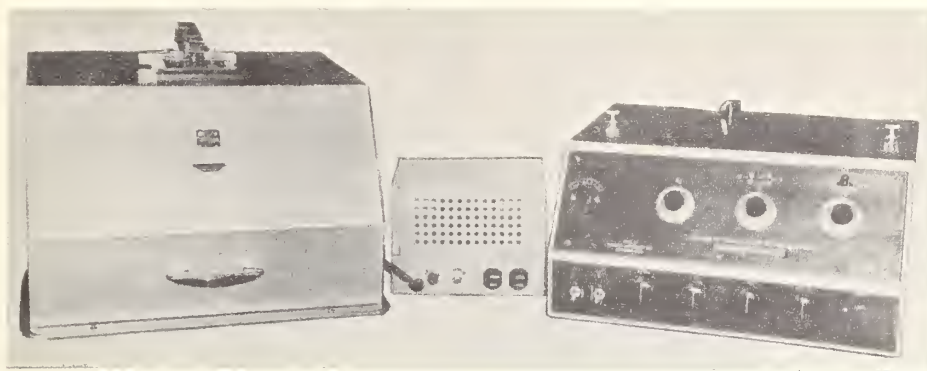


FIGURE 13a. *Tristimulus colorimeters by Gardner Laboratory.*



FIGURE 13b. *Tristimulus colorimeters by Hunter Associates Laboratory that give results in nearly uniform color spacing.*

sists of identifying the particular member of the system yielding a match for the color to be specified. We deal here both with systems of material standards of such scope that a considerable fraction of the colors possible in nonself-luminous objects are represented and with a few special small groups of material standards for particular purposes.

4.2. Transparent Media

Color systems based upon transparent media take advantage of the fact that it is possible with

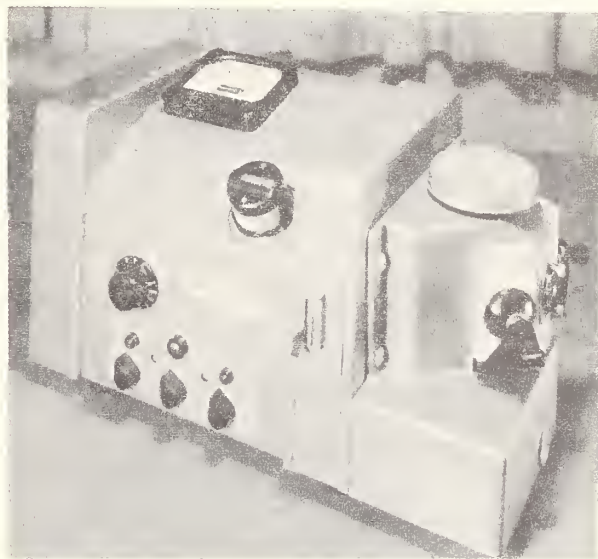


FIGURE 13c. *Tristimulus colorimeters by Instrument Development Laboratories that provide narrow-band interference filters to aid in evaluating the spectral character of a color.*

a fixed source to control the color of the transmitted light over a wide range by introducing varying amounts of three absorbing materials. This is done by permitting the light to pass through two or more elements of the absorbing medium instead of through a single element, and is called subtractive combination or mixture because the action of each element is to subtract a certain fraction of each part of the spectrum of the incident light. The color specification consists of the number of unit elements of each of the three absorbing components required to produce the color match.

a. Lovibond Glasses

The Lovibond color system consists of three sets of colored glasses, red, yellow, and blue [86, 87, 88], the principal coloring materials being gold, silver, and cobalt, respectively. The unit of the

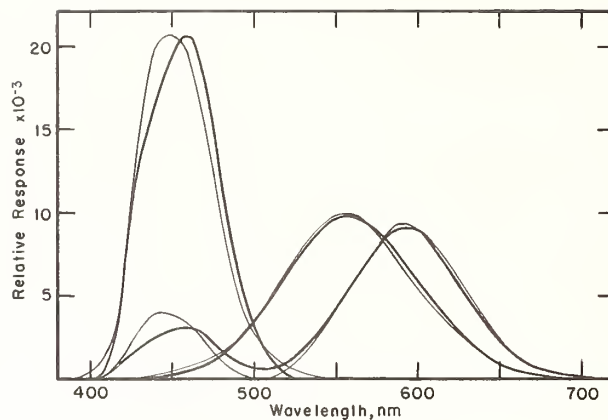


FIGURE 14. Curves showing the approach of the Hunter tristimulus filters [55] combined with incandescent lamp and barrier-layer cell to the CIE spectral tristimulus values for source C.

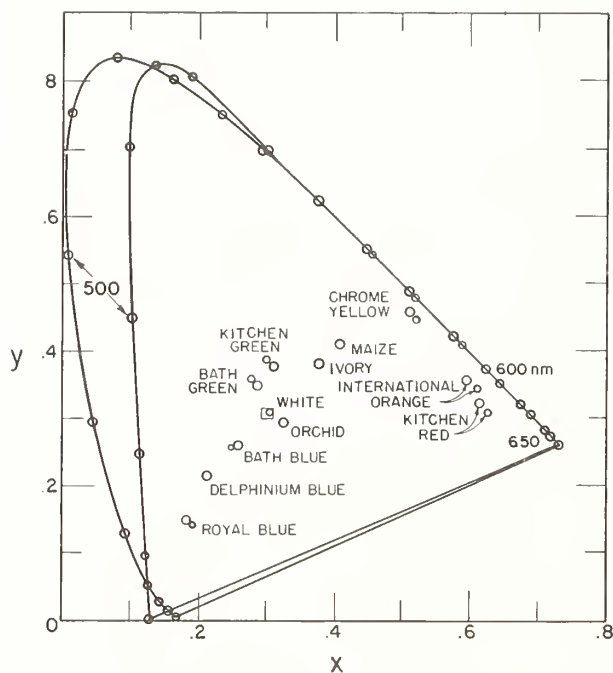


FIGURE 15. Chromaticity discrepancies expected from the use of the multipurpose reflectometer with Hunter tristimulus filters [55].

Measurements are all referred to magnesium oxide as the standard white. O, spectrophotometric colorimetry, o, multipurpose reflectometer.

scale defined by each set is arbitrary, but the three units are related by being adjusted so that, for observation by daylight, subtractive combination of one unit of each of the red, the yellow, and the blue scales results in a filter perceived as neutral or achromatic. Each scale is exemplified by many glasses, each glass being marked with the number of unit glasses to which it is equivalent. Although the original purpose of the Lovibond color system

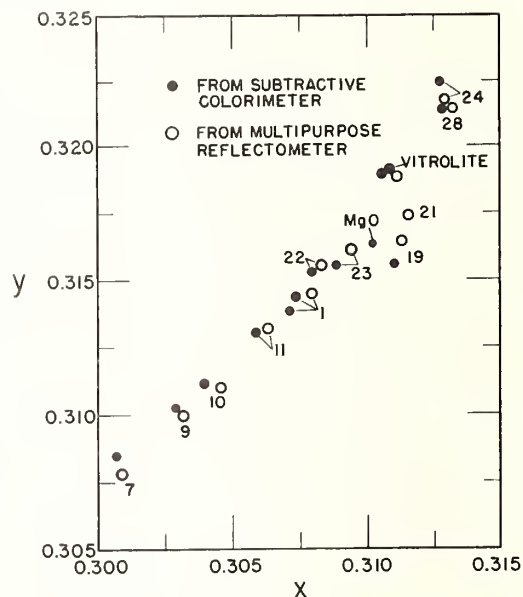


FIGURE 16. Agreement between multipurpose reflectometer [55] and the chromaticity-difference (subtractive) colorimeter for near-white porcelain-enamel specimens measured relative to magnesium oxide.

was to aid in the color control of beer, these glasses are widely used today for other products such as vegetable oils, lubricating oils, and paint vehicles.

A spectrophotometric analysis of the Lovibond color system was made by Gibson and Harris [32], and a scale of the red glasses used in combination with the 35-yellow glass has been constructed by Priest and Gibson [33, 50, 155], having the same unit as the original Lovibond red scale but embodying a closer approach to the principle that the

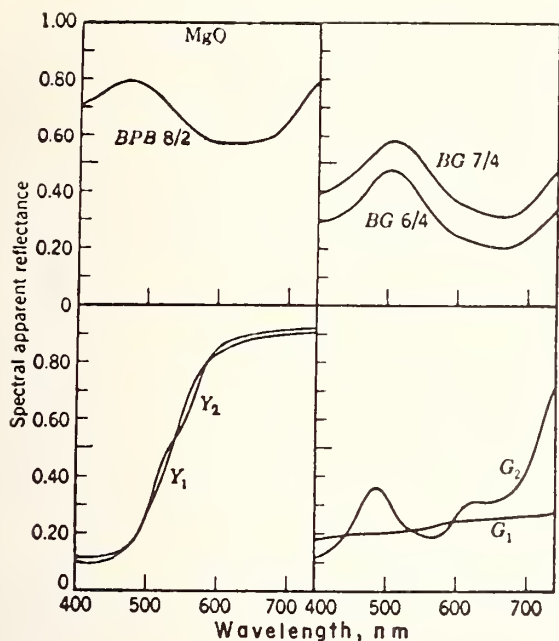


FIGURE 17. *Spectral reflectances of pairs of samples exhibiting various degrees of metamorphism.*

Upper two pairs differ considerably in color, but show little or no metamerism. Lower two pairs are near matches; left, moderately metameric; right, strongly.

Lovibond numeral should indicate the number of unit glasses to which the single glass bearing the numeral is equivalent. It is possible to adjust the glasses to slightly lower numerals, if desired, by reducing their thickness slightly [29]. Tintometer, Ltd., makers of the Lovibond glasses, have computed from the published spectrophotometric measurements [32] the chromaticity coordinates, x, y , of all the colors of the ideal Lovibond system [140] produced by illuminating the glasses with CIE standard sources B and C. Computation of these colors for source A has been done at the National Bureau of Standards [51]. Figure 18 shows the Lovibond network for source A. In addition to calibration of red glasses on the Lovibond scale, Tintometer Ltd. will also calibrate them on a scale, not precisely the same, set up in 1961 by agreement with the Color Committee of the American Oil Chemists' Society, based on the Priest-Gibson scale and known as the AOCS scale. Lovibond red glasses defining the AOCS scale are on deposit at Tintometer Ltd and at the National Bureau of Standards. The grading of vegetable oils in this country is carried out by means of this scale.

b. Army Solutions

The Army solutions consist of groups of solutions whose concentrations are adjusted to produce the color match. The required concentrations are the specifications of the color. The most used group is a triad consisting of half-normal aqueous solutions of cobalt chloride (red), ferric chloride (yellow), and copper sulfate (blue) in 1 percent hydro-

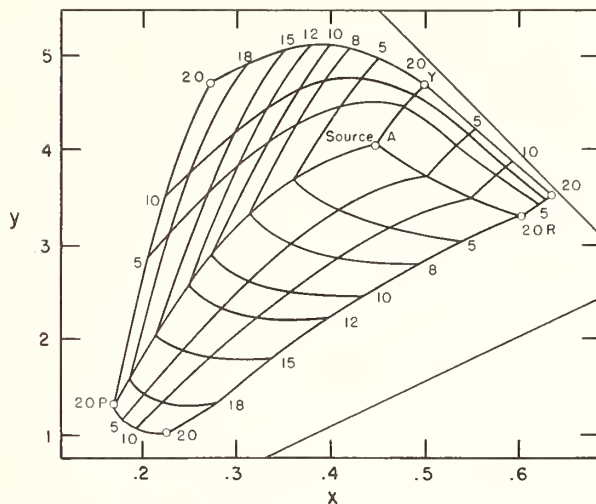


FIGURE 18. *The chromaticities of various two-part combinations of Lovibond red, yellow, and blue glasses for source A as computed by Haupt and Douglas [51].*

Two-part combinations of Lovibond glasses will produce many commercially-important colors.

chloric acid. This group produces all colors except deep blue and deep red; it is supplemented by a triad of ammoniated aqueous solutions of potassium permanganate and potassium dichromate [8, 9]. Mellon and Martin [101] have reported the spectral transmittances for a number of solutions for colorimetric standards, including the Army solutions at three or four concentrations, each for the spectral range 440 to 700 nm. By extrapolation of these data it is possible to find approximately the tristimulus values and chromaticity coordinates, x, y , on the standard coordinate system adopted in 1931 for these solutions, just as was done by Mellon [102] for the coordinate system used in America before the international agreement. In this way it is possible to transform color specifications from the standard system into the required concentrations of the Army solutions, and the reverse. The Army solutions are used in the 11th edition of the United States Pharmacopeia as standards for the color of cod-liver oil and in carbonization tests with sulfuric acid for 28 organic compounds.

4.3. Pigmented or Dyed Surfaces

If the material standards are pigmented or dyed surfaces, no automatically convenient notation, such as suggested by additive combination of lights or subtractive combination of absorbing elements, is available. Any systematic aspect of the color specification must be derived from the method of identifying the various members of the set of colored surfaces serving as standards.

a. Color Dictionaries

From a color dictionary are obtained definitions of color names in terms of material standards. The primary aim is therefore to provide an array of

named surface colors adequate for the purpose, and any arrangement or organization of the colors serves only the secondary purpose of assisting the user to find the one which most nearly matches.

The Maerz and Paul Dictionary of Color [97] is the foremost authority on color names. It contains about 7000 different samples of color printed on semiglossy paper, and there are listed about 4000 color names which are keyed to one or another of the color samples. These names are drawn from usage in many fields: paint, textile, ceramic, scientific, technical, and artistic. The samples are also identified by plate, column, and row, and because of their large number and fairly uniform color distribution it is usually possible to find among them a sample approaching what is called a "commercial color match" for any given uniform opaque surface. On this account the Maerz and Paul Dictionary finds a considerable application as a collection of color standards quite separate from its primary function of defining color names. There are noticeable color differences between corresponding samples in different copies of the Dictionary, but the differences have been held to a reasonably small amount by discarding the less satisfactory printed sheets.

The accepted authority for color names in the textile and allied industries is the Color Association of the United States. This association has issued nine editions of a standard color card since 1915, the current edition [149] containing 216 color samples of pure dye silk. Furthermore, the association issues to its members several seasonal cards each year. All colors of these standard and seasonal cards are identified by name and cable number. The standard colors have been measured by spectrophotometric and colorimetric procedures, and luminous reflectance, Y/Y_0 , relative to magnesium oxide, and chromaticity coordinates, x, y , for source C have been published [137].

A color dictionary much used for the specification of the colors of flowers, insects, and birds was prepared in 1921 by Ridgway [139]. This outstanding pioneer work contains about 1000 named color samples of paper painted by hand. Each chart shows columns of colors of the same dominant wavelength progressing from each chromatic color at the middle of the column toward white and the top, and toward black at the bottom; and there are five series of such columns, each one encompassing the entire hue circuit, but at different purities. Many of the names were coined at the time of publication to fill in gaps in popular color nomenclature and so have not much descriptive value. Each sample is arbitrarily identified by column, row, and series, however. In addition, there is an alphabetical list of the color names giving this identification.

b. Ostwald System

The notation of the Ostwald system is based on the properties of idealized pigment surfaces having spectral reflectance constant at a certain value

between two complementary wavelengths and reflectance constant at a certain other value at other parts of the spectrum [30, 128]. The full colors are those that have the low values of spectral reflectance equal to zero and the high ones equal to 100 percent. The difference between these two reflectances for other idealized pigment surfaces is the full color content, the value of the low reflectance is the white content, and the difference between the high reflectance and 100 percent is the black content. The complete Ostwald notation consists of a number and two letters. The number indicates dominant (or complementary) wavelength on an arbitrary but approximately uniform perceptual scale, and is called Ostwald hue number. The first letter indicates white content, a being a white content of 89.13 percent, which is as near to 100 percent as is practicable for usual pigment-vehicle combinations, and other letters in alphabetical sequence indicating decreasing white content on a logarithmic scale. The second letter indicates black content, a being a black content of 10.87 percent, which is as near to zero as is practicable, and other letters in alphabetical sequence indicating increasing black content on a logarithmic scale. The logarithmic scales were thought by Ostwald to insure uniform color scales, but this is true only to a rough approximation. Since the percent white content, black content, and full-color content must necessarily add up to 100, no explicit indication of the latter is required.

The Ostwald ideas have been a considerable aid in thinking about color relationships on the part of those who duplicate colors by mixtures of chromatic pigments with white and black pigments, and they have served as a guide in the selection of combinations of such colors to produce pleasing effects. However, the use of these idealized pigment surfaces as a basis for a system of colorimetry has been hampered by the fact that actual pigment surfaces approximate them rather poorly, and by the fact that not all actual pigment surfaces can be color matched by one of these ideal surfaces. Still, color charts made up more or less in accord with the Ostwald principles have been widely used for color standards and for the selection of harmonizing colors [63, 127, 129, 148]. Of these, the *Jacobson Color Harmony Manual* [63] is pre-eminent not only because of its technical excellence, but also because Foss [30] has given a clear statement of which of the somewhat contradictory Ostwald principles were followed in its construction, and Granville and Jacobson [47] have made a spectrophotometric study of the color chips and have published luminous reflectance, Y/Y_0 , relative to magnesium oxide, and chromaticity coordinates, x, y , for every chip. These chips are therefore valuable for use in colorimetry by difference from a working standard (see sec. 3), and the fact that the chip is a member of an orderly arrangement of colors facilitates the selection of a working standard for any particular purpose.

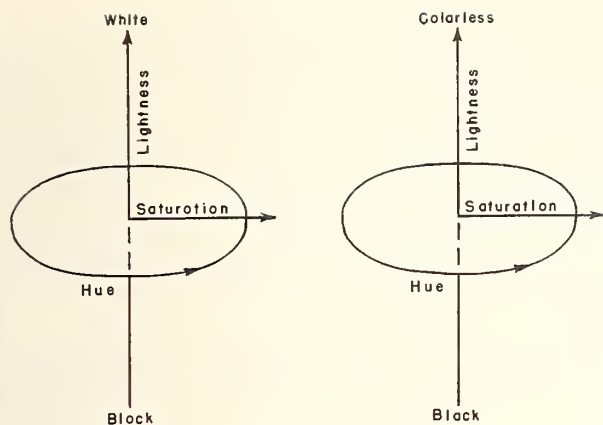


FIGURE 19. Dimensions of the surface-color solid:
(a) opaque surfaces (b) transparent volumes.

c. Munsell System

The basis of the Munsell system is description of colors perceived to belong to surfaces in terms of hue, lightness, and saturation. Each such tridimensional description can be represented by a point plotted in a space diagram known as the *surface-color solid*, as shown in figure 19. In the surface-color solid the central axis represents the grays extending from black at the bottom to white at the top. *Lightness* of a chromatic (nongray) color determines the gray to which it is equivalent on this scale. Lightness is represented in the color solid by distance above the base plane. *Hue* determines whether a color is perceived as red, yellow, green, blue, purple, or some intermediate; it is represented in the color solid by angle about the central axis. *Saturation* indicates the degree of departure of a surface-color perception from the gray of the same lightness; it is represented by distance from the central gray axis.

The Munsell color system specifies a surface color by giving for usual viewing conditions its position on more or less arbitrary hue, lightness, and saturation scales having perceptually nearly uniform steps [107]. The Munsell term corresponding to lightness is Munsell value, that for saturation is Munsell chroma, and that for hue is Munsell hue. Munsell value is zero for the ideal black surface having luminous reflectance equal to zero, and it is 10 for the ideal white diffusing surface having luminous reflectance equal to 1. Munsell chroma is expressed in arbitrary units intended to be perceptually of the same size regardless of value and hue. The strongest known pigment colors have chromas of about 16; neutral grays have zero chroma as do black and white. Munsell hue is expressed on a scale intended to divide the hue circuit (red, yellow, green, blue, purple, back to red) into 100 perceptually equal steps. According to one convention the 100 Munsell hues are identified simply by a number from 1 to 100, and on this scale hues that differ by 50 are nearly complementary. The

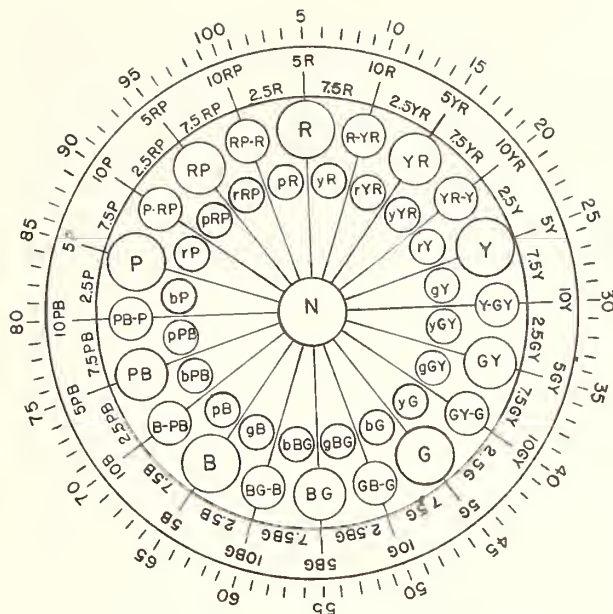


FIGURE 20. Diagram of hue circle with Munsell hue notation.

	RP	rRP	RP-R	pR	R	yR	R-YR	rYR	YR
BY LETTER	5RP	7.5RP	10RP	2.5R	5R	7.5R	10R	2.5YR	5YR
BY LETTER & NUMBER	6 7 8 9	1R 2 3 4	6 7 8 9	1YR 2 3 4					
NOTATION	96 97 98 99	1 2 3 4	6 7 8 9	11 12 13 14					
BY NUMBER	95	97.5	100	2.5	5	7.5	10	12.5	15

FIGURE 21. Alternate ways of expressing Munsell hue notation.

most common convention, however, is to divide these 100 hues into 10 groups of 10 hues each, and identify each group by initials indicating the central member of the group, thus: red R, yellow red YR, yellow Y, green yellow GY, green G, blue green BG, blue B, purple blue PB, purple P, and red purple RP. The hues in each group are identified by the numbers 1 to 10. Thus, the most purplish of the red hues (1 on the scale of 100) is designated as 1R, the most yellowish as 10R, and the central hue as 5R, or often simply as R; see figures 20 and 21. The transition points (10R, 10YR, 10Y, and so on) between the groups of hues are also sometimes designated by means of the initials of the two adjacent hue groups, thus: R-YR = 10R, YR-Y = 10YR, Y-GY = 10Y, and so on; but this convention is little used. The Munsell notation is commonly written: Hue value/chroma, this is, the hue notation, such as 6R, then the value, such as 7, and finally the chroma, such as 4, the latter two being separated by a solidus: 6R 7/4. More precise designations are given in tenths of the arbitrary steps of the scales, thus: 6.2R 7.3/4.4. The grays are indicated by the symbol N for neutral followed by the value notation, thus: N 7/ or N 7.3/; the chroma being zero for neutrals is not specifically noted.

Three representations of the Munsell system

have been published, the original *Atlas* in 1915 [105], now chiefly a collector's item, the *Munsell Book of Color* in 1929 and 1942 [106], and the *Munsell Color Standards in High-Gloss Surface* in 1957. The book consists of rectangles of mat-finish handpainted paper permanently mounted on charts in a loose-leaf binding. The high-gloss color standards are detachably mounted. The neutrals form a one-dimensional color scale extending from N 1/ to N 9/. Each chromatic sample, of which there are about 1000, takes its place on three color scales: a hue scale, a value scale, and a chroma scale; and the spacing of these scales is intended to be perceptually uniform. The pocket edition, adapted for determining Munsell notation of unknown colors by visual comparison, consists of forty constant-hue charts, so called because all the samples on each chart have the same Munsell hue. These samples are arranged in rows and columns, the rows being chroma scales at constant Munsell value, the columns being value scales at constant Munsell chroma. Comparison of an unknown color with these two families of scales gives by interpolation the Munsell value and Munsell chroma of the unknown. Interpolation between adjacent constant-hue charts gives the Munsell hue. Unknowns not too far outside the range of the Munsell charts may be evaluated with some reliability by extrapolation along the value and chroma scales. Table 9 gives *Munsell Book* notations of the four printing-ink specimens reproduced on figure 4. The greenish yellow specimen was evaluated by interpolation, the other three by extrapolation of varying degrees of uncertainty, the red-purple specimen being furthest outside the range of the Munsell charts and hence the least certainly evaluated by visual estimate.

To facilitate the comparison of the unknown color with those of the permanently mounted paper rectangles on the Munsell charts, either the unknown color must be brought into juxtaposition with the rectangle and held in nearly the same plane, or, if the form of the unknown prevents such juxtaposition, two masks of thin cardboard having a rectangular opening to fit the paper rectangles should be used. One mask should be placed over the unknown color; the other over one or another of the Munsell colors in succession to obtain the interpolated *Munsell Book* notation.

Kelly [69, 76] made effective use of a form of mask with three rectangular openings particularly adapted to comparisons involving powdered chemicals and drugs viewed through a cover glass. It is advantageous to have the color of the mask fairly close to that of the unknown, particularly in Munsell value; that is, if the unknown color is dark, the mask should be of a dark color also. Use of a light mask for a dark color prevents the observer from making as precise a visual estimate as he can make with a mask more nearly a color match for the unknown.

Because of the visual uniformity of the scales, the estimates of Munsell notation for unknown colors within the color range of the charts have a reliability corresponding to the use of a much larger collection of unequally spaced color standards. On this account the pocket edition of the *Munsell Book of Color* is widely used as a practical color standard for general purposes.

The standard or library edition shows the same colors as the pocket edition, but it shows them on constant-value charts in a polar-coordinate system, and on constant-chroma charts in a rectangular coordinate system, as well as on constant-hue charts. This edition has full explanatory matter in the text and is adapted particularly for teaching color. It is too bulky for convenient practical use in determining the Munsell notation of an unknown color.

The samples of the 1929 *Munsell Book of Color* have been measured by means of the spectrophotometer twice independently with generally concordant results [39, 78]. In both these studies luminous reflectance, Y/Y_0 , relative to magnesium oxide and chromaticity coordinates, x, y , of the roughly 400 samples were computed for source C. Glenn and Killian [39] have also published the dominant wavelength and purity for each of the colors; Kelly, Gibson, and Nickerson [78] have published specifications (Y, x, y) for three additional sources (source A, Macbeth daylight, and limit blue sky. Nickerson and Wilson [118] have extended them to nine sources. Furthermore, they have published [78] a series of (x, y) chromaticity diagrams showing the position of the Munsell colors for each of the Munsell values from 2/ through 8/. From these diagrams it is possible to find the chromaticity coordinates, x, y , correspond-

TABLE 9. *Munsell renotations, luminous reflectances, chromaticity coordinates, Munsell Book Notations, and ISCC-NBS color designations of four printing-ink specimens*

Hue designation of specimen	Luminous reflectance (Y/Y_0)	Chromaticity coordinates		Munsell renotation (hue value/ chroma)	Munsell book notation (hue value/ chroma)	ISCC-NBS color designation
		x	y			
Red Purple-----	0.221	0.430	0.239	5.5RP 5.2 ₅ /16.1	6.0RP 4.8/16	Vivid purplish red
Greenish Yellow-----	.704	.426	.476	8.3Y 8.6 ₀ /10.6	7.5Y 9.0/9.5	Brilliant greenish yellow
Greenish Blue-----	.242	.194	.248	5.6B 5.4 ₆ /8.5	3.0B 5.4/9	Strong greenish blue
Blue-----	.246	.190	.213	0.8PB 5.5 ₀ /9.6	10.0B 5.4/11	Strong blue

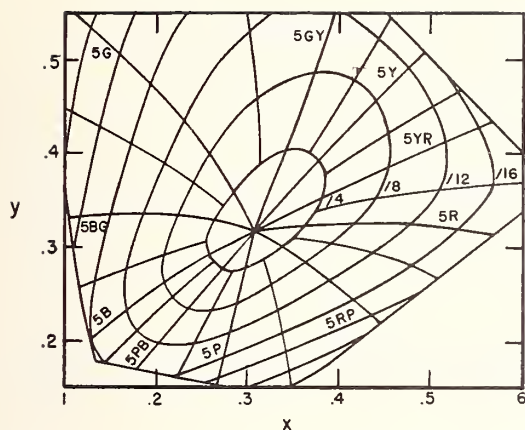


FIGURE 22. Chromaticities of ideal Munsell colors, value 5, shown on the (x,y) -diagram.

This chart serves to define Munsell renotation hue and chroma for colors having $Y/Y_0 = 0.198$. (Prepared by Color Measurement Laboratory, War Food Administration, U.S.D.A.)

ing to any *Munsell Book* notation; and the reverse transformation is also possible. The samples of the 1942 supplement to the *Munsell Book of Color* together with many special Munsell standards have been measured spectrophotometrically by Granville, Nickerson, and Foss [46] and by Nickerson, Tomaszewski, and Boyd [119]. These Munsell standards, together with those of the 1929 *Munsell Book of Color*, number nearly 1500 and comprise the largest systematic set of color standards of known luminous reflectance and chromaticity coordinates ever made. These standards are commercially available separately in disk form and on large sheets, and they make practical the general colorimetry of opaque specimens not only by disk mixture, but also by difference from a standard.

From the charts themselves, luminous reflectance and chromaticity coordinates of an unknown color may also be found quickly and with an accuracy sufficient for many purposes by obtaining first the *Munsell Book* notation of the unknown and then transforming it by reference to (x,y) interpolation charts based on the complete set of standards. Two methods by which to make sure transformations have been described. One, an ASTM method [7a], makes use of tables and charts of corresponding CIE chromaticity coordinates and Munsell notations. The other is an automatic-computer program [75] by which CIE notations are transformed to Munsell notations.

The Munsell color standards may also be used, though less conveniently, in the colorimetry of light-transmitting [69, 76] elements (gelatin films, crystal and glass plates, solutions, and so on); and, conversely, such elements may be given Munsell Book notations from their CIE specifications by means of interpolation charts. Table 12, to be discussed presently in another connection, shows *Munsell Book* notations so derived from the luminous transmittances, T , and chromaticity coordi-

nates, x,y , of the glass standards of the ASTM Union colorimeter. The last four book notations given are relatively uncertain because the colors to be specified are far outside the range of the Munsell standards.

The spacing of the Munsell colors has been examined in detail by a subcommittee of the Colorimetry Committee of the Optical Society of America [111]. This committee work confirmed the many local irregularities in spacing revealed by the spectrophotometric studies and established the need for some more general but minor adjustments to make the colors of the Munsell charts correlate more nearly perfectly under ordinary conditions (adaption to daylight, gray surrounding field, and so on) with the surface-color solid. The subcommittee found it possible from this study to recommend specifications (Y,x,y) on the standard coordinate system defining an ideal Munsell system [112]. It has also given to every Munsell standard a revised notation, called the *Munsell renotation*, indicating exactly in what way and how much each color chip deviates from the ideal. Furthermore, the recommended definition of the ideal Munsell system has been extended beyond the color ranges covered by the present Munsell charts so as to include all colors theoretically producible from nonfluorescent materials [83] under source C. The connection between luminous reflectance, Y , and Munsell renotation value is given in Table 10. Note that N 0/ corresponds to $Y = 0$, N 9.91/ corresponds to magnesium oxide ($Y = 1.00$), and N 5.0/ corresponds to $Y = 0.198$ (quite different from 0.50, the halfway point). Figure 22 shows one of the (x,y) -chromaticity charts (that for Munsell renotation value equal to 5.0) defining the ideal system. From these charts combined with the data given in table 10 it is possible to find the ideal Munsell notation for any color specified in terms of luminous reflectance, Y , and chromaticity coordinates, x,y . Table 9 shows these Munsell renotations for the four printing-ink specimens of figure 4. Note that they agree in a general way, though not perfectly, with the notations found by direct visual comparison with the *Munsell Book of Color*. Some of the discrepancies are ascribable to local irregularities of the color spacing of the Munsell charts, but most of them may be laid to the uncertainty of the visual estimates, all but that for the greenish yellow specimen requiring extrapolation over a considerable color range.

Munsell renotations, such as these, have a unique usefulness as color specifications. Because of their definition in terms of the standard CIE coordinate system they are capable of nearly the precision of the Y,x,y form of specification, and like that form they may be extended to apply to all object colors, both opaque and transparent objects. For opaque surfaces Y/Y_0 is luminous reflectance relative to magnesium oxide; for transparent objects Y/Y_0 is luminous transmittance relative to an equivalent thickness of air; for solutions Y/Y_0 is luminous

TABLE 10. CIE (Y') equivalents (in percent relative to MgO) of the recommended Munsell value scale (V) from 0/ to 10/

V	Y _V	V	Y _V	V	Y _V	V	Y _V	V	Y _V	V	Y _V	V	Y _V	V	Y _V	V	Y _V	V	Y _V	V	Y _V
10.00	102.56																				
9.99	102.30	9.49	89.77	8.99	78.45	8.49	68.20	7.99	58.92	7.49	50.52	6.99	42.92	6.49	36.07	5.99	29.94	5.49	24.48		
8	102.04	8	89.53	8	78.23	8	68.01	8	58.74	8	50.26	8	42.77	8	35.94	8	29.82	8	24.38		
7	101.78	7	89.30	7	78.02	7	67.81	7	58.57	7	50.00	7	42.63	7	35.81	7	29.71	7	24.28		
6	101.52	6	89.06	6	77.80	6	67.59	6	58.32	6	50.04	6	42.34	6	35.68	6	29.59	6	24.17		
5	101.25	5	88.82	5	77.59	5	67.33	5	58.22	5	49.88	5	42.34	5	35.56	5	29.48	5	24.07		
9.94	100.99	9.44	88.59	8.94	77.38	8.44	67.33	7.94	58.04	7.44	49.72	6.94	42.20	6.44	35.43	5.94	29.36	5.44	23.97		
8	100.73	8	88.35	8	77.16	8	67.04	8	57.87	8	49.56	8	42.06	8	35.30	8	29.25	8	23.87		
7	100.47	7	88.12	7	76.95	7	66.85	7	57.69	7	49.41	7	41.92	7	35.17	7	29.13	7	23.77		
6	100.21	6	87.88	6	76.74	6	66.66	6	57.52	6	49.25	6	41.77	6	35.04	6	29.02	6	23.67		
5	99.95	5	87.65	5	76.53	5	66.46	5	57.35	5	49.09	5	41.63	5	34.92	5	28.90	5	23.51		
9.89	99.69	9.39	87.41	8.89	76.32	8.39	66.37	7.89	57.17	7.39	48.93	6.89	41.49	6.39	34.79	5.89	28.79	5.39	23.47		
8	99.44	8	87.18	8	76.11	8	66.08	8	57.00	8	48.78	8	41.35	8	34.66	8	28.68	8	23.37		
7	99.18	7	86.95	7	75.90	7	65.89	7	56.83	7	48.62	7	41.21	7	34.54	7	28.57	7	23.27		
6	98.92	6	86.72	6	75.69	6	65.70	6	56.66	6	48.47	6	41.07	6	34.41	6	28.45	6	23.17		
5	98.66	5	86.48	5	75.48	5	65.51	5	56.48	5	48.31	5	40.93	5	34.28	5	28.34	5	23.07		
9.84	98.41	9.34	86.25	8.84	75.27	8.34	65.32	7.84	56.31	7.34	48.16	6.84	40.79	6.34	34.16	5.84	28.23	5.34	22.97		
8	98.15	8	86.02	8	75.06	8	65.13	8	56.14	8	48.00	8	40.65	8	34.03	8	28.12	8	22.87		
7	97.90	7	85.79	7	74.85	7	64.94	7	55.97	7	47.85	7	40.51	7	33.91	7	28.01	7	22.78		
6	97.64	6	85.56	6	74.64	6	64.76	6	55.80	6	47.69	6	40.37	6	33.78	6	27.90	6	22.68		
5	97.39	5	85.33	5	74.44	5	64.57	5	55.63	5	47.54	5	40.23	5	33.66	5	27.78	5	22.58		
9.79	97.14	9.29	85.10	8.79	74.23	8.29	64.38	7.79	55.46	7.29	47.38	6.79	40.09	6.29	33.54	5.79	27.67	5.29	22.48		
8	96.88	8	84.88	8	74.02	8	64.19	8	55.29	8	47.23	8	39.95	8	33.41	8	27.56	8	22.38		
7	96.63	7	84.65	7	73.82	7	64.01	7	55.12	7	47.08	7	39.82	7	33.29	7	27.45	7	22.29		
6	96.38	6	84.42	6	73.61	6	63.82	6	54.95	6	46.92	6	39.68	6	33.16	6	27.34	6	22.19		
5	96.13	5	84.19	5	73.40	5	63.63	5	54.78	5	46.77	5	39.54	5	33.04	5	27.23	5	22.09		
9.74	95.88	9.24	83.97	8.74	73.20	8.24	63.45	7.74	54.62	7.24	46.62	6.74	39.40	6.24	32.92	5.74	27.12	5.24	22.00		
8	95.63	8	83.74	8	72.99	8	63.26	8	54.45	8	46.47	8	39.27	8	32.80	8	27.02	8	21.90		
7	95.38	7	83.52	7	72.79	7	63.08	7	54.28	7	46.32	7	39.13	7	32.67	7	26.91	7	21.81		
6	95.13	6	83.29	6	72.59	6	62.89	6	54.11	6	46.17	6	38.98	6	32.55	6	26.80	6	21.71		
5	94.88	5	83.07	5	72.38	5	62.71	5	53.94	5	46.02	5	38.86	5	32.43	5	26.69	5	21.62		
9.69	94.63	9.19	82.84	8.69	72.18	8.19	62.52	7.69	53.78	7.19	45.87	6.69	38.72	6.19	32.31	5.69	26.58	5.19	21.52		
8	94.38	8	82.62	8	71.98	8	62.34	8	53.61	8	45.72	8	38.59	8	32.19	8	26.48	8	21.43		
7	94.14	7	82.39	7	71.78	7	62.16	7	53.45	7	45.57	7	38.45	7	32.07	7	26.37	7	21.33		
6	93.89	6	82.17	6	71.57	6	61.98	6	53.28	6	45.42	6	38.32	6	31.95	6	26.26	6	21.24		
5	93.64	5	81.95	5	71.37	5	61.79	5	53.12	5	45.27	5	38.18	5	31.83	5	26.15	5	21.14		
9.64	93.40	9.14	81.73	8.64	71.17	8.14	61.61	7.64	52.95	7.14	45.12	6.64	38.05	6.14	31.71	5.64	26.05	5.14	21.05		
8	93.15	8	81.50	8	70.97	8	61.43	8	52.79	8	44.97	8	37.92	8	31.59	8	25.94	8	20.96		
7	92.91	7	81.28	7	70.77	7	61.25	7	52.62	7	44.82	7	37.78	7	31.47	7	25.84	7	20.86		
6	92.66	6	81.06	6	70.57	6	61.07	6	52.46	6	44.67	6	37.65	6	31.35	6	25.73	6	20.77		
5	92.42	5	80.84	5	70.37	5	60.88	5	52.30	5	44.52	5	37.52	5	31.23	5	25.62	5	20.68		
9.59	92.18	9.09	80.62	8.59	70.17	8.09	60.70	7.59	52.13	7.09	44.38	6.59	37.38	6.09	31.11	5.59	25.52	5.09	20.59		
8	91.93	8	80.40	8	69.97	8	60.52	8	51.97	8	44.23	8	37.25	8	30.99	8	25.41	8	20.49		
7	91.69	7	80.18	7	69.78	7	60.35	7	51.81	7	44.08	7	37.12	7	30.87	7	25.31	7	20.40		
6	91.45	6	79.97	6	69.58	6	60.17	6	51.64	6	43.94	6	36.99	6	30.75	6	25.20	6	20.31		
5	91.21	5	79.75	5	69.38	5	59.99	5	51.48	5	43.79	5	36.86	5	30.64	5	25.10	5	20.22		
9.54	90.97	9.04	79.53	8.54	69.18	8.04	59.81	7.54	51.32	7.04	43.64	6.54	36.72	6.04	30.52	5.54	25.00	5.04	20.13		
8	90.73	8	79.31	8	68.99	8	59.59	8	51.16	8	43.50	8	36.59	8	30.40	8	24.89	8	20.03		
7	90.49	7	79.08	7	68.79	7	59.38	7	50.98	7	43.35	7	36.46	7	30.29	7	24.79	7	19.97		
6	90.25	6	78.88	6	68.59	6	59.18	6	50.80	6	43.21	6	36.33	6	30.17	6	24.69	6	19.82		
5	90.01	5	78.66	5	68.40	5	58.99	5	50.68	5	43.06	5	36.20	5	30.05	5	24.58	5	19.77		

TABLE 10. CIE (Y) equivalents (in percent relative to MgO) of the recommended Munsell value scale (V) from 0/ to 10/ (continued)

[illegible]

transmittance relative to the same thickness of distilled water or solvent. For any of these objects Munsell renotation value, V , may be found from Y/Y_0 in accord with table 10. Because of their close correlation with the color solid, Munsell renotations are capable of being quickly understood. Thus the renotation 8.3 Y 8.6 $_0$ /10.6 for the greenish yellow printing-ink specimen indicates from the letter Y that the specimen is a yellow, from the value 8.6 $_0$ that it is a relatively light color, being close to the top of the value scale, 0 to 10, and from the chroma 10.6, that it is a strong or nongrayish color, being more than 10 Munsell chroma steps away from the gray of the same Munsell value. Munsell renotation hue and chroma serve more adequately for object colors the purposes formerly served by dominant wavelength and purity. Munsell renotation hue correlates significantly better under ordinary conditions of daylight observation with the hue of the perceived color than does dominant wavelength, and Munsell renotation chroma is by far superior to purity in its correlation to saturation. This correlation with the color-perception solid does not, however, necessarily hold under all observing conditions. Ordinarily this printing-ink specimen will be perceived to have a light, strong greenish yellow color, but it is not so perceived under all conditions. If this specimen be viewed next to a brilliant yellow-green area such as is provided by a fluorescent fabric, it will be perceived to take on a darker color of yellowish orange hue and moderate saturation. Thus the lightness, hue, and saturation of the color perception depend upon the surroundings and upon the adaptive state of the eye; and lightness, hue, and saturation are taken correctly to be psychological terms. But the Munsell renotation refers only to the light that is reflected from the specimen and stays constant regardless of these changes in observing conditions. It is therefore a psychophysical characterization of the specimen according to the light reflected from it, just as are luminous reflectance, Y , and chromaticity coordinates, x, y from which it can be derived.

Another advantage of expressing spectrophotometric results in the form of the Munsell renotation is that the amount and kind of the color difference between two specimens can be found immediately from the two renotations in an easily understandable form. Thus, from the hue difference between the two blue printing-ink specimens (5.6 B compared to 0.8 PB), the former is seen to be more greenish (less purplish) by about five Munsell hue steps. From the value difference (5.4 $_6$ compared to 5.5 $_0$) the two are seen to be of the same value to the nearest one-tenth Munsell value step; and from the chroma difference (8.5 compared to 9.6) the greenish blue is seen to be more grayish by one Munsell chroma step. Such differences as these in terms of Munsell hue, value, and chroma may be combined into a single index, I , of color difference [115, 117]:

$$I = \frac{C}{5} (2\Delta H) + 6\Delta V + 3\Delta C \quad (8)$$

where C is Munsell chroma, and ΔH , ΔV , ΔC , are the differences between the two colors in Munsell hue, value, and chroma, respectively. The difference between the colors of the two blue printing-ink specimens would be found by this formula as:

$$\begin{aligned} I &= \frac{9.0}{5} (2 \times 5.2) + 6 \times 0.04 + 3 \times 1.1 \\ &= 18.7 + 0.2 + 3.3 = 22.2 \text{ units.} \end{aligned}$$

These units are of such size that color differences of less than one unit would ordinarily not be of commercial importance; that is, pairs of colors exhibiting such differences would be considered to be commercial matches. Note that these two blue printing-ink specimens are far from being a commercial match; also note that the hue difference is far more important than the chroma difference, and that the value equivalence is well within commercial toleration.

d. ISCC-NBS System

A method devised at the request of the American Pharmaceutical Association and the United States Pharmacopoeial Convention for designating the colors of drugs and chemicals is coming into use for general purposes. The general plan of the method was worked out by the Inter-Society Color Council, and the details were developed at the National Bureau of Standards; the method is therefore referred to as the ISCC-NBS method of designating colors [69, 79]. This method provides a designation for every color perceived as belonging to an object (either an opaque surface, or a light-transmitting layer), and it has been extended to the colors of self-luminous areas by Kelly [77]; see figure 3. The number of color designations was purposely made small, 267, for the sake of simplicity. Since about ten million surface colors can be distinguished by the normal human observer with optimum observing conditions, the ISCC-NBS method falls far short of supplying a different designation for each distinguishable color, or even for all colors (numbering perhaps half a million) considered to be commercially different.

The plan of the method is to divide the surface-color solid (see fig. 19) arbitrarily into 267 compartments, and assign a designation to each in as good conformity as possible to color nomenclature currently used in art, science, and industry. The compartments embracing the black-white axis are given the following designations: black, dark gray, medium gray, light gray, and white. The compartments adjacent to these are given similar designations formed by adding an adjective indicating the hue, such as yellowish white, dark purplish gray, or greenish black. All other compartments take designations consisting of a hue name (red, orange, yellow, green, blue, purple, pink,

white	light gray (l. Gy.)	medium gray (med. Gy.)	dark gray (d. Gy.)	black (Bl.)
-ish white	light-gray (l.-ish Gy.)	-ish gray (-ish Gy.)	dark-ish gray (d.-ish Gy.)	-ish black (-ish Bl.)
very pale (v. p.)	pale (p.)	grayish (gy.)	dark grayish (d. gy.)	blackish (bl.)
very light (v. l.)	light (l.)	moderate (m.)	dark (d.)	very dark (v. d.)
brilliant (brill.)				
vivid (v.)				
deep				
very deep (v. deep)				

Saturation (Munsell Chroma)

FIGURE 23. The modifiers used in the color designations of the ISCC-NBS system [75].

brown, olive) preceded by modifiers (light, dark, grayish, strong) indicating the lightness and saturation of the perceived color. Figure 23 shows the complete list of modifiers used in the ISCC-NBS system; note that *vivid* is a substitute for *very strong*; *pale*, a substitute for *light grayish*; *deep*, for *dark and strong*, and so on.

The boundaries between the groups of colors known by these designations have been adjusted to accord as closely as possible with common usage and have been expressed in terms of *Munsell Book* notation. Figure 23 also shows these boundaries for colors of purple hue.

The ISCC-NBS designations are not to be considered a substitute for numerical designation of color resulting from application of a suitable colorimetric method, but they do supply a certain precision to verbal color designations that has previously been lacking. Table 11 gives the ISCC-NBS designations of the four printing-ink specimens shown on figure 4 and the ISCC-NBS hue designations have been used throughout the text of this chapter. These color designations are used in the current edition (N.F. VII) of the National Formulary, and they are being inserted into the United States Pharmacopoeia and into a textbook of qualitative analysis [130]. They have been used for describing the colors of building stone [83] and soils [138], and for a considerable variety of research purposes such as the description of mica colors after heat treatment [53]. Kelly [80] has found the Munsell notation for the centroid color

TABLE 11. Abbreviations for the hue names used in the ISCC-NBS system

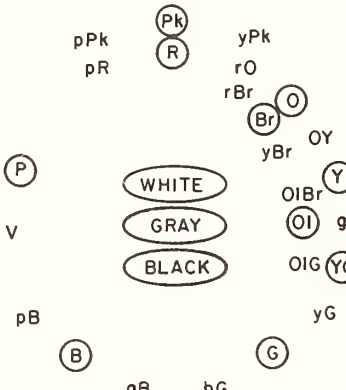
Name	Abbreviation	Name	Abbreviation
Red	R	Purple	P
Reddish orange	rO	Reddish purple	rP
Orange	O	Purplish red	pR
Orange yellow	OY	Purplish pink	pPk
Yellow	Y	Pink	Pk
Greenish yellow	gY	Yellowish pink	yPk
Yellow green	YG	Brownish pink	brPk
Yellowish green	yG	Brownish orange	brO
Green	G	Reddish brown	rBr
Bluish green	bG	Brown	Br
Greenish blue	gB	Yellowish brown	yBr
Blue	B	Olive brown	OlBr
Purplish blue	pB	Olive	Ol
Violet	V	Olive green	OlG



of each compartment assigned an ISCC-NBS color designation and has recommended a system of abbreviations; see table 11.

The ISCC-NBS designations are generally unsuited for use in sales promotion. The method has been approved for color description of drugs and chemicals by the delegates of nine national societies, and it has been recommended for general use by the United States of America Standards Institute [151a].

In 1958 the method was recommended by the Subcommittee on the Expression of Historical Color Usage [60], Inter-Society Color Council, for the statistical expression of color trends. The adaptation of the method for this purpose involved the tabulation of six distinct, but correlated, degrees or levels of accuracy in color description [82]. In the first level the color solid is divided into just 13 parts, ten described by a generic hue name and three neutrals, white, gray and black. In the second level, the color solid is divided into 29 parts, ten of the original 13 parts being further divided and assigned intermediate hue names. In the third level each part of the color solid described by a generic or intermediate hue name and the appropriate modifier descriptive of its lightness and saturation; that is, in the third level of accuracy, the 267 color designations of the ISCC-NBS method are used without abridgement. Level four is illustrated by the Munsell Book of Color [100] with its more than 1,000 uniformly spaced color samples. Level five is illustrated by the interpolated Munsell book notation by which about 100,000 different colors can be reliably specified. Level six, the level in which the greatest accuracy of color identification is possible, is illustrated by two basic methods: the CIE method, and the interpolated Munsell renotations. This degree of accuracy is realizable *only through the measurement of the color with a spectrophotometer*, and is equivalent to the division of the color solid into about 5,000,000 parts. Figure 24 gives further information about the six levels: (A) level of fineness of color identification, (B) number of divisions of color solid, (C) type of color designation, (D) example of color designation, and (E) alternate color-order systems usable in that level.

Through the cooperation of the Inter-Society

A	1	2	3	4	5	6
B	13	29	267	943 to 7056	About 100,000	About 5,000,000
C	Generic hue names and neutrals (circled)	All hue names and neutrals	ISCC-NBS All hue names and neutrals with modifiers (NBS C553)	Color-order system (Collections of color standards sampling the color solid systematically)	Visually interpolated Munsell notation from Munsell Book of Color	CIE (x, y, Y) $x = 0.395$ $y = 0.382$ $Y = 35.6$ or Interpolated Munsell renotation
D	brown	yellowish brown	light yellowish brown (centroid Na.76)	10YR 6/4 Munsell 1548	$9\frac{1}{2}$ YR 6.4/4 $\frac{1}{4}$	9.6YR 6.4 ₅ /4.3
E			TCCA (9th Std.) 216 70128 TTC-595 (1st Ed) 187 F2310 H.C.C. H407 800	M&P (1st Ed) 7056 12H6 Plochere 1248 180 O 5-d Ridgway III5 XXIX 13" b C.H.M. (3rd Ed) 943 3gc		

Direction for increased fineness of color definition 
 Direction for statistical expression of color usage (roll up) 

A. Level of fineness of color identification
 B. Number of divisions of color solid
 C. Type of color designation
 D. Example of color designation
 E. Alternate color-order systems usable in that level

FIGURE 24. *Basis of Universal Color Language.*

Color Council and the Tobey Color Card Co., St. Louis, Mo., the National Bureau of Standards commenced issuing the ISCC-NBS Centroid Color Charts [109] in February 1965. Each set consists of 18 charts displaying samples closely approximating 215 of the 267 centroid colors, and fair approximations of 36 others, making 251 color samples in all. The samples are in the form of one-inch squares of paper coated with glossy-finish paint affixed to a variable gray background so that each color is on a neutral background of approximately its own

lightness. The Munsell renotation of each color sample is supplied in the cover pages to the charts. Duplicates of each of the 251 ISCC-NBS centroid colors can be obtained in 9 by 12 in sheets from the Munsell Color Company, Inc. of Baltimore, Md. These centroid colors have been used as standards for a wide variety of purposes. They facilitate color description at level three, and have found their most important application so far in the recording of data for establishing trends of public acceptance of color in various lines of merchandise.

5. One-Dimensional Color Scales

5.1. General Principles

There are many tests analogous to the comparison of a solution of an unknown amount of a constituent with a series of suitably prepared standard solutions to find the concentration of the specimen. In these tests the colors of the unknowns exhibit a one-dimensional change with concentration; and, although this change may be complicated in terms of luminous transmittance and chromaticity coordinates [102], a suitably spaced series of standards over this range of colors will yield the desired con-

centration either by actual match with one of the standards, or by visual interpolation among them. Such a series of standards is said to constitute a *color scale*. The ideal material from which to make the standards is the constituent of the unknown itself; in this way there is guaranteed not only a perfect color match at some point along the scale, but also a perfectly nonmetameric match so that variation of the illuminant or individual-observer variations are generally unimportant.

However, if the unknown is impermanent, it may become necessary to try to duplicate the desired

colors in a more nearly permanent medium. Glass is a frequent choice because of its generally superior permanence. Some degree of metamerism has then to be tolerated because the standards have coloring constituents not a perfect spectral match for the unknown. It is also rare that a perfect job of color matching for any standard illuminant and observer is done. The observer is then faced with what is often a difficult, and sometimes an impossible, task. He must estimate the position of the unknown color on the scale, and often it will seem to him that the unknown color is not equal to any of the standard colors, nor intermediate between any two of them. The concepts in terms of which the observer perceives these color differences then come into play. If he judges the color difference between the two luminous areas presented to him in terms of hue, brightness, and saturation, as is fairly common, he could estimate the position of the unknown color on the color scale as the point on the scale yielding the same hue, or as that yielding the same brightness or the same saturation; or he could try to estimate the point on the scale yielding the closest color match; or he could disregard brightness differences and try to estimate the point on the scale yielding the closest chromaticity match. The determination becomes an estimate based on what criterion of equivalence is used by the observer, and it depends upon his mental capability in an essentially indescribable way. In spite of these drawbacks, a good color scale is a useful time-saver, as long as it is not used in attempts to provide a one-dimensional solution to what is essentially a multidimensional problem.

Judgments of position on the color scale according to equality of brightness can be expected to correspond to luminous transmittance. Judgments according to equality of hue agree well with the Munsell renotation hue; loci of constant hue for ordinary conditions of observation are indicated on the (x,y) diagram by the curved lines of figure 3 separating the areas corresponding to the various hue names. Judgments according to equality of saturation agree well with Munsell renotation chroma (see fig. 22). If there is only secondary brightness variation along the scale, judgments of nearest chromaticity match may be found approximately by taking the shortest distance on a uniform-chromaticity-scale diagram [14, 23, 55, 66, 141, 142]. Figure 8 shows the uniform-chromaticity-scale triangle according to Judd [66]. If the chromaticity coordinates of a color are (x,y) in the standard CIE system, the color would have chromaticity coordinates (r,g) in this uniform-chromaticity-scale triangle in accord with eq (5). On this diagram the ellipses of figure 7 would be equal tangent circles. If there is primary variation of both luminance and chromaticity, no reliable way of estimating the nearest color match has yet been developed. According to the OSA Colorimetry Committee [24], "The complete experimental clarification of this problem is one of the major pro-

grams yet to be undertaken in the field of colorimetric research."

5.2. Color Temperature

Perhaps the most widely used one-dimensional color scale is that of color temperature for classifying light sources. The *color temperature* of a light source is the temperature at which the walls of a furnace must be maintained so that light from a small hole in it will yield the chromaticity of the source to be specified. The color scale thus consists of the series of lights producible by closed-cavity radiation and is specified by temperature on the absolute scale (degrees Kelvin). Working standards of color temperature may consist of an incandescent lamp operating at a fixed voltage combined with a series of amber or blue glasses, like the Lovibond blue glasses; but by far the most common way of producing these chromaticities over moderate ranges of color temperature is by variation of the voltage applied to an incandescent lamp. The locus of these chromaticities (the so-called Planckian locus) is shown on figures 7 and 8. If the chromaticity of the light source is close to, but not exactly equal to, any of the Planckian chromaticities, still it is possible to correlate a color temperature with the source by taking the nearest chromaticity match. Figure 25 shows this correlation [67, 81]. The isotemperature lines, which cut the Planckian locus at varying angles, are all such as to be perpendicular to this curve on figure 26. The x,y and u,v chromaticity coordinates of a number of Planckian radiators ($c_2 = 14,380$) and standard sources in the 2° and 10° observer systems have been computed by Nimeroff [123]. These are listed in table 12. Since color temperature specifies only the chromaticity of a light, there are many spectral compositions corresponding to the same color temperature. Color temperature of a source is therefore an incomplete and unreliable indication of the rendering of the colors of objects illuminated by it or of the photographic effect of the source. To make color temperature a meaningful and useful basis for comparing two lights it must also be known that they are spectrally similar. Thus, incandescent lamps may be usefully intercompared by means of color temperature, and fluorescent lamps with about the same admixture of mercury spectrum may also be so intercompared, but incandescent lamps may not be intercompared with fluorescent.

5.3. Grading Systems

When the degree of refinement and quality of such products as oils, rosin, and sugars may be characterized on similar one-dimensional color scales which range from dark red through yellow to perfectly colorless, the development of a complex color specification for these products is redundant. These perceived color changes correlate

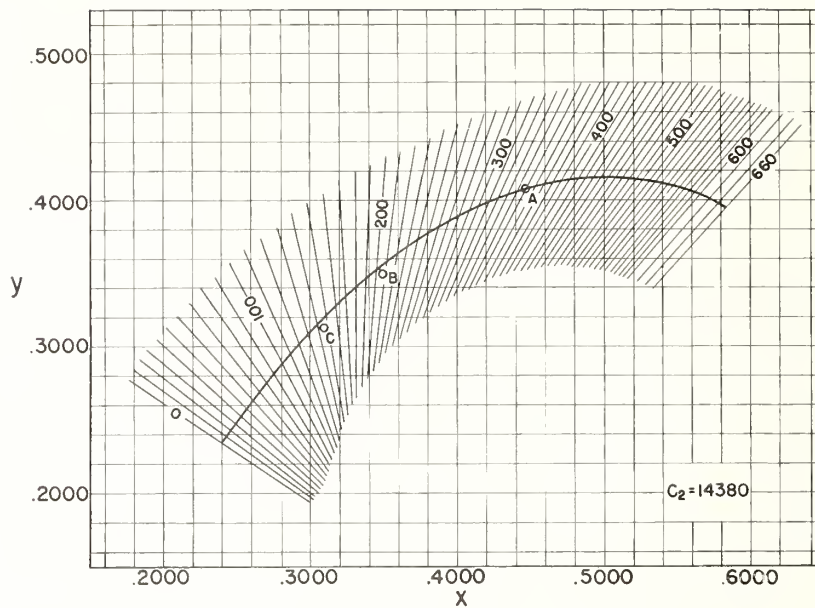


FIGURE 25. *Isotherm lines for evaluation of correlated color temperatures of non-Planckian colors.*
 These lines are perpendicular to the Planckian locus on figure 26.

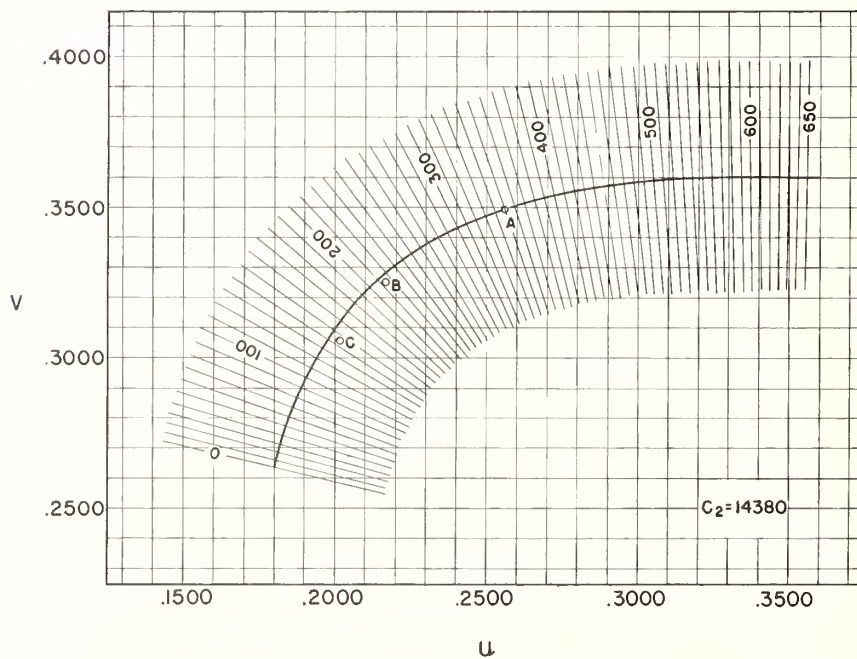


FIGURE 26. *Isotherm lines of correlated color temperature shown perpendicular on the Planckian locus on the u,v chromaticity diagram [81].*

TABLE 12. Chromaticity coordinates of Planckian radiators ($c_2 = 14,380$) and standard sources

Source (°K)	2° Observer system				10° Observer system			
	x_2	y_2	u_2	v_2	x_{10}	y_{10}	u_{10}	v_{10}
1000	0.6526	0.3447	0.4482	0.3547	0.6474	0.3504	0.4383	0.3555
2000	.5266	.4133	.3050	.3590	.5300	.4122	.3078	.3591
3000	.4368	.4041	.2506	.3476	.4403	.4026	.2532	.3476
4000	.3804	.3767	.2251	.3344	.3827	.3759	.2266	.3345
5000	.3450	.3516	.2115	.3231	.3464	.3515	.2119	.3234
6000	.3220	.3317	.2034	.3141	.3227	.3323	.2030	.3146
7000	.3063	.3165	.1982	.3070	.3066	.3176	.1973	.3078
8000	.2952	.3048	.1947	.3014	.2951	.3063	.1934	.3022
9000	.2869	.2956	.1922	.2970	.2866	.2975	.1905	.2979
10000	.2806	.2883	.1904	.2933	.2802	.2905	.1884	.2944
∞	.2399	.2342	.1800	.2636	.2394	.2366	.1786	.2648
A	.4476	.4075	.2560	.3495	.4512	.4059	.2588	.3496
B	.3485	.3517	.2137	.3235	.3498	.3527	.2138	.3241
C	.3101	.3163	.2009	.3073	.3104	.3191	.1994	.3086
E	.3333	.3333	.2105	.3158	.3333	.3333	.2105	.3158

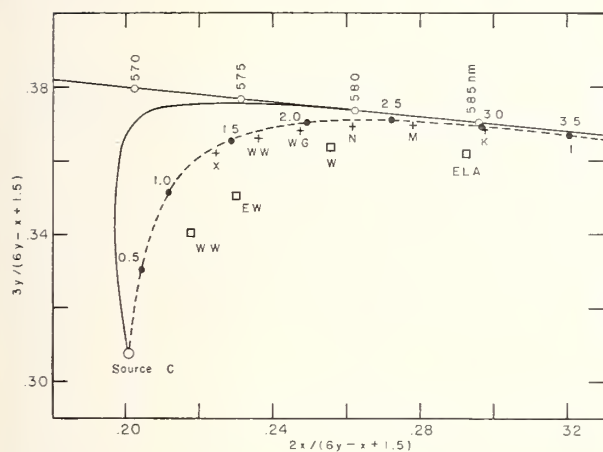


FIGURE 27. Chromaticities of some standards for grading petroleum products (solid circles), rosin (crosses), and honey (squares).

The solid curve is generated by an ideal absorption-band sweeping across the spectrum from the short-wave to the long-wave band [40]. The standards have broad absorption bands of different sharpness.

with a shift in the wide absorption band toward the ultraviolet as brown pigments are removed on refining. The differences in scales among these products correspond to differences in sharpness of the absorption band, with oils possessing the sharpest bands, rosins next, and sugars least sharp. Figure 27 compares, on the 1960 CIE-UCS [80] chromaticity diagram, the chromaticity locus generated by varying an ideal absorption band (40) with a few color standards used for petroleum (solid dots), rosin (crosses), and sugar (squares) products.

It is sometimes convenient to state the color of a product in terms of its equivalent on another scale and modified by an adjective that describes the off-locus position. Thus U. S. Rosin Standard X (extra water white), which is equivalent to ASTM petroleum color 1.4 but slightly on the purple side, will be designated 1.4_p. The letter g will be used to designate departures toward green.

a. Petroleum Products

For more than 30 years the color of lubricating oils and petrolatum has been graded by comparison with the colors of 12 glass standards [7]. The petroleum product in a 33-mm layer and the standard are illuminated by artificial daylight produced by combining an incandescent lamp of color temperature approximately 2750 °K with a filter of Corning Daylite glass specially selected to have spectral transmittances within specified tolerances and further to have for standard source A luminous transmittance, τ_v , and chromaticity coordinates, x, y, z , within the limits:

τ_v	0.107 to 0.160
x	.314 to .330
y	.337 to .341
z	.349 to .329

The specimen holder, the magazine containing the glass color standards, the artificial daylight assembly, and a viewing diaphragm defining the direction of view are mounted together to form a portable instrument known as the Union colorimeter.

Table 13 gives the Lovibond analysis of the glass color standards (6), the luminous transmittance, τ_v , and chromaticity coordinates, x, y , for source C [141], the color names used by the National Petroleum Association, and the nearest chromaticity match on the new ASTM color scale, which was adopted in 1957. This new scale consists of a set of glasses which define a scale that is closer to the range of petroleum colors and with improved step uniformity [48, 72].

Table 14 gives for these glasses the luminous transmittances and chromaticity coordinates for source C in the defining UCS (r, g)-system as well as in the standard CIE (x, y)-system. Also listed are the luminous transmittance tolerances for the glass standards and their equivalents on three other scales.

The grading of naphthas, kerosines, and so on, has for many years been carried out by comparison of the color of rather thick layers (up to 20 in) of the refined oil with the colors of a set of three color standards made of yellowish glass. The Saybolt

TABLE 13. ASTM Union colorimeter standards, Lovibond analysis, luminous transmittance, τ_v , chromaticity coordinates, x, y , for CIE source C, NPA color names, nearest chromaticity match on the new ASTM color scale, and Munsell Book notations

Union color Number	Lovibond analysis			Luminous transmittance, τ_v	Chromaticity coordinates		National Petroleum Association names	Nearest chromaticity match on ASTM scale	Munsell Book notations
	Red	Yellow	Blue		x	y			
1.0	0.12	2.4	-----	0.751	0.349	0.382	Lily white	0.55g	10Y 9.6/3.8
1.5	.60	8.0	-----	.654	.400	.446	Cream white	1.05g	8Y 8.8/8.0
2.0	2.5	26.0	-----	.443	.472	.476	Extra pale	1.85p	3Y 7.2/12
2.5	4.6	27.0	-----	.365	.498	.457	Extra lemon pale	2.35p	10YR 6.0/12
3.0	6.9	32.0	-----	.287	.525	.440	Lemon pale	2.85p	7YR 5.7/13.5
3.5	9.4	45.0	-----	.211	.556	.423	Extra orange pale	3.5p	4YR 5.0/14
4.0	14.0	50.0	0.55	.096	.591	.400	Orange pale	4.2p	2YR 3.2/12
4.5	21.0	56.0	.55	.065	.620	.376	Pale	4.9	1YR 2.6/12
5.0	35.0	93.0	-----	.036	.653	.347	Light red	5.8	1YR 2.2/9.5
6.0	60.0	60.0	.55	.017	.676	.323	Dark red	6.5	1YR 1.3/8.5
7.0	60.0	106.0	1.8	.0066	.684	.316	Claret red	6.75	1YR 0.6/4.5
8.0	166.0	64.0	-----	.0020	.714	.286	-----	7.8	1YR 0.2/1.4

TABLE 14. ASTM color standards for petroleum products. Chromaticity coordinates in the CIE and UCS system with tolerances, luminous transmittances with tolerances, and nearest chromaticity matches on the Union scale, the 35-yellow plus N-red Lovibond scale used for vegetable oils, and the Gardner scale used for paint vehicles

ASTM color number	Chromaticity coordinates				Luminous transmittance			Nearest chromaticity match on :		
	CIE-system		UCS-system*		τ_v			Union color scale	Lovibond 35Y + NR scale N	Gardner color scale
	x	y	r	g						
0.5	0.3469	0.3739	0.462	0.473	0.86	±	0.06	0.9 p	-----	5.3p
1.0	.3945	.4359	.489	.475	.77	±	.06	1.45p	-----	8.0p
1.5	.4473	.4781	.521	.454	.67	±	.06	1.8 g	1.5p	10.0
2.0	.4876	.4826	.552	.442	.55	±	.06	2.15g	3.3	11.4g
2.5	.5192	.4711	.582	.416	.44	±	.04	2.65g	5.3g	12.3g
3.0	.5437	.4517	.611	.388	.31	±	.04	3.1 g	7.7g	13.2
3.5	.5648	.4308	.640	.359	.22	±	.04	3.5 g	10.3g	13.9
4.0	.5855	.4102	.671	.328	.152	±	.022	3.85g	13.5g	14.8
4.5	.6053	.3906	.703	.296	.109	±	.016	4.2 g	17.5g	15.6
5.0	.6257	.3742	.736	.264	.081	±	.012	4.55	23 g	16.3
5.5	.6435	.3565	.770	.230	.058	±	.010	4.85	30	17.2
6.0	.6604	.3395	.805	.195	.040	±	.008	5.3	41	18.1
6.5	.6765	.3235	.841	.159	.026	±	.006	6.0	57	-----
7.0	.6913	.3086	.877	.123	.016	±	.004	7.2	82	-----
7.5	.7059	.2941	.915	.085	.0081	±	.0016	7.7	-----	-----
8.0	.7204	.2796	.956	.044	.0025	±	.0006	-----	-----	-----

*Tolerances on chromaticity coordinates, r, g, b ($b = 1 - r - g$), are ± 0.006 .

chromometer is a device for carrying out this comparison. It consists of an artificial daylight lamp meeting the same requirements as given above for the ASTM Union colorimeter, a graduated tube for the oil specimen, a holder for the glass color standards, and a prism eyepiece to bring into juxtaposition the two fields to be compared. There is an open ungraduated tube beneath the holder for the glass standards that serves to duplicate to a degree on the standard side of the instrument the effect of light multiply reflected within the specimen tube. The glass standards consist of two whole disks and one "one-half" disk; these standards must have luminous transmittances, τ_v , and chromaticity coordinates, x, y , for source C, as follows:

	Whole disks	One-half disks
τ_v -----	0.860 to 0.865	0.888 to 0.891
x -----	.342 to .350	.327 to .331
y -----	.367 to .378	.344 to .350
z -----	.272 to .291	.319 to .330

Oils having colors closely resembling that of distilled water are graded on the Saybolt chromometer by comparison with the half disk; more yellowish oils by one or two whole disks. The depth of oil yielding the closest color match with the glass disk is found by a prescribed procedure [5],

and the color of the oil specimen is designated by a number defined from the disc used and the required depth of oil. Table 15 gives this definition [5] in terms of Saybolt number, Union color, u , and ASTM color, a .

In spite of the fact that the prism eyepiece brings the fields to be compared into juxtaposition and so facilitates the detection of differences in brightness, the change in luminous transmittance with thickness of these refined oils is so slight that it is much less easily detected than the corresponding chromaticity change. The settings of the Saybolt chromometer therefore probably depend essentially on nearest chromaticity match. If the oil sample is turbid, however, not even an approximate match can be obtained, and the method may be inapplicable. In these cases a thickness of the specimen yielding a chromaticity match is much darker than the standard, and no reliable setting of depth of sample yielding nearest match can be found.

Figure 28 shows a portion of the (x,y) chromaticity diagram on which have been plotted the rectangles corresponding to the chromaticity tolerances for Saybolt half disks and whole disks (see above). It will be noted that the tolerances are fairly wide so that variations of ± 10 percent of the chromaticity change caused by introduction of the disk into the daylight beam are permissible. The relation between the Union color numbers, u , and

TABLE 15. Saybolt numbers and corresponding Union color, u , and ASTM color, a , defined in terms of specimen depth and disk number

Number of disks n	Depth of oil in	Saybolt number	Union color u	ASTM color a
$\frac{1}{2}$	20.00	+30	0.030	0.015
$\frac{1}{2}$	18.00	+29	.033	.017
$\frac{1}{2}$	16.00	+28	.037	.019
$\frac{1}{2}$	14.00	+27	.042	.022
$\frac{1}{2}$	12.00	+26	.049	.025
1	20.00	+25	.059	.030
1	18.00	+24	.066	.034
1	16.00	+23	.074	.038
1	14.00	+22	.084	.043
1	12.00	+21	.098	.051
1	10.75	+20	.110	.056
1	9.50	+19	.124	.064
1	8.25	+18	.143	.074
1	7.25	+17	.163	.084
1	6.25	+16	.189	.097
2	10.50	+15	.225	.116
2	9.75	+14	.243	.125
2	9.00	+13	.262	.135
2	8.25	+12	.286	.147
2	7.75	+11	.305	.167
2	7.25	+10	.326	.167
2	6.75	+9	.350	.180
2	6.50	+8	.364	.187
2	6.25	+7	.376	.194
2	6.00	+6	.393	.202
2	5.75	+5	.411	.211
2	5.50	+4	.430	.221
2	5.25	+3	.450	.231
2	5.00	+2	.474	.243
2	4.75	+1	.498	.256
2	4.50	0	.526	.270
2	4.25	-1	.557	.286
2	4.00	-2	.592	.304
2	3.75	-3	.631	.324
2	3.625	-4	.652	.335
2	3.50	-5	.677	.347
2	3.375	-6	.701	.360
2	3.25	-7	.727	.374
2	3.125	-8	.758	.388
2	3.00	-9	.789	.405
2	2.875	-10	.823	.422
2	2.75	-11	.860	.441
2	2.625	-12	.901	.462
2	2.50	-13	.947	.486
2	2.375	-14	.996	.511
2	2.25	-15	1.05	.539
2	2.125	-16	1.11	.571

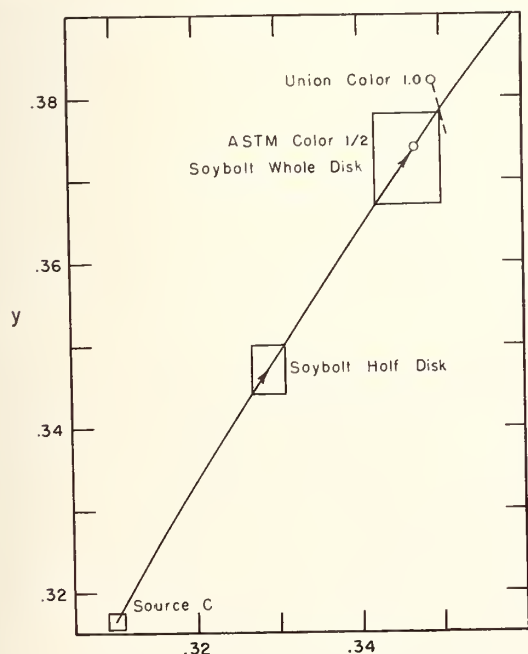


FIGURE 28. Chromaticities of the Saybolt color standards compared to that for Union No. 1 color, and to those of the yellowish wedge of the chromaticity-difference colorimeter (arrows).

the Saybolt color expressed in terms of number of whole disks, n , and depth, D , in inches,

$$u = \frac{1.182n}{D},$$

may be derived by reference to figure 28. Two vectors, each corresponding to 300 wedge units define the steps from source C to Saybolt $\frac{1}{2}$ disk to Saybolt whole disk, or 600 wedge units in all. The chromaticity point representing Union Color No. 1 does not fall exactly on the extended line. The intersection of the dotted line and the extended line gives the closest chromaticity match, and shows that the chromaticity of Union Color No. 1 approximates 1.10 Saybolt disk. An oil of Union Color No. 1 will therefore match 1.00 Saybolt disk in a thickness of 33.0/1.10 or 30.0 mm (1.182 in) and will match two whole disks in a thickness of 60.0 mm (2.365 in).

The corresponding relation for ASTM color number, a , is

$$a = 0.607n/D ,$$

based on the facts that the glass that defines ASTM color number 0.5 is closely equivalent to 1.03 Saybolt disk (as shown on fig. 28) and that thickness on the ASTM scale has been changed to 1.25 in from 33.0 mm [7].

b. Vegetable Oils

In addition to the brown pigments found in petroleum products, edible vegetable oils (corn, cottonseed, olive, peanut, rapeseed, sesame, soybean) usually contain some chlorophyll. Although metamerism exists between the oils and Lovibond glasses [100], the oils are graded, for commercial purposes, by the number (N) of Lovibond red units, added to Lovibond 35-yellow, that are required to match a 5.25-in layer of the oil. Table 16

TABLE 16. *Lovibond 35-yellow plus N-red scale used to color-grade edible vegetable oils. CIE chromaticity coordinates, x, y , and nearest chromaticity match on the ASTM and Gardner color scales*

Lovibond 35Y + NR scale	Chromaticity coordinates		Nearest chromaticity match on:	
	x	y	ASTM color scale	Gardner color scale
0	0.441	0.521	1.3g	9.2g
1	.455	.508	1.4g	9.9g
2	.469	.496	1.6g	10.7g
3	.482	.485	1.9	11.3
4	.494	.474	2.1p	11.7
5	.506	.464	2.4p	12.2p
6	.517	.455	2.6p	12.6p
7	.527	.446	2.8p	12.9p
8	.537	.438	3.0p	13.2p
9	.546	.431	3.3p	13.6p
10	.555	.424	3.5p	13.8p
12	.570	.412	3.8p	14.4p
14	.583	.402	4.0p	14.9p
16	.594	.393	4.3p	15.3p
18	.603	.385	4.5p	15.6p
20	.612	.378	4.7p	15.9p
24	.625	.367	5.1p	16.5p
28	.636	.358	5.4p	17.0p
32	.645	.351	5.6	17.4
36	.652	.345	5.8	17.7
40	.658	.340	6.0	18.0
45	.665	.334	6.1	-----
50	.670	.329	6.3	-----
60	.679	.321	6.6	-----
70	.685	.314	6.8	-----
80	.691	.309	7.0	-----
90	.695	.305	7.1	-----
100	.699	.301	7.3	-----

shows for values of N from 0 to 100 on the 35Y + NR scale the CIE source C chromaticity coordinates, x, y , and the nearest chromaticity match on the ASTM scale and on the Gardner scale (the Gardner scale is described in the next section).

Thomson [50] showed that spectral transmittance of oil at wavelengths 460, 550, 620, and 670 nm (τ_{460} , τ_{550} , τ_{620} , and τ_{670}) can be expressed in an index that gives excellent correlation with color grading by the Lovibond red units. This index,

$$1.29 \log (1/\tau_{460}) + 69.7 \log (1/\tau_{550}) \\ + 41.2 \log (1/\tau_{620}) - 56.4 \log (1/\tau_{670}),$$

has been found useful for commercial grading of vegetable oils.

c. Rosins and Paint Vehicles

Gum rosin has been graded by color for more than 50 years. Up to 1914 the color standards were made of rosin itself in spite of the relative impermanence of its color, and from 1914 to 1936 standards composed of combinations of Lovibond glasses were used. Brice [15] has described the selection of the present twelve official standards composed of two components of colored glass combined with one component of clear glass, all three cemented together with Canada balsam. The various combinations are given letter designations denoting the grades of rosin delimited by them and have legal status under the Naval Stores Act. The cemented face of the clear glass in each combination is fine ground so as to duplicate the slight turbidity characteristic of molded samples of rosin, which commonly contain traces of fine dirt. The chromaticity spacing was adjusted by means of the uniform-chromaticity-scale triangle of figure 8 so as to progress regularly from small steps for yellow rosins to steps of about four times the initial size for reddish orange rosins. Osborn and Kenyon have studied rosin colors spectrophotometrically [124]. Table 17 gives the names associated with letter designations, luminous transmittance, T , and chromaticity coordinates, x, y , for CIE source C, together with the nearest chromaticity match on the ASTM, 35Y + NR, and Gardner color scales.

Many special color scales have been set up for the specification of paint vehicles (varnishes, linseed oil, tung oil, etc.). A solution of nickel sulfate and iodine [16] is used to define the darkest color permissible for spar varnish. A color comparator having eighteen glass color standards made by Hellige, Inc., has been used for similar purposes. The Pfund color grader compares a variable thickness of the unknown specimen with a variable thickness of an amber colored glass [131]. The standard is wedge shaped, and the cell for the specimen is likewise wedge shaped. The Parlin (or Cargille) color standards consist of a set of thirty-five solutions. The first ten are Hazen platinum-cobalt solutions [52], developed originally to measure the color of natural water and still used for that purpose under the name of APHA (American Public Health Association) standards [4]. The remainder of the Parlin color standards are caramel solutions. They have been adopted by the ASTM (Designation D365-39) for testing the color of soluble nitrocellulose base solutions. The Pratt and Lambert color standards are varnish mixtures calibrated against the Pfund color grader. The DuPont colorimeter employs six plates as color standards, together with a wedge of the same glass permitting a continuous variation of color between the standards. The Gardner color standards consist of eighteen combinations of the red and yellow Army solutions. Gardner has determined the Army and

TABLE 17. U.S. standards for rosin. Luminous transmittance, τ_v , chromaticity coordinates, x, y , for CIE source C, and nearest chromaticity match on the ASTM color scale, the 35-yellow plus N-red Lovibond scale, and the Gardner color scale

Designation		Luminous transmittance	Chromaticity coordinates		Nearest chromaticity match on :		
Letter	Name	τ_v	x	y	ASTM color scale	35Y + NR scale N	Gardner color scale
X	Extra water white	0.609	0.4339	0.4663	1.4p	1.2p	9.7p
WW	Water white	.531	.4579	.4732	1.7p	2.2p	10.8p
WG	Window glass	.466	.4785	.4741	1.9p	3.2p	11.4p
N	Nancy	.396	.5001	.4704	2.2p	4.4	11.9
M	Mary	.322	.5212	.4619	2.6p	6.0g	12.5
K	Kate	.245	.5430	.4483	3.0p	7.8g	13.2p
I	Isaac	.178	.5649	.4310	3.5	10.3g	13.9
H	Harry	.1114	.5879	.4102	4.0	13.8g	14.9
G	George	.0723	.6116	.3874	4.6	18.7g	15.7
F	Frank	.0398	.6364	.3632	5.3	27 g	16.9
E	Edward	.0131	.6640	.3358	6.1	44	18.2
D	Dolly	.0021	.6943	.3057	7.2	88	-----

TABLE 18. The Gardner color standards. Chromaticity coordinates, x, y , for CIE source C, nearest chromaticity matches on the ASTM color scale and the 35-yellow plus N-red Lovibond scale, and both nearest lightness matches and nearest chromaticity matches on the Union color scale

Gardner color number	Chromaticity coordinates		Nearest chromaticity match on :			Nearest lightness match on Union color scale from Gardner (30)
	x	y	35Y + NR scale N	ASTM color scale	Union color scale	
1	0.313	0.324	-----	0.06g	0.12	-1
2	.318	.333	-----	.12g	.22	1+
3	.322	.339	-----	.18g	.33	1-1½
4	.330	.349	-----	.28	.51p	1-1½
5	.342	.371	-----	.47g	.86	1½-2
6	.354	.387	-----	.6	1.05p	1½-2
7	.372	.410	-----	.8	1.25p	2
8	.394	.440	-----	1.0 g	1.45	2-2½
9	.419	.462	0.7p	1.2	1.6	2½
10	.444	.475	1.4p	1.5	1.75	2½-3
11	.467	.482	2.3p	1.75	1.9 g	3-3½
12	.504	.471	4.6	2.3 p	2.45g	3½-4
13	.535	.454	7.1g	2.85	3.0 g	4-4½
14	.567	.427	10.6g	3.55	3.55g	4½+
15	.590	.407	14.2g	4.1	3.9 g	-5
16	.619	.380	21 g	4.8	4.45	6
17	.638	.361	28 g	5.35	4.75	7-8
18	.659	.340	40	6.0	5.25	8+

Lovibond specification for the nearest matches for all the above-mentioned sets of color standards and has also obtained the nearest equivalents in terms of potassium dichromate solutions [31]. From these nearest equivalents it is possible to express color specifications given by any of these means in terms of any other of them, as shown in Table 18 which gives the chromaticity coordinates, x, y , of each of the eighteen Gardner standards for CIE source C, nearest chromaticity match on the Lovibond 35Y + NR scale, the ASTM scale and the Union color scale, together with the nearest lightness match on the Union scale. The Gardner standards define a locus of chromaticities that agrees closely with that of the ASTM standards with only 6 of the 18 standards departing by more than 0.001 as indicated by the letters "p" and "g".

The British Paint Research Station has recommended [151] combinations of Lovibond glasses for color-grading oils and varnishes. Some of the combinations involve colorless or blue glasses to be combined with the oil to match red and yellow glasses, and a device facilitating the setting up of such combinations is also recommended. The Lovibond glasses are mounted in a slide, and the two photometric fields to be compared are brought into juxtaposition by mirrors.

d. Sugar Products

For regulatory purposes, the classification of sugar products (honey, maple syrup, molasses, sugarcane) is conducted visually by comparison with glass chromaticity standards. The source C chromaticity coordinates, x, y , are listed in table 19

TABLE 19. U.S. Department of Agriculture chromaticity standards for sugar products. Chromaticity coordinates, x, y , for CIE source C, and nearest chromaticity matches on the ASTM, the Lovibond 35Y + NR, and the Gardner color scales

Designation and Name	Chromaticity coordinates		Nearest chromaticity match on		
	x	y	ASTM color scale	Lovibond 35Y + NR scale	Gardner color scale
<i>Liquid sugar</i>					
WW(l _s), water white -----	0.3381	0.3529	0.35p	-----	4.2p
W(l _s), white -----	.3772	.3937	.75p	-----	7.0p
EW(h), light amber -----	.4169	.4245	1.3 p	-----	9.4p
<i>Extracted honey</i>					
WW(h), water white -----	0.3818	0.3982	0.8 p	-----	7.1p
EW(h), extra white -----	.4169	.4245	1.3 p	-----	9.4p
W(h), white ¹ -----	.4786	.4531	2.1 p	4.0p	11.7p
ELA(h), extra light amber ² -----	.5317	.4450	2.9 p	7.3	13.1p
LA(h), light amber ³ -----	.6141	.3845	4.7	19.5g	15.85
A(h), amber -----	.6711	.3279	6.3	51	-----
<i>Maple sirup</i>					
LA(ms), light amber -----	0.4947	0.4509	2.35p	4.9p	12.1p
MA(ms), medium amber -----	.5567	.4352	3.35	9.4g	13.7
DA(ms), dark amber -----	.6041	.3943	4.4	16.8g	15.5
<i>Sugarcane molasses</i>					
No. 1 -----	0.5183	0.4489	2.7 p	6.3	12.7p
No. 2 -----	.6301	.3691	5.15	25 g	16.6
No. 3 -----	.6815	.3179	6.65	64	-----

¹ Same as Sugarcane Sirup No. 1.

² Same as Sugarcane Sirup No. 2.

³ Same as Sugarcane Sirup No. 3.

for the glasses developed by Brice [15] and adopted by the U. S. Department of Agriculture for this purpose. The nearest chromaticity matches on three one-dimensional color scales (ASTM, 35Y+NR, and Gardner) are also given for each standard.

Increasing use is being made of one-dimensional photometric methods [36, 37, 162, 163] for more precise designation of sugar colors. Deitz [26] has developed a one-dimensional specification that is

defined as the color difference between the solvent and the sugar solution. The color difference is evaluated in Adams chromatic-value space [2] and is expressed in terms of a so-called NBS unit of sugar color. Measurements of the attenuation at two wavelengths (420 nm and 560 nm) are used in this evaluation to a good approximation. Brice [16] developed a similar one-dimensional specification that is based on chromaticity differences expressed in MacAdam units [19].

6. Limitations of the Methods

6.1. Spectrophotometric Colorimetry

Spectrophotometric colorimetry, the most fundamental color-measurement technique, suffers from two major sources of difficulty. First, the 1931 standard observer color-matching data were obtained for only some 17 observers [61] under 2° angular subtense and quite low illumination levels. Extrapolation of the use of this system for all observers and conditions may require justification. Secondly, there is usually some lack of precision and accuracy of spectral measurement of exitance, transmittance, and reflectance. Spectrophotometers are subject to errors of wavelength and photometric scales, stray-energy and slit-width effects, multiple reflections, and errors due to samples that are temperature dependent, wedge-shaped, and translucent. A compilation of these errors and recommendation for their accounting has been published by Gibson [35].

Although the CIE standard observer system for colorimetry has been generally satisfactory since its recommendation in 1931, there have been some

difficulties with the system. Most notable is its inability to resolve the chromaticity difference of anatase and rutile titanium dioxide [62, 71] usually viewed with angular subtense much greater than 2°. These difficulties have led the CIE to seek a new system (the 1964 CIE supplementary observer) through the Stiles and Burch [146] and Speranskaya [144] color-matching data of some 75 observers for a 10° photometric field. A complete analysis of these data permits the establishment of estimates of within- and between-observer variability of the system. Estimation of the variability in the color-matching functions, r (red), g (green), and b (blue) requires information about the between-observer deviations, σ_r , σ_g , σ_b , the within-observer deviations, $w\sigma_r$, $w\sigma_g$, $w\sigma_b$, and the covariances, σ_{rg} , σ_{rb} , σ_{bg} . The covariances, σ_{ij} , are the products of the correlation coefficients, ρ_{ij} , and the deviations, σ_i , σ_j ; thus $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$ where i and j refer to the color-matching functions. Figure 29 shows the between-observer deviations and the correlation coefficients determined by Nimeroff [123]

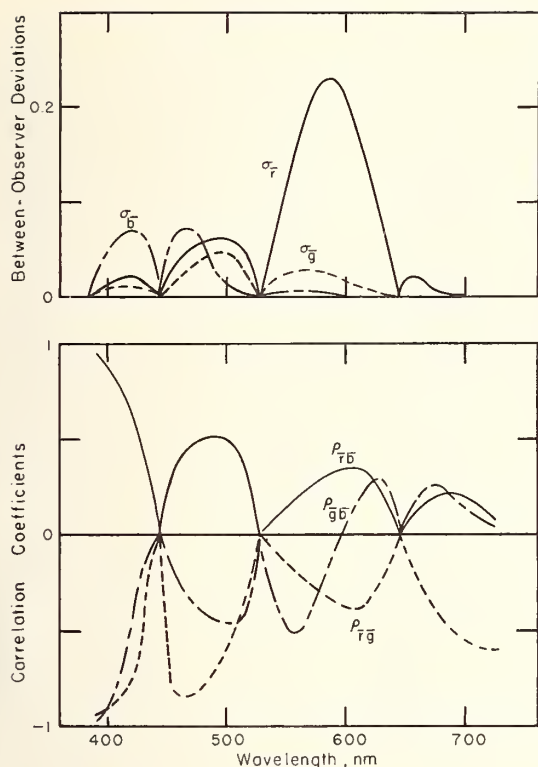


FIGURE 29. The standard between-observer deviations (upper graph) for 10°-field color-matching functions by Stiles and Burch [146] and Speranskaya [144] and the correlation-coefficients (lower graph) derived by Nimeroff [123].

from the Stiles-Burch and Speranskaya data. The average ratio of the between-observer deviations, σ_i , to within-observer deviations, $w\sigma_i$, for the three color-matching functions was estimated as 5.7.

Judd and Kelly [74] have transformed the average color-matching data, r, g, b , to the tristimulus values, x, y, z , by the transformation equations:

$$\begin{aligned}\bar{x} &= k_1\bar{r} + k_2\bar{g} + k_3\bar{b} \\ \bar{y} &= k_4\bar{r} + k_5\bar{g} + k_6\bar{b} \\ \bar{z} &= k_7\bar{r} + k_8\bar{g} + k_9\bar{b}.\end{aligned}\quad (9)$$

The variances $V(u)$ and the covariances $C(uv)$ in the transformed system are computed by applying the propagation of error theory, thus:

$$\begin{aligned}V(u) &= \sum \left(\frac{\partial u}{\partial i} \right)^2 \sigma_i^2 + 2 \sum \left(\frac{\partial u}{\partial i} \right) \left(\frac{\partial u}{\partial j} \right) \sigma_{ij} \\ C(u) &= \sum \left(\frac{\partial u}{\partial i} \right) \left(\frac{\partial v}{\partial i} \right) \sigma_i^2 + \sum \left[\left(\frac{\partial u}{\partial i} \right) \left(\frac{\partial v}{\partial j} \right) + \left(\frac{\partial u}{\partial j} \right) \left(\frac{\partial v}{\partial i} \right) \right] \sigma_{ij}.\end{aligned}\quad (10)$$

In table 20 are listed the tristimulus values derived by Judd and Kelly. Also listed are the between-observer variances and covariances in these values derived by Nimeroff [123] on the as-

sumption that the variance and covariance terms involving the constants k_i of the transformation are zero. These data permit estimation of the ranges of color within which matches set by all observers for any one color will fall. Use of the factor 0.175 may be useful in estimating ranges by one observer from the ranges by many observers.

Propagation of error theory has also been applied [120] to solve the problem of finding the limitations of spectral measurements used in evaluating chromaticity coordinates. On the assumption that the standard observer is free from error, uncertainty ellipses for specimens illuminated by CIE source C may be derived from the equation

$$T\epsilon = \frac{(x-x_n)^2\sigma_y^2 + (x-x_n)(y-y_n)\sigma_{xy} + (y-y_n)^2\sigma_x^2}{\sigma_x^2\sigma_y^2 - \sigma_{xy}^2} \quad (11)$$

where $T\epsilon$ depends on the level of significance, x and y are coordinates of the ellipse, x_n and y_n are the chromaticity coordinates computed from the mean spectral data of a specimen, σ_x^2 and σ_y^2 , are the variances, and σ_{xy} is the covariance of the chromaticity coordinates. The results of this investigation are shown in figure 30 compared with the perceptibility ellipses of MacAdam [91]. This error theory has been applied also to the problem of estimating the uncertainty ellipses resulting from imprecision in measuring spectral exitance of fluorescent lamps [122].

Errors introduced by using summation as in eq (2) instead of integration to evaluate tristimulus values X, Y, Z , have been treated by Nickerson [114] and by De Kerf [27]. Integration is not possible with the present CIE system because the spectral tristimulus values are specified for small but finite intervals. De Kerf compares colorimetric results from summation with intervals of 1, 5, 10, and 20 nm. The conclusion reached by both investigators is that the size of the interval is determined by the type of problem. The most spectrally selective specimens require the smallest summation intervals.

6.2. Photoelectric Tristimulus Colorimeters

Errors in measurement by photoelectric tristimulus colorimeters result mainly from three failings [58, 153]: (1) to duplicate the geometric conditions of the real situation, (2) to illuminate the specimen with light of applicable spectral character, and (3) to duplicate the required filter-detector spectral sensitivity for the color-matching functions.

a. Angular Conditions

Many photoelectric colorimeters use the standard 45°0° conditions, or the equivalent 0°45° conditions. Errors expected for nonglossy specimens by substitution of 0°-hemispherical condition for the standard 45°0° conditions have been analyzed by Budde and Wyszecki [18].

TABLE 20. Spectral tristimulus values and between-observer variances and covariances

Wave-length nm	\bar{x}_{10}	\bar{y}_{10}	\bar{z}_{10}	$V(\bar{x})$ $\times 10^4$	$V(\bar{y})$ $\times 10^4$	$V(\bar{z})$ $\times 10^4$	$O(\bar{xy})$ $\times 10^4$	$O(\bar{xz})$ $\times 10^4$	$O(\bar{yz})$ $\times 10^4$
400	0.0191097	0.0020044	0.0860109	1.26	0.012	25.58	0.10	0.68	0.47
410	.084736	.008756	.389366	6.61	.11	132.11	.71	29.38	3.21
420	.204492	.021391	.972542	9.36	.26	193.19	.73	42.06	3.50
430	.314679	.038676	1.55348	7.37	.67	154.07	1.06	32.93	5.09
440	.383734	.062077	1.96728	3.85	.45	97.76	.82	18.93	3.55
450	.370702	.089456	1.99480	3.53	.56	81.53	.41	16.14	1.85
460	.302273	.128201	1.75437	10.93	1.00	206.07	.15	45.40	1.36
470	.195618	.185190	1.31756	11.04	2.72	180.40	1.01	42.31	1.32
480	.080507	.253589	.772125	7.16	6.02	60.63	1.08	18.24	.08
490	.016172	.339133	.415254	6.74	10.45	23.38	.63	9.73	.76
500	.003816	.460777	.218502	4.14	12.87	5.93	.006	2.79	2.45
510	.037465	.606741	.112044	3.25	8.29	1.91	.88	1.12	1.00
520	.117749	.761757	.060709	1.82	4.57	.65	1.09	.32	.18
530	.236491	.875211	.030451	1.43	2.53	.27	1.00	.055	.020
540	.376772	.961988	.013676	6.22	5.90	.43	3.01	.13	.28
550	.529826	.991761	.003988	14.31	6.68	1.05	5.28	.53	.70
560	.705224	.997340	.000000	28.28	8.47	1.00	9.60	1.02	.51
570	.878655	.955552		43.72	9.98	.94	14.20	1.48	.11
580	1.01416	.868934		56.88	11.25	.78	18.12	1.84	.38
590	1.11852	.777405		58.85	9.47	.65	19.41	1.94	.81
600	1.12399	.658341		49.31	7.31	.38	16.98	1.48	.72
610	1.03048	.527963		32.43	4.75	.25	11.63	1.02	.52
620	.856297	.398057		15.87	2.40	.12	6.01	.45	.22
630	.647467	.283493		5.75	.92	.059	2.26	.12	.059
640	.431567	.179828		.75	.13	.003	.30	.001	.0007
650	.268329	.107633		.47	.079	.0005	.19	.001	.0003
660	.152568	.060281		.46	.071	.0004	.18	.002	.0009
670	.0812606	.0318004		.13	.019	.0002	.049	.001	.0004
680	.0408508	.0159051		.043	.006	.00002	.016	.0001	.0001
690	.0199413	.0077488		.009	.001	.000004	.003	.00003	.00002
700	.00957688	.00371774		.004	.0005	.000002	.001	9×10^{-6}	5×10^{-6}
710	.00455263	.00176847		.001	.0002	4×10^{-7}	.0004	1×10^{-6}	9×10^{-7}
720	.00217496	.00084619		.0002	.00003	4×10^{-8}	7×10^{-5}	3×10^{-8}	5×10^{-8}

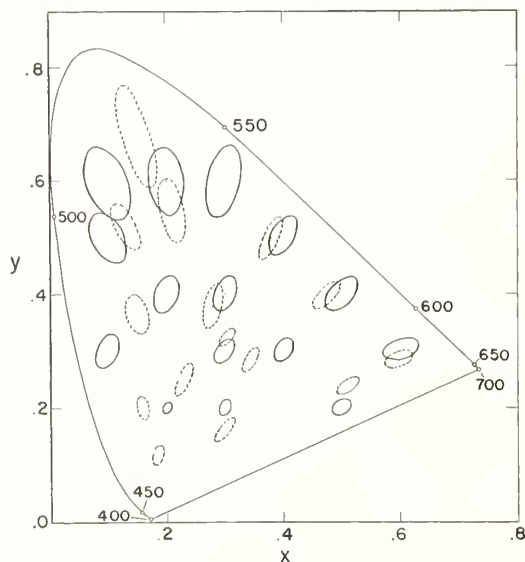


FIGURE 30. Uncertainty ellipses compared with MacAdam perceptibility ellipses.

Correlated uncertainty, solid line ellipses; MacAdam perceptibility, dotted line ellipses. (All ellipses are plotted on a times-ten scale.)

b. Source

Most photoelectric colorimeters are designed for one standard source, usually CIE source C. Such colorimeters do not indicate how closely specimen and standard match for other sources, which de-

pend on the degree of metamerism. Several colorimeters now have provision for inserting an auxiliary filter to approximate a source other than the one supplied.

c. Filter-detector combinations

Most photoelectric tristimulus colorimeters use three filter-detector combinations and approximate the short-wave lobe of the x -function by a suitable fraction of the z -function. Errors encountered in this type of colorimeter have been treated by Van den Akker [152].

There is some degree of failure to duplicate the CIE color-matching functions even in colorimeters with four filters, one for y , one for z , and two for the two-lobed x . This failure restricts utility of these colorimeters to measurement of the color of specimens of spectral characteristics similar to that of the standard.

If the limitations are appreciated, methods using photoelectric tristimulus colorimeters are useful in product-control colorimetry of nonfluorescent specimens. The National Bureau of Standards has developed three sets of color standards, Non-selective standards, KB-Chromatic standards, and S-Chromatic standards, for use with these methods. These standards are now available, either individually or in sets, from the Gardner Laboratory, Inc., Bethesda, Md. 20014, and Hunter Associates Laboratory, Inc., Fairfax, Va. 22030.

6.3. Material Standards of Color

Limitations of visual comparisons result from inability to make precise interpolation between two colors. Use of the Munsell color system requires interpolation of hue, value, and chroma. Experience shows that the best a trained observer does in such interpolations is correct within 0.5 hue step for chromas 4 or more, within 0.2 value step, and 0.4 chroma step.

Visual comparisons are limited also because of the difficulty of choosing permanent materials that have suitable spectral characteristics. The most permanent of these are vitreous enamel and glass, which, however, are so limited in variety of spectral characteristics that metamerism cannot be avoided. This limitation requires control of the spectral character of the light source and disqualifies observers with abnormal vision.

Material standards of glass used in one-dimensional color scales cause difficulty because the luminous-transmittance match with the specimen may not match in chromaticity; see, for example, table 17. Hence, one-dimensional scales as represented by material standards are not always an adequate solution to a problem.

Gross errors in visual judgment may result from marked dissimilarity of surrounds. For example, the orchid standard of the KB-chromatic set issued by the NBS is perceived to have a moderately saturated light reddish-purple color when viewed against a middle gray surround. When it is viewed against a brilliant magenta surround it is seen as greenish gray. Spectrophotometric colorimetry does not account for the influence of surround.

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The Ideal Lovibond Color System

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Lovibond red, yellow, and blue glasses, widely used as color standards in industry, are assigned numerals in accord with the basic plan of marking each glass with the number of unit glasses of the same type through which light must be passed to produce its color. It is possible to compute from the spectral transmittances of the unit glasses defining the Lovibond scales the CIE specification of the color produced by all combinations of any number of unit glasses. Such specifications were computed in 1939 not only for all ideal red, yellow, and blue Lovibond glasses illuminated by CIE sources *B* (representing noon sunlight) or *C* (representing average daylight) but also for two-part (red-yellow, yellow-blue, or blue-red) combinations thereof. The present paper gives the results of such computations for CIE source *A* (representing gas-filled incandescent lamps). Although actual Lovibond glasses must unavoidably depart somewhat from this definition of the ideal Lovibond system, the computed color specifications serve to indicate with good reliability not only the CIE specification of the color produced by single glasses and two-part combinations, but also the choice of Lovibond glasses required to produce a color of any desired chromaticity within the gamut of the system.

1. Introduction

Lovibond red, yellow, and blue glasses [1]¹ are widely used in color grading the various materials and articles of commerce (vegetable oils, petroleum products, Naval stores, paint vehicles, and so forth). The basic plan of the system is that each glass is marked with the number of unit glasses to which it is equivalent colorimetrically; that is, the number of unit glasses through which light from the source must be passed to produce the same color. Furthermore, the unit glasses of the red, yellow, and blue scales are related so that combinations of all three kinds of glasses of equal numbers on the Lovibond scales give a nearly neutral filter; that is, produce little or no change in chromaticity from that of the source. Because of the relation between the three scales, the chromaticity of a color matched by Lovibond glasses of all three sorts, say by glasses of numerals, *R*, *Y*, *B*, where *R* and *Y* are both greater than or equal to *B*, corresponds closely to that of the two-part combination (*R_c*, *Y_c*), where *R_c* = *R* - *B*, *Y_c* = *Y* - *B*. Thus all Lovibond chromaticities may be closely identified with those of two-part combinations, and no separate consideration of three-part combinations is required for chromaticity.

The numbers engraved on the glasses by the maker throughout the years have followed the basic plan closely, but some of the uses to which the glasses have been put require a more precise grading than has always been maintained by the maker. The present paper gives the derivation of an ideal Lovibond color system consisting of colorimetric definitions in fundamental terms against which any actual glass can be compared and a new numeral in the ideal system assigned. This system is based upon spectrophotometric determinations of 20 glasses of

each series (1, 2, 3, . . . 20). Some of these data are those [2] obtained at the National Bureau of Standards for the glasses of the set (BS9940) purchased by this Bureau in 1912; others are those obtained by Tintometer Ltd. for glasses retained there as standards.

2. Method

The chromaticities of the colors of the ideal Lovibond system have been specified in the standard colorimetric coordinate system recommended in 1931 by the International Commission on Illumination (CIE) [3] in a way similar to that used by Schofield [4], and later by Haupt and Douglas [5] to express the chromaticities of two-part (red-yellow, yellow-blue, or blue-red) combinations of Lovibond glasses. The spectral transmittances, *T_N*, for glasses of each series (red, yellow, blue) for *N* = 1, 2, 3, . . . 20, were reduced to negative logarithms of the internal transmittances, *T_{i,N=1}*, of the unit glasses by correcting *T_N* for reflection losses, taking the negative logarithm of the internal transmittance so found, and dividing by the maker's numeral, *N*, thus:

$$-\log (T_{i,N=1}) = (1/N)[-\log (T_N/0.92)]. \quad (1)$$

The adopted negative logarithms for each unit glass were found by Schofield [4] by taking an average of these values weighted in accord with the numeral, *N*, engraved on the glass by the maker, and are those mentioned by Fawcett [6], to compute the chromaticities and transmittances of two-part combinations of Lovibond glasses for CIE standard sources *B* (representative of average noon sunlight), and *C* (representative of average daylight). Table 1 gives the spectral internal transmittances of the red, yellow, and blue unit glasses defining the ideal Lovibond color system.

The computations by Schofield of the chromaticities and transmittances of two-part combinations of

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¹ Figures in brackets indicate the literature references at the end of this paper.

TABLE 1. Spectral internal transmittances of the Lovibond unit red, yellow, and blue glasses

Wave-length $m\mu$	$T_{i,r}$	$T_{i,y}$	$T_{i,b}$
	Red	Yellow	Blue
380-----	0.90258	0.02889	0.99815
390-----	.90352	.12593	.99809
400-----	.90439	.25435	.99788
410-----	.90603	.39957	.99711
420-----	.90737	.52037	.99573
430-----	.90824	.61634	.99363
440-----	.90886	.70289	.99111
450-----	.90858	.77822	.98800
460-----	.90722	.84481	.98338
470-----	.90444	.89471	.97459
480-----	.89819	.92976	.96004
490-----	.88633	.95277	.94109
500-----	.86526	.96755	.92316
510-----	.83257	.97738	.89900
520-----	.79598	.98364	.87326
530-----	.77392	.98754	.84574
540-----	.78952	.99040	.83553
550-----	.83317	.99195	.85049
560-----	.87817	.99236	.86792
570-----	.91300	.99247	.85702
580-----	.93628	.99179	.81808
590-----	.95268	.99073	.77002
600-----	.96362	.98933	.76498
610-----	.97109	.98768	.77420
620-----	.97648	.98599	.77827
630-----	.98053	.98438	.77386
640-----	.98348	.98333	.76119
650-----	.98572	.98287	.76656
660-----	.98753	.98279	.78191
670-----	.98892	.98330	.83172
680-----	.99012	.98405	.88572
690-----	.99117	.98449	.93507
700-----	.99194	.98510	.96744
710-----	.99247	.98627	.98466
720-----	.99303	.98789	.99228
730-----	.99336	.98912	.99587
740-----	.99365	.99014	.99719
750-----	.99402	.99108	.99770
760-----	.99420	.99160	.99790
770-----	.99430	.99210	.99800

ideal Lovibond glasses for CIE sources *B*[4] and *C*[6] have been extended in the present paper to CIE source *A* (color temperature 2,854 °K, representative of incandescent lamps). The calculations were carried out on an automatic digital computer by the Division of Applied Mathematics of the National Bureau of Standards, as follows:

The tristimulus values, *X*, *Y*, *Z*, of the color of the light transmitted by a combination of *R* Lovibond red unit glasses, *Y* Lovibond yellow unit glasses, and *B* Lovibond blue unit glasses in a medium of refractive index equal to that of the glasses were computed

for two-part combinations by setting one of the exponents, *R*, *Y*, *B*, equal to zero in the following expressions:

$$\left. \begin{aligned} X &= \sum_{380}^{770} T_{i,r}^R T_{i,y}^Y T_{i,b}^B H_A \bar{x} \Delta\lambda \\ Y &= \sum_{380}^{770} T_{i,r}^R T_{i,y}^Y T_{i,b}^B H_A \bar{y} \Delta\lambda \\ Z &= \sum_{380}^{770} T_{i,r}^R T_{i,y}^Y T_{i,b}^B H_A \bar{z} \Delta\lambda \end{aligned} \right\} \quad (2)$$

where H_A is the spectral irradiance of CIE source *A*, \bar{x} , \bar{y} , \bar{z} are the tristimulus values of the spectrum of unit irradiance defining the 1931 CIE standard observer [3], $T_{i,r}$, $T_{i,y}$, $T_{i,b}$ are the spectral internal transmittances (table 1) by which Tintometer Ltd. defines the ideal Lovibond red, yellow, and blue units, and $\Delta\lambda$ is taken at 10 m μ .

Then the chromaticity coordinates, *x*, *y*, of the color were computed as:

$$x = X/(X+Y+Z), \quad y = Y/(X+Y+Z)$$

The chromaticity coordinate, *z* = *Z*/(*X* + *Y* + *Z*), may be computed, if desired, from *x* and *y* as 1 - *x* - *y*.

The luminous internal transmittance, $T_{i,A}$, for CIE source *A*, of combinations of ideal Lovibond glasses identified by the numbers, *R*, *Y*, *B*, one or two of which are zero, was computed as Y/Y_0 , where Y_0 is the value of *Y* found from eq (2) by setting

$$R=Y=B=0; \text{ that is } Y_0 = \sum_{380}^{770} H_A \bar{y} \Delta\lambda.$$

The values of luminous internal transmittance, $T_{i,A}$, for CIE source *A* and chromaticity coordinates, *x*, *y*, so found, are listed in table 2 for various ideal single Lovibond glasses, and for various two-glass combinations of ideal Lovibond glasses. The tristimulus values, *X*, *Z*, of these colors may be computed, if desired, as follows:

$$X = x T_{i,A} Y_0 / y, \quad Z = z T_{i,A} Y_0 / y.$$

If it is desired to compute the transmittance (including reflection losses) of a single ideal Lovibond glass from its internal transmittance given in table 2, the expression 0.92 $T_{i,A}$ should be evaluated; if that of a two-part combination is desired, use the expression: (0.92)² $T_{i,A}$.

TABLE 2. Luminous internal transmittances in percent, 100 $T_{i,A}$, and chromaticity coordinates, *x*, *y*, of single glasses and two-part combinations of ideal Lovibond glasses illuminated by CIE Source *A*

The first three columns (*R*, *Y*, *B*) give the number of unit red, yellow, and blue glasses, respectively.

<i>R</i>	<i>Y</i>	<i>B</i>	$T_{i,A}$	<i>x</i>	<i>y</i>	<i>R</i>	<i>Y</i>	<i>B</i>	$T_{i,A}$	<i>x</i>	<i>y</i>	<i>R</i>	<i>Y</i>	<i>B</i>	$T_{i,A}$	<i>x</i>	<i>y</i>
0	0	0	100.00	0.44759	0.40754	10	0	0	43.610	0.55068	0.34978	20	0	0	26.281	0.61066	0.33254
1	0	0	89.879	.46022	.39834	11	0	0	41.048	.55830	.34703	22	0	0	24.202	.61917	.33068
2	0	0	81.286	.47235	.39007	12	0	0	38.744	.56550	.34461	24	0	0	22.378	.62681	.32903
3	0	0	73.949	.48396	.38266	13	0	0	36.662	.57231	.34246	26	0	0	20.763	.63367	.32753
4	0	0	67.647	.49504	.37606	14	0	0	34.772	.57874	.34056	28	0	0	19.323	.63985	.32615
5	0	0	62.202	.50559	.37021	15	0	0	33.047	.58483	.33886	30	0	0	18.029	.64542	.32483
6	0	0	57.470	.51560	.36503	16	0	0	31.468	.59058	.33734	32	0	0	16.861	.65045	.32358
7	0	0	53.332	.52510	.36047	17	0	0	30.016	.59603	.33597	34	0	0	15.801	.65499	.32236
8	0	0	49.693	.53410	.35644	18	0	0	28.675	.60118	.33472	36	0	0	14.834	.65911	.32117
9	0	0	46.474	.54262	.35290	19	0	0	27.434	.60605	.33359	38	0	0	13.950	.66284	.32001

TABLE 2. *Luminous internal transmittances in percent, 100 T_{i,A}, and chromaticity coordinates, x,y, of single glasses and two-part combinations of ideal Lovibond glasses illuminated by CIE Source A—Continued*

R	Y	B	T _{i,A}	x	y	R	Y	B	T _{i,A}	x	y	R	Y	B	T _{i,A}	x	y
40	0	0	13.138	0.66623	0.31886	36	0	2	9.1195	0.65161	0.31789	32	0	4	6.4673	0.62643	0.31301
45	0	0	11.374	.67346	.31609	38	0	2	8.5670	.65639	.31703	34	0	4	6.0410	.63402	.31282
50	0	0	9.9158	.67926	.31345							36	0	4	5.6565	.64090	.31255
55	0	0	8.6956	.68399	.31093	40	0	2	8.0617	.66072	.31615	38	0	4	5.3079	.64713	.31219
60	0	0	7.6638	.68791	.30856	45	0	2	6.9701	.66985	.31391	40	0	4	4.9906	.65276	.31177
						50	0	2	6.0738	.67704	.31162						
65	0	0	6.7840	.69121	.30634	55	0	2	5.3272	.68278	.30934	45	0	4	4.3094	.66457	.31042
70	0	0	6.0283	.69403	.30426	60	0	2	4.6980	.68742	.30712	50	0	4	3.7543	.67375	.30876
75	0	0	5.3752	.69648	.30232							55	0	4	3.2945	.68093	.30692
80	0	0	4.8077	.69863	.30052	65	0	2	4.1628	.69123	.30499	60	0	4	2.9085	.68661	.30499
90	0	0	3.8774	.70228	.29729	70	0	2	3.7037	.69442	.30297	65	0	4	2.5809	.69116	.30305
100	0	0	3.1567	.70528	.29448	75	0	2	3.3073	.69713	.30106						
						80	0	2	2.9629	.69947	.29927	70	0	4	2.3004	.69486	.30115
0	0	1	82.614	.43007	.40197	90	0	2	2.3983	.70334	.29604	75	0	4	2.0584	.69794	.29932
1	0	1	74.080	.44257	.39284	100	0	2	1.9605	.70645	.29323	80	0	4	1.8482	.70053	.29758
2	0	1	66.751	.45468	.38464							90	0	4	1.5035	.70470	.29440
3	0	1	60.544	.46637	.37730	0	0	3	56.986	.39327	.38571	100	0	4	1.2356	.70796	.29160
4	0	1	55.222	.47762	.37078	1	0	3	50.773	.40513	.37683						
						2	0	3	45.515	.41681	.36887	0	0	5	39.880	.35554	.36262
5	0	1	50.633	.48840	.36501	3	0	3	41.044	.42827	.36178	1	0	5	35.347	.36627	.35407
6	0	1	46.652	.49873	.35993	4	0	3	37.220	.43948	.35552	2	0	5	31.517	.37701	.34644
7	0	1	43.178	.50861	.35546	5	0	3	33.932	.45042	.35002	3	0	5	28.265	.38771	.33969
8	0	1	40.130	.51803	.35154	6	0	3	31.091	.46107	.34522	4	0	5	25.490	.39836	.33378
9	0	1	37.440	.52702	.34811	7	0	3	28.620	.47141	.34105						
						8	0	3	26.462	.48144	.33745	5	0	5	23.111	.40892	.32864
10	0	1	35.052	.53557	.34512	9	0	3	24.565	.49115	.33436	6	0	5	21.059	.41938	.32421
11	0	1	32.922	.54371	.34250	10	0	3	22.889	.50054	.33172	7	0	5	19.281	.42971	.32042
12	0	1	31.012	.55146	.34021	11	0	3	21.401	.50961	.32947	8	0	5	17.733	.43990	.31723
13	0	1	29.290	.55883	.33821	12	0	3	20.074	.51835	.32757	9	0	5	16.378	.44992	.31456
14	0	1	27.731	.56583	.33645	13	0	3	18.883	.52677	.32596	10	0	5	15.186	.45977	.31236
						14	0	3	17.810	.53487	.32460	11	0	5	14.131	.46943	.31057
15	0	1	26.313	.57248	.33490	15	0	3	16.839	.54267	.32347	12	0	5	13.195	.47889	.30914
16	0	1	25.017	.57880	.33353	16	0	3	15.957	.55015	.32252	13	0	5	12.359	.48813	.30803
17	0	1	23.829	.58480	.33231	17	0	3	15.152	.55733	.32172	14	0	5	11.609	.49715	.30719
18	0	1	22.735	.59050	.33122	18	0	3	14.415	.56422	.32105	15	0	5	10.934	.50595	.30658
19	0	1	21.724	.59592	.33024	19	0	3	13.737	.57082	.32048	16	0	5	10.323	.51450	.30617
						20	0	3	13.112	.57714	.32000	17	0	5	9.7690	.52282	.30592
20	0	1	20.787	.60106	.32934	21	0	3	12.531	.58397	.31923	18	0	5	9.2638	.53088	.30580
22	0	1	19.104	.61058	.32776	22	0	3	11.996	.58897	.31863	19	0	5	8.8018	.53870	.30579
24	0	1	17.634	.61917	.32639	23	0	3	11.031	.59977	.31812	20	0	5	8.3778	.54626	.30587
26	0	1	16.338	.62691	.32515	24	0	3	10.187	.60961	.31764	21	0	5	7.9268	.55395	.30621
28	0	1	15.185	.63390	.32399	25	0	3	9.4422	.61856	.31724	22	0	5	7.4812	.56167	.30670
						26	0	3	8.7804	.62668	.31716	23	0	5	7.0426	.56927	.30722
30	0	1	14.154	.64020	.32289	27	0	3	8.1882	.63403	.31665	24	0	5	6.6128	.57686	.30773
32	0	1	13.225	.64590	.32183	28	0	3	7.6553	.64069	.31611	25	0	5	6.2025	.58433	.30827
34	0	1	12.384	.65104	.32078	29	0	3	7.1391	.64672	.31562	26	0	5	5.8018	.59186	.30886
36	0	1	11.620	.65570	.31974	30	0	3	6.7349	.65217	.31519	27	0	5	5.4194	.59939	.30947
38	0	1	10.922	.65991	.31871	31	0	3	6.3349	.65710	.31421	28	0	5	5.0508	.60682	.30987
						32	0	3	5.9736	.66146	.31326	29	0	5	4.7025	.61433	.31038
40	0	1	10.282	.66373	.31768	33	0	3	5.6349	.66556	.31236	30	0	5	4.3682	.62186	.31087
45	0	1	8.8953	.67183	.31513	34	0	3	5.3272	.66934	.31151	31	0	5	4.0478	.62939	.31136
50	0	1	7.7530	.67826	.31264	35	0	3	5.0489	.67297	.31066	32	0	5	3.7543	.63692	.31185
55	0	1	6.7992	.68345	.31022	36	0	3	4.7990	.67655	.31004	33	0	5	3.4929	.64445	.31234
60	0	1	5.9941	.68769	.30791	37	0	3	4.5649	.67997	.30949	34	0	5	3.2544	.65198	.31283
						38	0	3	4.3491	.68329	.30894	35	0	5	3.0309	.65951	.31332
65	0	1	5.3083	.69123	.30573	39	0	3	4.1428	.68650	.30840	36	0	5	2.8174	.66704	.31381
70	0	1	4.7197	.69421	.30367	40	0	3	3.9459	.68961	.30786	37	0	5	2.6149	.67457	.31430
75	0	1	4.2111	.69678	.30174	41	0	3	3.7584	.69262	.30732	38	0	5	2.4224	.68210	.31479
80	0	1	3.7693	.69902	.29995	42	0	3	3.5809	.69553	.30678	39	0	5	2.2399	.68963	.31528
90	0	1	3.4051	.70277	.29671	43	0	3	3.4134	.69834	.30624	40	0	5	2.0674	.69716	.31577
100	0	1	2.4838	.70583	.29390	44	0	3	3.2559	.70105	.30570	41	0	5	1.9049	.70469	.31626
						45	0	3	3.1084	.70376	.30516	42	0	5	1.7424	.71222	.31675
0	0	2	68.492	.41190	.39471	46	0	3	2.9709	.70647	.30462	43	0	5	1.5800	.71975	.31724
1	0	2	61.196	.42415	.38569	47	0	3	2.8334	.70918	.30408	44	0	5	1.4175	.72728	.31773
2	0	2	55.016	.43611	.37759	48	0	3	2.7059	.71189	.30354	45	0	5	1.2550	.73481	.31822
3	0	2	49.754	.44775	.37037	49	0	3	2.5884	.71460	.30300	46	0	5	1.0925	.74234	.31871
4	0	2	45.249	.45904	.36396	50	0	3	2.4709	.71731	.30246	47	0	5	0.9300	.74987	.31920
						51	0	3	2.3534	.72002	.30192	48	0	5	0.7675	.75740	.31969
5	0	2	41.369	.46997	.35831	52	0	3	2.2359	.72273	.30138	49	0	5	0.6050	.76493	.32018
6	0	2	38.010	.48051	.35355	53	0	3	2.1184	.72544	.30084	50	0	5	0.4425	.77246	.32067
7	0	2	35.086	.49067	.34902	54	0	3	2.0009	.72815	.30030	51	0	5	0.2800	.78000	.32116
8	0	2	32.524	.50045	.34525	55	0	3	1.8834	.73086	.30000	52	0	5	0.1175	.78753	.32165
9	0	2	30.269	.50984	.34197	56	0	3	1.7659	.73357	.30000	53	0	5	0.0550	.79506	.32214
						57	0	3	1.6484	.73628	.30000	54	0	5	0.0000	.80259	.32263
10	0	2	28.273	.51884	.33914	58	0	3	1.5309	.73900	.30000	55	0	5	0.0000	.81012	.32312
11	0	2	26.496	.52748	.33669	59	0	3	1.4134	.74171	.30000	56	0	5	0.0000	.81765	.32361
12	0	2	24.906	.53574	.33458	60	0	3	1.2959	.74442	.30000	57	0	5	0.0000	.82518	.32410
13	0	2	23.477	.54365	.33275							58	0	5	0.0000	.83271	.32459
14	0	2	22.186	.55122	.33118	10	0	4	18.604	.48078	.32280	59	0	5	0.0000	.84024	.32508
						11	0	4	17.354	.49020	.32077	60	0	5	0.0000	.84777	.32557
15	0	2	21.015	.55844	.32982	12	0	4	16.241	.49934	.31910	61	0	5	0.0000	.85530	.32606
16	0	2	19.948	.56535	.32863	13	0	4	15.245	.50822	.31773	62	0	5	0.0000	.86283	.32655

TABLE 2. Luminous internal transmittances in percent, $100 T_{i,A}$, and chromaticity coordinates, x, y , of single glasses and two-part combinations of ideal Lovibond glasses illuminated by CIE Source A—Continued

R	Y	B	$T_{i,A}$	x	y	R	Y	B	$T_{i,A}$	x	y	R	Y	B	$T_{i,A}$	x	y
30	0	6	4.3814	0.56644	0.30217	30	0	8	2.8083	0.56843	0.28673	30	0	10	1.8354	0.53431	0.26665
32	0	6	4.0722	.60718	.30305	32	0	8	2.6037	.58192	.28891	32	0	10	1.6974	.55062	.27029
34	0	6	3.7967	.61701	.30379	34	0	8	2.4227	.59441	.29087	34	0	10	1.5763	.56599	.27369
36	0	6	3.5497	.62597	.30437	36	0	8	2.2616	.60591	.29260	36	0	10	1.4692	.58035	.27680
38	0	6	3.3271	.63412	.30478	38	0	8	2.1173	.61646	.29408	38	0	10	1.3741	.59368	.27960
40	0	6	3.1254	.64150	.30505	40	0	8	1.9874	.62608	.29532	40	0	10	1.2889	.60597	.28208
45	0	6	2.6959	.65698	.30511	45	0	8	1.7130	.64640	.29742	45	0	10	1.1108	.63226	.28690
50	0	6	2.3489	.66894	.30448	50	0	8	1.4936	.66211	.29832	50	0	10	0.96996	.65280	.28991
55	0	6	2.0633	.67816	.30336	55	0	8	1.3142	.67413	.29831	55	0	10	.85580	.66850	.29149
60	0	6	1.8246	.68531	.30193	60	0	8	1.1651	.68331	.29766	60	0	10	.76139	.68038	.29199
65	0	6	1.6227	.69090	.30032	65	0	8	1.0393	.69034	.29661	65	0	10	.68202	.68933	.29175
70	0	6	1.4500	.69594	.29865	70	0	8	0.93199	.69578	.29531	70	0	10	.61439	.69609	.29103
75	0	6	1.3011	.69892	.29697	75	0	8	.83948	.70004	.29389	75	0	10	.55611	.70126	.29002
80	0	6	1.1718	.70186	.29533	80	0	8	.75910	.70345	.29244	80	0	10	.50541	.70526	.28886
90	0	6	0.95955	.70642	.29226	90	0	8	.62686	.70852	.28959	90	0	10	.42177	.71095	.28638
100	0	6	.79412	.70985	.28952	100	0	8	.52339	.71214	.28698	100	0	10	.35593	.71479	.28401
0	0	7	28.324	.31880	.33361	0	0	9	20.420	.28493	.30052	0	0	12	12.864	.24240	.24836
1	0	7	24.990	.32807	.32544	1	0	9	17.949	.29260	.29278	1	0	12	11.261	.24785	.24139
2	0	7	22.176	.33747	.31819	2	0	9	15.863	.30048	.28593	2	0	12	9.9070	.25353	.23525
3	0	7	19.789	.34698	.31182	3	0	9	14.094	.30856	.27995	3	0	12	8.7591	.25945	.22989
4	0	7	17.755	.35660	.30627	4	0	9	12.589	.31683	.27477	4	0	12	7.7818	.26562	.22527
5	0	7	16.014	.36629	.30150	5	0	9	11.302	.32529	.27035	5	0	12	6.9465	.27204	.22133
6	0	7	14.517	.37604	.29745	6	0	9	10.196	.33393	.26663	6	0	12	6.2297	.27870	.21804
7	0	7	13.222	.38582	.29405	7	0	9	9.2417	.34273	.26337	7	0	12	5.6119	.28563	.21536
8	0	7	12.098	.39563	.29126	8	0	9	8.4147	.35168	.26110	8	0	12	5.0774	.29281	.21323
9	0	7	11.116	.40544	.28901	9	0	9	7.6946	.36078	.25918	9	0	12	4.6130	.30024	.21162
10	0	7	10.256	.41524	.28724	10	0	9	7.0648	.37000	.25776	10	0	12	4.2080	.30792	.21050
11	0	7	9.4978	.42500	.28591	11	0	9	6.5117	.37933	.25679	11	0	12	3.8534	.31586	.20981
12	0	7	8.8270	.43472	.28496	12	0	9	6.0238	.38875	.25623	12	0	12	3.5417	.32404	.20954
13	0	7	8.2307	.44336	.28436	13	0	9	5.5916	.39825	.25603	13	0	12	3.2667	.33245	.20964
14	0	7	7.6983	.45191	.28405	14	0	9	5.2071	.40781	.25615	14	0	12	3.0232	.34109	.21008
15	0	7	7.2210	.46036	.28399	15	0	9	4.8638	.41740	.25655	15	0	12	2.8067	.34995	.21084
16	0	7	6.7913	.47268	.28415	16	0	9	4.5561	.42702	.25720	16	0	12	2.6137	.35901	.21188
17	0	7	6.4031	.48187	.28449	17	0	9	4.2792	.43662	.25806	17	0	12	2.4410	.36826	.21318
18	0	7	6.0510	.49090	.28499	18	0	9	4.0292	.44620	.25911	18	0	12	2.2859	.37768	.21471
19	0	7	5.7304	.49976	.28561	19	0	9	3.8027	.45574	.26031	19	0	12	2.1462	.38724	.21645
20	0	7	5.4377	.50845	.28633	20	0	9	3.5968	.46521	.26163	20	0	12	2.0200	.39694	.21837
22	0	7	4.9230	.52522	.28797	22	0	9	3.2372	.48386	.26456	22	0	12	1.8017	.41663	.22264
24	0	7	4.4858	.54114	.28977	24	0	9	2.9345	.50200	.26771	24	0	12	1.6204	.43657	.22737
26	0	7	4.1104	.55614	.29160	26	0	9	2.6771	.51949	.27095	26	0	12	1.4682	.45653	.23239
28	0	7	3.7848	.57019	.29338	28	0	9	2.4559	.53621	.27416	28	0	12	1.3392	.47632	.23754
30	0	7	3.5001	.58325	.29504	30	0	9	2.2642	.55207	.27725	30	0	12	1.2289	.49572	.24270
32	0	7	3.2492	.59533	.29654	32	0	9	2.0965	.56698	.28015	32	0	12	1.1336	.51456	.24776
34	0	7	3.0264	.60645	.29785	34	0	9	1.9489	.58090	.28281	34	0	12	1.0508	.53267	.25263
36	0	7	2.8273	.61663	.29897	36	0	9	1.8179	.59381	.28521	36	0	12	0.97815	.54990	.25722
38	0	7	2.6485	.62591	.29988	38	0	9	1.7010	.60571	.28732	38	0	12	.91404	.56615	.26148
40	0	7	2.4869	.63435	.30060	40	0	9	1.5961	.61662	.28915	40	0	12	.85708	.58133	.26538
45	0	7	2.1442	.65211	.30160	45	0	9	1.3755	.63980	.29253	45	0	12	.73917	.61446	.27344
50	0	7	1.8687	.66581	.30166	50	0	9	1.2000	.65778	.29441	50	0	12	.64709	.64079	.27913
55	0	7	1.6428	.67633	.30104	55	0	9	1.0572	.67153	.29513	55	0	12	.57314	.66106	.28279
60	0	7	1.4544	.68441	.29996	60	0	9	0.93880	.68197	.29501	60	0	12	.51233	.67634	.28485
65	0	7	1.2952	.69067	.29860	65	0	9	.83907	.68990	.29433	65	0	12	.46138	.68773	.28574
70	0	7	1.1593	.69557	.29709	70	0	9	.75402	.69596	.29329	70	0	12	.41801	.69617	.28583
75	0	7	1.0421	.69946	.29553	75	0	9	.68074	.70065	.29206	75	0	12	.38062	.70246	.28541
80	0	7	0.94029	.70262	.29397	80	0	9	.61704	.70433	.29073	80	0	12	.34804	.70719	.28467
90	0	7	.77302	.70742	.29099	90	0	9	.51210	.70970	.28805	90	0	12	.29405	.71361	.28275
100	0	7	.64242	.71095	.28831	100	0	9	.42976	.71343	.28554	100	0	12	.25120	.71765	.28073
0	0	8	24.004	.30140	.31744	0	0	10	17.438	.26954	.28317	0	0	14	9.6392	.22034	.21505
1	0	8	21.137	.30987	.30947	1	0	10	15.303	.27642	.27567	1	0	14	8.4244	.22460	.20871
2	0	8	18.716	.31852	.30242	2	0	10	13.501	.28353	.26904	2	0	14	7.3974	.22908	.20312
3	0	8	16.664	.32734	.29624	3	0	10	11.974	.29087	.26326	3	0	14	6.5258	.23379	.19824
4	0	8	14.917	.33631	.29088	4	0	10	10.674	.29843	.25826	4	0	14	5.7834	.23873	.19403
5	0	8	13.422	.34541	.28629	5	0	10	9.5622	.30621	.25400	5	0	14	5.1485	.24392	.19043
6	0	8	12.137	.35464	.28241	6	0	10	8.6079	.31420	.25044	6	0	14	4.6035	.24936	.18742
7	0	8	11.027	.36398	.27919	7	0	10	7.7850	.32240	.24751	7	0	14	4.1340	.25507	.18495
8	0	8	10.064	.37342	.27657	8	0	10	7.0722	.33080	.24518	8	0	14	3.7278	.26104	.18300
9	0	8	9.2249	.38293	.27450	9	0	10	6.4523	.33939	.24339	9	0	14	3.3751	.26729	.18152
10	0	8	8.4901	.39250	.27292	10	0	10	5.9106	.34817	.24209	10	0	14	3.0679	.27381	.18048
11	0	8	7.8438	.40211	.27178	11	0	10	5.4355	.35711	.24125	11	0	14	2.7991	.28063	.17986
12	0	8	7.2728	.41174	.27105	12	0	10	5.0169	.36620	.24082	12	0	14	2.5633	.28772	.17963
13	0	8	6.7661	.42138	.27066	13	0	10	4.6467	.37544	.24076	13	0	14	2.3555	.29511	.17976
14	0	8	6.3146	.43100	.27058	14	0	10	4.3180	.38480	.24104	14	0	14	2.1720	.30278	.18023
15	0	8	5.9107	.44059	.27077	15	0	10	4.0249	.39426	.24161	15	0	14	2.0092	.31074	.18102
16	0	8	5.5479	.45013	.27120	16	0	10	3.7627	.40381	.24244	16	0	14	1.8644	.31897	.18209
17	0	8	5.2208	.45969	.27182	17	0	10	3.5273	.41343	.24350	17	0	14	1.7351	.32748	.18344
18	0	8	4.9248	.46986	.27260	18	0	10	3.3151	.42308	.24476	18	0	14	1.6195	.33625	.18504
19	0	8	4.6560	.47822	.27352	19	0	10	3.1232	.43276	.24619	19	0	14	1.5156	.34527	.18687
20	0	8	4.4111	.48735	.27456	20	0	10	2.9492	.44244	.24776	20	0	14	1.4221	.35453	.18890</

TABLE 2. *Luminous internal transmittances in percent, 100 T_{i,A}, and chromaticity coordinates, x,y, of single glasses and two-part combinations of ideal Lovibond glasses illuminated by CIE Source A—Continued*

R	Y	B	T _{i,A}	x	y	R	Y	B	T _{i,A}	x	y	R	Y	B	T _{i,A}	x	y
30	0	14	0.84686	0.45560	0.21664	32	0	18	0.40933	0.40538	0.17427	30	0	25	0.20646	0.30893	0.11092
32	0	14	.77952	.47631	.22285	34	0	18	.42791	.18190	.32972	32	0	25	.18996	.32935	.11819
34	0	14	.72143	.49663	.22899	36	0	18	.35167	.45067	.18967	34	0	25	.17616	.35121	.12610
36	0	14	.67092	.51637	.23495	38	0	18	.32871	.47335	.19743	36	0	25	.16453	.37432	.13456
38	0	14	.62667	.53532	.24063	40	0	18	.30874	.49567	.20505	38	0	25	.15467	.39842	.14347
40	0	14	.58764	.55334	.24597	45	0	18	.26874	.54814	.22284	40	0	25	.14623	.42319	.15268
45	0	14	.50773	.59356	.25751	50	0	18	.23873	.59345	.23782	45	0	25	.12976	.48568	.17606
50	0	14	.44616	.62632	.26629	55	0	18	.21532	.63010	.24949	50	0	25	.11782	.54448	.19812
55	0	14	.39717	.65187	.27245	60	0	18	.19640	.65826	.25796	55	0	25	.10870	.59516	.21709
60	0	14	.35714	.67117	.27644	65	0	18	.18066	.67907	.26375	60	0	25	.10140	.63566	.23212
65	0	14	.32371	.68545	.27877	70	0	18	.16726	.69405	.26748	65	0	25	.095310	.66611	.24329
70	0	14	.29526	.69589	.27991	75	0	18	.15561	.70465	.26973	70	0	25	.090062	.68798	.25117
75	0	14	.27070	.70351	.28025	80	0	18	.14534	.71210	.27096	75	0	25	.085416	.70320	.25651
80	0	14	.24925	.70909	.28008	80	0	18	.12791	.72103	.27166	80	0	25	.081222	.71357	.26002
90	0	14	.21346	.71631	.27891	100	0	18	.11353	.72561	.27127	90	0	25	.073823	.72519	.26366
100	0	14	.18476	.72055	.27736							100	0	25	.067392	.73036	.26499
0	0	16	7.3378	.20311	.18471	0	0	20	4.4616	.18052	.13535	0	0	30	1.6958	.16163	.06862
1	0	16	6.4085	.20646	.17907	1	0	20	3.9008	.18272	.13114	1	0	30	1.5013	.16289	.06707
2	0	16	5.6216	.21000	.17408	2	0	20	3.4238	.18506	.12743	2	0	30	1.3329	.16424	.06572
3	0	16	4.9529	.21374	.16973	3	0	20	3.0167	.18756	.12418	3	0	30	1.1869	.16568	.06455
4	0	16	4.3826	.21770	.16595	4	0	20	2.6681	.19023	.12135	4	0	30	1.0598	.16723	.06355
5	0	16	3.8944	.22188	.16273	5	0	20	2.3687	.19306	.11892	5	0	30	0.94904	.16889	.06273
6	0	16	3.4750	.22630	.16001	6	0	20	2.1107	.19609	.11686	6	0	30	.85224	.17068	.06206
7	0	16	3.1135	.23096	.15778	7	0	20	1.8876	.19932	.11516	7	0	30	.76744	.17259	.06154
8	0	16	2.8006	.23588	.15600	8	0	20	1.6942	.20276	.11378	8	0	30	.69301	.17466	.06118
9	0	16	2.5290	.24107	.15464	9	0	20	1.5259	.20642	.11272	9	0	30	.62754	.17688	.06096
10	0	16	2.2924	.24654	.15367	10	0	20	1.3791	.21033	.11196	10	0	30	.56984	.17927	.06088
11	0	16	2.0856	.25229	.15309	11	0	20	1.2507	.21448	.11149	11	0	30	.51888	.18184	.06094
12	0	16	1.9042	.25834	.15286	12	0	20	1.1380	.21891	.11129	12	0	30	.47381	.18461	.06114
13	0	16	1.7446	.26469	.15296	13	0	20	1.0389	.22361	.11135	13	0	30	.43387	.18759	.06148
14	0	16	1.6037	.27135	.15338	14	0	20	0.95152	.22861	.11167	14	0	30	.39841	.19080	.06197
15	0	16	1.4791	.27832	.15410	15	0	20	.87422	.23391	.11223	15	0	30	.36688	.19424	.06260
16	0	16	1.3683	.28561	.15510	16	0	20	.80570	.23953	.11304	16	0	30	.33881	.19795	.06337
17	0	16	1.2697	.29322	.15637	17	0	20	.74480	.24548	.11407	17	0	30	.31377	.20193	.06430
18	0	16	1.1817	.30114	.15790	18	0	20	.69056	.25177	.11534	18	0	30	.29140	.20620	.06538
19	0	16	1.1028	.30937	.15966	19	0	20	.64213	.25841	.11684	19	0	30	.27140	.21078	.06662
20	0	16	1.0320	.31792	.16165	20	0	20	.59879	.26541	.11855	20	0	30	.25347	.21569	.06802
22	0	16	0.91076	.33589	.16625	22	0	20	.52499	.28050	.12261	22	0	30	.22295	.22654	.07132
24	0	16	.81149	.35498	.17158	24	0	20	.46513	.29707	.12749	24	0	30	.19824	.23889	.07532
26	0	16	.72943	.37502	.17751	26	0	20	.41617	.31509	.13313	26	0	30	.17814	.25284	.08006
28	0	16	.66098	.39586	.18393	28	0	20	.37580	.33452	.13947	28	0	30	.16168	.26848	.08555
30	0	16	.60336	.41727	.19070	30	0	20	.34224	.35522	.14643	30	0	30	.14813	.28585	.09180
32	0	16	.55446	.43900	.19769	32	0	20	.31411	.37702	.15392	32	0	30	.13690	.30496	.09881
34	0	16	.51260	.46080	.20476	34	0	20	.29037	.39970	.16182	34	0	30	.12753	.32573	.10655
36	0	16	.47648	.48241	.21179	36	0	20	.27015	.42299	.17000	36	0	30	.11967	.34804	.11495
38	0	16	.44509	.50356	.21867	38	0	20	.25282	.44658	.17834	38	0	30	.11302	.37168	.12394
40	0	16	.41760	.52402	.22529	40	0	20	.23783	.47015	.18668	40	0	30	.10735	.39638	.13339
45	0	16	.36193	.57088	.24019	45	0	20	.20810	.52692	.20674	45	0	30	.096354	.46045	.15812
50	0	16	.31962	.61016	.25216	50	0	20	.18606	.57736	.22435	50	0	30	.088429	.52296	.18243
55	0	16	.28629	.64132	.26107	55	0	20	.16901	.61899	.23855	55	0	30	.082394	.57849	.20410
60	0	16	.25921	.66502	.26725	60	0	20	.15530	.65134	.24922	60	0	30	.077552	.62386	.22182
65	0	16	.23665	.68252	.27126	65	0	20	.14390	.67533	.25676	65	0	30	.073486	.65845	.23531
70	0	16	.21745	.69519	.27365	70	0	20	.13416	.69254	.26182	70	0	30	.069944	.68348	.24504
75	0	16	.20084	.70427	.27490	75	0	20	.12566	.70462	.26506	75	0	30	.066769	.70091	.25177
80	0	16	.18627	.71077	.27539	80	0	20	.11812	.71300	.26703	80	0	30	.063684	.71272	.25629
90	0	16	.16176	.71885	.27513	90	0	20	.10518	.72277	.26869	90	0	30	.058642	.72579	.26118
100	0	16	.14185	.72326	.27414	100	0	20	.094331	.72751	.26888	100	0	30	.053995	.73142	.26317
0	0	18	5.6759	.19009	.15808	0	0	25	2.6218	.16693	.09384	0	0	40	.90106	.15986	.04389
1	0	18	4.9578	.19277	.15316	1	0	25	2.3039	.16847	.09119	1	0	40	.80955	.16087	.04346
2	0	18	4.3485	.19561	.14882	2	0	25	2.0314	.17013	.08886	2	0	40	.72889	.16196	.04311
3	0	18	3.8298	.19864	.14501	3	0	25	1.7971	.17189	.08652	3	0	40	.65768	.16313	.04285
4	0	18	3.3866	.20184	.14171	4	0	25	1.5950	.17378	.08505	4	0	40	.59470	.16438	.04267
5	0	18	3.0066	.20525	.13888	5	0	25	1.4203	.17581	.08354	5	0	40	.53891	.16574	.04257
6	0	18	2.6798	.20887	.13649	6	0	25	1.2688	.17798	.08227	6	0	40	.48942	.16720	.04256
7	0	18	2.3977	.21271	.13451	7	0	25	1.1370	.18030	.08124	7	0	40	.44546	.16877	.04262
8	0	18	2.1534	.21679	.13292	8	0	25	1.0221	.18280	.08042	8	0	40	.40635	.17047	.04277
9	0	18	1.9412	.22112	.13170	9	0	25	0.92169	.18548	.07980	9	0	40	.37152	.17231	.04300
10	0	18	1.7563	.22571	.13082	10	0	25	.83372	.18835	.07940	10	0	40	.34046	.17429	.04333
11	0	18	1.5946	.23057	.13028	11	0	25	.75647	.19143	.07918	11	0	40	.31273	.17643	.04374
12	0	18	1.4528	.23571	.13005	12	0	25	.68847	.19474	.07916	12	0	40	.28794	.17875	.04424
13	0	18	1.3281	.24115	.13012	13	0	25	.62848	.19828	.07933	13	0	40	.26576	.18125	.04484
14	0	18	1.2181	.24690	.13047	14	0	25	.57546	.20207	.07969	14	0	40	.24589	.18394	.04554
15	0	18	1.1208	.25296	.13110	15	0	25	.52849	.20614	.08022	15	0	40	.22808	.18685	.04635
16	0	18	1.0345	.25935	.13200	16	0	25	.48681	.21049	.08095	16	0	40	.21209	.18999	.04726
17	0	18	0.95777	.26607	.13315	17	0	25	.44974	.21514	.08186	17	0	40	.19773	.19338	.04829
18	0	18	.88935	.27312	.13455	18	0	25	.41671	.22010	.08295	18	0	40	.18481	.19702	.04945
19	0	18	.82820	.28052	.13618	19	0	25	.38723	.22540	.08423	19	0	40	.17318	.20095	.05073
20	0	18	.77340	.28827	.13804	20	0	25	.36087	.23105	.08570	20	0	40	.16270	.20517	.05214
22	0	18															

TABLE 2. Luminous internal transmittances in percent, 100 $T_{i,A}$, and chromaticity coordinates, x, y , of single glasses and two-par combinations of ideal Lovibond glasses illuminated by CIE Source A—Continued

<i>R</i>	<i>Y</i>	<i>B</i>	$T_{i,A}$	<i>x</i>	<i>y</i>	<i>R</i>	<i>Y</i>	<i>B</i>	$T_{i,A}$	<i>x</i>	<i>y</i>	<i>R</i>	<i>Y</i>	<i>B</i>	$T_{i,A}$	<i>x</i>	<i>y</i>
30	0	40	0.099785	0.26731	0.07502	30	1	0	17.667	0.64810	0.32779	30	3	0	17.202	0.65238	0.33098
32	0	40	.092931	.28477	.08176	32	1	0	16.516	.65272	.32625	32	3	0	16.079	.65646	.32907
34	0	40	.087191	.30400	.08926	34	1	0	15.471	.65690	.32479	34	3	0	15.060	.66016	.32728
36	0	40	.082354	.32496	.09750	36	1	0	14.518	.66068	.32339	36	3	0	14.131	.66352	.32559
38	0	40	.078249	.34749	.10641	38	1	0	13.647	.66412	.32204	38	3	0	13.281	.66658	.32398
40	0	40	.074741	.37139	.11591	40	1	0	12.847	.66726	.32073	40	3	0	12.501	.66937	.32244
45	0	40	.067907	.43513	.14143	45	1	0	11.109	.67395	.31764	45	3	0	10.807	.67539	.31890
50	0	40	.062946	.49974	.16749	50	1	0	9.6739	.67937	.31476	50	3	0	9.4073	.68032	.31571
55	0	40	.059137	.55917	.19157	55	1	0	8.4733	.68382	.31209	55	3	0	8.2372	.68442	.31281
60	0	40	.056050	.60913	.21188	60	1	0	7.4587	.68754	.30962	60	3	0	7.2487	.68790	.31017
65	0	40	.053427	.64803	.22774	65	1	0	6.5941	.69069	.30732	65	3	0	6.4066	.69089	.30775
70	0	40	.051114	.67658	.23940	70	1	0	5.8519	.69341	.30520	70	3	0	5.6839	.69350	.30554
75	0	40	.049015	.69662	.24759	75	1	0	5.2109	.69579	.30323	75	3	0	5.0600	.69580	.30352
80	0	40	.047074	.71025	.25317	80	1	0	4.6542	.69789	.30141	80	3	0	4.5184	.69786	.30165
90	0	40	.043530	.72531	.25933	90	1	0	3.7429	.70147	.29816	90	3	0	3.6321	.70139	.29834
100	0	40	.040322	.73171	.26193	100	1	0	3.0380	.70443	.29536	100	3	0	2.9469	.70433	.29551
0	0	50	.59264	.16104	.03355	0	2	0	97.368	.46417	.42917	0	4	0	94.897	.47446	.44214
1	0	50	.53782	.16193	.03353	1	2	0	87.433	.46783	.41941	1	4	0	85.206	.48728	.43196
2	0	50	.48308	.16204	.03321	2	2	0	79.034	.48907	.41042	2	4	0	77.023	.49965	.42248
3	0	50	.42704	.16088	.03244	3	2	0	71.876	.50075	.40225	3	4	0	70.048	.51139	.41379
4	0	50	.37816	.15988	.03173	4	2	0	65.733	.51182	.39485	4	4	0	64.063	.52245	.40589
5	0	50	.33704	.15935	.03122	5	2	0	60.429	.52228	.38820	5	4	0	58.894	.53282	.39872
6	0	50	.30158	.15907	.03081	6	2	0	55.821	.53213	.38222	6	4	0	54.403	.54253	.39225
7	0	50	.27033	.15889	.03047	7	2	0	51.792	.54138	.37687	7	4	0	50.477	.55160	.38640
8	0	50	.24266	.15879	.03016	8	2	0	48.250	.55007	.37207	8	4	0	47.025	.56005	.38113
9	0	50	.21811	.15875	.02989	9	2	0	45.116	.55821	.36778	9	4	0	43.971	.56793	.37637
10	0	50	.19627	.15877	.02966	10	2	0	42.329	.56584	.36394	10	4	0	41.254	.57526	.37208
11	0	50	.17679	.15884	.02945	11	2	0	39.836	.57300	.36049	11	4	0	38.823	.58209	.36820
12	0	50	.15938	.15894	.02927	12	2	0	37.594	.57970	.35739	12	4	0	36.638	.58846	.36458
13	0	50	.14381	.15907	.02911	13	2	0	35.568	.58598	.35459	13	4	0	34.663	.59439	.36149
14	0	50	.12985	.15924	.02897	14	2	0	33.728	.59187	.35206	14	4	0	32.869	.59993	.35858
15	0	50	.11734	.15943	.02885	15	2	0	32.050	.59740	.34977	15	4	0	31.233	.60510	.35592
16	0	50	.10611	.15966	.02875	16	2	0	30.513	.60259	.34767	16	4	0	29.734	.60993	.35348
17	0	50	.096030	.15992	.02867	17	2	0	29.100	.60747	.34575	17	4	0	28.356	.61445	.35123
18	0	50	.086971	.16020	.02861	18	2	0	27.795	.61205	.34399	18	4	0	27.084	.61868	.34915
19	0	50	.078828	.16053	.02857	19	2	0	26.587	.61637	.34236	19	4	0	25.905	.62265	.34723
20	0	50	.071503	.16089	.02855	20	2	0	25.464	.62043	.34084	20	4	0	24.810	.62638	.34543
22	0	50	.058008	.16087	.02814	22	2	0	23.441	.62788	.33810	22	4	0	22.837	.63318	.34217
24	0	50	.047797	.16163	.02814	24	2	0	21.665	.63452	.33567	24	4	0	21.105	.63922	.33927
26	0	50	.039645	.16273	.02830	26	2	0	20.093	.64045	.33347	26	4	0	19.572	.64460	.33666
28	0	50	.033044	.16409	.02859	28	2	0	18.691	.64577	.33146	28	4	0	18.204	.64941	.33428
30	0	50	.027677	.16571	.02901	30	2	0	17.432	.65056	.32959	30	4	0	16.976	.65374	.33209
32	0	50	.023304	.16765	.02957	32	2	0	16.295	.65488	.32785	32	4	0	15.866	.65765	.33005
34	0	50	.019734	.16995	.03029	34	2	0	15.264	.65878	.32619	34	4	0	14.860	.66119	.32815
36	0	50	.016680	.17213	.03098	36	2	0	14.323	.66233	.32463	36	4	0	13.942	.66440	.32636
38	0	50	.014307	.17530	.03210	38	2	0	13.462	.66555	.32313	38	4	0	13.103	.66733	.32466
40	0	50	.012376	.17912	.03349	40	2	0	12.672	.66849	.32169	40	4	0	12.332	.67002	.32305
45	0	50	.0089179	.19178	.03833	45	2	0	10.957	.67480	.31834	45	4	0	10.659	.67581	.31936
50	0	50	.0067682	.21023	.04567	50	2	0	9.5396	.67993	.31529	50	4	0	9.2771	.68058	.31606
55	0	50	.0054080	.23644	.05633	55	2	0	8.3543	.68418	.31249	55	4	0	8.1218	.68458	.31308
60	0	50	.0045264	.27222	.07108	60	2	0	7.3529	.68776	.30992	60	4	0	7.1461	.68798	.31038
65	0	50	.0039365	.31853	.09035	65	2	0	6.4996	.69082	.30755	65	4	0	6.3149	.69092	.30793
70	0	50	.0035257	.37445	.11374	70	2	0	5.7673	.69348	.30538	70	4	0	5.6019	.69349	.30569
75	0	50	.0032259	.43655	.13982	75	2	0	5.1349	.69581	.30338	75	4	0	4.9863	.69578	.30364
80	0	50	.0029956	.49943	.16628	80	2	0	4.5858	.69789	.30154	80	4	0	4.4520	.69782	.30176
90	0	50	.0026530	.60634	.21141	90	2	0	3.6871	.70143	.29826	90	4	0	3.5779	.70133	.29843
100	0	50	.0023925	.67324	.23972	100	2	0	2.9921	.70438	.29544	100	4	0	2.9024	.70427	.29558
0	1	0	98.660	.45696	.41986	0	3	0	96.115	.46987	.43640	0	5	0	93.709	.47823	.44679
1	1	0	88.597	.46946	.41042	1	3	0	86.300	.48261	.42642	1	5	0	84.140	.49110	.43643
2	1	0	80.085	.48159	.40178	2	3	0	78.012	.49493	.41715	2	5	0	76.058	.50350	.42678
3	1	0	72.830	.49320	.39396	3	3	0	70.947	.50665	.40869	3	5	0	69.171	.51524	.41791
4	1	0	66.606	.50426	.38692	4	3	0	64.885	.51773	.40101	4	5	0	63.260	.52628	.40982
5	1	0	61.231	.51475	.38062	5	3	0	59.650	.52815	.39407	5	5	0	58.156	.53661	.40248
6	1	0	56.561	.52468	.37500	6	3	0	55.102	.53792	.38782	6	5	0	53.721	.54625	.39582
7	1	0	52.479	.53404	.36999	7	3	0	51.126	.54708	.38219	7	5	0	49.844	.55524	.38980
8	1	0	48.889	.54287	.36553	8	3	0	47.629	.55564	.37713	8	5	0	46.435	.56359	.38436
9	1	0	45.714	.55118	.36157	9	3	0	44.536	.56364	.37258	9	5	0	43.418	.57136	.37943
10	1	0	42.889	.55900	.35804	10	3	0	41.784	.57111	.36849	10	5	0	40.735	.57858	.37498
11	1	0	40.362	.56636	.35490	11	3	0	39.323	.57809	.36480	11	5	0	38.335	.58529	.37094
12	1	0	38.091	.57329	.35209	12	3	0	37.110	.58461	.36147	12	5	0	36.176	.59152	.36728
13	1	0	36.038	.57980	.34958	13	3	0	35.110	.59070	.35845	13	5	0	34.225	.59733	.36394
14	1	0	34.174	.58593	.34732	14	3	0	33.294	.59640	.35571	14	5	0	32.454	.60273	.36090
15	1	0	32.474	.59171	.34529	15	3	0	31.637	.60173	.35321	15	5	0	30.838	.60776	.35811
16	1	0	30.917	.59715	.34344	16	3	0	30.119	.60672	.35092	16	5	0	29.357	.61246	.35554
17	1	0	29.485	.60227	.34176	17	3	0	28.724	.61140	.34882	17	5	0	27.996	.61685	.35318
18	1	0	28.163	.60711	.34023	18	3	0	27.436	.61579	.34688	18	5	0	26.739	.62095	.35099
19	1	0	26.939	.61167	.33881	19	3	0	26.242	.61992	.34508	19	5	0	25.575	.62480	.34896
20	1	0	25.802	.61597	.33750	20	3	0	25.134	.62380	.34341	20	5	0	24.493	.62	

TABLE 2. *Luminous internal transmittances in percent, 100 T_{i,A}, and chromaticity coordinates, x, y, of single glasses and two-part combinations of ideal Lovibond glasses illuminated by CIE Source A—Continued*

R	Y	B	T _{i,A}	x	y	R	Y	B	T _{i,A}	x	y	R	Y	B	T _{i,A}	x	y
32	5	0	15.657	0.65855	0.33085	30	8	0	16.106	0.65670	0.33496	30	15	0	14.706	0.65800	0.33769
34	5	0	14.663	.66196	.32886	32	8	0	15.049	.66017	.33261	32	15	0	13.732	.66118	.33512
36	5	0	13.756	.66506	.32699	34	8	0	14.090	.66333	.33044	34	15	0	12.848	.66408	.33273
38	5	0	12.927	.66789	.32523	36	8	0	13.216	.66621	.32841	36	15	0	12.044	.66674	.33052
40	5	0	12.166	.67049	.32356	38	8	0	12.416	.66884	.32651	38	15	0	11.308	.66918	.32845
45	5	0	10.513	.67611	.31975	40	8	0	11.682	.67127	.32472	40	15	0	10.633	.67144	.32652
50	5	0	9.1488	.68076	.31636	45	8	0	10.090	.67656	.32066	45	15	0	9.1707	.67642	.32216
55	5	0	8.0082	.68467	.31332	50	8	0	8.7753	.68097	.31709	50	15	0	7.9654	.68064	.31836
60	5	0	7.0450	.68802	.31058	55	8	0	7.6773	.68473	.31392	55	15	0	6.9600	.68427	.31502
65	5	0	6.2247	.69092	.30809	60	8	0	6.7506	.68797	.31108	60	15	0	6.1127	.68744	.31204
70	5	0	5.5210	.69347	.30583	65	8	0	5.9619	.69080	.30851	65	15	0	5.3926	.69024	.30937
75	5	0	4.9137	.69573	.30376	70	8	0	5.2857	.69331	.30619	70	15	0	4.7761	.69274	.30697
80	5	0	4.3867	.69776	.30186	75	8	0	4.7024	.69555	.30408	75	15	0	4.2450	.69499	.30478
90	5	0	3.5246	.70127	.29851	80	8	0	4.1965	.69757	.30215	80	15	0	3.7850	.69702	.30279
100	5	0	2.8586	.70422	.29565	90	8	0	3.3695	.70108	.29875	90	15	0	3.0342	.70058	.29929
0	6	0	92.548	.48136	.45060	100	8	0	2.7312	.70403	.29585	100	15	0	2.4559	.70358	.29632
1	6	0	83.098	.49428	.44012	0	10	0	88.141	.48986	.46076	0	20	0	78.391	.49900	.47198
2	6	0	75.116	.50669	.43032	1	10	0	79.140	.50283	.44994	1	20	0	70.384	.51182	.46086
3	6	0	68.314	.51842	.42130	2	10	0	71.538	.51523	.43976	2	20	0	63.618	.52401	.45035
4	6	0	62.475	.52943	.41306	3	10	0	65.058	.52689	.43036	3	20	0	57.848	.53542	.44059
5	6	0	57.436	.53972	.40556	4	10	0	59.497	.53778	.42173	4	20	0	52.894	.54602	.43159
6	6	0	53.055	.54930	.39876	5	10	0	54.694	.54790	.41384	5	20	0	48.613	.55581	.42332
7	6	0	49.225	.55821	.39259	6	10	0	50.519	.55727	.40664	6	20	0	44.892	.56483	.41575
8	6	0	45.857	.56648	.38701	7	10	0	46.870	.56594	.40010	7	20	0	41.637	.57313	.40883
9	6	0	42.878	.57415	.38195	8	10	0	43.600	.57395	.39414	8	20	0	38.772	.58075	.40251
10	6	0	40.228	.58127	.37736	9	10	0	40.820	.58134	.38872	9	20	0	36.237	.58775	.39673
11	6	0	37.856	.58787	.37320	10	10	0	38.293	.58816	.38379	10	20	0	33.981	.59417	.39145
12	6	0	35.724	.59400	.36941	11	10	0	36.032	.59445	.37929	11	20	0	31.961	.60007	.38661
13	6	0	33.797	.59969	.36596	12	10	0	33.998	.60026	.37518	12	20	0	30.144	.60550	.38218
14	6	0	32.047	.60498	.36281	13	10	0	32.160	.60563	.37143	13	20	0	28.502	.61049	.37811
15	6	0	30.450	.60990	.35991	14	10	0	30.491	.61061	.36798	14	20	0	27.010	.61510	.37436
16	6	0	28.988	.61449	.35725	15	10	0	28.968	.61522	.36481	15	20	0	25.649	.61935	.37090
17	6	0	27.642	.61877	.35479	16	10	0	27.573	.61950	.36188	16	20	0	24.401	.62329	.36770
18	6	0	26.401	.62276	.35251	17	10	0	26.289	.62348	.35917	17	20	0	23.254	.62694	.36473
19	6	0	25.251	.62650	.35039	18	10	0	25.105	.62718	.35666	18	20	0	22.195	.63033	.36197
20	6	0	24.182	.63001	.34841	19	10	0	24.007	.63064	.35431	19	20	0	21.214	.63349	.35939
22	6	0	22.255	.63638	.34482	20	10	0	22.988	.63388	.35213	20	20	0	20.303	.63645	.35698
24	6	0	20.565	.64203	.34163	22	10	0	21.149	.63975	.34814	22	20	0	18.660	.64180	.35259
26	6	0	19.068	.64705	.33875	24	10	0	19.536	.64493	.34400	24	20	0	17.219	.64652	.34808
28	6	0	17.733	.65155	.33614	26	10	0	18.108	.64954	.34143	26	20	0	15.945	.65072	.34517
30	6	0	16.534	.65559	.33375	28	10	0	16.835	.65366	.33854	28	20	0	14.808	.65449	.34199
32	6	0	15.451	.65923	.33153	30	10	0	15.691	.65736	.33590	30	20	0	13.789	.65790	.33909
34	6	0	14.469	.66254	.32946	32	10	0	14.659	.66072	.33347	32	20	0	12.869	.66099	.33642
36	6	0	13.573	.66555	.32753	34	10	0	13.722	.66377	.33122	34	20	0	12.035	.66382	.33396
38	6	0	12.754	.66831	.32571	36	10	0	12.868	.66656	.32912	36	20	0	11.276	.66642	.33167
40	6	0	12.002	.67084	.32400	38	10	0	12.088	.66911	.32716	38	20	0	10.582	.66881	.32954
45	6	0	10.370	.67632	.32009	40	10	0	11.372	.67147	.32531	40	20	0	9.9466	.67104	.32754
50	6	0	9.0224	.68088	.31663	45	10	0	9.8173	.67663	.32114	45	20	0	8.5690	.67595	.32305
55	6	0	7.8962	.68472	.31354	50	10	0	8.5354	.68095	.31749	50	20	0	7.4354	.68014	.31915
60	6	0	6.9454	.68802	.31075	55	10	0	7.4648	.68465	.31426	55	20	0	6.4909	.68376	.31572
65	6	0	6.1357	.69089	.30824	60	10	0	6.5616	.68785	.31137	60	20	0	5.6958	.68693	.31268
70	6	0	5.4414	.69343	.30595	65	10	0	5.7931	.69067	.30877	65	20	0	5.0208	.68975	.30995
75	6	0	4.8422	.69568	.30387	70	10	0	5.1546	.69317	.30642	70	20	0	4.4435	.69227	.30750
80	6	0	4.3223	.69771	.30196	75	10	0	4.5668	.69540	.30429	75	20	0	3.9466	.69454	.30527
90	6	0	3.4721	.70121	.29859	80	10	0	4.0744	.69742	.30234	80	20	0	3.5166	.69660	.30324
100	6	0	2.8154	.70416	.29571	90	10	0	3.2700	.70094	.29890	90	20	0	2.8158	.70020	.29968
0	8	0	90.301	.48625	.45647	100	10	0	2.6494	.70391	.29598	100	20	0	2.2768	.70325	.29665
1	8	0	81.080	.49920	.44579	0	15	0	83.068	.49568	.46773	0	30	0	70.012	.50219	.47724
2	8	0	73.292	.51162	.43577	1	15	0	74.585	.50860	.45670	1	30	0	62.857	.51478	.46609
3	8	0	66.654	.52333	.42652	2	15	0	67.418	.52091	.44629	2	30	0	56.807	.52674	.45553
4	8	0	60.957	.53428	.41806	3	15	0	61.308	.53245	.43665	3	30	0	51.645	.53790	.44569
5	8	0	56.038	.54448	.41033	4	15	0	56.063	.54318	.42777	4	30	0	47.210	.54825	.43661
6	8	0	51.763	.55395	.40330	5	15	0	51.532	.55312	.41963	5	30	0	43.376	.55780	.42824
7	8	0	48.025	.56273	.39691	6	15	0	47.594	.56230	.41219	6	30	0	40.041	.56659	.42056
8	8	0	44.738	.57085	.39111	7	15	0	44.150	.57076	.40540	7	30	0	37.122	.57456	.41352
9	8	0	41.830	.57837	.38584	8	15	0	41.120	.57854	.39921	8	30	0	34.553	.58206	.40708
10	8	0	39.242	.58532	.38105	9	15	0	38.439	.58570	.39355	9	30	0	32.278	.58884	.40118
11	8	0	36.927	.59175	.37670	10	15	0	36.053	.59228	.38840	10	30	0	30.253	.59507	.39577
12	8	0	34.846	.59770	.37272	11	15	0	33.918	.59833	.38368	11	30	0	28.440	.60078	.39081
13	8	0	32.964	.60321	.36909	12	15	0	31.998	.60391	.37937	12	30	0	26.809	.60602	.38626
14	8	0	31.255	.60832	.36577	13	15	0	30.262	.60905	.37541	13	30	0	25.334	.61085	.38206
15	8	0	29.696	.61306	.36271	14	15	0	28.685	.61379	.37177	14	30	0	23.994	.61530	.37819
16	8	0	28.268	.61747	.35990	15	15	0	27.246	.61818	.36842	15	30	0	22.771	.61941	.37462
17	8	0	26.954	.62158	.35729	16	15	0	25.928	.62225	.36532	16	30	0	21.651	.62321	.37130
18	8	0	25.742	.62541	.35488	17	15	0	24.716	.62602	.36245	17	30	0	20.621	.62623	.36822
19	8	0	24.619	.62899	.35263	18	15	0	23.596	.62953	.35977	18	30	0	19.670	.63001	.36535
20	8	0	23.575	.63234	.35053	19	15	0	22.559	.63280	.35728	19	30	0	18.789	.63307	.36267
22	8	0	21.693	.63842	.34671	20	15	0	21.596	.63585	.35495	20					

TABLE 2. Luminous internal transmittances in percent, 100 $T_{i,A}$, and chromaticity coordinates, x, y , of single glasses and two-part combinations of ideal Lovibond glasses illuminated by CIE Source A—Continued

<i>R</i>	<i>Y</i>	<i>B</i>	$T_{i,A}$	x	y	<i>R</i>	<i>Y</i>	<i>B</i>	$T_{i,A}$	x	y	<i>R</i>	<i>Y</i>	<i>B</i>	$T_{i,A}$	x	y
30	30	0	12.136	0.65678	0.34147	30	60	0	8.3332	0.65126	0.34801	0	2	5	38.645	0.38107	0.40487
32	30	0	11.314	.65982	.33868	32	60	0	7.7423	.65439	.34497	0	2	6	32.462	.36352	.39524
34	30	0	10.570	.66260	.33610	34	60	0	7.2091	.65728	.34215	0	2	7	27.365	.34607	.38403
36	30	0	9.8936	.66517	.33371	36	60	0	6.7259	.65996	.33953	0	2	8	23.150	.32892	.37138
38	30	0	9.2760	.66755	.33148	38	60	0	6.2864	.66246	.33709	0	2	9	19.655	.31227	.35745
40	30	0	8.7104	.66976	.32940	40	60	0	5.8854	.66479	.33481	0	2	10	16.748	.29629	.34244
45	30	0	7.4873	.67468	.32472	45	60	0	5.0232	.67001	.32966	0	2	12	12.292	.26692	.31023
50	30	0	6.4834	.67889	.32066	50	60	0	4.3213	.67453	.32520	0	2	14	9.1542	.24167	.29769
55	30	0	5.6490	.68256	.31709	55	60	0	3.7425	.67849	.32128	0	2	16	6.9186	.22084	.24406
60	30	0	4.9483	.68580	.31393	60	60	0	3.2600	.68200	.31780	0	2	18	5.3073	.20427	.21348
65	30	0	4.3547	.68867	.31111	65	60	0	2.8542	.68513	.31469	0	2	20	4.1328	.19149	.18596
70	30	0	3.8482	.69125	.30856	70	60	0	2.5101	.68795	.31188	0	2	25	2.3620	.17209	.13215
75	30	0	3.4130	.69359	.30625	75	60	0	2.2163	.69051	.30934	0	2	30	1.4800	.16376	.09715
80	30	0	3.0371	.69571	.30416	80	60	0	1.9640	.69283	.30703	0	2	40	0.73833	.16020	.06126
90	30	0	2.4261	.69942	.30047	90	60	0	1.5571	.69691	.30297	0	2	50	.46197	.16153	.04605
100	30	0	1.9576	.70256	.29734	100	60	0	1.2482	.70037	.29953	0	3	0	96.115	.46987	.43640
0	40	0	62.705	.50321	.48078	0	100	0	33.551	.49699	.49557	0	3	1	79.327	.45515	.43582
1	40	0	56.292	.51556	.46969	1	100	0	30.090	.50806	.48526	0	3	2	65.691	.43976	.43401
2	40	0	50.865	.52728	.45917	2	100	0	27.148	.51862	.47539	0	3	3	54.583	.42380	.43084
3	40	0	46.231	.53822	.44936	3	100	0	24.626	.52852	.46612	0	3	4	45.509	.40738	.42619
4	40	0	42.248	.54836	.44027	4	100	0	22.450	.53776	.45745	0	3	5	38.074	.39063	.41498
5	40	0	38.802	.55772	.43189	5	100	0	20.561	.54634	.44938	0	3	6	31.964	.37369	.41218
6	40	0	35.804	.56633	.42419	6	100	0	18.912	.55428	.44189	0	3	7	26.928	.35675	.40280
7	40	0	33.175	.57423	.41712	7	100	0	17.465	.56164	.43494	0	3	8	22.765	.33998	.39190
8	40	0	30.866	.58148	.41063	8	100	0	16.189	.56844	.42850	0	3	9	19.312	.32354	.37959
9	40	0	28.818	.58813	.40467	9	100	0	15.056	.57472	.42254	0	3	10	16.441	.30763	.36604
10	40	0	26.995	.59423	.39921	10	100	0	14.047	.58053	.41701	0	3	12	12.042	.27793	.33602
11	40	0	25.362	.59983	.39419	11	100	0	13.143	.58592	.41187	0	3	14	8.9463	.25182	.30370
12	40	0	23.892	.60498	.38957	12	100	0	12.329	.59091	.40710	0	3	16	6.7419	.22979	.27100
13	40	0	22.564	.60972	.38530	13	100	0	11.593	.59554	.40266	0	3	18	5.1545	.21188	.23955
14	40	0	21.356	.61409	.38137	14	100	0	10.925	.59985	.39852	0	3	20	3.9986	.19777	.21051
15	40	0	20.255	.61812	.37772	15	100	0	10.316	.60386	.39466	0	3	25	2.2597	.17569	.15178
16	40	0	19.246	.62186	.37434	16	100	0	9.7598	.60761	.39104	0	3	30	1.3974	.16581	.11220
17	40	0	18.318	.62534	.37119	17	100	0	9.2489	.61112	.38765	0	3	40	0.67884	.16128	.07058
18	40	0	17.462	.62857	.36825	18	100	0	8.7784	.61442	.38446	0	3	50	.41579	.16270	.06278
19	40	0	16.670	.63158	.36551	19	100	0	8.3439	.61751	.38145	0	4	0	94.897	.47446	.44214
20	40	0	15.934	.63440	.36293	20	100	0	7.9416	.62043	.37862	0	4	1	78.301	.46041	.44263
22	40	0	14.600	.63953	.35822	22	100	0	7.2202	.62579	.37339	0	4	2	64.823	.44571	.44201
24	40	0	13.448	.64407	.35402	24	100	0	6.5924	.63061	.36868	0	4	3	53.844	.43046	.44017
26	40	0	12.423	.64814	.35024	26	100	0	6.0418	.63498	.36439	0	4	4	44.875	.41473	.43697
28	40	0	11.511	.65181	.34681	28	100	0	5.5553	.63896	.36048	0	4	5	37.528	.39864	.43231
30	40	0	10.695	.65514	.34367	30	100	0	5.1230	.64262	.35688	0	4	6	31.490	.38232	.42613
32	40	0	9.9594	.65818	.34079	32	100	0	4.7366	.64600	.35355	0	4	7	26.513	.36591	.41840
34	40	0	9.2944	.66098	.33813	34	100	0	4.3896	.64913	.35045	0	4	8	22.400	.34957	.40913
36	40	0	8.6904	.66357	.33566	36	100	0	4.0768	.65205	.34757	0	4	9	18.989	.33347	.39840
38	40	0	8.1396	.66597	.33333	38	100	0	3.7937	.65477	.34487	0	4	10	16.153	.31775	.38632
40	40	0	7.6358	.66821	.33119	40	100	0	3.5366	.65733	.34234	0	4	12	11.810	.28806	.35875
45	40	0	6.5485	.67321	.32635	45	100	0	2.9883	.66309	.33662	0	4	14	8.7548	.26147	.32801
50	40	0	5.6584	.67750	.32215	50	100	0	2.5471	.66810	.33164	0	4	16	6.5808	.23859	.29594
55	40	0	4.9205	.68126	.31847	55	100	0	2.1871	.67251	.32725	0	4	18	5.0165	.21961	.26422
60	40	0	4.3023	.68458	.31520	60	100	0	1.8901	.67644	.32335	0	4	20	3.8785	.20438	.23420
65	40	0	3.7798	.68753	.31227	65	100	0	1.6426	.67996	.31985	0	4	25	2.1699	.17986	.17142
70	40	0	3.3349	.69019	.30964	70	100	0	1.4347	.68313	.31670	0	4	30	1.3262	.16847	.12758
75	40	0	2.9535	.69259	.30726	75	100	0	1.2587	.68600	.31383	0	4	40	0.62890	.16295	.08028
80	40	0	2.6247	.69478	.30509	80	100	0	1.1087	.68862	.31122	0	4	50	.37779	.16450	.05981
90	40	0	2.0915	.69860	.30128	90	100	0	0.86949	.69323	.30664	0	5	0	93.709	.47823	.44679
100	40	0	1.6840	.70185	.29805	100	100	0	.69032	.69714	.30275	0	5	1	77.304	.46174	.44813
0	60	0	50.611	.50246	.48625	0	1	0	98.660	.45696	.41986	0	5	2	63.980	.45064	.44851
1	60	0	45.423	.51436	.47538	0	1	1	81.476	.44053	.41634	0	5	3	53.128	.43600	.44778
2	60	0	41.028	.52565	.46504	0	1	2	67.519	.42342	.41128	0	5	4	44.263	.42090	.44580
3	60	0	37.266	.53621	.45537	0	1	3	56.148	.40577	.40458	0	5	5	37.001	.40542	.44248
4	60	0	34.030	.54600	.44639	0	1	4	46.858	.38775	.39620	0	5	6	31.034	.38968	.43771
5	60	0	31.228	.55505	.43808	0	1	5	39.244	.36957	.38614	0	5	7	26.116	.37381	.43145
6	60	0	28.786	.56339	.43041	0	1	6	32.987	.35143	.37447	0	5	8	22.052	.35794	.42368
7	60	0	26.647	.57106	.42334	0	1	7	27.827	.33355	.36132	0	5	9	18.683	.34222	.41443
8	60	0	24.761	.57811	.41684	0	1	8	23.561	.31617	.34687	0	5	10	15.882	.32678	.40377
9	60	0	23.090	.58459	.41086	0	1	9	20.022	.29946	.33135	0	5	12	11.593	.29733	.37872
10	60	0	21.602	.59054	.40534	0	1	10	17.078	.28363	.31502	0	5	14	8.5768	.27055	.34986
11	60	0	20.268	.59602	.40026	0	1	12	12.564	.25506	.28109	0	5	16	6.4322	.24712	.31884
12	60	0	19.068	.60107	.39556	0	1	14	9.3830	.23113	.24724	0	5	18	4.8903	.22734	.28734
13	60	0	17.983	.60573	.39122	0	1	16	7.1150	.21189	.21524	0	5	20	3.7696	.21119	.25683
14	60	0	16.997	.61004	.38720	0	1	18	5.4789	.19695	.18624	0	5	25	2.0901	.18451	.19088
15	60	0	16.098	.61403	.38346	0	1	20	4.2850	.18569	.16081	0	5	30	1.2639	.17168	.14317
16	60	0	15.275	.61774	.37998	0	1	25	2.4807	.16914	.11278	0	5	40	0.58630	.16521	.09028
17	60	0	14.518	.62119	.37674	0	1	30	1.5775	.16236	.08260	0	5	50	.34599	.16694	.06711
18	60	0	13.820	.62441	.37370	0	1	40	0.81067	.15973	.05236	0	6	0	92.548	.48136	.45060
19	60	0	13.175	.62742	.37055	0	1	50	.51933	.16099	.03965	0	6	1	76.331	.46836	.45266
20	60	0	12.576														

TABLE 2. *Luminous internal transmittances in percent, 100 T_{i,A}, and chromaticity coordinates, x,y, of single glasses and two-part combinations of ideal Lovibond glasses illuminated by CIE Source A—Continued*

<i>R</i>	<i>Y'</i>	<i>B</i>	<i>T_{i,A}</i>	<i>x</i>	<i>y</i>	<i>R</i>	<i>Y'</i>	<i>B</i>	<i>T_{i,A}</i>	<i>x</i>	<i>y</i>	<i>R</i>	<i>Y'</i>	<i>B</i>	<i>T_{i,A}</i>	<i>x</i>	<i>y</i>
0	6	5	36.492	0.41122	0.45094	0	15	5	32.480	0.43998	0.48912	0	40	5	24.314	0.46045	0.51249
0	6	6	30.595	.39603	.44740	0	15	6	27.163	.42811	.49158	0	40	6	20.277	.45185	.51820
0	6	7	25.735	.38067	.44244	0	15	7	22.784	.41611	.49820	0	40	7	16.953	.44333	.52357
0	6	8	21.719	.36527	.43602	0	15	8	19.168	.40402	.49391	0	40	8	14.210	.43494	.52854
0	6	9	18.390	.34995	.42813	0	15	9	16.173	.39193	.49361	0	40	9	11.940	.42670	.53307
0	6	10	15.623	.33485	.41882	0	15	10	13.685	.37987	.49224	0	40	10	10.057	.41867	.53710
0	6	12	11.387	.30580	.39627	0	15	12	9.8850	.35615	.48608	0	40	12	7.1861	.40334	.54345
0	6	14	8.4099	.27906	.36943	0	15	14	7.2220	.33337	.47526	0	40	14	5.1822	.38926	.54726
0	6	16	6.2941	.25532	.33975	0	15	16	5.3367	.31200	.45989	0	40	16	3.7709	.37666	.54822
0	6	18	4.7738	.23497	.30887	0	15	18	3.9885	.29245	.44045	0	40	18	2.7684	.35863	.54613
0	6	20	3.6698	.21810	.27827	0	15	20	3.0151	.27504	.41773	0	40	20	2.0505	.35683	.54088
0	6	25	2.0182	.18956	.20999	0	15	25	1.5762	.24188	.35304	0	40	25	1.0074	.34397	.51415
0	6	30	1.2087	.17541	.15883	0	15	30	0.88827	.22298	.28919	0	40	30	0.52632	.34588	.47125
0	6	40	0.54946	.16805	.10054	0	15	40	.35472	.21479	.19596	0	40	40	.17870	.38863	.36958
0	6	50	.31899	.17004	.07465	0	15	50	.18580	.22451	.14807	0	40	50	.083866	.45450	.29516
0	8	0	90.301	.48625	.45647	0	20	0	78.391	.49900	.47198	0	60	0	50.611	.50246	.48625
0	8	1	74.452	.47404	.45964	0	20	1	64.552	.48918	.47778	0	60	1	41.656	.49475	.49289
0	8	2	61.580	.46132	.46211	0	20	2	53.313	.47910	.48335	0	60	2	34.373	.48700	.49948
0	8	3	51.097	.44813	.46377	0	20	3	44.160	.46877	.48862	0	60	3	28.436	.47923	.50598
0	8	4	42.534	.43453	.46447	0	20	4	36.685	.45824	.49350	0	60	4	23.583	.47150	.51234
0	8	5	35.521	.42058	.46411	0	20	5	30.565	.44755	.49791	0	60	5	19.606	.46384	.51853
0	8	6	29.759	.40635	.46258	0	20	6	25.539	.43673	.50175	0	60	6	16.340	.45628	.52450
0	8	7	25.012	.39194	.45977	0	20	7	21.400	.42585	.50496	0	60	7	13.650	.44888	.53020
0	8	8	21.090	.37744	.45562	0	20	8	17.983	.41494	.50744	0	60	8	11.429	.44167	.53558
0	8	9	17.839	.36294	.45010	0	20	9	15.154	.40405	.50912	0	60	9	9.5912	.43468	.54061
0	8	10	15.138	.34857	.44318	0	20	10	12.806	.39323	.50993	0	60	10	8.0667	.42796	.54525
0	8	12	11.006	.32060	.42529	0	20	12	9.2199	.37202	.50869	0	60	12	5.7430	.41544	.55317
0	8	14	8.1033	.29437	.40257	0	20	14	6.7101	.35168	.50337	0	60	14	4.1228	.40438	.55906
0	8	16	6.0428	.27056	.37609	0	20	16	4.9360	.33259	.49383	0	60	16	2.9835	.39502	.56266
0	8	18	4.5641	.24965	.34721	0	20	18	3.6697	.31507	.48021	0	60	18	2.1762	.38758	.56374
0	8	20	3.4920	.23186	.31739	0	20	20	2.7575	.29940	.46289	0	60	20	1.5998	.38229	.56213
0	8	25	1.8931	.20652	.24665	0	20	25	1.4156	.26940	.40778	0	60	25	0.76789	.37974	.54556
0	8	30	1.1146	.18416	.18993	0	20	30	0.78034	.25286	.34633	0	60	30	.38974	.39423	.51146
0	8	40	0.48880	.17537	.12158	0	20	40	.29717	.25100	.24487	0	60	40	.12440	.46852	.41232
0	8	50	.27566	.17814	.09030	0	20	50	.15024	.27000	.18822	0	60	50	.056520	.55951	.32823
0	10	0	88.141	.48986	.46076	0	30	0	70.012	.50219	.47724	0	100	0	33.551	.49699	.49557
0	10	1	72.650	.47827	.46472	0	30	1	57.626	.49326	.48362	0	100	1	27.646	.49004	.50198
0	10	2	60.069	.46622	.46812	0	30	2	47.565	.48415	.48988	0	100	2	22.835	.48313	.50832
0	10	3	49.823	.45376	.47083	0	30	3	39.371	.47489	.49598	0	100	3	18.905	.47629	.51456
0	10	4	41.454	.44092	.47276	0	30	4	32.679	.46553	.50183	0	100	4	15.688	.46955	.52067
0	10	5	34.600	.42777	.47377	0	30	5	27.199	.45610	.50738	0	100	5	13.047	.46296	.52661
0	10	6	28.971	.41436	.47375	0	30	6	22.699	.44665	.51257	0	100	6	10.875	.45654	.53233
0	10	7	24.333	.40077	.47262	0	30	7	18.995	.43722	.51732	0	100	7	9.0835	.45033	.53781
0	10	8	20.501	.38707	.47027	0	30	8	15.937	.42786	.52157	0	100	8	7.6028	.44436	.54302
0	10	9	17.327	.37335	.46666	0	30	9	13.407	.41859	.52525	0	100	9	6.3762	.43867	.54792
0	10	10	14.690	.35970	.46173	0	30	10	11.307	.40947	.52831	0	100	10	5.3579	.43329	.55248
0	10	12	10.656	.33297	.44793	0	30	12	8.1037	.39183	.53233	0	100	12	3.8046	.42355	.56046
0	10	14	7.8256	.30760	.42912	0	30	14	5.8656	.37523	.53323	0	100	14	2.7210	.41538	.56674
0	10	16	5.8178	.28421	.40603	0	30	16	4.2869	.35966	.53074	0	100	16	1.9594	.40896	.57112
0	10	18	4.3786	.26331	.37974	0	30	18	3.1634	.34626	.52470	0	100	18	1.4204	.40453	.57340
0	10	20	3.3365	.24518	.35153	0	30	20	2.3568	.33434	.51513	0	100	20	1.0365	.40228	.57338
0	10	25	1.7869	.21220	.28072	0	30	25	1.1789	.31341	.47732	0	100	25	0.48572	.40768	.56204
0	10	30	1.0368	.19427	.22018	0	30	30	0.62958	.30599	.42585	0	100	30	.23889	.43124	.53295
0	10	40	0.44075	.18467	.14299	0	30	40	.22400	.32575	.32159	0	100	40	.070943	.52792	.43224
0	10	50	.24245	.18871	.10652	0	30	50	.10811	.36914	.25394	0	100	50	.030968	.63142	.33697
0	15	0	83.068	.49568	.46773	0	40	0	62.705	.50321	.48078	0	40	0			
0	15	1	68.430	.48517	.47289	0	40	1	51.603	.49482	.48736	0	40	1			
0	15	2	56.542	.47430	.47771	0	40	2	42.582	.48631	.49388	0	40	2			
0	15	3	46.861	.46313	.48208	0	40	3	35.233	.47772	.50027	0	40	3			
0	15	4	38.955	.45167	.48592	0	40	4	29.230	.46908	.50649	0	40	4			

3. Relation Between the Ideal and Actual Lovibond Scales

The ideal Lovibond system specified by calculation from the spectral internal transmittances of the ideal unit glasses given in table 1 necessarily yields perfectly smooth loci of constant numbers of red, yellow,

or blue units (see figs. 2–6) on the chromaticity diagram, and also necessarily meets perfectly the additivity condition that a glass designated *R* on the ideal Lovibond red scale have the same chromaticity for the specified source as the light from that source after transmission through *R* ideal Lovibond red unit glasses in succession. The actual Lovibond scales identify the glasses issued by Tintometer Ltd.

by the numbers, N_r , N_y , N_b , engraved on the red, yellow, and blue glasses, respectively. These numbers are assigned to the glasses by visual comparison with the master standards of the Lovibond system, and, in general, the glasses to which they are assigned deviate slightly from the spectral characteristics of the ideal Lovibond glasses given in table 1. This deviation arises from the fact that slight variations in the spectral characteristics of the glasses have to be tolerated to permit Lovibond glasses to be distributed at prices making feasible their application to the practical problems of color grading. As a result of these deviations, a plot of the spectrophotometrically determined chromaticity points for either the red, the yellow, or the blue series does not yield a perfectly smooth locus. Furthermore, as a result of errors in the visual grading, the spacing of points along the locus is irregular. If the portion of the glass measured spectrophotometrically is different from that graded visually rather large irregularities may result [5]. However, this error due to portion measured which is introduced when grading glasses visually refers to glasses purchased in 1912 which are 2 in. \times $\frac{3}{4}$ in. in size. For the past 10 years rectangles $\frac{3}{8}$ in. \times $\frac{3}{4}$ in. in size only have been supplied by the maker and deviations between visual and spectrophotometric measurements due to portion of glass should be eliminated. The manufacturing tolerances adopted are such that deviations between the ideal (R, Y, B) notation and the nominal (N_r, N_y, N_b) are undetectable, or nearly so, in a viewing field subtending 2° or less at the eye of the observer.

4. Uses of the Ideal Lovibond Scales

By means of the relation shown in table 2 between the CIE x - and y -scales and the ideal Lovibond R -, Y - and B -scales for CIE source A , it is possible to obtain a close estimate of the nominal N_r -, N_y -, N_b -values of the two-part combination of Lovibond glasses required to change the chromaticity of incandescent-lamp light to any chromaticity (x, y) within the Lovibond gamut. To facilitate such determinations, the loci of chromaticity points corresponding to two-part combinations have been plotted on the CIE chromaticity diagram. Each locus corresponds to variations of one of R , Y , or B , with one of the other two held constant. Figure 1 shows a few of these loci to a small scale, and figures 2 to 6 show more of the loci for five sections of the chromaticity gamut shown completely in figure 1.

These chromaticity networks would appear also to be useful to assign regrade numerals to Lovibond glasses on the ideal Lovibond system which would be more precise than the nominal Lovibond grades. If the chromaticity point of the glass to be regraded falls precisely on the corresponding single-glass locus, a one-number regrade value may be assigned by one-dimensional interpolation; for example, a nominal 16.0 Lovibond red glass ($N_r=16.0$) might by spectrophotometric measurement be found to correspond to the chromaticity point for $R=15.8$, and might there-

fore be regraded as 15.8 on the ideal Lovibond red scale. This regrade could be taken as valid in combinations with other Lovibond glasses of whatever color (red, yellow, blue) or number, because the agreement of the spectrophotometrically determined chromaticity point with the single-glass locus corroborates the correctness of the spectral character of the glass. If, on the other hand, the spectrophotometrically determined chromaticity point fails to fall precisely on the corresponding single-glass locus, the regrade would have to consist of two numerals found by two-dimensional interpolation on the chromaticity network (figs. 2 to 6). For example, a nominally 16.0 Lovibond red glass ($N_r=16.0$) might be regraded as $R=16.0$, $Y=0.10$. This two-numeral regrade is approximately valid in combinations of this glass with other single Lovibond glasses of any color (red, yellow, or blue). The failure of this two-numeral regrade to apply strictly arises from the fact that the off-locus glass must depart somewhat (perhaps within manufacturing tolerances) from the intended spectral character, and also may, and almost certainly will, depart from the spectral character implied by the two-numeral regrade. This failure constitutes a restriction on this use of the chromaticity network of the ideal Lovibond system (figs. 2 to 6) until such time as improvements in glass-making techniques make possible the provision by the Tintometer Company of Lovibond glasses conforming to the ideal. A practical restriction on this use is that the cost of determining the chromaticity coordinates (x, y) accurately by spectrophotometric measurements is likely to be several times the cost of the Lovibond glass itself.

5. Summary

The basic definitions of the ideal Lovibond color scales are given; see table 1. These definitions permit, for any defined source, the correlation of the ideal Lovibond color scales with the internationally recognized CIE coordinate system for colorimetry. Such correlations are available from Tintometer Ltd. in the form of large scale graphs [4,6] for CIE sources B and C , and are supplied in the present paper for CIE source A ; see table 2 and figures 1 to 6.

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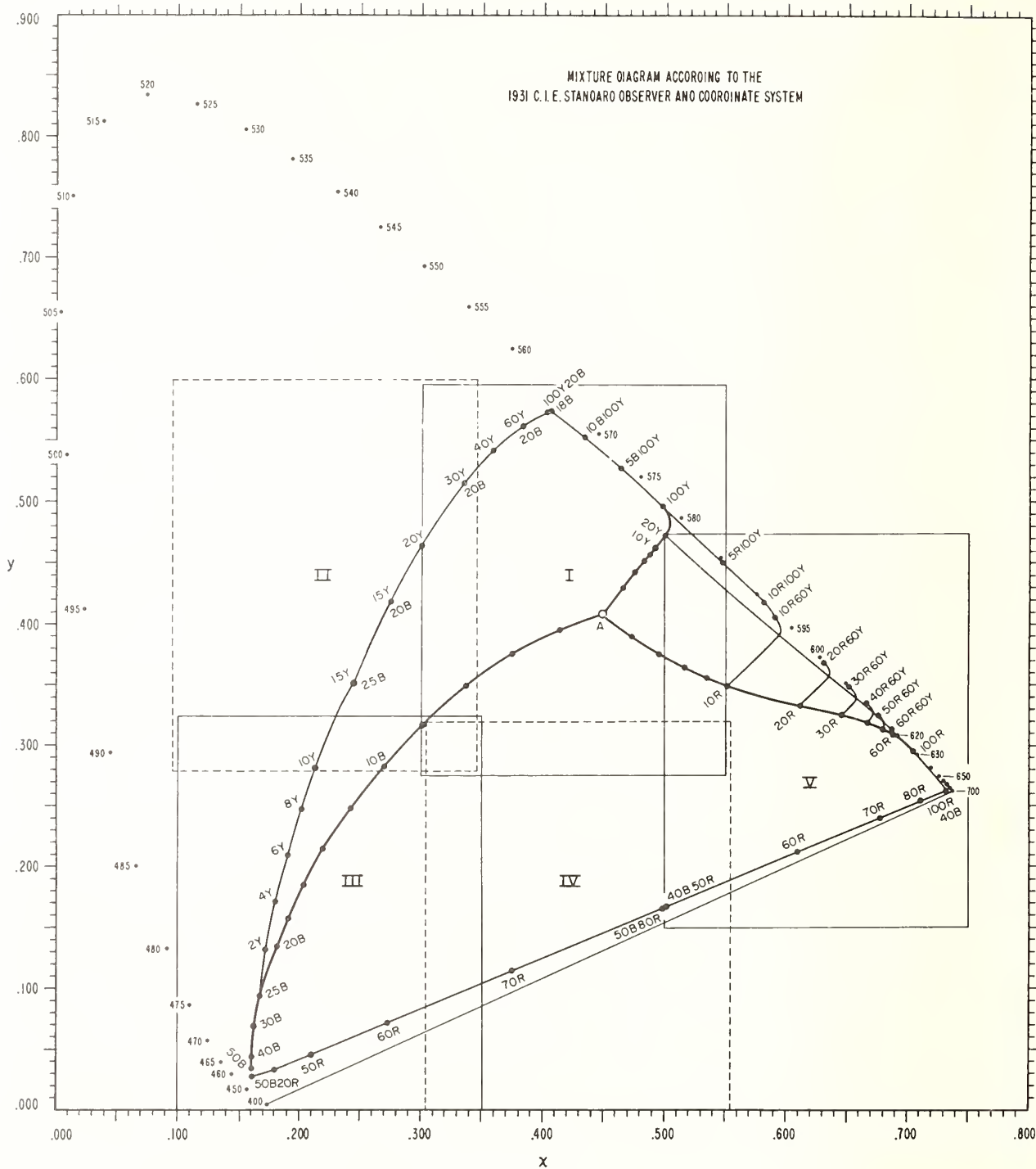


FIGURE 1. Chromaticity gamut of the ideal Lovibond color system showing the relation between the x - and y -scales and the ideal Lovibond R -, Y -, and B -scales for CIE Source A.

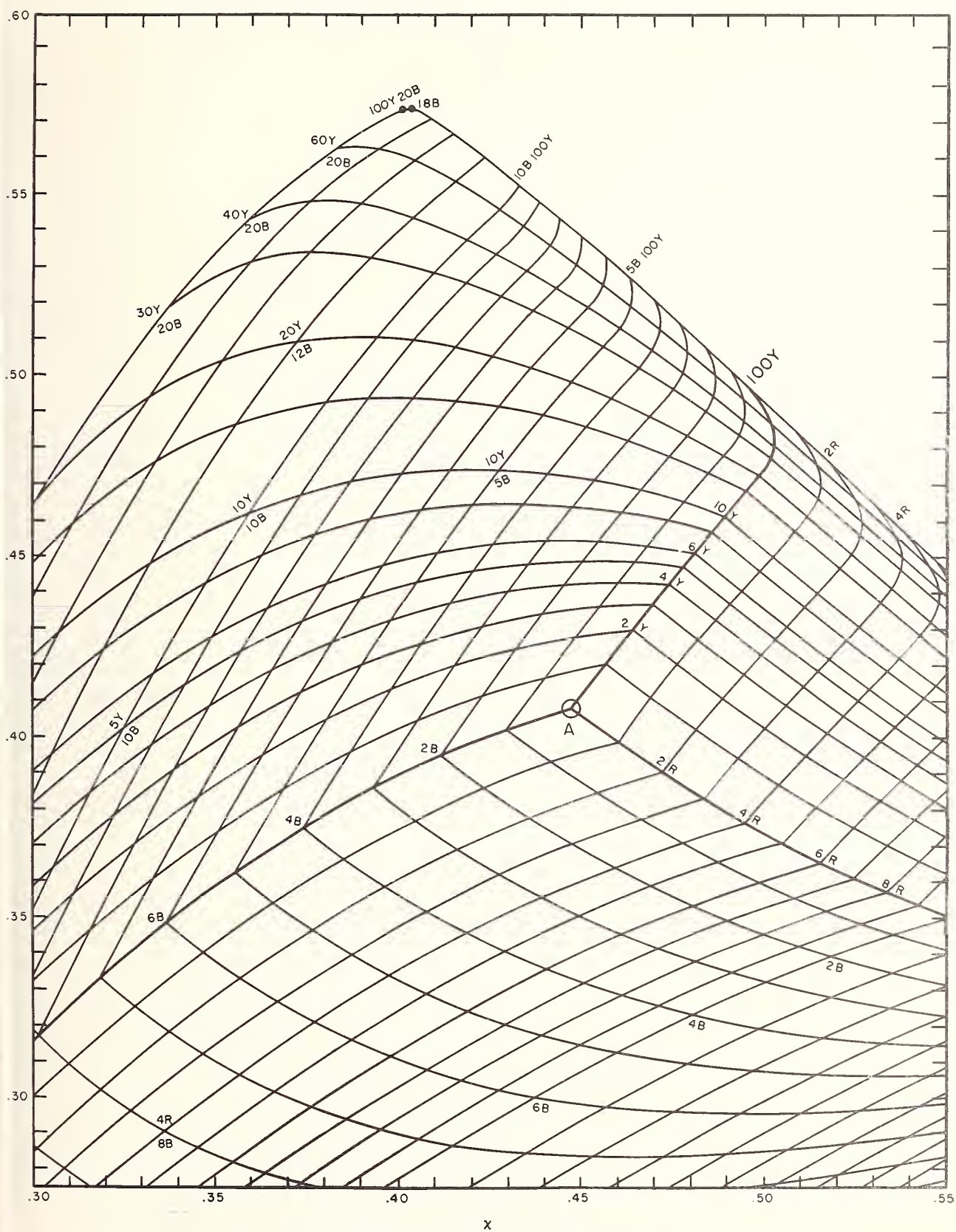


FIGURE 2. Enlarged graph of section I of figure 1.

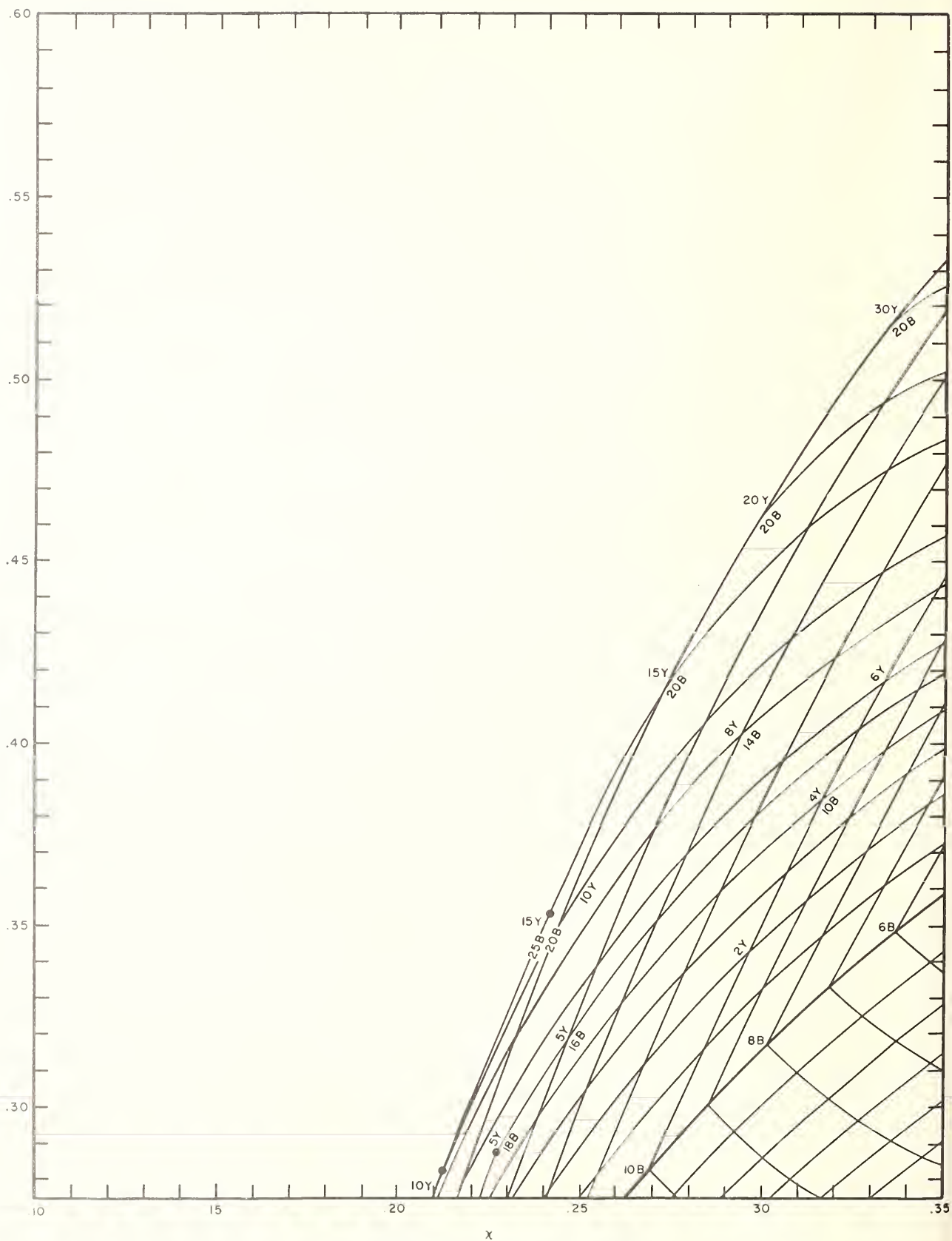


FIGURE 3. Enlarged graph of section II of figure 1.

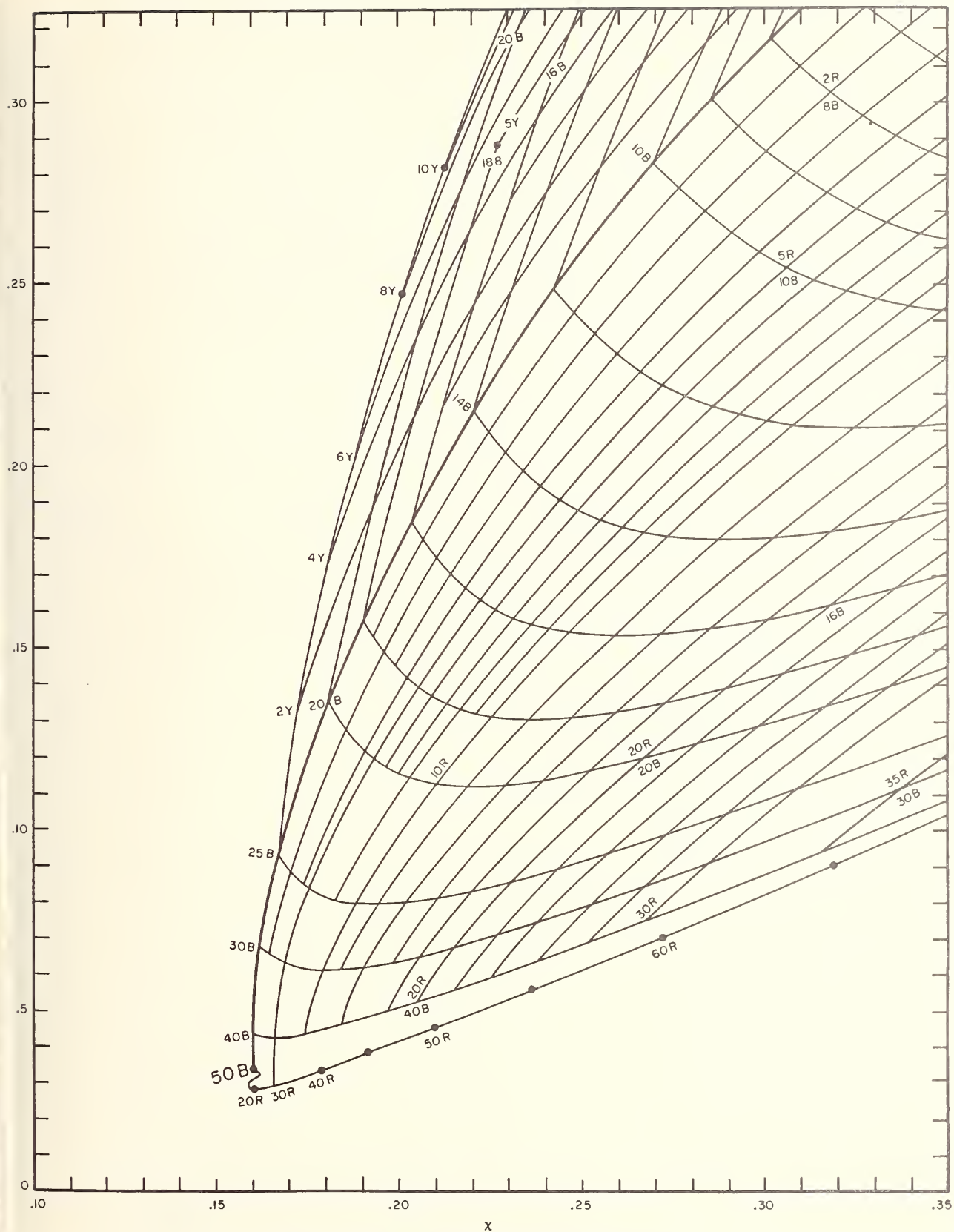


FIGURE 4. Enlarged graph of section III of figure 1.

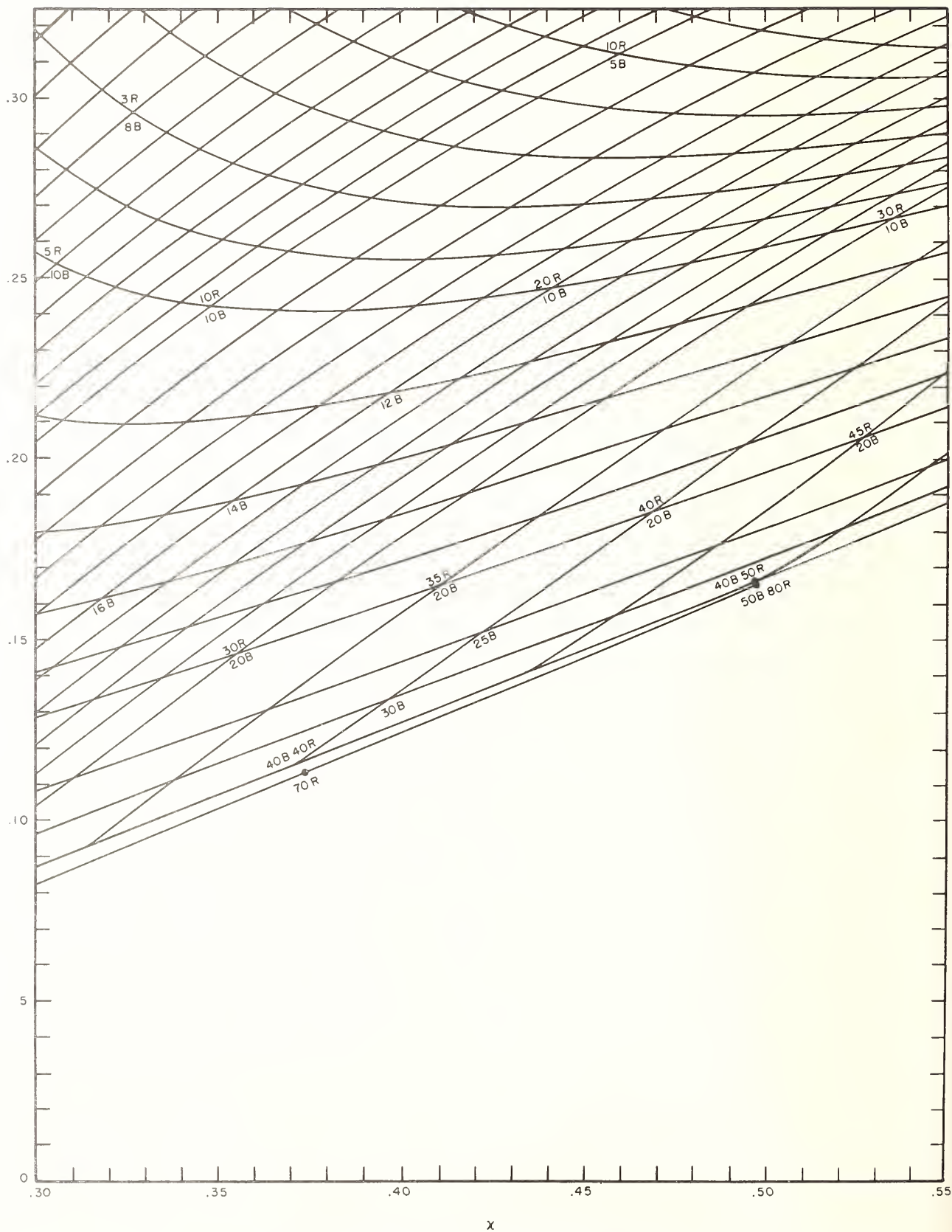


FIGURE 5. Enlarged graph of section IV of figure 1.

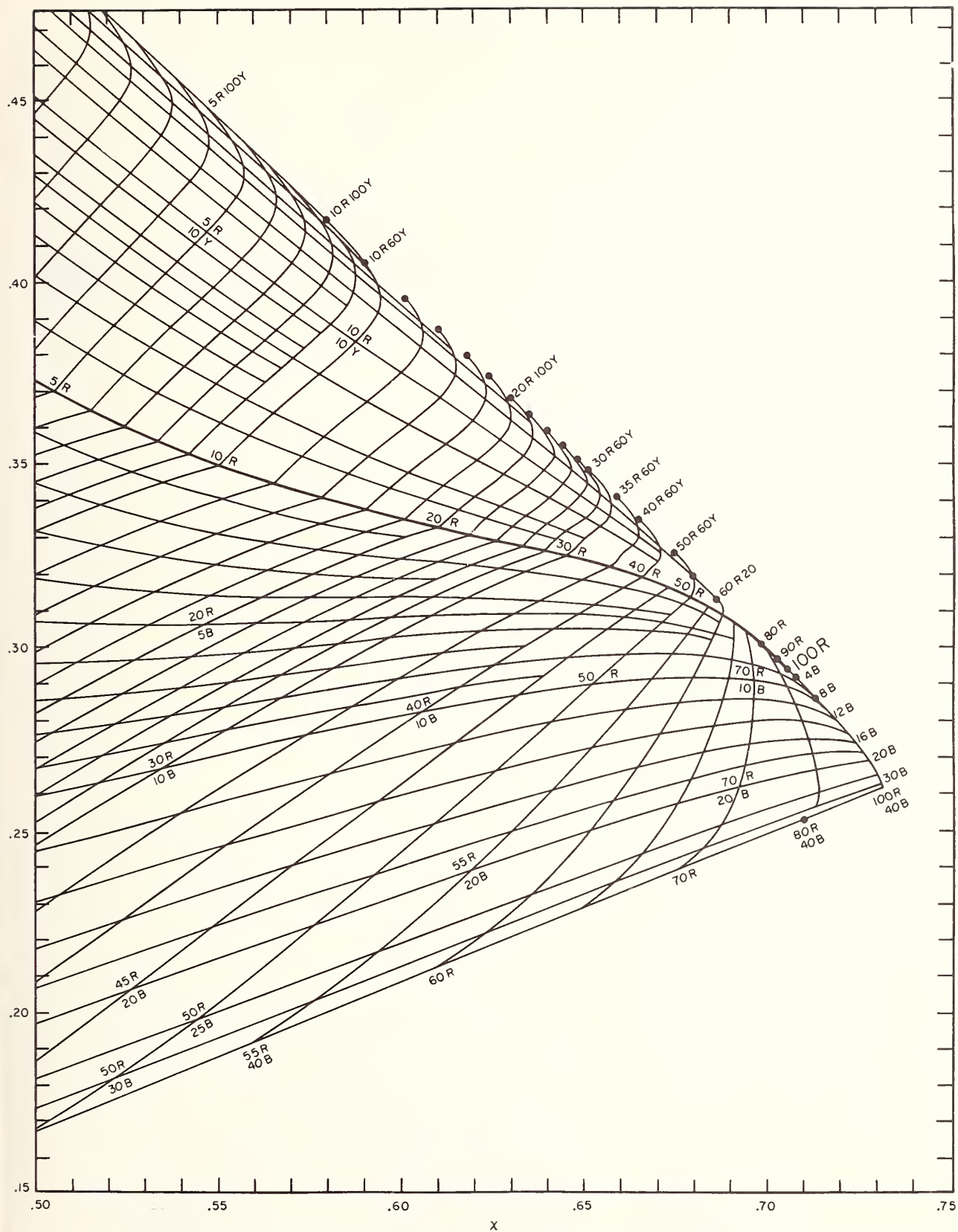


FIGURE 6. Enlarged graph of section V of figure 1.

(Paper 66C2-93)

A Universal Color Language

by Kenneth L. Kelly

Color
ENGINEERING

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A method of color designation which provides a universal means of color communication for people of varying technical and occupational backgrounds, using a newly available, basically different type of color chart.

Ever since the language of man began to develop, words or expressions have been used first to indicate and then to describe colors. Today, the color names for certain colors may vary remarkably with the geographic location of their origin, with the ethnic group using them, or with the trade or occupation involved. Many of these color designations become very limited in use and therefore are not meaningful in neighboring fields of color application. Many exotic color names are developed every year in the field of sales promotion whose intrinsic value would be lost entirely if used in the field of, say, paleontology. For instance, the color described as Happy Day by Plochere (33, 47), would be described as Ingenue by Maerz and Paul (31, 40), as Glaucous by Ridgway (16, 50), as Greige in the Descriptive

Color Names Dictionary (51) [Color Harmony Manual (13, 14, 20)], and as Stramonius by Dade (10). In simple terminology this color is a light purplish gray.

The latter color name, although having little sales appeal, would be understood by the whole public at least in a general way. Such a color language, by which users in different fields of color application can communicate with each other, has been urgently needed for many years. In fact, its lack has hindered development in art, science, and industry. Such a color language is now available as the result of a massive cooperative effort of organizations and individuals all interested in realizing the utmost benefits from color in every field of its application. To use a mechanical simile, it can be likened to a bridge over which flows color information be-

tween the salesman and customer, for example, or between the scientist and the manufacturer. At this point, let us consider this color language as made up of color designations, that is, color names or letter or numeral notations.

Method of Color Designation

The Inter-Society Color Council (30) and the National Bureau of Standards (ISCC-NBS) have cooperated for nearly 30 years in the development and application of a method of designating colors which was by definition to be accurate, and at the same time convenient and easily understood. A basic solution to this problem was published in 1939 (22) and was revised in 1955 under the title "The ISCC-NBS Method of Designating Colors and a Dictionary of Color Names," NBS Circular 553 (28).

For convenience this publication is called the Dictionary of Color Names or the Color Names Dictionary. This method is expressed in a set of color-name charts dimensioned according to the Munsell scales (34) of hue, value, and chroma. The application of this method in science and industry has been increasing rapidly. One drawback to its use had been the lack of colored samples illustrating the color names of the color-name blocks.

The ISCC-NBS method also has the ability to designate colors with different degrees of accuracy. This fact was utilized in its application to the solution of the problem of designating drug and chemical colors (1, 28, 53) for which it was originally developed. However, it was not until 1958, when the newly formed ISCC Subcommittee on Problem 23, the Expression of Historical Color Usage (18), adapted this method to the statistical expression of color trends, that this inherent ability to describe colors with different degrees of accuracy was finally tabulated in five distinct but correlated steps or levels (26). This original division into five levels has through usage been refined into six levels where each higher numbered level represents a higher degree of accuracy of color designation; or to say it in another way, it represents a finer division of the whole color solid (43) into a greater number of smaller divisions.

Centroid Colors

The ISCC-NBS colors have just become available as NBS Standard Sample Number 2106 (19) which can be obtained for \$3.00 per set from the National Bureau of Standards. These standard colors, which already give promise of many interesting applications, are in the form of one-inch square glossy paint-on-paper samples affixed to a variable gray background so that each color is on a neutral background of approximately its own lightness. Under each color is the abbreviation of the appropriate ISCC-NBS color name with the corresponding centroid or color-name block number. Duplicates of each of 251 ISCC-NBS colors can be obtained in 9 by 12 inch sheets from the Munsell Color Company, Inc., of Baltimore, Md.

The ISCC-NBS colors supply another powerful tool for use with this

six level scale of fineness of color designation. They illustrate the centers of the color-name blocks of the ISCC-NBS Method of Designating Colors (28); that is, each color illustrates a typical color described by the color name of that particular block. For the first time we can now see what the average person is supposed to mean when he uses one of the ISCC-NBS color designations or color names. However, these colors go further in usefulness. They have been manufactured to match very closely the mathematically determined center of gravity (25) or centroid of each of the color-name blocks. Their block numbers, the corresponding ISCC-NBS color designations and Munsell notations are listed in the table attached to each set of these charts. Thus the color chips of these charts can serve as color standards of pin-pointing accuracy in their own right.

Now what does this mean in dollars and cents? Suppose that, for instance, you are an official of an organization and you have been instructed to select two colors to be used as the flag colors of a university or of a new country, or as the corporate colors of a large company. By selecting your colors from among the chips of the centroid charts you will save considerable money and time since these colors have already been measured and specified as stated above. Also, separate sheets of these colors can be purchased at any time with the assurance that they will not only match each other sheet to sheet, but that repaints will also match the originals.

There is also a dividend to be had in the corresponding color-name block or centroid numbers. Since each number will always be connected with the same ISCC-NBS color designation, we can jot down this number when in the field and enter the full color designation in our notebook when we get back in the comfort of the laboratory. Later, if we wish to list the color of an item on an IBM card such as an acquisition or inventory card, we need only list the centroid number which will require three columns, 1 to 267. The remaining possibilities from 268 to 999 are open for use in listing the type of specimen, such as wood, metal, paint, textile, ceramic, jewel, glossy, iridescent and so forth. It is also possible to code the color of an item on

a data-processing card on each of our six-levels; in fact, this has already been done on a commercial basis.

There is cause for concern that a considerable number of purchasers will misinterpret the low price of \$3 (19) for these ISCC-NBS centroid color charts, as compared with other color-order systems costing from \$25 to several hundred dollars. This very low price results from the cooperative nature of the large amount of research which went into the development and production of these centroid color charts. The Inter-Society Color Council (30), the Munsell Color Company, the Tobey Color Card Company (producer of the charts), the National Bureau of Standards, and many organizations, companies and individuals have over the years given most generously of their time and talents to bring to fruition the wedding of the Color Names Dictionary and the Centroid Color Charts. This wedding had been anticipated for nearly a quarter of a century but did not come to pass until after ISCC Subcommittee 23 asked for help. Then the organizations and people listed above went back to work together. Consequently we now have the ISCC-NBS Centroid Color Charts as the Supplement to the Color Names Dictionary (28).

Color Communication

First, let us dispose of another question which may have concerned you since you read the title of this article: What has all this to do with color communication? Have you ever tried to describe the color, say, of the plumage of a newly acquired bird specimen from the Congo by using the long-used and well-known Ridgway (16, 50) Latin color names, only to realize your color language was not understood? Or did you have a similar feeling when your wife used color names from Plochere (47, 33) in describing the new color scheme the interior decorator had sold her?

Fortunately, sales promotion is not a factor in everyday color communication. Therefore, an accurate and easily understood color language which can be applied and comprehended by the largest number of people will be the most useful. As a special dividend, this color language has, for the first

time, the ability to translate the color languages of Ridgway (ornithology, geology, botany, biology), Plocher (interior decorating), Color Harmony Manual (mass marketing), the Nicker-son Color Fan (41) (flower colors) or Maerz and Paul (historical color names), into these easily understood color designations or from one of these special color languages into one of the others. It also has the ability to express itself appropriately in each of the six levels of accuracy of color designation. The rapid growth in its application is due in great part to its utilization in every phase of color designation by the Color Marketing Group (7). The Color Marketing Group is a direct outgrowth of the work of ISCC Subcommittee on Problem 23, mentioned above, and consists of representatives from many organizations in industry and commerce where color is an important part of the product.

Six-Level System

It is basic to any understanding of this six-level system of accuracy or fineness of color description, to know that in each level the whole color solid (43) is divided into a stated number of divisions, that the boundaries of each such division are accurately specified, and that all six levels are related through the ISCC-NBS Method of Designating Colors. Table I, which is summarized from NBS Technical Note 152, Coordinated Color Identifications for Industry (26), lists the six levels, the corresponding number of divisions of the color solid in each level, the type of color designation used in each level, a typical example of the designation of a color in each of the six levels, the equivalent designations of this color in the other color-order systems and a circular diagram showing the relation of the 13 and 29 hue-name-and-neutral breakdown in levels 1 and 2. Two arrows are shown, one indicating the direction for increased accuracy of color designation, the other indicating the direction for statistical expression of color trends.

The First Level

In the first level, the color solid is divided into just 13 parts, ten described by a generic hue name and

three neutrals, white, gray and black. These generic hue names are pink, red, orange, brown, yellow, olive, yellow-green, green, blue and purple; they are circled in the diagram. Thus, in this level any color may be described by using one of 13 color designations, such as a brown carpet or a yellow plastic.

The Second Level

In the second level, the color solid is divided into 29 parts, ten of the original 13 parts being further divided and assigned intermediate hue names. These intermediate hue names are yellowish pink, reddish orange, reddish brown, orange yellow, yellowish brown, olive brown, greenish yellow, olive green, yellowish green, bluish green, greenish blue, purplish blue, violet, reddish purple, purplish red and purplish pink. These intermediate hue names are shown but not circled in the diagram. The complete list of hue names with their abbreviations is found on page 4, Table I of the Color Names Dictionary. In this level, the brown carpet might better be described perhaps as a yellowish brown, and the plastic as greenish yellow.

The Third Level

In the third level, each part of the color solid described by a generic or intermediate hue name is subdivided. To each such subdivision is assigned that hue name and the appropriate modifier descriptive of its lightness and saturation, as shown in the table of modifiers on page 3 of the Color Names Dictionary. These modifiers include vivid, brilliant, strong, deep, very deep, very light, light, moderate, dark, very dark, very pale, pale, grayish, dark grayish and blackish. The part of the color solid described by the color name gray, is subdivided into three parts to each of which is assigned a modifier descriptive of its lightness; light, medium, and dark. This is the ISCC-NBS Method of Designating Colors, using the 267 color-name blocks in the Color Names Dictionary. Now we can describe our yellowish brown carpet still more accurately as, for example, a light yellowish brown (centroid 76 or color-name block 76) and the plastic as a strong greenish yellow (centroid 99).

This level of accuracy of color designation is suited for a variety of scientific and industrial uses (3, 2). These designations have been used in descriptions of drugs and chemicals (28, 1, 53), in qualitative chemical analysis (32, 46), in dermatology (4), and in the descriptions of mica (17), building materials (29, 6), soils (36) and rocks (37). The centroid charts are well suited for use in statistical studies of trends in industrial color usage, and in planning lines of merchandise intended to have coordinated colors. Besides being used to designate the colors of manufactured items, chips of the centroid color charts can also be used as the standards upon which the colors of these items are based.

The Fourth Level

Level four of our coordinated series is illustrated by the Munsell Book of Color (35, 11). There are about 1000 color samples in this book which have been prepared with great care to exemplify equally-spaced scales of hue, value, and chroma, on which the boundaries of the ISCC-NBS color-name blocks are based. That is, the Munsell color system (42, 27, 15, 44, 38, 24, 49) is a true color-appearance system and lends itself to interpolation among its colored samples. This level is retained in our system because other color-order systems can be used in it in place of the Munsell Book of Color. These other color-order systems are Maerz and Paul (first edition), Ridgway, Plocher, and the Color Harmony Manual (third edition). The number of divisions of the color solid in this level, 943 to 7056, refers to the number of colors in these color-order systems. Our light yellowish brown rug could be more accurately described by the Munsell notation 10YR 6/4 in this level.

The Fifth Level

Level five is illustrated by the interpolated Munsell book notation. Since the Munsell color system is a true color-appearance system, it is possible through comparison of a color with the Munsell scales of hue, value, and chroma to interpolate visually its Munsell notation to a tenth of a value step, to a quarter of a chroma step, and to anywhere from one hue step at

TABLE I

A	1	2	3	4	5	6
B	13	29	267	943 to 7056	About 100,000	About 5,000,000
C	Generic hue names and neutrals (circled)	All hue names and neutrals	ISCC-NBS All hue names and neutrals with modifiers (NBS C553)	Color-order systems (Collections of color standards sampling the color solid systematically)	Visually interpolated Munsell Book of Color	CIE (x, y, Y) x = 0.3 9 5 y = 0.3 8 2 Y = 3 5 . 6 or
Direction for increased fineness of color definition					Interpolated	
Direction for statistical expression of color usage (roll up)					Munsell renotation	
D	brown	yellowish brown	light yellowish brown (centroid #76)	10YR 6/4 Munsell 1548	9½ YR 6.4/4¼	9.6YR 6.4⅕/4.3
E				TCCA (9th Std.) 216 70128 TTC-595 (1st Ed) 187 F2310 H.C.C. 800 H407	M&P (1st ed) 7056 12H6 Plochere 1248 180 0 5-d Ridgway 1115 XX1X 13''b C.H.M. (3rd Ed) 943 3 gc	
A. Level of fineness of color identification B. Number of divisions of color solid C. Type of color designation D. Example of color designation E. Alternate color-order systems usable in that level						

Adapted from COORDINATED COLOR IDENTIFICATION FOR INDUSTRY, National Bureau of Standards Technical Note 152 (From Interim Report of ISCC Sub-committee on Problem 23, Expression of Historical Color Usage. Also used in Color Fair of Color Marketing Group).

chroma /2 to as little as one-quarter of a hue step at chroma /10 and above. It is estimated that under these conditions, this accuracy is equivalent to the division of the color solid into about 100,000 divisions. On this basis, the color of our light yellowish brown rug can now be very accurately specified as $9\frac{1}{2}$ YR 6. $4/4\frac{1}{4}$.

The Sixth Level

Level six, the level in which the greatest accuracy of color identification is possible, is illustrated by two methods of color designation which are related in our system. The basic method of color designation is that known as the CIE method (21, 23, 45) in which the color is specified in terms of chromaticity coordinates x , y and

daylight reflectance Y . This degree of accuracy is realizable only through the measurement of the color with a spectrophotometer, and is equivalent to the division of the color solid into about 5,000,000 divisions.

The second method of color designation in level six is illustrated by the interpolated Munsell notation (38) in which the hue may be interpolated to a tenth of a hue step, the value to a twentieth of a value step and the chroma to a tenth of a chroma step. The Munsell AAA tolerance, that is, the accuracy within which a repaint of a Munsell standard must color match its predecessor, is based on this level of accuracy of color identification. Now, the color of this light yellowish brown rug can be specified with the greatest accuracy as 9.6YR

6.4 \pm 4.3, or $x = 0.395$, $y = 0.382$ and $Y = 35.6\%$. Further details on the different levels of fineness of color designation will be found in NBS Technical Note 152.

From the descriptions of levels four and six, it can be seen that there is a considerable variation of fineness of designation for a color in each of these levels. In level four, for instance, the Munsell Book of Color contains more than 1000 color samples, but this figure can be multiplied by approximately 100 through the use of interpolation alone, while the other color-order systems which can also be used in this level contain from 943 to 7056 color samples. In level six, the variation in fineness of color designation is accomplished by increasing the number of decimal places used in the numeri-

cal notation. This flexibility is most useful and fortunate in these levels since they will be used more and more in industry and science in the specification of colors and color differences.

Maximum Accuracy

Now, let us summarize the six levels of fineness in reverse order. For maximum accuracy, a color should be measured by a spectrophotometer and the results expressed numerically either in terms of the CIE method or a Munsell renotation (level six). If this accuracy is not required, a carefully interpolated Munsell book notation may suffice (level five). When a color notation is given in or desired to be in, either Munsell or in one of the other color-order systems mentioned above, level four is used. If one wants a quick understandable color designation such as that of a moist soil sample before its color changes due to drying, a description in terms of the ISCC-NBS color names can be read from the centroid charts and recorded (level three). There are other collections of color samples, smaller than the Munsell Book of Color which can be used in level three; these are the ninth edition of the Standard Color Card (52, 48), the Horticultural Colour Chart (5, 39), the Nickerson Color Fan (41), Federal Specification TTC-595, (first edition) (12), the molded urea (8) and polystyrene plastic colors (9), the Rock Color Chart (37), and the Soil Color Chart (36).

If one is interested in the color of a thin section of a sample under the microscope (28) where the thickness of the section is not easily controlled, the hue name of the color in question is usually sufficient (level two). Level one is useful when a quick approximate designation of a color is sufficient or when color changes or trends are being plotted.

To assist in coordinating the color designations derived from these many collections of color samples, there are listed in the third part of NBS Circular 553, the Color Names Dictionary, all of the color names used in these color-order systems with their equivalent ISCC-NBS color designations. In the second part of Circular 553, all of these color names are again listed, each under its equivalent ISCC-

NBS color name. Opposite each of these color names in part two is listed the numerical or letter designation used in each collection to identify the illustrating color sample. Thus, it is possible to get from part two, a color name in one color-order system which is synonymous or nearly synonymous with a color name in one of the other color-order systems. Here it should be emphasized that the centroid numbers, besides being a shorthand designation of the centroids, are most useful in that they are used in, and tie together, the color names of the dictionary part of Circular 553 (part three), the synonyms and near-synonyms (part two), the color-name blocks in the color-name charts in the first part and the centroid colors themselves.

Conclusion

In this article, attention has been called to a method of designating colors in simple, easily understood but accurately defined color designations in definite, correlated degrees of accuracy of color designation. These designations, to use the inclusive name, all fit together as parts of a master plan and form of themselves a universal color language either spoken or written. They are not time bound but are founded upon basic measurements and observations over many years. It is a method or language that utilizes all of the well-known color-order systems and relates them. It contains of itself only one small very specialized set of color standards, relying on the other color-order systems for the larger sets of color samples. For the highest accuracy of color designation, it uses the fundamental CIE numerical system or the Munsell renotation system, which interestingly enough can be used separate from the Munsell color samples.

Actually this method welds together into a unified system all of the best known tools of color measurement and designation, some of which have been in use for many years. It is really this unity with its unlimited flexibility and applicability in so many fields of color endeavor, that makes use of the system so rewarding. The availability of the ISCC-NBS centroid color standards supplies the last missing link in our now complete universal color language.

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Three years later, Mr. Kelly and Dr. Deane B. Judd prepared research paper RP1239, Method of Designating Colors, which was approved by the ISCC for use in pharmaceutical literature. In 1955, again with Dr. Judd, he prepared the ISCC-NBS Method of Designating Colors and a Dictionary of Color Names. Three years later he authored the research paper, Central Notations for the Revised ISCC-NBS Color-Name Blocks.

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UNITED STATES DEPARTMENT OF COMMERCE

Alexander B. Trowbridge, *Acting Secretary*

NATIONAL BUREAU OF STANDARDS · A. V. Astin, *Director*

Colors of Signal Lights

Their Selection, Definition, Measurement, Production,
and Use

F. C. Breckenridge

Institute for Basic Standards

National Bureau of Standards

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Foreword

The safety of the traveler, whether by rail, highway, air, or sea, depends on the reliability with which signal lights of various colors are seen and their meaning correctly interpreted. In August 1964, the United States Standard for the Colors of Signal Lights, developed under the auspices of the U.S. National Committee on Color for Signal Lights, was issued as National Bureau of Standards Handbook 95. The colors selected, the method of their specification, and the methods of testing signal ware for compliance with the specifications were all based upon technical information, much of it unpublished.

It is the purpose of this Monograph to make available a complete analysis of the problem of signaling by lights of various colors, combined with a detailed reproduction of the basic data used to derive the solution of this problem given in Handbook 95. An effort has been made to present the material in such a manner as to facilitate comparison between the technically established information and the requirements.

This Monograph is designed as a guide to the engineer who must implement the United States Standard, the psychophysicist who is concerned with the meaningful presentation of research in this field, and the administrator who is responsible for the use of signal lights. Those who seek, in years to come, to improve on the current signaling systems, should find their task simplified by the availability of this Monograph.

A. V. ASTIN, *Director.*

Preface

The work of many investigators, authors, and colleagues has provided the material from which this Monograph has been prepared so that it is not possible to call each by name, but the assistance derived from many discussions with Deane B. Judd have contributed so much to the analysis of the problem that the writer desires especially to acknowledge his indebtedness to him and to express his appreciation for his collaboration. Other highly valuable contributions to the work have come out of correspondence with J. G. Holmes of Great Britain and from Lorenzo Plaza, of the Institute of Optics, Madrid, Spain, who, before computer codes were available, very kindly carried out several of the more laborious tasks of transforming diagrams, which had been available only in the C.I.E. system, into the RUCS system. Other contributions have been acknowledged in connection with the figures and tables. Finally, I would like to thank Mrs. R. L. Meyers for assembling the references, Mr. R. E. Lee, Jr., for assistance with many of the computations and drawings, and especially Mrs. Ellen B. Way for her devotion to the task of completing the myriad of details that are essential to the final correction and completion of any manuscript.

F. C. BRECKENRIDGE.

Colors of Signal Lights: Their Selection, Definition, Measurement, Production, and Use

F. C. Breckenridge

This Monograph is intended to serve as a reference work for all those concerned with the selection, specifications, and use of signal-light colors. It discusses the nature of the problem, and the mathematical representation, recognition, production, control, and use of such colors. The characteristics of different types of chromaticity boundaries and the purpose and effect of the requirement for similarity of chromaticity characteristics are given special consideration. The treatment is varied according to the intended use. The discussion of the control of colors and the section on the use of colors are nontechnical, whereas the section on the production of signal colors is designed for the colorimetrist who is faced with the problem of selecting limit filters or drafting a specification.

1. Introduction

All forms of organized transportation depend to a very considerable extent upon signal lights of different colors for safe operation. Whether one travels by aircraft or railroad, automobile or ocean liner, his safety requires the correct interpretation of signal colors. Radio, radar, and other electronic aids have supplemented the signal lights in some forms of transportation, but in no form of transportation have the lights been abandoned. Other methods of identifying lights exist, such as flashing them and arranging them in distinctive shapes and configuration. These are also used by the transportation services, but they do not eliminate the need for color identification. On the other hand, the increasing density of traffic is constantly increasing the dependence upon signal lights. This situation seems to warrant additional attention to the speci-

fication and control of their colors.

This Monograph, which is an outgrowth of an informal publication,¹ has been prepared to furnish those interested in specifications for the colors of light signals with the technical information necessary for preparing and appraising such specifications. It is intended to provide a basis for interpreting the recommendations of the International Commission on Illumination (Commission Internationale de l'Eclairage, or C.I.E.) which were set forth in the Proceedings of the Thirteenth Session, held in Zürich in 1955 [CIE 1955]² and later published as C.I.E. Publication No. 2 [1959], *Colours of Light Signals*. It also explains the background of the U.S. Standard [NBS 1964]. Most of the C.I.E. recommendations have been incorporated into this Monograph at appropriate places.

2. Outline of Problem

2.1. The Three Essential Steps

As one examines the specifications and recommendations for the colors of light signals, he finds a confusing assortment of requirements. Some of the specifications are lengthy with a diversity of provisions; some are relatively brief. Some depend entirely on numerical values, while others make reference to standard filters. Some of the provisions are directly related to what the observer sees. Other provisions refer only to items of equipment. All of these recommendations and specifications aim at similar final results, namely, the insurance that those who use the signal lights will be shown the most reliably distinctive colors practicable for their particular requirements.

To understand how the various requirements and provisions contribute to this end, it is necessary to recognize that the effective control of the colors of signal lights always involves three steps: the selection of the colors to be used, the selection of color standards,³ and the application of those standards to control the colors of the signals. All three steps may not be immediately evident. The selection of the colors and the standards may, for example, be done at the same time. In other cases the controlling agency may define the acceptable

¹ NBS Letter Circular 484, no longer available.

² Brackets enclosing a date indicate a reference listed in Appendix 1.

³ "standard(s)" with lower case "s" in this monograph refers to a filter in combination with a stipulated light source. When a publication is meant it will be called a "Standard" with capital "S".

colors and leave the selection of the standards and their application to the manufacturers who produce the equipment. In general the three steps are accomplished by basic chromaticity definitions, master specifications for color standards, and procurement specifications, although these may appear under other titles or may be combined into a single specification.

If instruments suitable for measuring the signal colors directly in colorimetric terms were available, it might be possible to eliminate the need for color standards selected for specific applications, but such instruments are not as yet available. The possible use of such instruments is discussed in sec. 9, Instrumentation.

2.2. Basic Chromaticity Definitions

Basic chromaticity definitions are statements of chromaticity limits within which a signal is acceptable for use in a system of signal colors. Such a system may be established by a government agency, an association of state or local authorities, or by a national or international technical body. For example, two very similar international systems have been adopted for signal-light colors, the recommendations of the International Commission on Illumination [1955, 1959] and the Standard of the International Civil Aviation Organization [1964].

Most specifications for signal-light colors in use in the United States contain statements of basic chromaticity requirements. The most complete systems are those contained in part I of the "U.S. Standard for the Colors of Signal Lights," the two government specifications for aeronautical lights, and the two specifications of the Association of American Railroads, Signal Section. These are all listed in Appendix 3 to this Monograph.

Other specifications used in the United States have paragraphs devoted to basic chromaticity requirements. Some of these, like the two railroad specifications mentioned above, are valid in Canada as well. These also are listed in Appendix 3.

For basic definitions, mathematically expressed chromaticity limits applicable to the emitted signal are used. Such limits are reproducible at any time and place if competent personnel and suitable instruments are available for their establishment. This makes them especially adaptable for international agreements and agreements between different agencies within a single country, since such agreements are expected to remain in force over long periods and may be used by many different laboratories. Basic definitions are also suitable for governmental regulations because such definitions apply to light signals as shown and allow the maximum freedom of means of accomplishment.

In principle the basic chromaticity definitions are the foundations upon which detailed specifications are built. For this reason it is desirable that these definitions be based entirely upon established prin-

ciples of color recognition. The necessary researches, however, have turned out to be so laborious and expensive that thus far it has been necessary to base these definitions upon experience interpreted in the light of the research results. These will be discussed in section 4, Recognition of Signal Colors.

2.3. Master Specifications for Color Standards

To bridge the gap between the basic definitions and the purchase specifications, the U.S. Standard for the Colors of Signal Lights includes master specifications to control physical standards so that equipment purchased on the basis of such standards will provide signals in accordance with the basic definitions. These master specifications for standards for the colors of light signals are specifications giving chromaticity tolerances and other requirements for filters to be used in combination with specified light sources to constitute standards for the control of equipment, or the light-filtering components of equipment, to insure that such equipment will give a satisfactory color when placed in service.⁴ If there are basic chromaticity definitions which are applicable to a type of signal lights, then the requirements for standards must be such as to insure that the colors of those lights will conform to those definitions. If there are no appropriate basic definitions, standards are chosen on the basis of experience with other types of signal lights.

Historically, the relationship of the standards and the basic chromaticity definitions has generally been the reverse of the ideal one described in the last section. In most cases the standards were chosen first by those responsible for the colors of the lights and the definitions were later devised to describe what was giving satisfactory service.

The master specifications are prepared for the use of laboratories, which may be presumed to have suitable spectrophotometric and colorimetric equipment as well as light sources calibrated for color temperature. They are applicable to flat filters, the spectral transmittances of which are readily measured. Such specifications, therefore, have generally been stated as mathematically expressed chromaticity limits for the filter-source combinations which are to serve as physical standards; but spectrophotometry of adequate accuracy is difficult and costly, whereas the required accuracy can be obtained with the expenditure of much less time by differential colorimetry. This makes it

⁴ Another approach readily occurs to the spectrophotometrist. Two spectrophotometric curves may be drawn with the requirement that the filters used in lights have spectrophotometric transmissions that remain between these limits for all wavelengths between specified limits. For an example of this method see Otyřoký [1937]. This method presents a difficulty arising from the nature of the filters used in practical equipment. Very few signal light filters are in such a form that they can be spectrophotometered on American spectrophotometers. To sort acceptable from non-acceptable signal glasses by this method on a practical production scale would require an elaborate automatic spectrophotometer with an electronic evaluator which does not exist in this country. Furthermore, this approach unnecessarily restricts the contractor since a variety of spectrophotometric curves can produce the same chromaticity and all may be acceptable.

simpler in some instances to select primary⁵ standards, and control duplicates by setting limits for the maximum colorimetric differences. No examples of specifications for limit standards issued apart from basic chromaticity definitions or procurement specifications are known. Part II of the U.S. Standard for Colors of Signal Lights is designed to be a separate master specification for physical standards to control the colors of light signals, but at present it is available only in the complete Standard, NBS Handbook 95 [1964]. The most complete provisions for such standards to be found in specifications previously in use are those in Federal Standard No. 3 [GSA 1951],⁶ and those in the appendix of each of the two specifications for signal light colors issued by the Signal Section of the Association of American Railroads [1960, 1962]. These specifications contain chromaticity tolerances and transmittance requirements for filters to be used as limit standards, but they leave to the manufacturer the responsibility of seeing that the filters are satisfactory in other respects.

2.4. Procurement Specification

As the name implies, procurement specifications are specifications governing the purchase of equipment. Since they are to be used for distinguishing between satisfactory items, which are to be paid for, and unsatisfactory items, which constitute a loss to one of the parties to the contract, they should be as definite as it is practicable to make them. Most of those in use in the United States are based on standards. If such standards are not stipulated in the specification, whoever is responsible for compliance should provide himself with standards unless the item and available instruments are such that it is practicable to measure the chromaticity of *each* item to make certain that it is satisfactory. This could mean measuring the spectral transmittance of each item with a spectrophotometer, computing the corresponding chromaticity coordinates from the spectral transmittances, and checking these mathematically for compliance with the equations of basic chromaticity definitions. A possible compromise might be the measurement of a few representative items with a spectrophotometer and the acceptance or rejection of the remaining items by comparison with the measured items.

3. Mathematical Representation of Colors

3.1. Three Attributes of Color

Several references have been made to the mathematical expression of chromaticity and as the discussion proceeds there will be occasion to repre-

In this case the measured items become standards for the purpose of checking the unmeasured ones.

If suitable colorimeters were available, it might be feasible to use mathematical limits for procurement purposes. Such a procedure, however, would involve the necessity for maintaining accurate calibrations, and the use of more elaborate and expensive equipment than is required when standard filters are used. Moreover, it is not clear that the use of mathematical limits would provide any economies or other advantages to offset the difficulties that would be involved in their use for practical inspection procedures.

Examples of procurement specifications are well known but it should be pointed out that only a part of such specification as the Military Specification for Colors of Aeronautical Lights and Lighting Equipment [MIL SPEC 1963] or one of the railroad specifications mentioned in sec. 2.3 [AAR-1960, 1962] is to be considered as a procurement specification in the sense in which the term is being used in this publication. Generally the supplier is responsible only for the production of signal-light equipment and not for the manner in which it is used. The manner of use, however, may affect the color of the signal, as when a wrong lamp is installed in signaling equipment or a wrong voltage is applied to a signal lamp. Procurement specifications, therefore, should apply to the equipment, and not the light signal, unless the procurement contract includes the maintenance and operation of the equipment.

2.5. Regulatory Requirements

The application of legal requirements such as safety regulations generally involves all of the three steps of controlling signal-light colors that have been discussed above. The initial laws or regulations are generally written in the form of chromaticity definitions applying to the light emitted as signals, but if the field inspectors check the colors they must carry out their work by testing the signal equipment in comparison with physical standards. This is generally most easily done at the plant of the equipment manufacturer. Between the law and the inspector comes the colorimetric laboratory, which provides the necessary physical standards, referees doubtful cases, and tests the signal equipment when unusual problems are involved.

sent chromaticities in diagrams. To understand the equations and diagrams, it is necessary to realize that the perception of color⁷ is essentially three-dimensional, that is, three numbers are required to describe a color. The most familiar of these three attributes is hue. The gradual variation from red to violet from one side to the other of a

⁵ The term "primary" is used here in a general sense to differentiate between two types of filters, each of which is composed of more than one class. For a general discussion of the naming of classes of standards see McNish [1958]. For a classification directly applying to signal lights see the "U.S. Standard for Colors of Signal Lights," NBS Handbook 95 [1964], Part I, Definitions 2.12 to 2.19 inclusive.

⁶ Sec. 5.3.2. and table IV.

⁷ In this Monograph the word *color* is used as a general word equally applicable to the stimulus and the perception.

rainbow is familiar to everyone. The same transition is even more clearly discernible in continuous spectra. This is the first attribute of color. When hues of all wavelengths are combined in the necessary proportions, the result is white light. The rainbow colors, while differing from each other in hue, differ from the more vivid colors which can be obtained in spectra because there is usually some white light shining through the rainbow and the resulting chromaticity is paler than the rainbow itself. This variation from pure colors to white constitutes the second attribute of color and is called saturation. The more the color departs from white or gray, the greater the saturation. The third attribute is the quantitative aspect to which one refers when he observes that one spectrum is brighter than another. While the most common name for this attribute is *brightness*, two other terms are also used, *lightness* and *brilliance*. Lightness refers to variations from black to white. It is perceived by comparing unknown objects or surfaces with each other in an environment that contains familiar elements. It is applicable to

opaque surfaces and transparent volumes but not to signal lights, which are always seen either as self-luminous areas or as point sources. Brilliance is applicable only to point sources and is, therefore, used mostly for signal lights and stars.

Thus far we have described the attributes of color in psychological terms.⁸ Such terms, however, cannot be numerically evaluated. The radiant energy by means of which the signal lights and surfaces are seen could, in theory at least, be described by giving its energy distribution with respect to wavelength in each case, but in practice this has turned out to be too cumbersome for use. To avoid this difficulty, psychophysical terms and relationships have been developed. These terms correlate with the physical terms on the one hand and with the psychological terms on the other. This correlation is shown in table 1. The correlation of the psychophysical with the physical is accomplished by deriving the psychophysical values from the physical quantities through the medium

⁸ For a more extensive treatment of this aspect of the subject, the reader is referred to ASTM Special Technical Publication No. 297, Deane B. Judd [1961].

TABLE 1. *Correlation of terms*

The table below shows the relationship of photometric and colorimetric terms used in signal lighting (upper sections) to similar terms used in describing transmitted light (third section) and reflected light (lowest section). Each line shows the correlation between the terms of physics, psychophysics, and psychology.

Application	Physics	Psychophysics (color stimulus)		Psychology (color perception)	
		Quantitative	Qualitative	Magnitude	Quality
Point source, descriptive	Spectral intensity distribution $(P_\lambda/\omega)/P_{\lambda_0}/\omega$	Luminous intensity $\int (K_\lambda P_\lambda/\omega) d\lambda$	Chromaticity = Dominant wave- length and purity $\int (V_\lambda P_\lambda/\omega) d\lambda^*$	Brilliance or magnitude	Chromaticness = Hue and saturation
Point source, quality					
Point source, quantity	Radiant intensity $\int (P_\lambda/\omega) d\lambda$	Illuminance $\int (K_\lambda P_\lambda/A) d\lambda$			
Point source, visibility	Irradiance $\int (P_\lambda/A) d\lambda$				
Self luminous source, descriptive	Spectral flux distribution P_λ/P_{λ_0}	Luminous flux $\int K_\lambda P_\lambda d\lambda$	Chromaticity = Dominant wave- length and purity $\int V_\lambda P_\lambda d\lambda^*$	Brightness	Chromaticness = Hue and saturation
Self luminous, quality					
Self luminous, quantity	Radiant flux $\int P_\lambda d\lambda$	Luminance (Photometric brightness) $\int (K_\lambda P_\lambda/\omega) d\lambda/A$			
Self luminous, quantity/area	Radiance $\int (P_\lambda/\omega) d\lambda/A$				

TABLE 1. Correlation of terms—Continued

Application	Physics	Psychophysics (color stimulus)		Psychology (color perception)		
		Quantitive	Qualitative	Magnitude	Quality	
Transparent volumes (Filters), descriptive Filter and source, quality	Spectral transmittance $(P_2/P_1)_\lambda$		Chromaticity = Dominant wave- length and purity $\int V_\lambda \tau_\lambda P_\lambda d\lambda^*$		Chromaticness = Hue and saturation	
Filter and source, quantity	Transmitted radiant flux $\int \tau_\lambda P_\lambda d\lambda$	Transmitted luminous flux $\int K_\lambda \tau_\lambda P_\lambda d\lambda$		Clear to dense		
Filter and source, transmittance	Radiant transmittance $\int \tau_\lambda P_\lambda d\lambda / \int P_\lambda d\lambda$	Luminous transmittance $\int K_\lambda \tau_\lambda P_\lambda d\lambda / \int K_\lambda P_\lambda d\lambda$				
Reflecting surfaces, descriptive Surface and source, quality	Spectral reflectance $(P_2/P_1)_\lambda$		Chromaticity = Dominant wave- length and purity $\int V_\lambda R_\lambda P_\lambda d\lambda^*$		Chromaticness = Hue and saturation	
Surface and source, quantity	Reflected radiant flux $\int R_\lambda P_\lambda d\lambda$	Reflected luminous flux $\int K_\lambda R_\lambda P_\lambda d\lambda$		Lightness (black to white)		
Surface and source, reflectance	Radiant reflectance $\int R_\lambda P_\lambda d\lambda / \int P_\lambda d\lambda$	Luminous reflectance $\int K_\lambda R_\lambda P_\lambda d\lambda / \int K_\lambda P_\lambda d\lambda$				
Surface and source, quantity per unit area	Radiance $\int (R_\lambda P_\lambda / \omega) d\lambda / A$	Luminance $\int (K_\lambda R_\lambda P_\lambda / \omega) d\lambda / A$		Brightness (invisible to dazzling)		

*Symbolic formula, actual mathematical relationship is too complex for inclusion in table.

of the C.I.E. Standard Observer, which is numerically represented by three functions of wavelength known as the C.I.E. tristimulus distribution functions.

The symbols in table 1 have the following meanings:

P_λ radiant energy per second of wavelength λ ,
 ω solid angle,

K_λ luminosity function for relative effect of different wavelengths,

V_λ chromaticity function, a three component function relating radiant energy to visual quality,

τ_λ transmittance for wavelength λ ,

R_λ reflectance for wavelength λ .

Table 1 is composed of four sections applying respectively to point-source signal lights, self-luminous sources, transparent volumes and opaque

surfaces. "Self-luminous sources", for the purposes of this correlation, must be considered as including all lighting units containing optical systems or diffusing surfaces which cause the observer to see an appreciable bright area. This is the usual case with traffic signal lights and intervehicular signals. There is no sharp line of division between point sources and self-luminous sources. "Transparent volumes" are assumed to have nondiffusing surfaces and against black backgrounds they are seen by their surface reflections, some of which acquire color by being transmitted through the volume. Opaque surfaces are seen by light reflected from them.

The derivations of the psychophysical correlates from the corresponding physical quantities, by the inclusion of the luminosity function for quantitative

purposes and all three of the standard observer functions for qualitative purposes, are shown by the formulas following the terms. For qualitative purposes, that is, chromaticity computations, it is not necessary to know the actual energy distribution, but the relative energy distribution of the source must be known.

Although the psychophysical values are numerically derived from physical quantities, their definitions were constructed to correlate with the psychological terms. Thus the hues of the spectrum are naturally associated with the wavelengths of the radiations which give rise to them. Chromaticities which can be matched by a mixture of white light with monochromatic light of pure spectrum chromaticity are associated with the wavelengths of the spectral components, and these wavelengths are called the dominant wavelengths of the mixtures. All of the chromaticities now being used in signal lighting can be matched by such combinations of spectrum light and white light. Chromaticities exist, however, which can not be matched by any combination of monochromatic light and white light. In such cases it is possible to match white light by adding monochromatic light of suitable wavelength to the given sample. In this case the wavelength of the monochromatic light is called the complementary wavelength. In either case the ratio of the monochromatic light to the total light in the matching mixture is called the purity. As in the case of the saturation, the purity increases as the proportion of spectral light increases.

While the associations pointed out above are the basis for the correlations given in table 1, it must not be supposed that the hue of the response is determined solely by the qualitative aspects of the light from a source, or object, being viewed. In addition, the apparent size of a source, the illuminance received from a small source or the luminance of a larger one, and the characteristics of the environment in any case have important influences on the hue perceived. On the other hand, even when these factors remain the same, a change in the purity, that is, a change in the ratio of monochromatic to white light in the mixture received from a source, will also cause the observer to select a different monochromatic light as being the same in hue as the mixture.

Of the three psychological terms for the intensity attribute, *lightness*, *brightness*, and *brilliance*, only the latter two are applicable to signal lights. The choice of these is determined by the concentration of flux on the retina of the eye. If the light from the target is spread over an appreciable area of the retina, the signal is evaluated for brightness. If, however, the flux from the target appears to come from a point, its magnitude is evaluated for brilliance. As indicated in table 1, brightness correlates with luminance, which is sometimes called photometric brightness, while brilliance correlates with illuminance.

Colors may be defined by any three numbers which give their hue, saturation, and brightness or brilliance as, for example, the dominant wavelength, purity, and transmittance for the combination of a filter and a light source. These might be considered as cylindrical coordinates in which the dominant wavelength is plotted as the angle, the purity as a fraction of a fixed radius, and the transmittance as a distance perpendicular to the plane of the polar diagram. Experience has shown, however, that another form of diagram is more useful. This type of diagram results from the fact that it is possible to so arrange chromaticities that the chromaticity represented by any point on a line joining the points representing two given chromaticities can be produced by a certain mixture of lights having the given chromaticities. These diagrams are sometimes called color mixture diagrams because they are based upon this property. There are an infinite number of coordinate systems, derivable from each other by projective transformations, all of which give rise to diagrams having this characteristic. Two of these will be used in this circular, namely, the C.I.E. system and the RUCS system.

3.2. C.I.E. Coordinate System

The C.I.E. system of chromaticity coordinates was adopted as a device for the numerical description of chromaticities by the International Commission on Illumination at its meetings in 1931 [C.I.E., 1931; Judd, 1933]. Since that date it has been regularly used as the standard means of specifying chromaticities. This coordinate system rests on the C.I.E. Standard Observer of 1931 [Judd, 1933], which is a set of three functions of wavelength each expressed as a set of eighty-one values correlated with the 5 $m\mu$ intervals from 0.380 μ to 0.780 μ . One of these functions is the relative luminosity function for cone vision, that is, a set of values proportional to the relative contribution which light of different wavelengths makes to the brightness of a surface under daylight conditions. The other two functions which complete the standard observer express the results of color matching researches. An infinite number of coordinate systems are consistent with the standard observer, but any one of these can be derived from any other by a linear transformation, which in turn requires the assignment of eight coordinates to completely determine it. As defining coordinates, the C.I.E. selected values for the x and y coordinates for the spectrum colors of wavelengths 0.4358, 0.5461, and 0.7000 microns and C.I.E. standard light source B . The first two spectrum colors are to be found in the spectrum of mercury; the third can be produced by an incandescent lamp with a suitable red filter. Source B represents mean noon sunlight but it is defined in terms of a lamp and filter. By choosing the values in this way the calculation of the luminous transmittance or reflectance of a sample could be found incidental to the calculation of the chroma-

ticity coordinates of the sample. The selection of the above cardinal stimuli also resulted in a set of primaries (X , Y , Z) which corresponds under usual conditions of observation to colors which may be considered as unitary red, green, and blue.

3.3. RUCS Coordinate System

The Rectangular-Uniform-Chromaticity-Scale Coordinate system was first proposed by Breckenridge and Schaub in 1937 [1939].⁹ At the time when the C.I.E. coordinate system was adopted, little was known about the perceptibility of chromaticity differences. At that time it seemed reasonable to place the white point centrally with respect to the spectrum locus. It was believed that this would avoid the exaggerated extension or compression of any parts of the color field, that is, it would give approximately uniform chromaticity scales. Later research, however, showed that the spacing of the chromaticities in the C.I.E. system is far from uniform. The purpose of plotting chromaticities on a diagram is to assist in the visualization of the relationship between different chromaticities. The distortions of the C.I.E. diagram tend to defeat the purpose of such visual representations of chromaticity and a number of uniform chromaticity scale diagrams have been developed to reduce the distortions.

The limitations of the C.I.E. and RUCS systems are discussed in sec. 3.6. As compared with other available uniform chromaticity systems the RUCS has four advantages for the present use. Its derivation is based on sources of small angular size (2° or less), a transformation table is available for converting C.I.E. values into it, it has already been used in two international publications and there is a convenient relationship between arrangement of chromaticities and our ordinary conceptions of colors.

3.4. Chromaticity Diagrams

Each coordinate system gives rise to a characteristic chromaticity diagram in which every chromaticity is represented by a point. Figures 3-1 and 3-2 are C.I.E. and RUCS diagrams. In each of these the curved boundary represents colors such as those seen in spectra. It is called the spectrum locus and the numbers along the boundary are the wavelengths of the corresponding light. As already stated in sec. 3.1, to the observer with normal color vision these spectrum colors differ from each other in hue. If white light, which generally contains light of all the spectrum colors, is added to a single spectrum color, a new chromaticity results which does not match any color in the spectrum. Such chromaticities are represented by points inside the spectrum locus. The more white added, the fur-

ther from the boundary will lie the point corresponding to the resulting chromaticity.

It is a basic property of both of these chromaticity diagrams that when two samples of light, which are represented by two points in the diagram, are mixed they produce a color which is represented by some point on the straight line joining the other two points. From this it follows that the points on the straight segment of the boundary connecting the ends of the spectrum locus represent mixtures of the extreme red and extreme violet. The usual purples lie close to this straight boundary at the short wave end, while near the other end will be found purplish-red chromaticities.

The curved line within the figure is called the Planckian locus. It represents the chromaticities of blackbodies at different temperatures. In practical applications, it is accepted as representing the chromaticities of incandescent bodies such as lamp filaments. The numbers along this curve on the C.I.E. diagram, and the corresponding numbers in the margin of the RUCS diagram, indicate the color temperature in degrees Kelvin, sometimes called degrees absolute, corresponding to the marked points.

3.5. The z Coordinate

The initial computation of the C.I.E. coordinates gives three values one of which is an intensity factor, usually in the case of signal-light filters the transmittance. Since only two independent values can be plotted in a plane diagram, it is customary to reduce the three values to three proportional numbers totaling 1. This eliminates the intensity attribute and results in three coordinates which represent a point in the plane through the three points on the axes at $x=1$, $y=1$, and $z=1$, respectively. As may be seen from figure 3-3, the C.I.E. diagram shown in figure 3-1 is the projection on the x,y plane of the true x,y,z diagram.

3.6. Relative Distortion of Diagrams

A system of numerical values derivable from physical measurements has great advantages over names as a means of specifying colors in that the values can be made independent of the observer. Even a colorblind individual can determine the correct color coordinates if he is provided with suitable instrumentation. Moreover, a color once measured can be reproduced at another time or place if its coordinates in any well-designed system are known. The C.I.E. coordinate system served this purpose quite well, and since it was the first system that appeared likely to be generally utilized, it rapidly came into general use. The only purpose served by plotting the coordinates of chromaticities, however, is to visualize the chromaticity relationships. For this purpose every distortion means a false visualization. The distortions of the C.I.E. system have already been mentioned in sec. 3.3. It

⁹The coordinates of the RUCS system may be computed directly from tables, derived by a linear transformation, or closely approximated from a transformation table. The constants for the first two methods are given in the referenced paper. For the tables see Breckenridge [1948] in the list of references. It is also possible to obtain the uniform spacing, but not the other advantages described in sec. 3.7 by plotting C.I.E. coordinates on a distorted C.I.E. diagram as proposed by Holmes [1940].

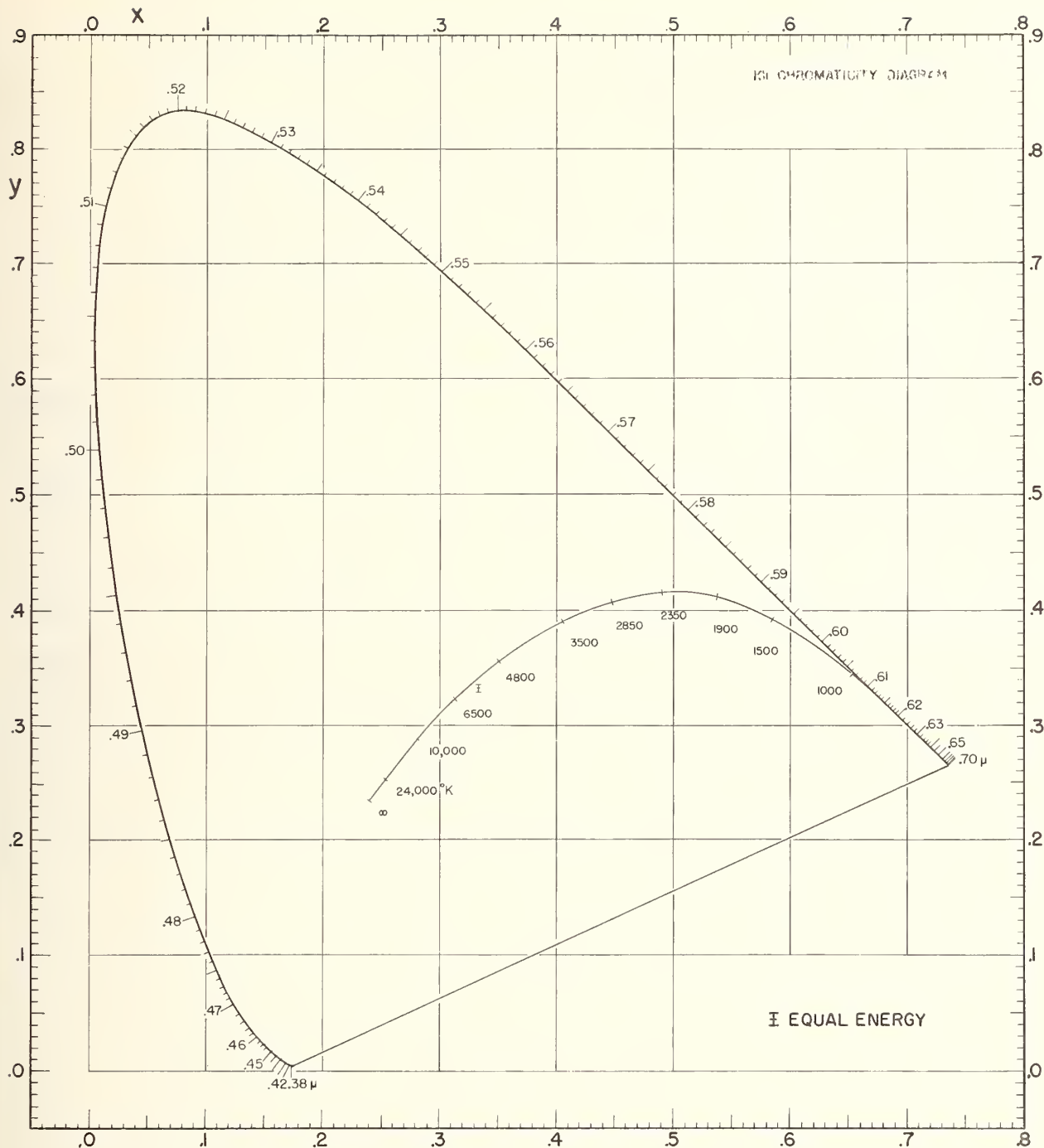


FIGURE 3-1. C.I.E. chromaticity diagram.

This diagram shows the spectrum locus and Planckian locus derived from the standard observer defined by the International Commission on Illumination in 1931 [C.I.E. 1931].

remains, however, to consider in this section the extent of those distortions.

The first attempt to estimate the extent of the distortions of the C.I.E. diagram was made by D. B. Judd in 1934 [1935]. Judd transformed the C.I.E. diagram into a system of trilinear coordinates de-

signed to give uniformity of spacing in accordance with the best information then available. He then filled the area of real chromaticities on this Uniform Chromaticity Scale diagram with a group of equal tangent circles and transformed these back into the C.I.E. system. The result was the diagram of ellipses shown in figure 3-4 [Judd 1936].

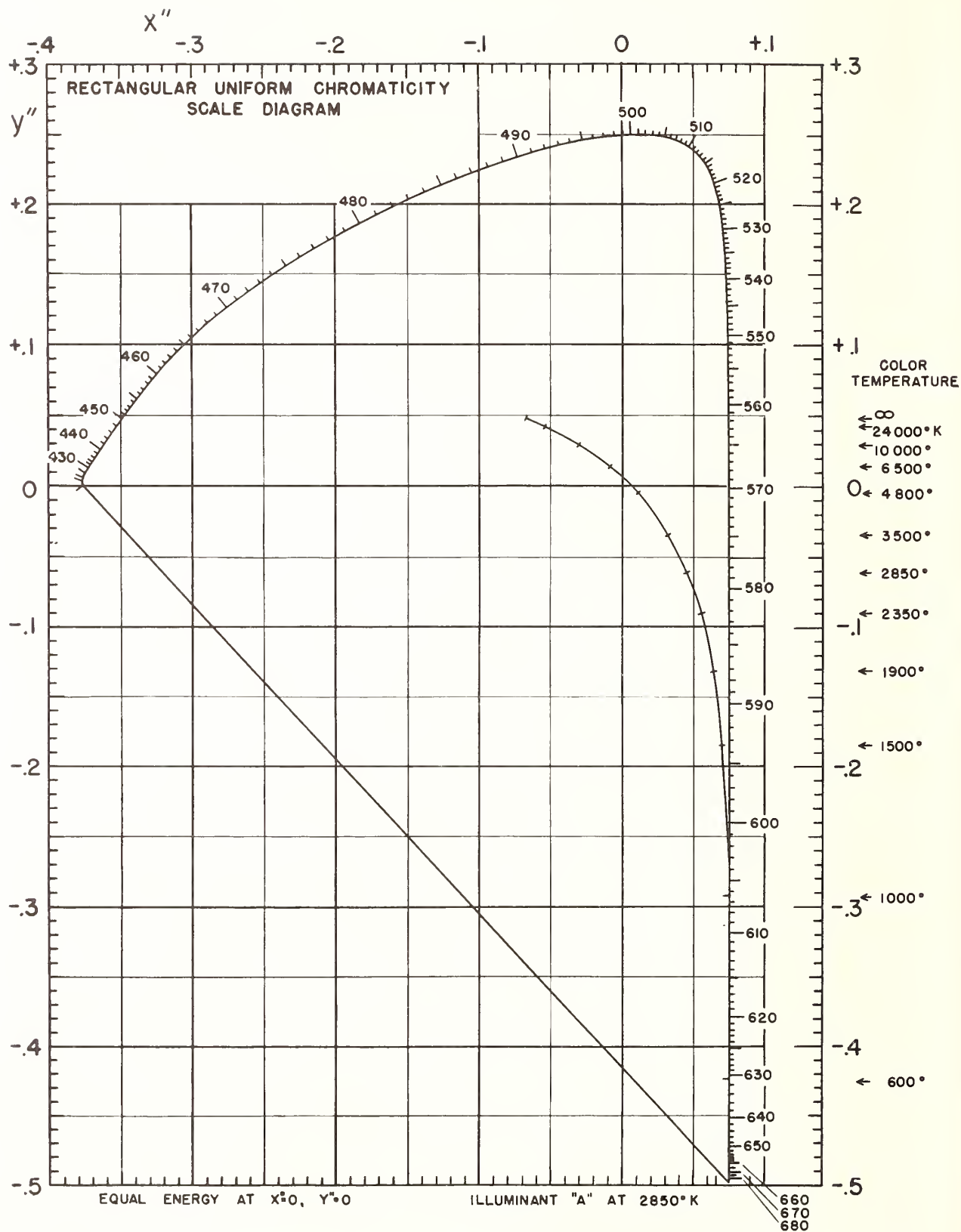


FIGURE 3-2. *RUCS chromaticity diagram.*
A transformation of the C.I.E. diagram designed to space the chromaticities of small sources uniformly in rectangular coordinates.

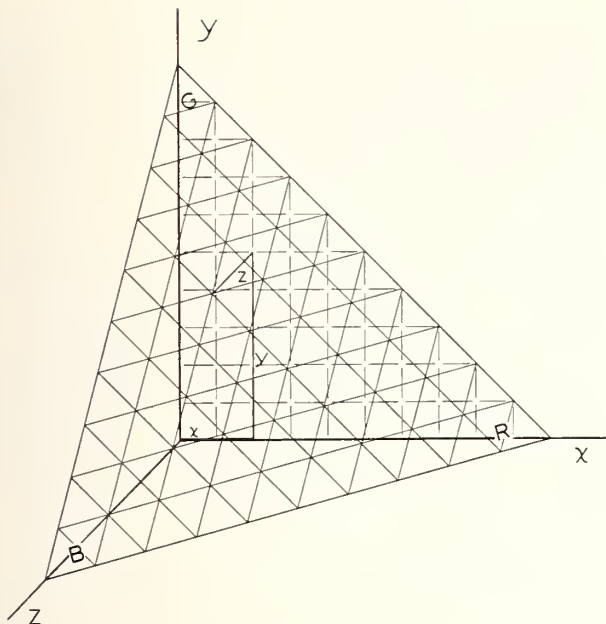


FIGURE 3-3. The "z" coordinate.

This diagram shows the relationship of the two-dimensional C.I.E. chromaticity diagram to the three-dimensional C.I.E. chromaticity coordinates. The coordinates plot in the sloping RGB plane. The C.I.E. diagram is the vertical plane at the back.

Numerous efforts have now been made to develop uniformly spaced diagrams, but most of these have been directed towards a diagram representative of the spacing for large color samples.¹⁰ Two studies have been made, however, which are applicable to the signal lighting problem. The results of one of these researches are shown in figure 3-5 for the C.I.E. diagram, and corresponding results for the RUCS system are shown in figure 3-6. Both of these figures are taken from a paper by W. D. Wright [1941]. In each of them, the bars represent small changes in chromaticity that are equally perceptible at different locations on the diagram.¹¹

Wright's study was made with a 2° field, that is, one with an angular diameter four times that of the full moon. Even this angular size is smaller than the apparent size of the samples preferred for accurate color matching of panels. On the other hand, it is a larger target than an ordinary 8" traffic light at a distance of 20 feet, a little more than a car length away. This is about the largest diameter at which a signal light ever has to be recognized, whereas in aviation and marine uses such lights frequently

¹⁰ In 1963 the International Commission on Illumination at its 15th Plenary Session adopted the CIE-UCS coordinate system to provide a generally acceptable system of representing surface colors with uniform spacing. This is applicable only to areas subtending at least 1° for the observer. See C.I.E. Proc 1963 or Judd [1963].

¹¹ Although most of the other researches are of little concern for signal light colors, three may be of interest to readers concerned with chromaticity diagrams. MacAdam [1943] made a study of the representation of small color differences and later [1959] he published the results of a comparison of small field and large field observations which show clearly that the diagrams most desirable for the representation of surface colors are not suitable for signal lights. The other study is that by Burnham and Newhall [1953]. Although this is a study of small fields, the criterion of uniformity used was the spacing of the Munsell diagram which has been carefully developed to approximate uniform spacing for large areas. The results shown are consequently quite misleading.

¹² Subtended diameter, 5'.

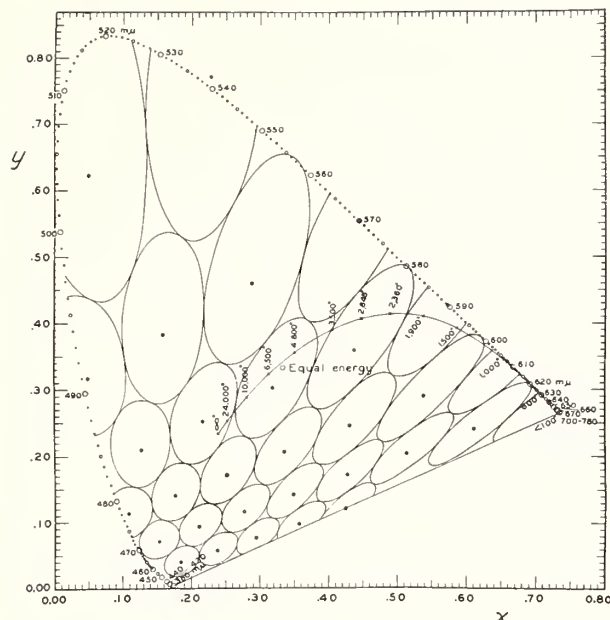


FIGURE 3-4. The Judd ellipses.

Circles drawn on the Judd Uniform Chromaticity Scale Diagram of 1934 [Judd 1935] presumably represent chromaticities that are equally different from the chromaticities at their centers. A group of such circles were transformed to the C.I.E. chromaticity diagram by Judd [1936]. The ratios of the diameters both within an ellipse and between ellipses presumably indicate the degree of nonuniformity throughout the C.I.E. diagram.

appear as "point" sources, lights often less than 1 minute in observed diameter.

Another study has been made by G. N. Rautian and M. K. Guryeva [1957]. In their study paths of least chromaticity difference were determined between chromaticities in different areas of the diagram and the number of distinguishable steps between these chromaticities was considered to be a measure of the chromaticity difference. The study was carried out with point sources¹² which adds to its significance as a test for the uniformity of a diagram intended for signal-light use. Figure 3-7 shows the results of the Rautian-Guryeva research on a C.I.E. diagram and figure 3-8 the same chromaticities on an RUCS diagram.

The results of the studies which have been shown in figures 3-4 to 3-8 can be expressed numerically by measuring the longest and shortest distances corresponding to equal perceptual differences and computing the ratios. These ratios are as follows:

Judd, C.I.E. in terms of U.C.S.	10/1
Judd, RUCS in terms of U.C.S.	1.1/1
Wright, C.I.E. for 2° field	20/1
Wright, RUCS for 2° field	4/1
Rautian and Guryeva, C.I.E. for point sources (5')	11/1
Rautian and Guryeva, RUCS for point sources (5')	6/1

These ratios indicate that the C.I.E. diagram is unsatisfactory and that the RUCS diagram, although

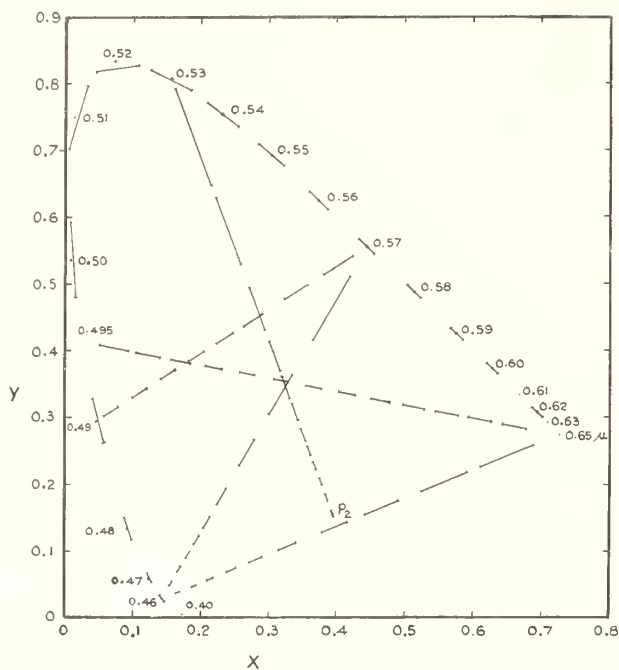


FIGURE 3-5. Color steps for the mean of four observers plotted in the C.I.E. diagram.

Each dash represents a chromaticity difference estimated to be the same as the others as determined by Wright [1941]. A comparison with figure 3-4 shows general agreement both as to the degree and the extent of the nonuniformity of the C.I.E. diagram.

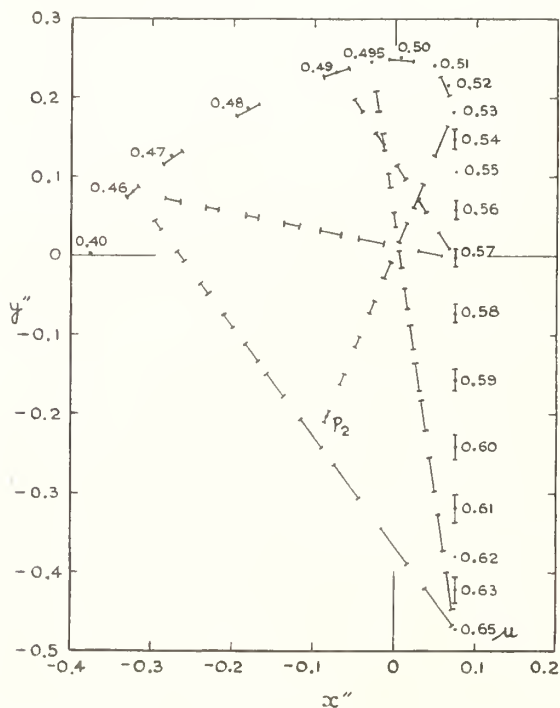


FIGURE 3-6. Color steps for the mean of four observers plotted in the RUCS diagram.

Each dash represents a chromaticity difference estimated equivalent to the others, as determined by Wright [1941]. A comparison of this diagram with figure 3-5 indicates that the RUCS is more uniform than the C.I.E. for small sources. It is also evident that the RUCS is not as uniform as would be desirable.

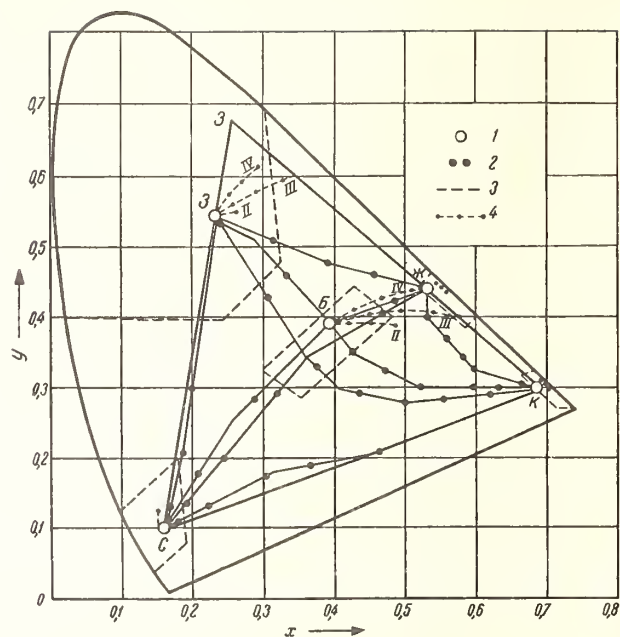


FIGURE 3-7. Chromaticity differences for signal light colors in C.I.E.

The dots indicate the distinguishable color steps for signal lights as determined by Rautian and Guryeva [1957]. Some similarities to the results shown in figure 3-5 are evident.

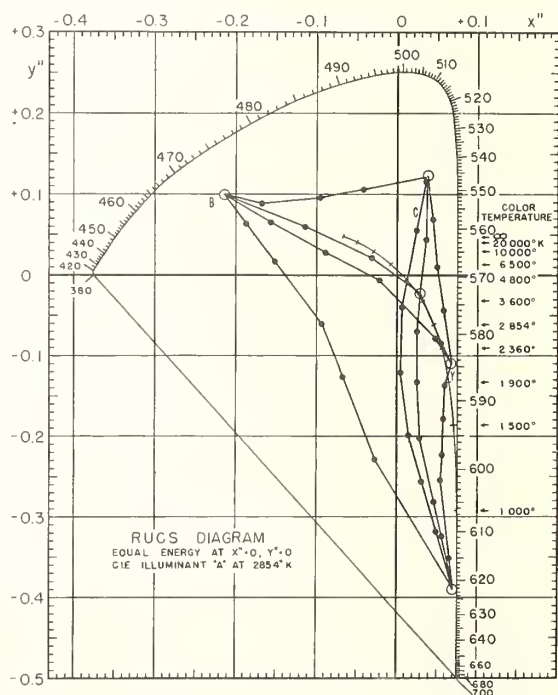


FIGURE 3-8. Chromaticity differences for signal light colors in RUCS.

The dots indicate the same points shown in figure 3-7 after transformation to the RUCS diagram. The comparison of figures 3-8 and 3-7 with figures 3-6 and 3-5 shows about the same degree of improvement. Both RUCS figures, 3-6 and 3-8, suggest that no linear transformation could make all the chromaticity differences shown equal. This is in agreement with MacAdam [1942].

leaving much to be desired, is not nearly as non-uniform as the C.I.E. While the close conformity of the RUCS to U.C.S. indicated by the ratio 1.1/1 merely reflects the derivation of the RUCS from the U.C.S., it is an indication of RUCS uniformity since the U.C.S. approximates uniformity by independent criteria.

The numerical values given above are not in themselves sufficient for estimating the distortions. It is also necessary to consider the degree to which the different results agree as to which areas of the diagrams are overextended and which too much compressed. All three agree in indicating the C.I.E. exaggerates the blue-green differences in the extreme green region. All three agree in indicating that the blue corner of the C.I.E. is the most compressed area, and that red-blue differences are exaggerated in the red corner.

An examination of the two RUCS diagrams indicates that the overextension of the green region has been corrected and the exaggeration of the red-blue differences at the red vertex has been reduced. The Wright data makes it appear that the blue vertex of the RUCS diagram should be further from the origin whereas the Rautian-Guryeva results indicate this vertex should be nearer the origin. This difference, however, should be expected to result from the difference between a 2° source and a point source as we shall see in the next section.¹³

4. Recognition of Signal Colors

4.1. General Principles

The first, and in some respects, the most important determinant for the limits of signal colors is the ability of the eye to recognize them. Unless there is a high degree of certainty that the signal will be recognized for the color it is intended to be, the signal may be a hazard instead of an aid to safety. The problem is a difficult one, however, because it is a statistical problem with nine parameters. The probability of correct recognition of a signal-light color depends upon: (1) the number of colors in the system, (2) the observer's familiarity with the system of colors, (3) the opportunity to compare colors if such is present, (4) the degree of concentration which the observer can devote to the recognition of the color, (5) the normality of the observer's vision, (6) the state of his visual adaptation, (7) the luminance of the background, (8) the solid angle subtended by the signal at the observer's eye, and (9) the illuminance, or the fixed-light equivalent illuminance at his eye.

¹³ See fig. 4-12 and discussion in text

¹⁴ Hues slightly on the blue side of the y'' -axis are, however, classified as green by many observers and are used satisfactorily as green signals.

3.7. Properties of RUCS Diagram

Although the chromaticity relationships in the Uniform Chromaticity Scale Diagram and the Rectangular Uniform Chromaticity Scale Diagrams are very nearly identical, in developing the RUCS diagram convenience of use was taken into consideration as well as uniformity of chromaticity spacing. From this standpoint it seemed desirable to use rectangular coordinates and to place a suitable white point, technically known as the achromatic point, at the origin of the coordinate system. If this is done, all radial lines from the origin become loci of approximately constant hue terminating in the corresponding dominant wavelength at the spectrum locus. Circles concentric with the origin approximate loci of constant saturation. By properly orienting the figure, it was found possible to make the y -axis correspond to red-green differences, while the x -axis corresponds, in a general way, to yellow-blue differences, although more specifically it connects violet to greenish yellow. As a result of this, the four quadrants represent green, yellow-orange-red, purple, and blue colors respectively, to a fairly satisfactory approximation.¹⁴

As will be seen in the next section, no one diagram can express chromaticity differences uniformly for all conditions of observation, but, so far as present knowledge makes it possible to judge, the deformation of the RUCS diagram to fit different conditions of observation is simpler than the corresponding deformation of the C.I.E. diagram.

4.2. Results of Research

No one has found it possible to investigate the problem of signal color recognition with respect to all the variables listed in sec. 4.1, but H. J. McNicholas [1936], J. G. Holmes [1941 and 1949], N. E. G. Hill [1947], and R. M. Halsey [1959] have studied important segments of it.¹⁵ A correlation of these four studies has been made and figures 4-1 to 4-12 and table 2 are from this correlation [Breckenridge, 1960]. The table gives most of the more important characteristics of these researches, but it should be added that only the observations of McNicholas Series II and Halsey Series III were properly recognition tests, that is, tests of signal colors separated by regions of unused chromaticities. All the other tests included in these four researches were naming tests to determine within what regions of chromaticity, and with what consistency, observers would

¹⁵ Many other investigations furnish support to the results of the four studies mentioned. Of these, four are of especial interest: Guild [1928], Ornstein, Eymers, and Vermeulen [1934], Halsey and Chapanis [1954], and Bedford and Wyszecki [1958]. In the Halsey and Chapanis study, many chromaticities were visible at the same time. This may be particularly significant for signal lights seen against a background of other lights. Guild's work was confined to the selection of a range of yellows, but it is of especial interest for the unusual method used.

apply the typical signal color names to the members of well-distributed sets of chromaticities.

To understand the correlation of the different tests, it is necessary to take account of some additional observations on the appearance of point sources of light near the threshold of visibility.

When a light is just visible it is nearly colorless. Some observers state that red lights are recognizable as such even at threshold but not all observers have found this to be true. We must conclude that the recognition of red lights at threshold is too uncertain for signal purposes, and hence at threshold we have only one signal color. As the intensity of the light is increased, or the observer approaches

closer to it, red lights may be distinguished from lights of other colors, but these continue to look alike. At still higher levels of illuminance at the eye of the observer, green lights become recognizable giving us the common three-color system composed of red, green, and an intermediate color which at still higher illuminances becomes differentiable into yellow, white, and blue. To get a sixth color reliably with point sources, it is necessary to use dichroic purple lights which usually appear as red lights surrounded by blue rays since the lenses of the eyes cannot focus for both colors at once. With sources of appreciable size, trained observers, and favorable conditions of observation, it is possible to insert additional colors.

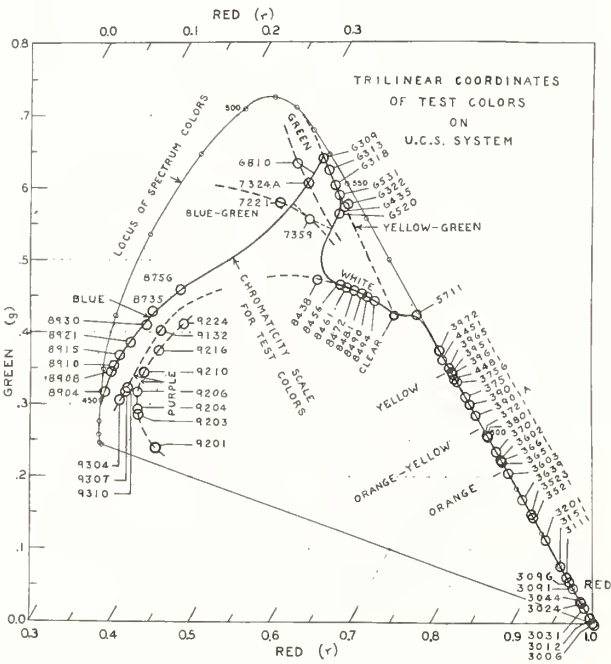


FIGURE 4-1. McNicholas test colors. This diagram, reproduced from a paper by McNicholas [1936], shows how McNicholas undertook to arrange his test colors so as to correlate his results. The numbers identify the filters he used to obtain the test colors.

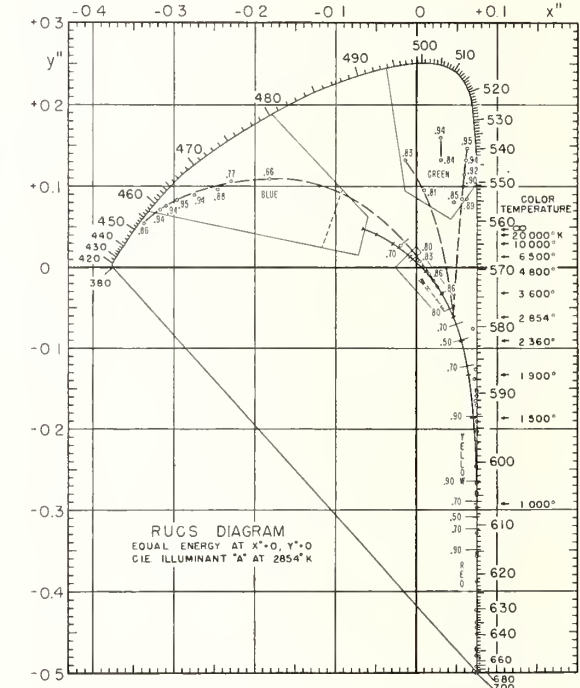


FIGURE 4-3. McNicholas results plotted on RUCS diagram. This diagram was prepared to facilitate a comparison of the results obtained by McNicholas with those obtained in later researches. The numbers on the curves show the fraction of answers agreeing with the color shown.

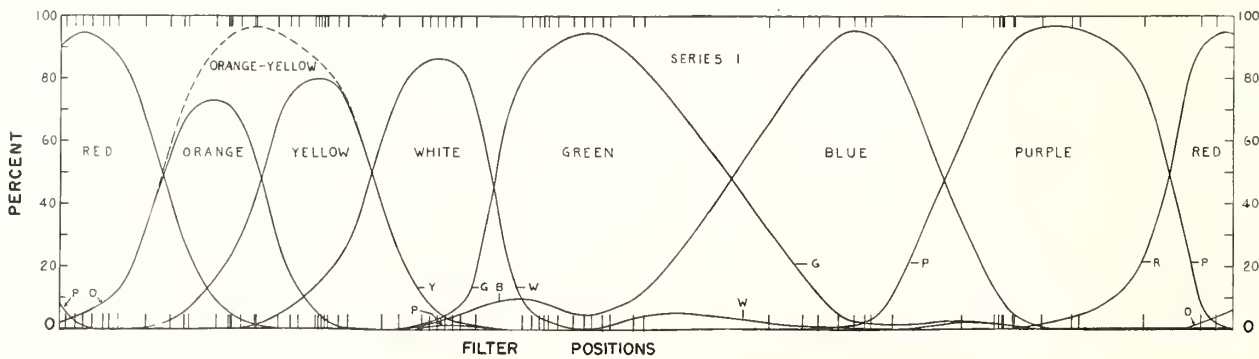


FIGURE 4-2. Results of signal color recognition tests by McNicholas [1936]. Colors are arranged in the order shown on the path drawn in figure 4-1. The ordinates are the percentage of answers indicated for the names shown under the curves. It would appear from this figure that none of McNicholas' colors were completely reliable under the conditions of observation, but these were relatively severe.

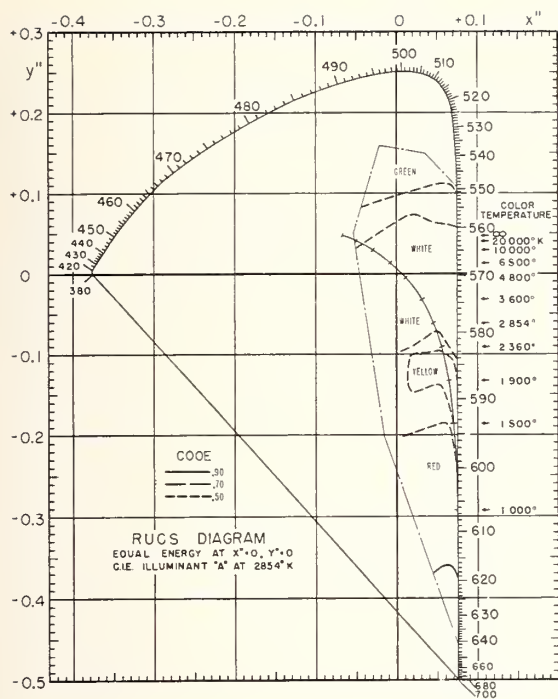


FIGURE 4-4. Signal color identification with 1 sea mile candle illuminance.

Determination from report by Holmes [1941]. Note that only red signal colors reach as high as 90 percent reliability under the conditions of observation.

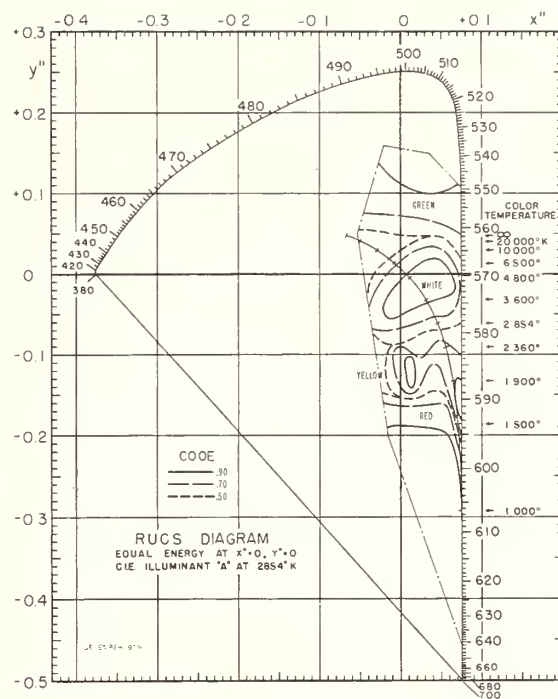


FIGURE 4-6. The effect of increasing the number of observers.

From report by Holmes [1941]. This figure represents results obtained under the same conditions as figure 4-5, but with 50 observers instead of 6 "average" observers. It was the author's conclusion that reliable results were being obtained with the smaller number of selected observers.

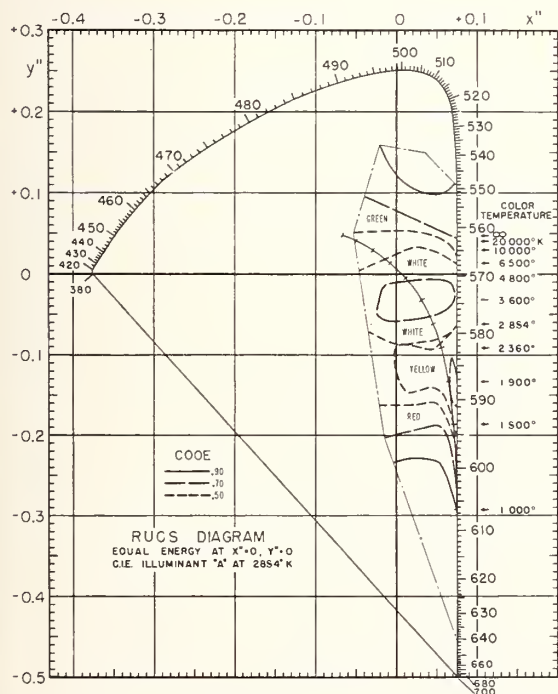


FIGURE 4-5. Signal color identification with 10 sea mile candles illuminance.

From report by Holmes [1941]. The increase in certainty of color identification as compared with figure 4-4 is quite evident.

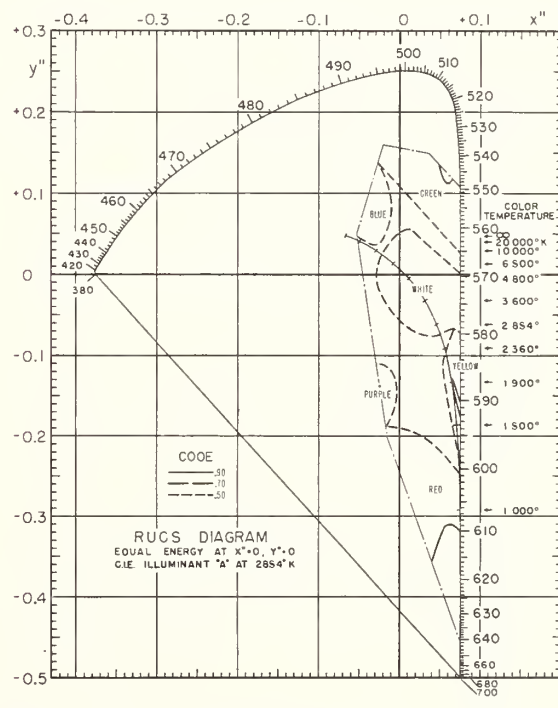


FIGURE 4-7. Identification of signal colors with 100 sea mile candles illuminance.

These results from Holmes' report show no further improvement in the reliability of red and green signals, but indicate for the first time the distinguishability of blue signals.

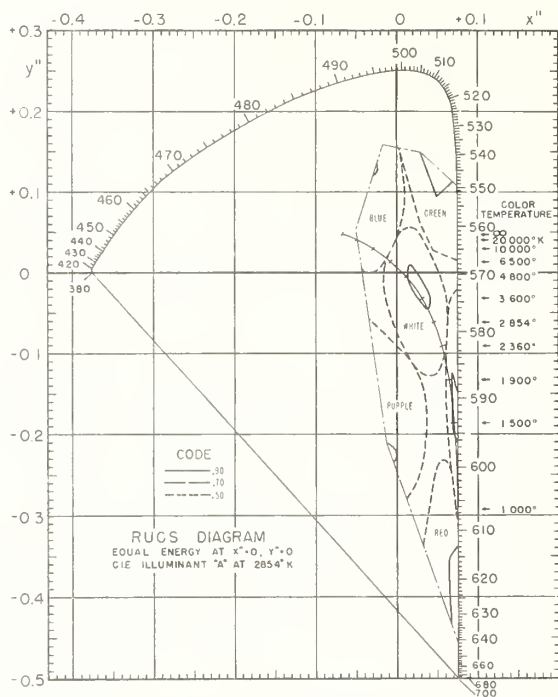


FIGURE 4-8. Identification of signal light colors with 3400 sea mile candles of illuminance.

This figure, the last from Holmes' report, shows a further increase in the distinguishability of blue signal lights and also in purple signals.

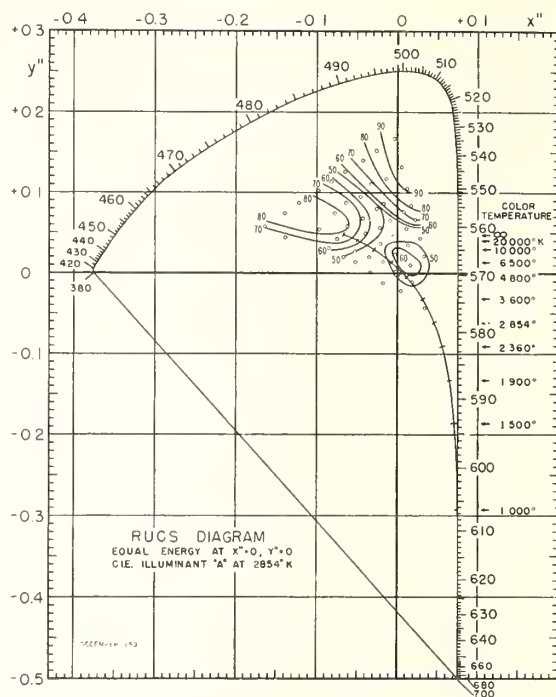


FIGURE 4-10. Distinguishability of blue, green, and white signals.

These results taken from a paper by Halsey [1959] were made with a relatively uniform distribution of target chromaticities throughout the test region as indicated by the small circles. For illuminance levels see text.

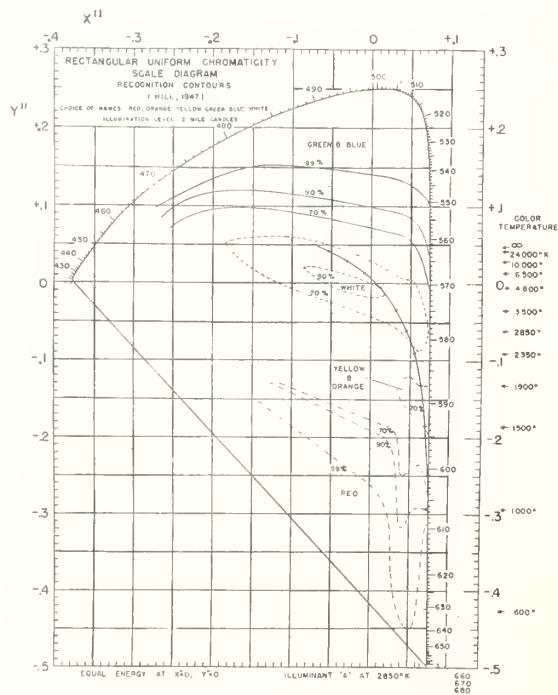


FIGURE 4-9. Signal color identification at 2 mile candles illuminance.

These results were obtained by Hill in 1947. An attempt to distinguish blue from green signals at this illuminance proved impracticable. Apparently the distinction of blue from green fades at a higher illuminance than the other distinctions which are represented in this figure. (See also figure 4-12 and discussion in sec. 4.2.)

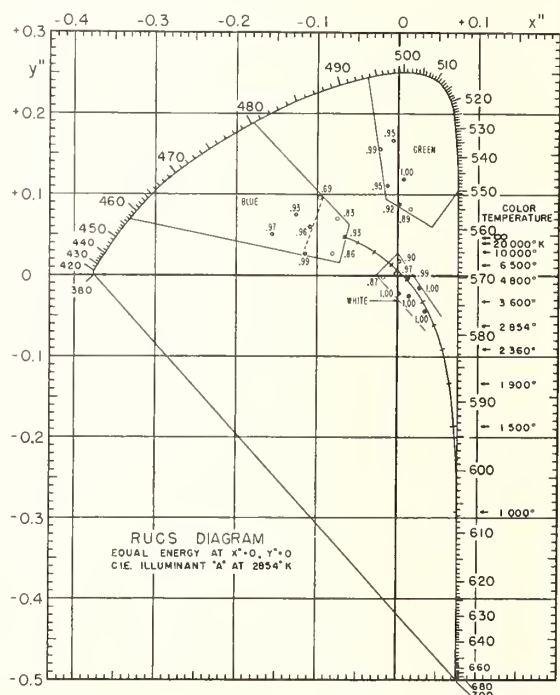


FIGURE 4-11. Effect of confining target colors to normally acceptable chromaticities.

This test, also by Halsey [1959] (not published), discloses the effect of omitting target chromaticities which were considered unreliable so that one might hope that the remaining chromaticities would be identified correctly 100 percent of the time. It also indicates the effect of giving observers some training.

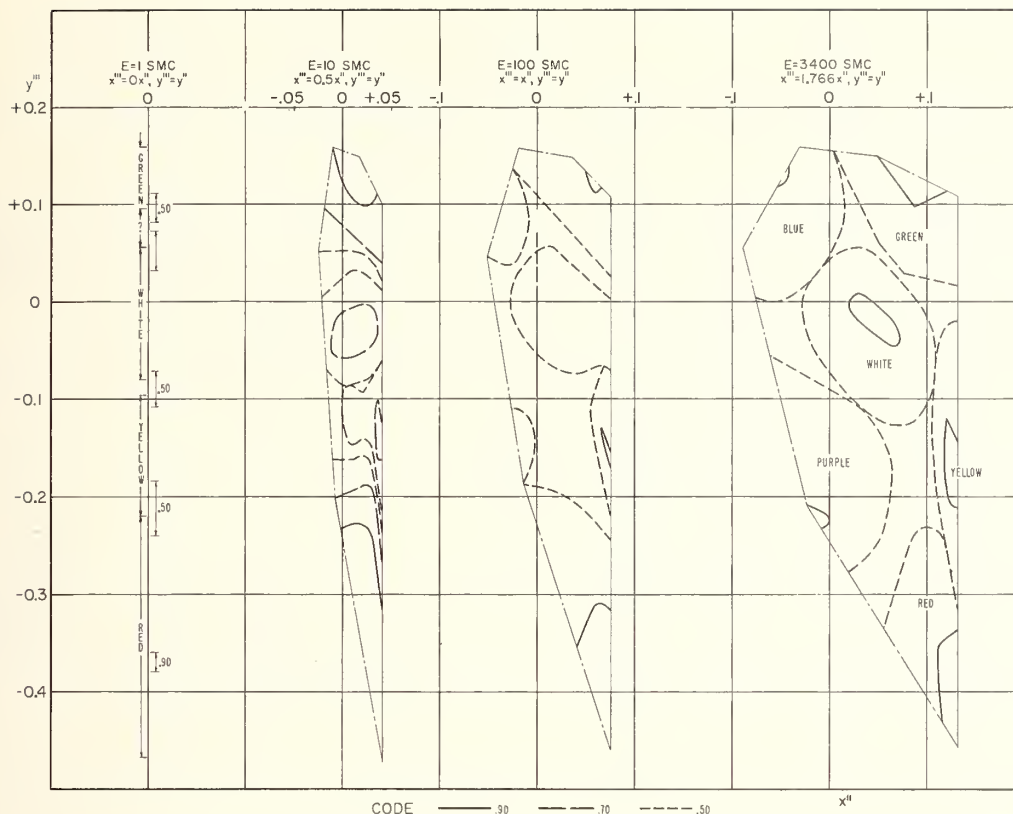


FIGURE 4-12. Correlation of Holmes' results.

The results shown in figures 4-4, 4-5, 4-7, and 4-8 have been compressed or expanded along the x'' axis to compensate for the effects of low-illuminance tritanopia. It should be noted that in the case of all except the leftmost figure, the white contours are approximately circular around a point near the origin. The leftmost figure has shrunk to a vertical line, signifying that the yellow-blue discrimination has been lost because of the tritanopic effect of low illuminance.

From the above discussion it becomes clear that if we were to represent signal colors in diagrams prepared to correspond to different levels of illuminance at the observer's eyes, we should make the one for threshold illuminance consist of a single point. At the next level of illuminance we shall need a short line with red represented at one end and non-red at the other. As the illuminance increases the line must be made longer and finally the figure takes on width and develops into something approximating the RUCS diagram.

An attempt has been made to prepare a series of diagrams on this principle by using the contours of Holmes' extensive investigation as a guide to the factor for the yellow-blue contraction, or expansion, of the RUCS diagram required for each level of illuminance. The results of this study are shown in figure 4-12 [Breckenridge, 1960]. The diagrams of this figure are those of figures 4-4, 4-5, 4-7, and 4-8 with the x'' coordinate contracted, or expanded by the factor indicated for x'' above each figure. Only that portion of the RUCS diagram necessary to include Holmes' test chromaticities has been included in each case. If these transformations had resulted in diagrams entirely in accord with the RUCS intent, the inner white contours

would have been circles around the origin in each case.

4.3. Results of Experience

While the contributions of experience to our knowledge of the recognition of signal colors are less definite than those of research, their influence upon the actual selection of limits for the signal colors in use has been greater. It is, of course, possible that a definition of a signal color might be used for many years without the realization that it permits certain chromaticities that could be mistaken under some conditions. Considerable areas of chromaticity which are within the definitions are not used in service. Chromaticities near the extremes of those represented by the standard limits presumably do not occur frequently and it may be much more rare to find such a borderline filter in a situation in which color recognition is particularly difficult. Finally, not every error in color identification causes results that reveal the mistake. Nevertheless the long and successful use of the same colors for marine and railroad signals is a strong indication that chromaticities within the limits of those used in these services are satis-

TABLE 2. Summary of procedures for color identification researches*

Research by	Illuminance	Source target size	Exposure	Number	Colors		Grouping of colors	Observers		Observations		Adaptation
					Coloration	Other lights		Number × runs	Ages	Total number	Names allowed	
McNicholas Series I	<i>smc</i>	<i>Minutes</i>	<i>Seconds</i>						<i>Years</i>			
Series II Holmes	0.5/8.2	0.4/0.04	5	69	Glass	Street	Partly					
	1.0/2800	"	"	"	Filters	Lights	Grouped	38*	20*/60	5800*	RYOGBPW*	Below horizon on dark background of woodland and southern sky
	1	0.7	1 on/1 off	256	Synthesized from 11 glass filter primaries	None	Continuous	6 × 3	22/42	4608	RYOGBW*	5 minutes before test
	6	1.7	"	"	do.	"	"	13 × 1	"	3328	do.	
	10	2.2	"	"	do.	"	"	6 × 1	"	1536	do.	
	10	2.2	"	"	do.	"	"	50 × 1	18/63	12800	do.	
	10	2.2	"	"	do.	"	"	5 × 3	22/42	3840	RYGW	
	10	2.2	"	"	do.	"	"	5 × 3	"	3840	RYGN	
	10	2.2	"	"	do.	"	"	5 × 3	"	3840	RGW	
	10	2.2	"	"	do.	Desk lamp*	"	4 × 3	"	3072	RYGBPW	Reading by desk lamp between observations
	100	7.0	"	"	do.	None	"	6 × 3	"	4608	do.	
	3400	40.0	"	"	do.	"	"	4 × 3	"	3072	do.	
Hill	1.3	0.6	1.5	73	17 Wratten filters + white	Dim reddish glow	Continuous	9 × 20#	20-40#	30420	R(YO)(G+B)W#	10 Minutes before testing General .0001 c/ft. ² with 15' disc illuminated to .0005 c/ft. for 1.5" before signal
	2.6	0.6	1.5	73				9 × 20#	20-40#		do.	.001 ftL on data pads for 15 minutes before test and during it
Halsey	6.2/280	4.1/7.2	2	50	4 Wratten filter primaries	Red & yellow fixation lights	Continuous	100 × 1	17-35	5000	RYGBPW	Dimly illuminated room
II _L	1.8/83	"	"	"	do.	do.	"	100 × 1	"	5000	do.	
II _H	6.2/280	"	"	"	do.	do.	"	100 × 1	17-28	5000	RYGBW	
II _L	1.8/83	"	"	"	do.	do.	"	100 × 1	"	5000	do.	
III _A	2.3/104	"	"	22	do.	do.	3	100 × 1	approx.	2200	RYGBPW	
III _B	"	"	"	"	do.	do.	Groups	100 × 1	as	2200	RYGBW	
III _C	"	"	"	"	do.	do.	do.	100 × 1	above	2200	GBW	
III _D	"	"	"	"	do.	do.	do.	100 × 1	do.	2200	GBW	
Notes			*Flashed as long as desired			*Between observations		*In two series #Some 30 times	*3 < 20 #17 > 40	*In the two series	*Note orange included in one test #orange recorded as yellow	

*[Breckenridge, 1960]

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factory for their purposes. It does not follow that chromaticities outside of those limits are unsafe.

While the limits for the red and green signals used in marine and railroad practice were selected before the researches mentioned in the last section were carried out, their selection was not casual. The earlier limits used for railroad signal colors were not satisfactory and at its second meeting the newly organized Railway Signal Club appointed a committee in April 1895 [AAR, p. 74, 1953] to investigate the question of colors for night signaling. At first the problem was which colors should mean clear, go ahead, and which danger, stop. At that time the usual colors for railroad signals were white for safety, red for danger, and green for caution. This was the practice in most countries in accordance with an agreement reached at a congress of railroad men in Birmingham, England in 1841. The system had been developed in France and it is interesting to note that the originators had reached the conclusion in the course of their experimenting that "the visibility of a red light was but one-third that of a white light of the same intensity; that of a green light one-fifth; and that of a blue light one-seventh" [AAR, p. 73, 1953]. A comparison of these values with present information evidences the progress which has been made by the glass manufacturers in producing signal filters of higher transmittance.¹⁶

The 1841 standard proved defective because a signal with a broken glass frequently appeared white, converting a danger or caution signal into a safety signal. The same mistake occurred if an engineer mistook an ordinary white light for a signal. On this account English practice was changed to use green for the safety signal with no caution signal. The first use of the present red-yellow-green system in this country was by the New York, New Haven, and Hartford Railroad in 1899.

Starting about this time interest began to develop in the scientific aspects of signal-light recognition. Laboratory work was started at Yale University and later continued at the Corning Glass Works by Dr. William Churchill to select the most effective colors for signal lights [Churchill, 1914]. Between 1914 and 1930 several committees appointed by the Signal Section of the Association of American Railroads made field tests with the cooperation of the Corning Glass Works to determine the most favorable limits for the colors. These tests had the advantage that they were made by the men who used the lights in service and they were made under conditions approximating somewhat the conditions under which such lights are normally used.

¹⁶ This assumes that the "same intensity" refers to the lamps without their filters and that the relative visibilities were measured by the distances at which the lights became visible.

The original selection of colors was made at a time when all railroad signal lights used kerosene wick lamps. The selection of glasses for use with electric lamp sources was carried out under the technical leadership of Dr. H. P. Gage at the Corning Glass Works. Dr. Gage [1928] reviewed the entire problem systematically. He found that in addition to meeting the requirements of recognizability, the railroad representatives considered it desirable that lights of the primary signal colors (red, yellow, and green) should be about equally visible. Since the green glass was thought to give the least conspicuous signal, the limits for green were determined first. Yellow and red glasses

were then selected to obtain signals that were regarded as comparable in conspicuity as well as safely recognizable.

The ranges of acceptable color variation were identified by filters representing the palest acceptable ware and the minimum acceptable transmittances. These filters were called limits. The selection of limits for marine, aviation, and highway signal lights in the United States has been based to a considerable degree on the limits selected for railroad signals and such consistency as exists between these different systems results largely from the influence of Dr. Gage in the selection of all of these limits.

5. Production of Signal Colors

5.1. Source and Filter

The second determinant which controls the selection of limits for signal colors is the practicability of producing initially, and maintaining in service, signals that will be within the proposed limits. This is a problem with many aspects, as indicated by the subdivisions of this section.

The color of the signal seen by the user depends upon the chromatic qualities of the source of light and those of the filtering elements interposed between it and the observer.¹⁷ Some signal colors can be obtained from autochromatic sources that require no filters, but most signaling is done with units that derive their light from incandescent lamps which for signal purposes are regarded as white and require glass, or plastic, filters when other colors are required. In some installations lamps have been used which contribute a weak coloration that is strengthened by a filtering element. In the following section, we will consider first, and chiefly, signals produced by filtered incandescent light, with a brief discussion of gaseous discharge lamps in the last subsection.

5.2. Effect of Colorants

If a desired signal color is to be obtained by filtering light from an incandescent lamp, the first question is whether suitable glass, plastic, or other material of the necessary spectral transmittance is available. The curves of figure 5-1 show chromaticities that are obtainable with different types of glass. In many cases slight changes in the composition of the colorant would move the corresponding curve to one side or the other of the path shown. It is evident from the distribution of these curves over the green and most of the blue quadrants that a considerable variety of chromaticities is avail-

able for these colors. In the yellow-red quadrant any desired degree of redness is available with high purity but this may not be apparent from the diagram since for wavelengths greater than $\lambda = 0.585\mu$ the curve for chromaticities obtainable from the practically available selenium glasses virtually coincides with the spectrum locus and is not represented by a separate line. In the purple quadrant only one type of glass is shown, but the

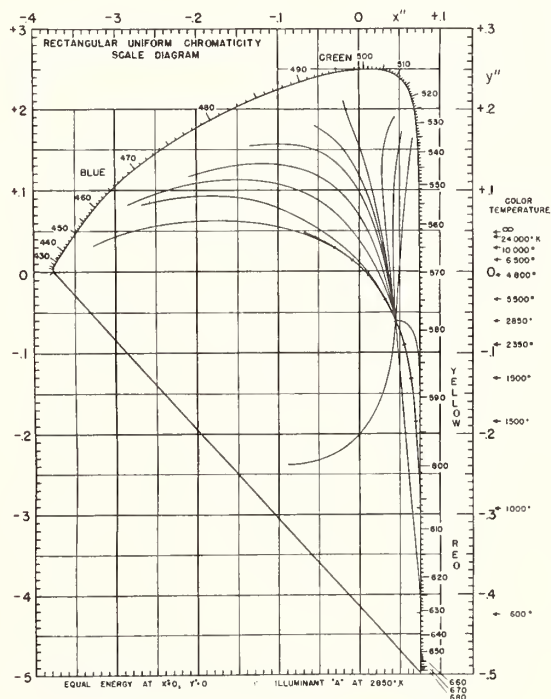


FIGURE 5-1. The effect of colorant selection.

With the exception of the Planckian locus, which is distinguished by the cross bars indicating the different color temperatures, each of these curves is the locus of chromaticities which result from changing the density of the colorant. In three cases, two types of glass have been combined in fixed proportions to obtain curves in the regions not commonly used for signal light purposes, but all the other loci correspond to actual filters.

¹⁷ For a somewhat parallel discussion of this aspect of the problem from a different point of view, see "Coloured Glasses for Lighthouse Purposes," Holmes [1937]. There is also some parallel information in Holmes' "Colorimetry in the Glass Industry" [1947].

chromaticities in this quadrant are not generally used for signal lighting purposes.

Unfortunately no data for plastic filters comparable to that shown in figure 5-1 for glass filters are available to the author, but red and yellow plastics with spectral transmittances very similar to those of the red and yellow glass have been measured at the National Bureau of Standards as well as green plastics of suitable chromaticity for signal purposes.

The fundamental difference between filters of different colors derives from the different degrees with which light of different wavelengths is transmitted. Figures 5-2 and 5-3 have been included to show typical differences in spectral transmittances for filters of different colors.¹⁸ The curves have been labeled to assist in correlating the curves of these figures with those of figure 5-1. For each color, two curves are shown with the same label, the lower curve representing a thicker filter than the upper curve. The filters represented by the curves of spectral transmittance are not the same as those of figure 5-1 but they are similar to some of them and would give chromaticity loci in the regions indicated.

5.3. Effect of Filter Thickness

One of the means of controlling the color of a signal light is to vary the thickness of filter used, or if this is not feasible, to change the concentration of the colorant in the filter. In figure 5-1 it will be seen that all the curves for the filter chromaticities converge to a single point. This point corresponds to filters of zero thickness which do not change the color of the incident light at all and consequently transmit light that is the color of the source. Theoretically any filter that is not neutral can be increased in thickness or concentration until its transmittance becomes virtually monochromatic. In the figure this would be represented by prolonging each curve until it reached the spectrum locus. There are practical reasons why this is not feasible. Even at the outer ends of most of the curves shown in figure 5-1 the transmittance is below the minimum practicable for signal filters. In production there may also be chemical difficulties. There may be reactions between the colorant and the body material that will change the spectral transmittance characteristics of the colorant; or such a reaction may have a deleterious effect on the physical qualities of the glass.

In the practical production of colored glassware there is always a variation of thickness from piece to piece, and from lot to lot there may be variations of the composition and concentration of the colorant

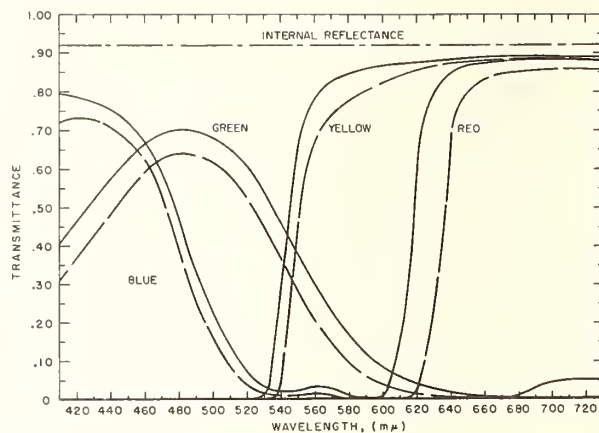


FIGURE 5-2. Spectral transmittance curves for National Standard filters.

Two filters of each of four colors are included. The upper full line curves represent pale limits. A lower broken curve in each case represents a denser filter of similar type.

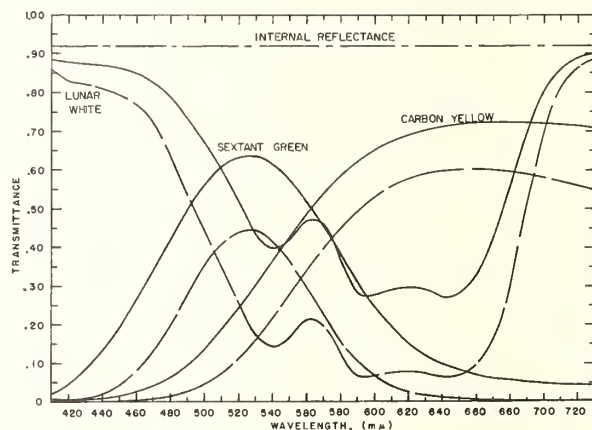


FIGURE 5-3. Spectral transmittance curves for three types of glass available for signal ware.

Lunar-white and carbon-yellow glasses have been used for railroad signal-light ware. Sextant green glass could be used if a yellow green signal were desirable. Broken curves are for thicker filters than those represented by the full line curves.

which cause corresponding variations in the chromaticity of the signal. In the case of green and blue ware, the possible variation is considerable. It is, in fact, so considerable that it is difficult to set up chromaticity limits for some ware that will keep within the limits regarded as safe for the recognition of the signal and at the same time allow enough variation to permit reasonably priced production. In the case of red and yellow selenium ware, the variation of chromaticity with thickness is much less and is mainly parallel to the spectrum locus. Similar, but larger, changes are produced by heat treating selenium glass. Since these sharp cutoff selenium red and yellow filters, excepting some too pale for signal purposes, produce colors close to the spectrum locus, there is a negligible change of

¹⁸ The spectrophotometric transmittance of the national standard filters are given in the U.S. Standard for the Colors of Signal Lights [NBS 1964] and also in Gibson, Haupt, and Keegan [1945] and [1946].

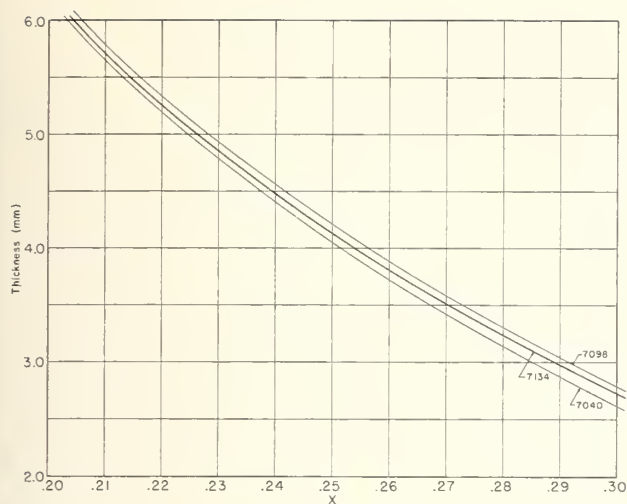


FIGURE 5-4. Variation of chromaticity with thickness.

The center curve shows the variation in the x coordinate for the chromaticity of a green filter transmitting light from a source 2854 °K as the thickness of the filter is varied thru a range of approximately 1 to 2. This type of curve is useful in estimating the change of thickness necessary to produce a required change in chromaticity when filters are being ground to come within a given tolerance. The central curve is for a national standard filter and the upper and lower curves are for other filters of similar types of glass. It is to be noted that the tangents for all three curves are nearly the same for any given value of x . In adjusting chromaticities by grinding, it is the rate of change that is important. This enables us to use a standard curve even though the particular piece of glass to be adjusted may have a slightly displaced curve.

x'' (or z) not only with changes of thickness but with changes in any of the other parameters we shall consider. For this reason the changes of chromaticity of the red and yellow selenium glasses may be represented diagrammatically by using y'' or y as an index of redness.

Variations in the chromaticity of plastic ware are less than those for glass but similar in character because the methods of production permit more rigid controls of thickness and composition. There is, on the other hand, greater risk of change of chromaticity through fading with age in the case of plastics. Some plastic colorants, however, are resistant to fading.

Curves showing the variation of chromaticity as represented by one of the coordinates as a function of the filter thickness are useful in the duplication of standard filters. Figure 5-4 shows this variation for three similar green glasses. To prepare the curve it is necessary to know the spectral transmittance of the filter, but after the curve is drawn all other chromaticity measurements may be made with a differential colorimeter (see sec. 9.1). An ordinary spectral transmittance curve does not give values that enable one to compute chromaticity coordinates with the accuracy suggested by the curves of figure 5-4, and for this reason the first grinding of the blank should be to a thickness that

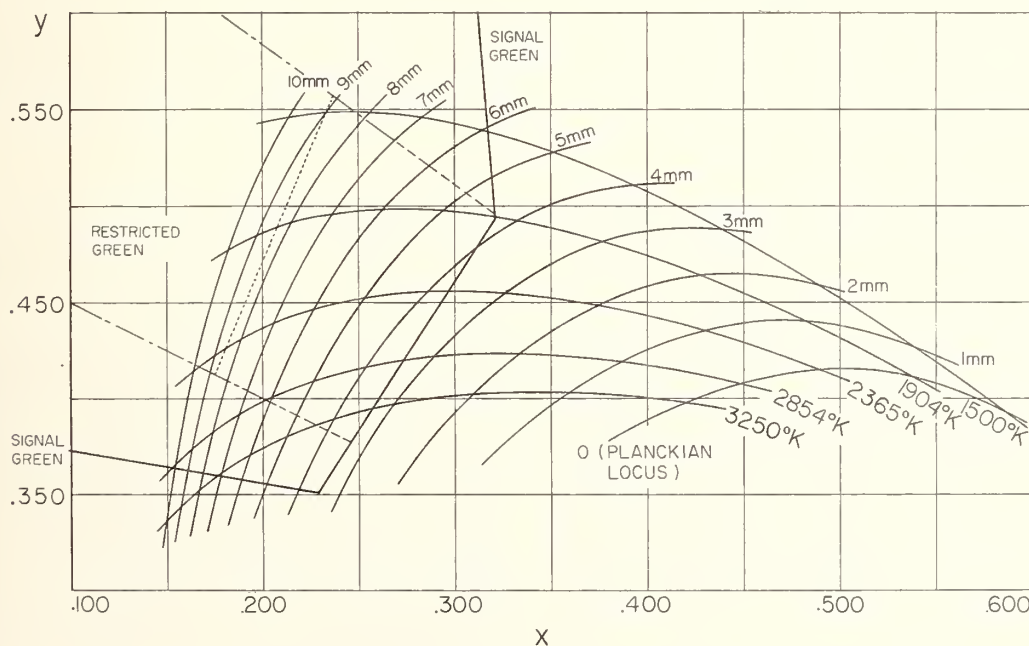


FIGURE 5-5. Variation (C.I.E.) of chromaticity in service for green glass filters.

This is the type of diagram which must be prepared to show that the signal color will remain within the basic chromaticity definitions in service. The curves in this figure are based on U.S. Standard Filter 7.134. Those labeled with color temperatures show the variation with thickness using light sources of the labeled color temperatures. Curves labeled with thicknesses show the variation with light-source color temperature for filters of the labeled thicknesses. The straight lines show the boundaries of the basic chromaticity definitions for signal green, intermediate green, and restricted green. To be acceptable, the chromaticities for the proposed colorant must remain inside the applicable basic chromaticity definitions for all points within the area bounded by the minimum and maximum light source color temperatures to be expected in service and by the curves for two thicknesses representative of the minimum and maximum thickness to be expected in the filter ware.

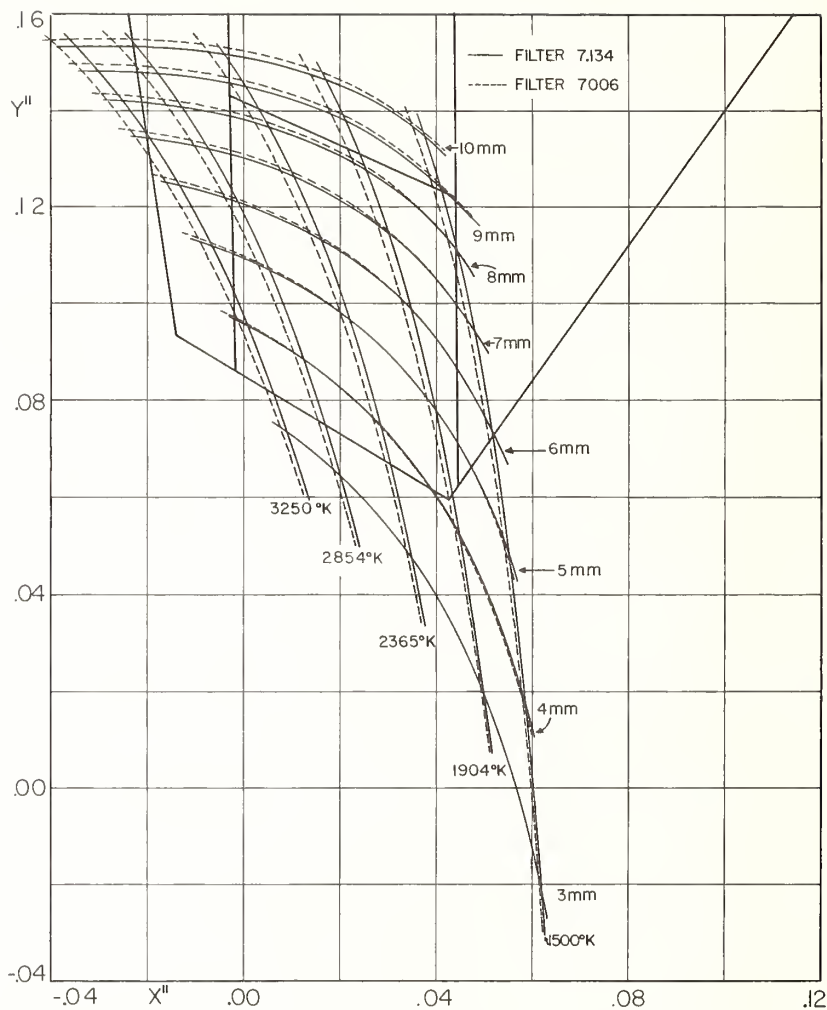


FIGURE 5-6. Variation (RUCS) of chromaticity in service for green filters.

The full line curves here represent the same variations as in figure 5-5. The dotted line curves represent the chromaticities to be expected on the basis of a green filter manufactured by a different company.

allows for a possible error of at least the estimated uncertainty. The differential measurement provides a more accurate value of the chromaticity, and if this were plotted on a diagram corresponding to figure 5-4 it would probably not fall on the curve. However, we now know how much change in x or y we desire, and we know the slope of the curve for the chromaticity as a function of thickness with sufficient accuracy so that we can compute the reduction in thickness required to bring the filter within the tolerances.

5.4. Effect of Color Temperature of Source

It has been stated in sec. 5.1 that not only the characteristics of the filtering element but also the color of the source affects the color of the signal.

Most signal lights today operate with incandescent lamps, but signal lights using kerosene wick flames, acetylene flames, and mantle burners may still be in use, and each of these has its characteristic spectral energy distribution. The incandescent lamps also differ in spectral energy distribution because of differences in the operating temperatures of the filaments. It is customary to specify the energy distribution of an incandescent lamp by giving its color temperature. This is not the physical temperature of the filament but the temperature of the blackbody or complete radiator which would most nearly match the filament in chromaticity.

The differences in filament temperature result in part from differences in lamp design and in part from variations in operating voltage and the latter may be either incidental or intentional. Lamps for

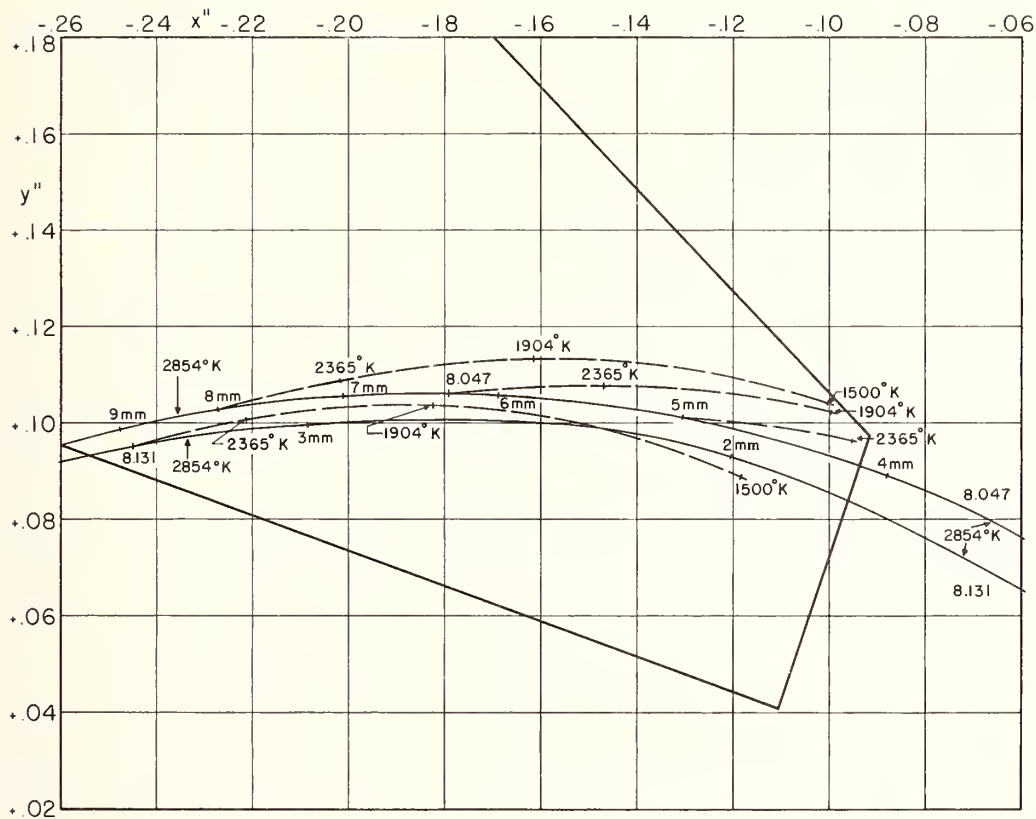


FIGURE 5-7. Variations (RUCS) of chromaticity in service for blue filters.

This figure is analogous to figure 5-6. In the case of blue signal filter ware, the variations caused by change of light source color temperature and those caused by change in the thickness of the filter tend in nearly the same direction, with the result that the entire range of colors actually used is compressed within a relatively narrow belt across the region defined by the basic chromaticity definitions. For this reason it is not practicable to show as many lines as in the case of green.

different applications are designed for different periods of life and this means different filament temperatures. Lamps now in use for signal lighting vary from about 2300 °K to about 3100 °K in color temperature at designed voltage. This variation should be compensated by a proper selection of colorant density in the filtering element, and in selecting the limit standards it is necessary to take account of the color temperature of the lamps to be used with the ware to be controlled.

It is difficult to estimate the variations in supply voltage for signal light installations in general but one survey of traffic signal lights showed that the lamps in the installations studied were operating at a minimum of 100 volts and a maximum of 130 volts. This could cause color temperatures varying through an interval of 200 °K. The largest color temperature variations are those caused by variations in lamp current which are made intentionally in order to adapt the intensity of the signal to prevailing conditions. Approach and runway lights designed for use at airports under conditions varying from daytime fog to clear night darkness are

designed to have their lamp filaments vary from 3100 °K at maximum intensity to 1500 °K at minimum intensity.

While it is not feasible in most signal-light applications to make provision for varying the intensity of the lights to compensate for the difference between fog and clear air, it would be feasible in some cases to make allowance for the variation between night and day requirements. With the increasing intensities required for traffic signals on high-speed roads, such compensation may soon be imperative for traffic signals. The increasing intensities used for vehicular signals make them glaring at night and it has been proposed to cut these intensities when headlights are used. Tests made in connection with the development of approach and runway lighting for airports have shown that approximately 1000 times the intensity is needed in the daytime to obtain the conspicuity equivalent to that considered necessary at night with the same atmospheric transmissivity.

Figures 5-5 to 5-9 show the effect of light-source color temperature on the chromaticity of the signal

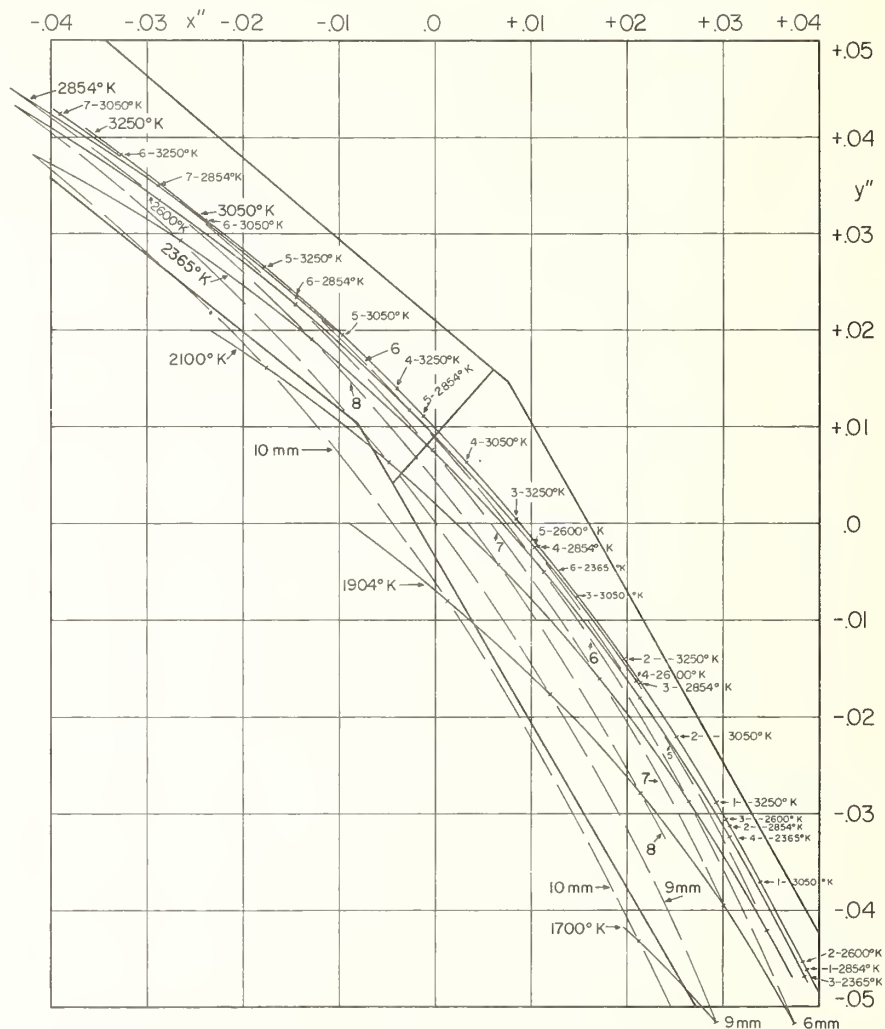


FIGURE 5-8. Variations (RUCS) of chromaticity in service for lunar-white and blue-white filters.

This figure corresponds to the preceding figures for green and blue filters. It shows how similar the changes of chromaticity caused by variations of thickness are to those resulting from changes of light-source color temperature in the case of lunar-white filters. The straight lines are boundaries for U.S. Standard Lunar White and Blue White.

for glasses of different colorant "densities." For green glasses, figures 5-5 and 5-6 also show the loci of constant colorant "densities." These curves and those for constant color temperature follow quite different paths so that it is feasible to show both sets on the same diagram as functions of x'' and y'' . Similar curves are shown for blue glasses in figure 5-7, but these may be found somewhat confusing because in this case the two types of loci lie much closer to each other, the change of chromaticity with source color temperature and with colorant density being similar. This similarity of variation is shown even more clearly in figure 5-8 which shows the corresponding variations of chromaticity for lunar white and blue white filters. In figure 5-9, however, we have plotted the change of the y'' coordinate as a function of the color temperature of the source since the chromaticity of red and

yellow filters can be represented to a good approximation by a single parameter.

To simplify the discussion the term "density" has been given a generalized meaning above. In the case of blue glass the change of "density" can be made either by changing the concentration of the colorant or by changing the thickness. In the case of green glass the change in concentration is liable to have undesirable chemical effects and the change of "density" must be considered as referring to changes of thickness primarily. With red and yellow glass of the selenium, sharp cutoff type a change of thickness has little effect upon the chromaticity. For these glasses the chromaticity is largely determined by the wavelength at which the cutoff occurs. This is to a considerable extent determined by the heat treatment given the glass. In the case of the blue and green glasses we have

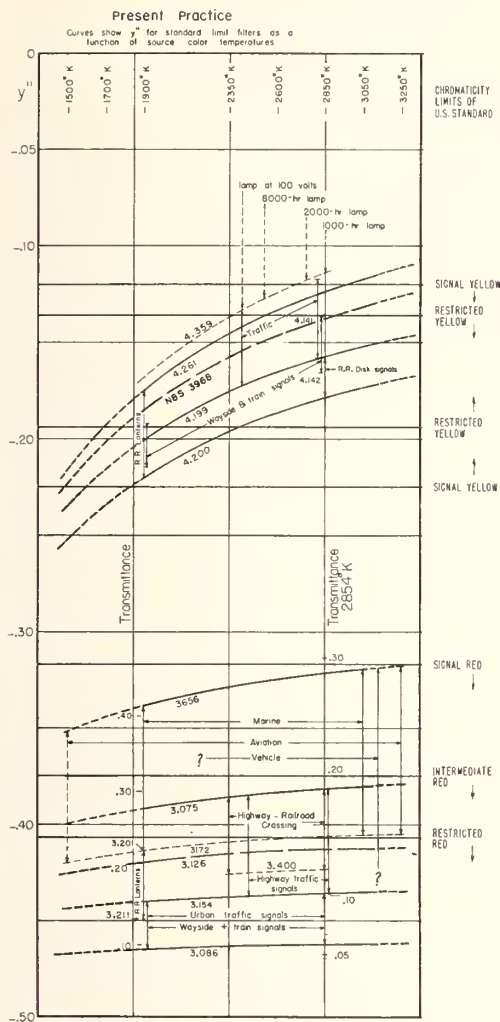
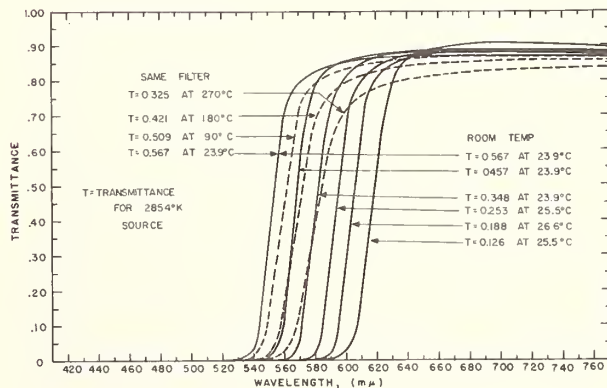


FIGURE 5-9. Variation (RUCS) of chromaticity in service for red and yellow filters.

In this figure the value of y'' has been used as an index of redness and yellowness. Since all these chromaticities lie so close to the spectrum locus that the distance between them and the spectrum locus is not significant, it is feasible to represent the chromaticities by a value for y'' alone. Limits shown by vertical arrows are for specifications in use prior to the adoption of the U.S. Standard. The chromaticity limits of the Standard are shown by horizontal lines as labeled.

FIGURE 5-10. Effect of physical temperature on spectral transmittance.

Spectral transmittances of six red and yellow selenium glass filters as measured by Leberknight and Stone [1955] compared with the spectral transmittance of one of them at four different physical temperatures. The filters are identified by giving their approximate transmittances at the temperature nearest room temperature.



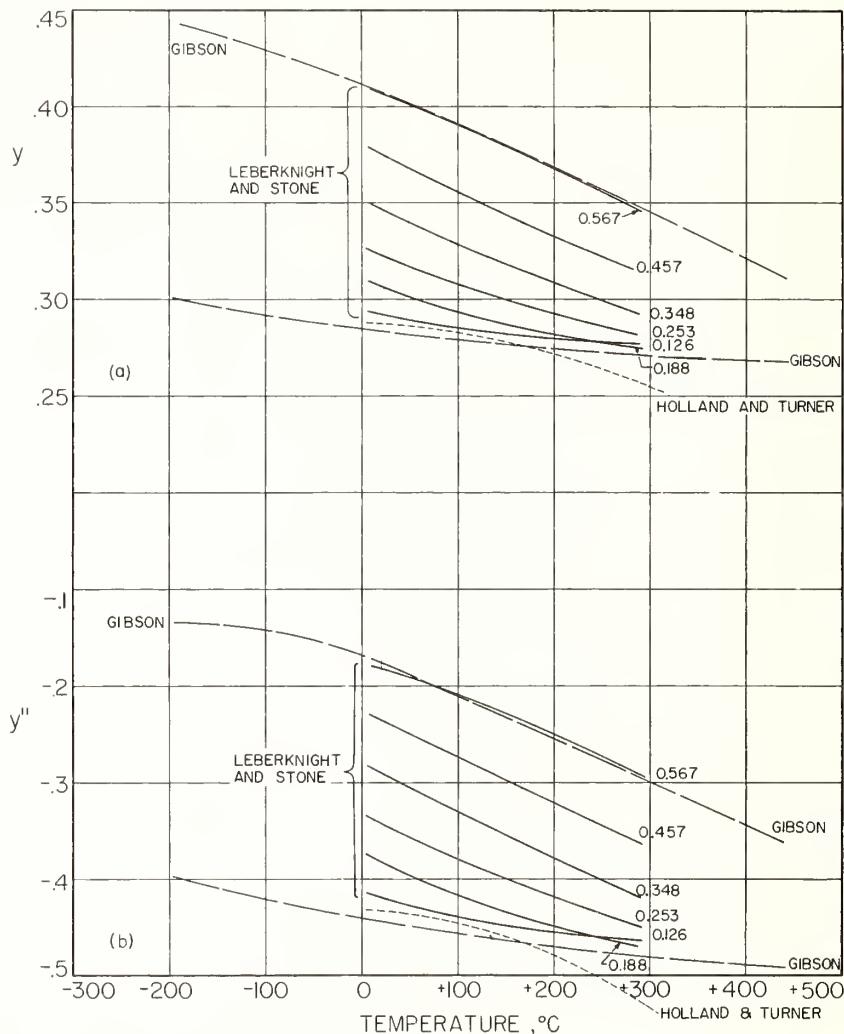


FIGURE 5-11. *Effect of physical temperature on the chromaticity for selenium filters.*

(a) These curves show how the chromaticity of the transmitted light, as measured by the y coordinate, varies with the physical temperature. Six of the curves are for the same filters shown in figure 5-10 and are identified by their transmittances at room temperature as shown in that figure; two are from measurements made by K. S. Gibson [1916]; and one is from the researches of Holland and Turner [1941].

(b) These curves are based on the same red and yellow filters shown in the upper curves (a), but y'' (RUCS) is used in place of y (C.I.E.) as the index of redness.

used thickness as a parameter of colorant "density" and in the case of the red and yellow glasses we have selected filters of different cutoff as the equivalent of different "densities" in the other cases.

From figures 5-5, 5-7 and 5-9, it is evident that there must be a correlation between the variations of colorant "density" required for a practical manufacturing tolerance, the changes of source color temperature required by the conditions of service, and the variation of chromaticity allowed by the basic chromaticity definitions. Especially in the green and the yellow it is necessary to consider the manufacturing and service limitations in adopting the definitions and in all cases it is necessary to consider them in adopting limits for the control of production.

5.5. Effect of Filter Temperature

Another source of variation which must be considered is the change in the chromaticity of the signal resulting from changes in the temperature of the filtering element. This effect is most marked in the case of selenium glassware. To understand the nature of the change in transmittance with the temperature of the glass, it is well to compare the curves of spectral transmittance for different glasses at different temperatures shown in figure 5-10. These curves, which are due to Leberknight and Stone [1955], show six glasses at room temperature and one of these glasses at three additional temperatures. There is a considerable similarity between the changes from glass to glass and those from

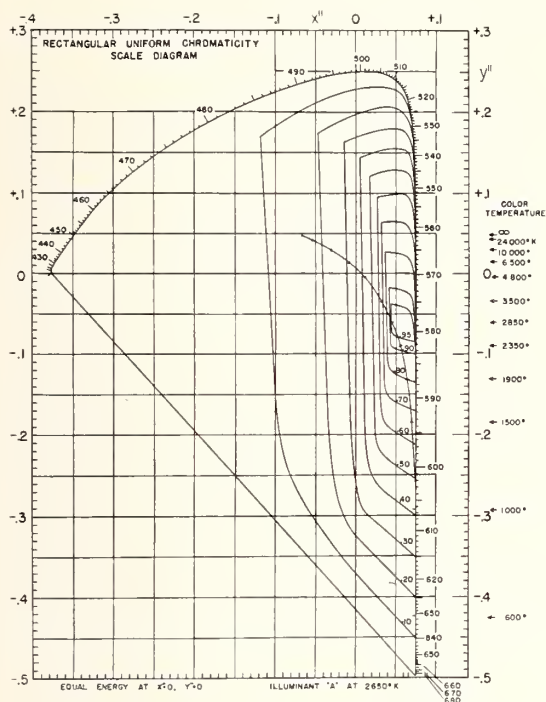


FIGURE 5-12. Contours of maximum transmittance.

The curves are contours of maximum possible transmittance for assumed source as computed by D. L. MacAdam [1935]. Source is at 2850 °K. These curves were computed by assuming 100 percent transmittance throughout one or two sections of the spectrum and zero transmittance for all other wavelengths. The original diagram in C.I.E. coordinates has been transformed to RUCS coordinates by Plaza.

temperature to temperature. In both cases the curves differ from others of the same set in the wavelength region in which the cutoff occurs, and the effect of increasing the temperature of the filter is much the same as that of substituting a somewhat redder filter except that heating reduces the transmittance as well as moves the cutoff.

Figures 5-11a and 5-11b show the change in redness as measured by the y (C.I.E.) and y'' (RUCS) coordinates for the chromaticities of the six filters of figure 5-10 as a function of physical temperature. The light source is at 2854 °K. These figures include, in addition to the filters measured by Leberknight and Stone, two measured by K. S. Gibson [1916] and one measured by Holland and Turner [1941]. The filters are all selenium red glasses, which indicates that they are all sharp cutoff filters. Although some of the measurements used for these curves were published as early as 1916 and some as late as 1955, although some were made in this country and some in Great Britain, and although the glass of three different glassmakers was measured, the results are in substantial agreement. Glass of other colors changes slightly with temperature but not sufficiently to constitute a problem.

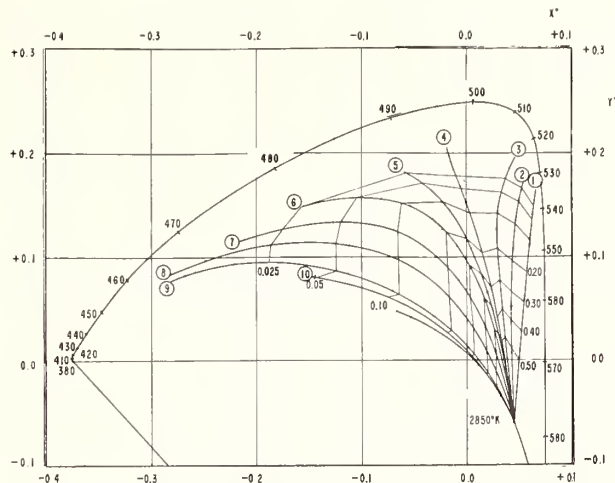


FIGURE 5-13. Transmittance obtainable with blue and green glasses.

The smooth curves which fan out from the point 2850 °K are the loci resulting from changing the thickness with the same type of filter. The irregular lines are contours of equal transmittance for the filters included. The diagram gives some indication of the maximum transmittance it is reasonable to expect for different chromaticities, but service glassware will have somewhat lower transmittances for the same chromaticities because of losses incidental to the designs. Types of glasses are shown in the following table by designation in use at the time the glasses were received at the Bureau.

Types of Glasses Corning Glass Works Designations

1 G 401 CZ	7 Combination 2/3 thickness G 403 ED
2 G 408 N	1/3 thickness 552 PY
3 G 408 F	8 Combination 1/3 thickness G 403 ED
4 G 406 AY	2/3 thickness 552 PY
5 G 40 D	9 552 PY
6 G 403 ED	10 557

5.6. Relation of Transmittance to Chromaticity

All those variations which affect the chromaticity of a signal light also affect the transmittance of the filtering element and so increase or decrease the intensity of the signal.^{19, 20} There are, moreover, limits to the transmittance which it is possible to obtain for any given degree of saturation in a selected hue. This is evident from the fact that the selective coloration of the light transmitted by the filtering element is obtained by absorbing the undesirable wavelengths. These limits have been computed by D. L. MacAdam [1935].²¹ One of his fig-

¹⁹ For similar information on glass made by a different manufacturer, see Holmes, [1937] and [1946].

²⁰ It has now been found, as was long suspected, that the luminous intensity of a colored signal light computed from the transmittance of the filter is not an accurate measure of the relative brilliance of the light as seen at long range. (See Middleton [1937] and Dressler [1953].)

²¹ Later MacAdam [1950] extended his investigation of this problem to the case of the maximum attainable luminous efficiencies with any possible sources. He found these to be monochromatic sources or pairs of such sources in every case.

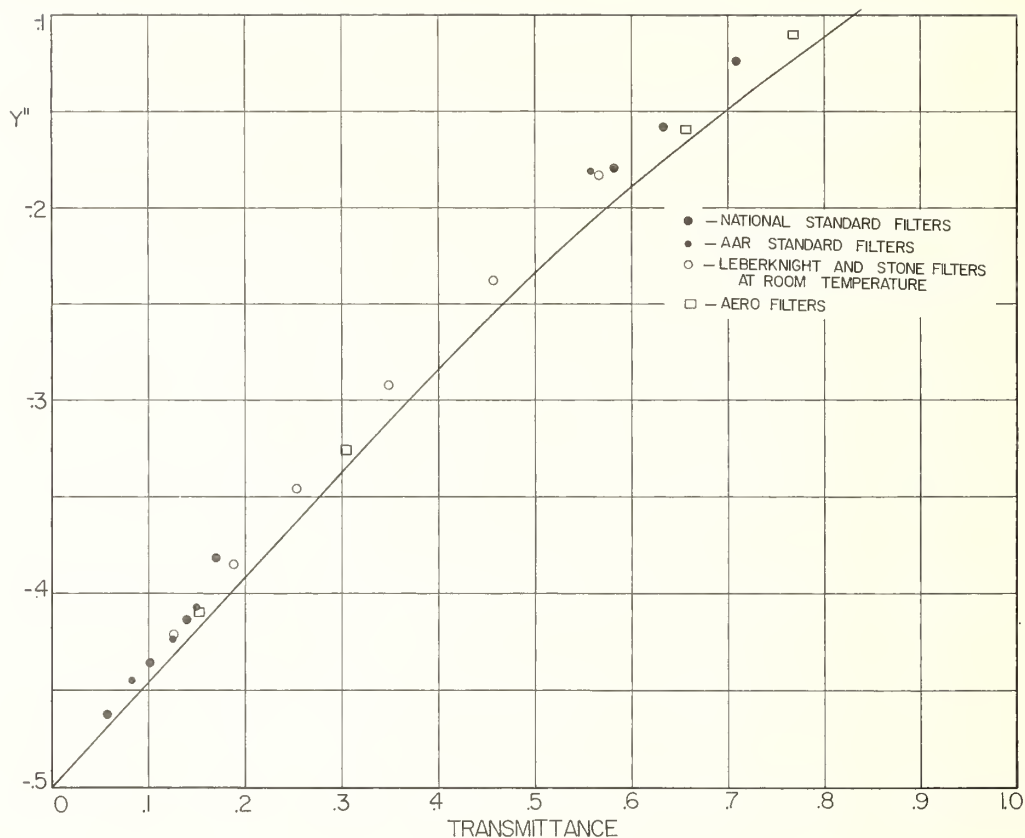


FIGURE 5-14. *Maximum and practical transmittance for red and yellow filters.*

The continuous line represents the theoretical limit of transmittance (T) as a function of redness as indicated by y'' . Allowance has been made for surface losses of 0.08. The points marked by the several symbols represent actual filters of selenium glass.

ures, transformed into the RUCS system by Plaza,²² is shown in figure 5-12. In practice the available transmittance is always somewhat lower than the theoretical maximum. Figure 5-13 shows the transmittances obtainable in different parts of the green and blue areas through the use of different types of glass in different thicknesses. These curves were computed for some of the same filters selected for figure 5-1.²³

Figure 5-14 shows the relation of the transmittance to the y'' coordinate, representing redness, for typical selenium glass filters transmitting light from sources at 2854 °K color temperature. The closer y'' approaches -0.5, the redder the chromaticity represented. The theoretical limit for the maximum transmittance possible for each value of y'' with a light source at this color temperature is

also shown on this diagram. The values for the typical filters are shown by open and solid circles. The reader can readily draw a curve to represent the maximum transmittances obtainable in practice for any desired redness. In using this curve as a basis for estimating the characteristics of glassware, a further allowance must be made for the differences between polished filters and practical filter ware, that is, for losses due to scattered light and additional internal reflections. This allowance may be more closely estimated when the transmittances of similar glassware of a somewhat different color are known, in which case it is only necessary to multiply the transmittance for the known ware by the ratio of the transmittances given by the curve for the desired and the known chromaticities. The ratios obtainable from this curve can be very useful in revising transmittance requirements to allow for changes in chromaticity requirements.

Figure 5-15 shows the variation of transmittance with the color temperature of the light source for typical filters of the more commonly used signal colors.

²² Lorenzo Plaza, of the Institute of Optics of Madrid, assisting as a guest associate at the National Bureau of Standards.

²³ It is customary to consider that the transmittance determined spectrophotometrically can be used to determine range and apparent brightness (brilliance) of a signal light, notwithstanding the experience of signal-light engineers that red lights often appeared brighter than this assumption warranted. Middleton and Gottfried [1957] have made tests confirming the qualitative observations. A corresponding effect with large sources has been found by several investigators. For a review see Dressler [1953]. Another practical consideration is the differential transmittance of smoke and haze. See Breckenridge [1931].

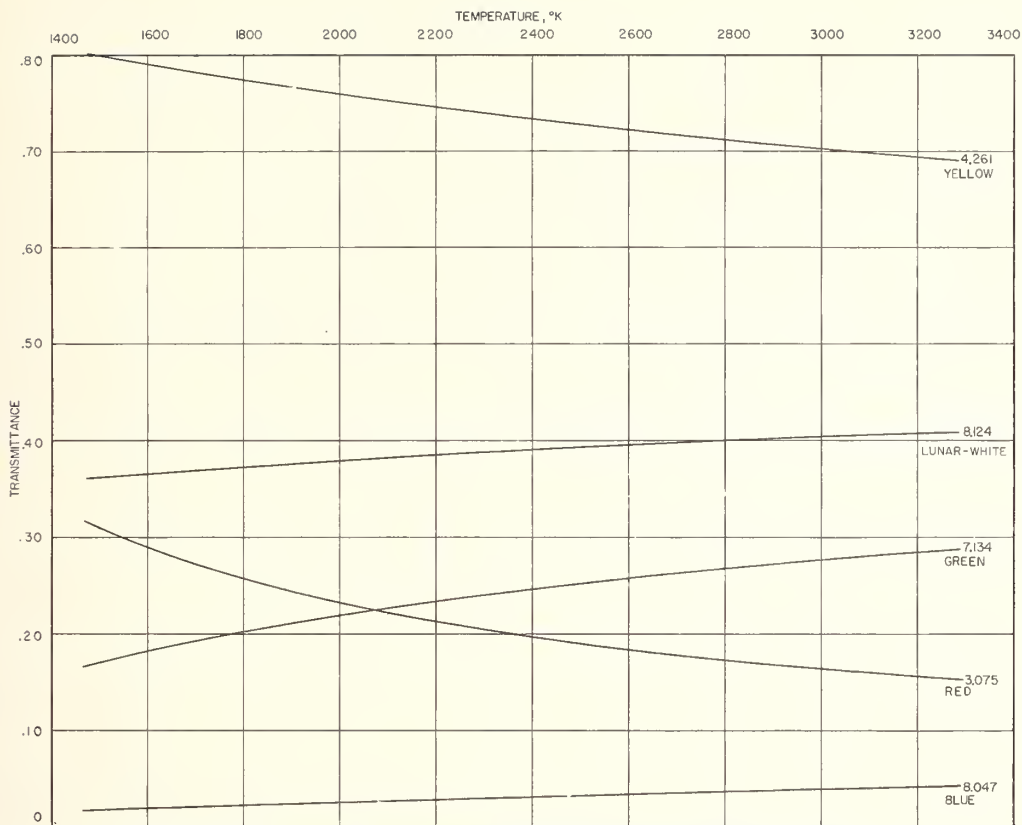


FIGURE 5-15. Variation of transmittance with color temperature of light source.

The figure shows the relationship between the transmittance (T) and the color temperature of the light source for five of the standard filters included in the U.S. Standard N.B.S. [1964]. It should be noted that while the blue, green, and lunar-white filters increase in transmittance with the increase in color temperature of the source, the red and yellow filters decrease.

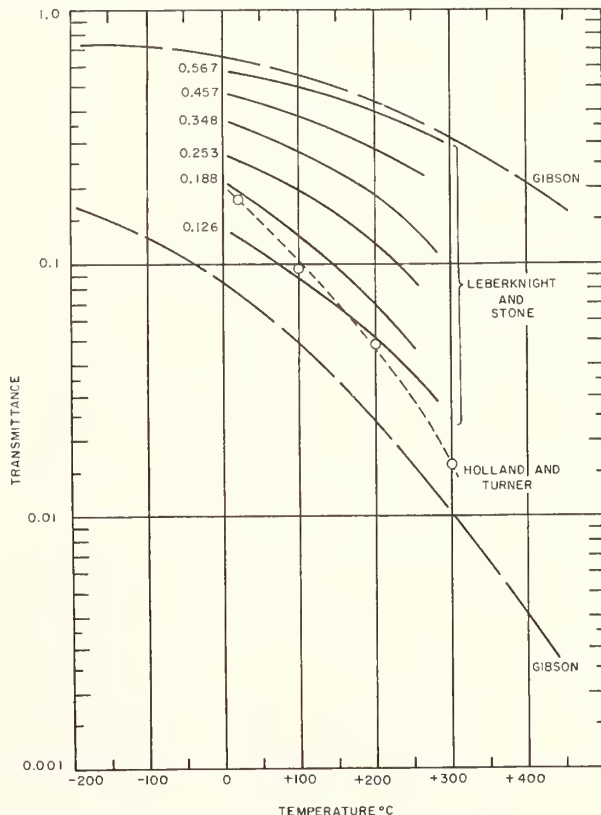


FIGURE 5-16. Effect of physical temperature on transmittance of selenium filters.

The curves show the decrease in transmittance as the physical temperature of the filter increases. Three different researches are included. They are distinguished by the names of the investigators: Gibson [1916]; Holland and Turner [1941]; and Leberknight and Stone [1955]. The numbering of the curves is the same as in figures 5-10 and 5-11. The two sets of measurements made in the United States appear consistent, notwithstanding that Gibson made his measurements on Corning glass filters and Leberknight and Stone made theirs on Kopp glass filters. The results by Holland and Turner indicate a more rapid change of transmittance with temperature than was found by either of the other researches.

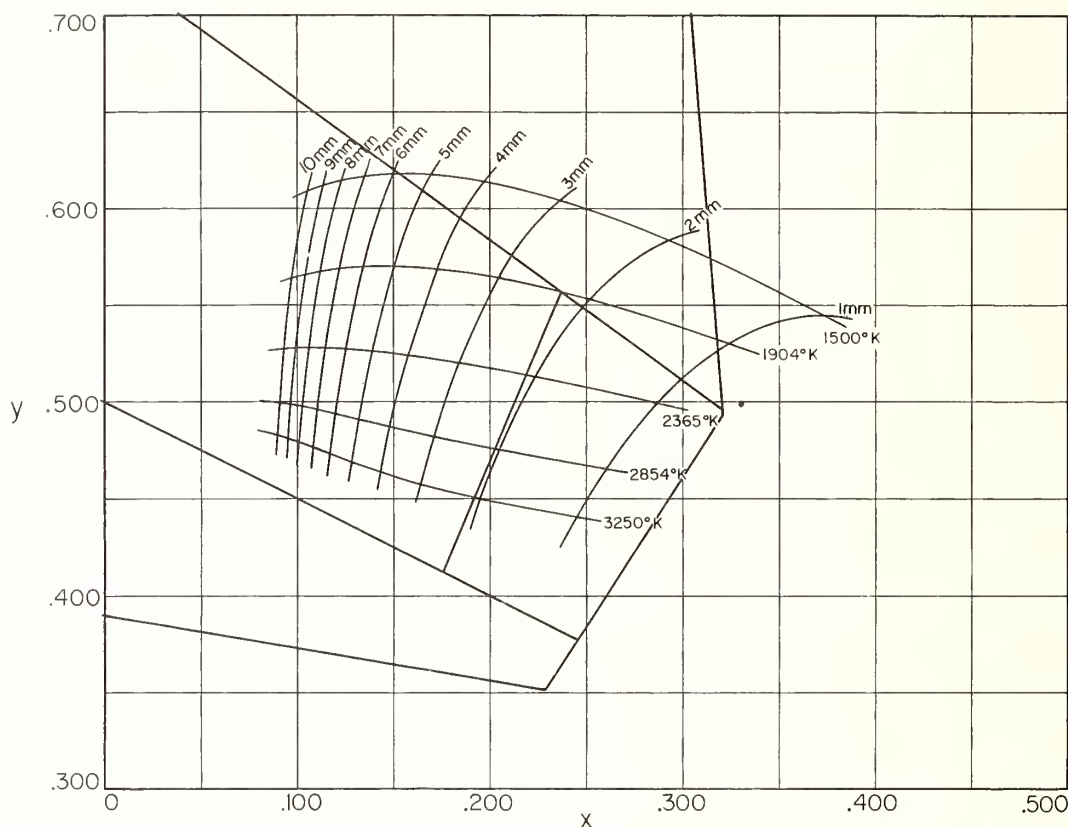


FIGURE 5-17. Chromaticities of green plastic filters (C.I.E.).

These curves correspond to those shown in figure 5-5. The more nearly horizontal curves (labeled with color temperatures) show the variations of chromaticity with change of thickness for the filter used with a light source of the color temperature indicated for each curve. The more nearly vertical curves (labeled with the thickness of the filter) show variations of chromaticity for changes of light source color temperature. The straight lines are chromaticity boundaries from the U.S. Standard, N.B.S. [1964].

5.7. Effect of Filter Temperature on Transmittance

The changes in the chromaticity of selenium filters with the temperature of the glass are accompanied by changes in their transmittances as shown in figure 5-16. The glasses for which values are given in these curves are the same filters for which the chromaticity characteristics are given in figures 5-11a and 5-11b. Some other types of filters show changes in transmittance with change of filter temperature but such changes are important only if the change of temperature amounts to several hundred degrees and the requirements are exacting.

5.8. Plastic Filters

Filters of any color required for signal lighting can be made of plastic material and spectral transmittance data are available for a few red, yellow, and green plastic filters. Unless care is taken to choose color-fast dyes, however, blue and green filters are liable to fade in service. Figures 5-17 and 5-18 show the chromaticities for light sources of color temperatures ranging from 1500 to 3250 °K

for a plastic filter which has been found to have a resistance to fading that was adjudged satisfactory for use in a beacon cover. This same figure also shows the variation in chromaticity with thickness for this filter for all these light sources.

Red and yellow plastic filters are available with sharp cutoffs that give as high purity of color as is available in glass. Yellow filters are also available which have chromaticities very similar to those of carbon-yellow glass although the spectral transmittance characteristics are somewhat different. Some of these red and yellow filters are fluorescent which interferes with making reliable spectrophotometric measurements for them on some of the common types of spectrophotometers.

5.9. Gaseous Discharge and Fluorescent Lamps

Gaseous discharge lamps offer the possibility of obtaining signal-light colors without the use of filters although such lamps may also be used with filters to improve their chromaticity. Such lamps

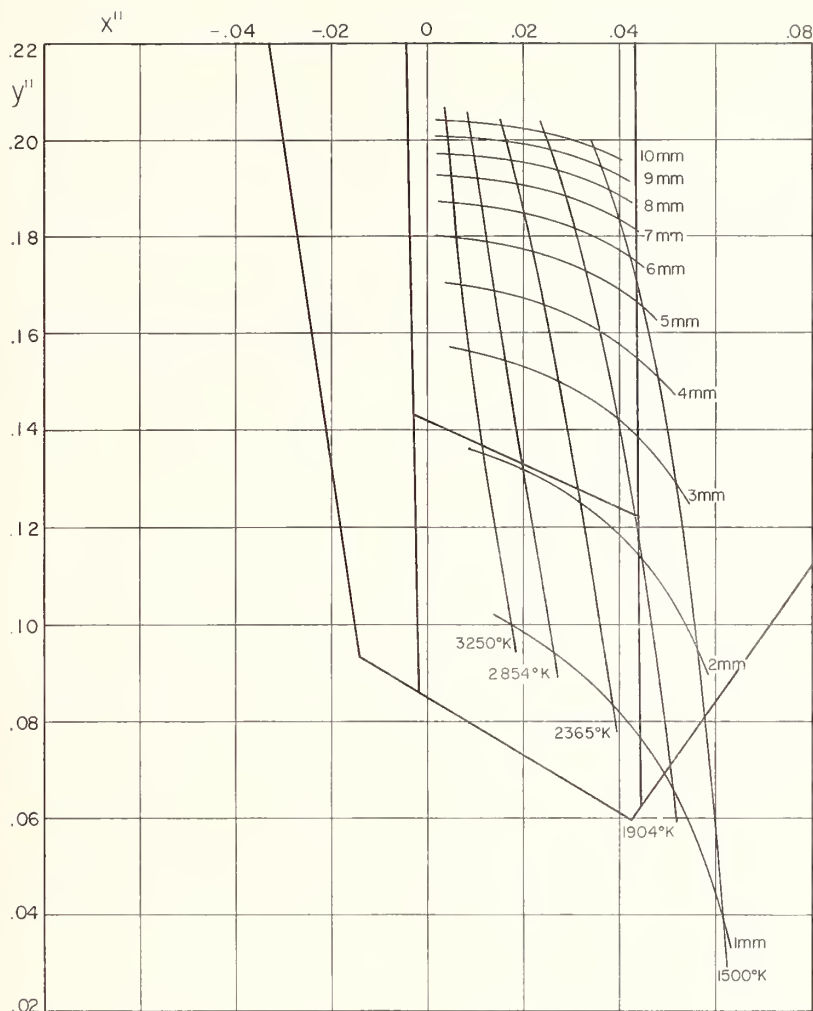


FIGURE 5-18. *Chromaticities of green plastic filters (RUCS).*
This figure is a transformation into RUCS coordinates of the curves shown in figure 5-17.

are sometimes assumed to have characteristic colors that are dependable. This is not an entirely safe assumption. The chromaticity of neon lamps varies with the pressure as indicated in figures 5-19a and 5-19b. When the lamps are new the pressures used in them are not normally low enough to give chromaticities paler than the limit allowed in the U.S. Standard [NBS 1964] and C.I.E. Recommendations [1955]. In use, however, the neon lamps decrease in pressure and this may cause the chromaticity to become paler than the specified limit.

In the case of discharge lamps which contain mercury, there may be a large change of chromaticity as the ambient temperature varies. At lower temperatures the vapor pressure of the mercury is reduced with consequent decrease in the intensity of the mercury spectrum while the spectral distribution of the light from any rare gases in the lamp is little affected by the change in temperature.

Similar difficulties have been experienced with sodium lamps which contain neon. At low temperatures the sodium is not volatilized and the neon gas which is used to start the discharge contributes most of the light. At such temperatures, the light is nearly red enough to meet the definition of aviation red. When the lamp is fully heated the color is slightly redder than the sodium *D* lines. One lamp measured at the National Bureau of Standards was found to have a chromaticity approximating $x=0.578$, $y=0.422$ ($x''=+0.075$, $y''=-0.160$) which would make it an acceptable yellow under any of the signal-color specifications in common use in the United States.

Fluorescent lamps are seldom used as signal lamps because of their low luminance. Figure 5-20, however, shows the chromaticities for a number of common types of fluorescent lamps. To distinguish those lamps which lie within the boundaries of the U.S. Standard [NBS 1964] the boundaries for the signal colors are also shown.

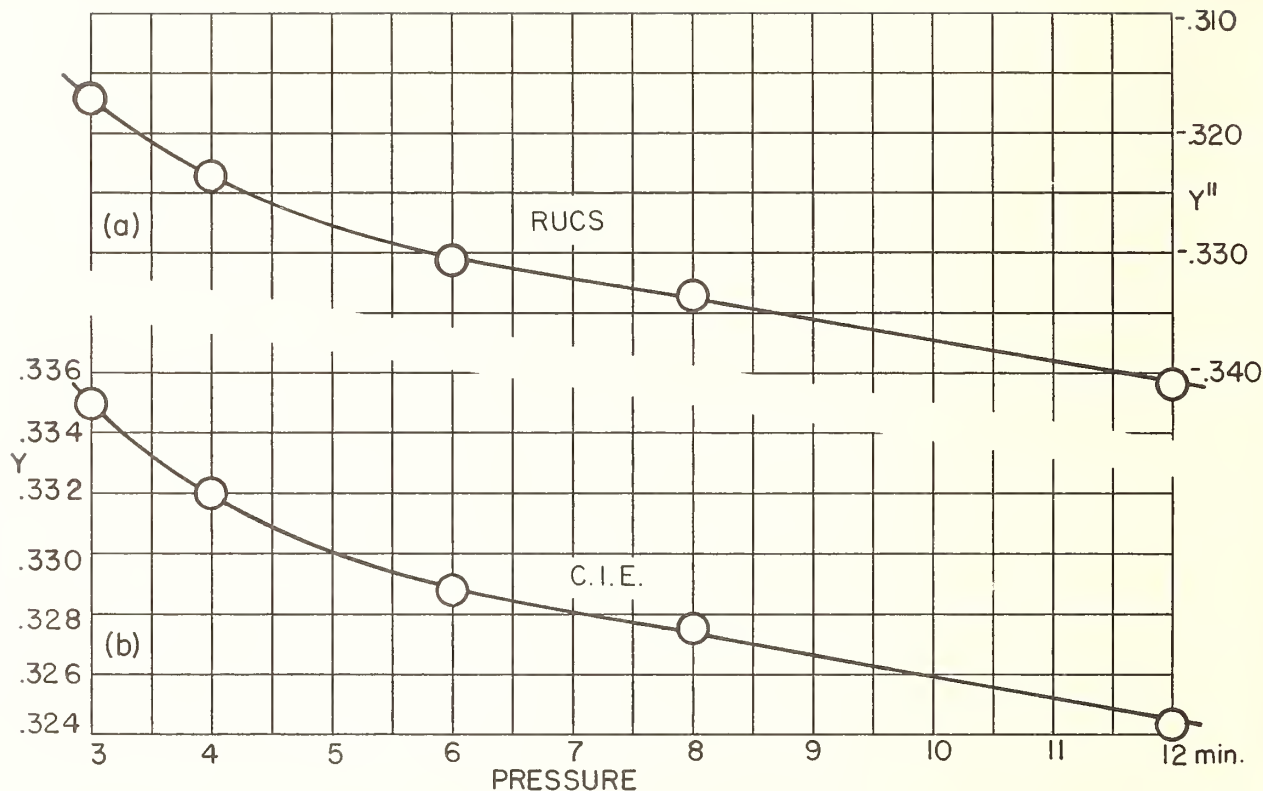
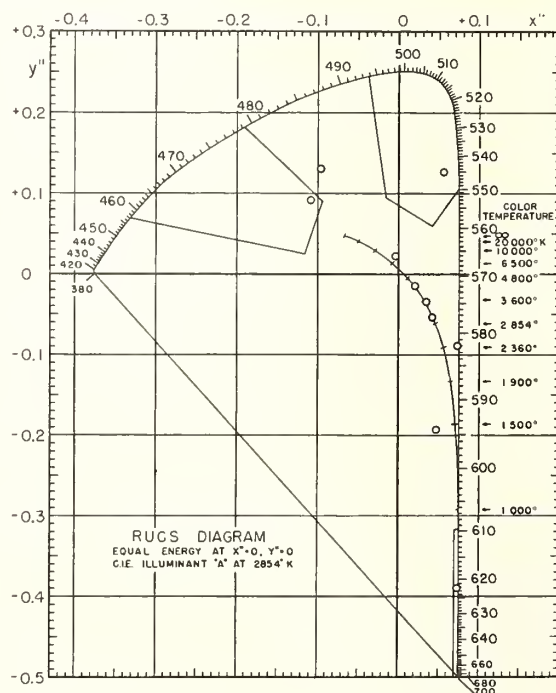


FIGURE 5-19. *Chromaticity of neon lamps.*

The curve shows the variation in the chromaticity in a group of neon lamps which had been filled with different pressures by the manufacturer. There was no way to determine how accurately the pressures had been measured. Inaccuracies in adjusting the pressures to the nominal values indicated may account for the irregularities in the values plotted. The pressure is expressed in mm of Hg. (a) Redness indicated by y'' (RUCS). (b) Redness indicated by y (C.I.E.).

FIGURE 5-20. *Chromaticities of fluorescent lamps (RUCS).*

The circles show chromaticities obtainable with the use of commercially available fluorescent lamps, plotted in RUCS coordinates. Additional chromaticities are obtainable with mercury, neon, and the sign-lighting type of lamps. Those shown in the figure along the Planckian locus are known as daylight, cool-white, white, and warm-white. Those with more saturated colors are blue, deep-blue (it is nearer the spectrum locus), green, gold, pink, and red.



6. Control of Signal Colors

6.1. Selection of Standards

In addition to making sure that specifications and regulations controlling signal-light colors describe colors which are recognizable and which may be produced in service with practicable means, it is also essential to be certain that the requirements be such that they may be applied most effectively in the various situations for which they are intended. Usually the control of signal-light colors involves the use of chromaticity standards which are represented by filters in combination with stipulated light sources.

6.2. Types of Color Standards

Most of the signal-light applications have so much in common that it is possible to have a single set of filters which, with a light source of 2854 °K, can be used as national standards for most purposes. A set of 18 such national standards is now in process of adoption. Fourteen suitable glasses are available among the thirty filters which the Signal Section of the Association of American Railroads adopted between 1931 and 1935.²⁴ The chromaticity characteristics for the other four have been approximately defined, but glasses have not yet been selected. All of the national standards have been checked in relation to the basic chromaticity definitions and found satisfactory.

The national standards²⁵ have been selected to provide for the colors that are at present required by more than one organization. There are a few other applications such as those for which the railroads use lunar white, and perhaps the walk signals used at pedestrian crossings, which did not appear to be sufficiently general to warrant the establishment of national standards. Provision is made in the U.S. Standard for the Colors of Signal Lights [1964] for organization standards to meet these needs. Organization standards must be checked against the basic chromaticity definitions to make sure that the signals will remain within these definitions under all the conditions of use.

Neither the national standard filters nor the filters of organization standards are available for inspection purposes. Duplicates are required for such purposes and the U.S. Standard provides for such duplicates by establishing tolerances for them.

Even the duplicates are likely to be considered too valuable for routine operations, and manufacturers and laboratories will generally make their own duplicates for use as working standards. No standard tolerances have been provided for these since they are solely the responsibility of the manufacturers.

6.3. Inspection Testing

The final step required to assure that the signal colors conform to the selected standards is the inspection testing of the equipment. This inspection may be made on completely assembled equipment, but more often it is made on the filtering element, for example, on a piece of glassware or a plastic cover. In either case the problem is to compare the light from the unit, or the light transmitted by its filter, with light from a source of known color transmitted through a duplicate standard and to decide from that comparison that the light from the unit is, or is not, within the specified chromaticity limits.

Several researches have been carried out to develop a photoelectric colorimeter for this purpose. The objective was to build an instrument which would give a direct reading of the chromaticity coordinates for the light intercepted by the meter. Few, if any, colorimetric laboratories have such equipment today. At this time, we know of no manufacturer who has an instrument of this type that is suitable for use on routine inspections. It would appear practicable to design photoelectric equipment that could be adjusted to distinguish between acceptable and unacceptable ware made with a predetermined colorant, but such equipment is not yet available.

An instrument has recently become available which measures small color differences photoelectrically and which is proving useful for some inspection purposes. Unless such instrumentation is available, visual comparisons must still be relied upon.

Since visual comparisons must be used for carrying out inspection tests, it is important to take into consideration the types of comparison which can be made by the usual inspector. These comparisons are limited to discriminations of hue, saturation, and brightness. In view of this, it is essential to draft specifications so that only direct comparisons of hue and saturation are required in testing equipment for conformity with the chromaticity requirements. Specifications worded in this manner are most easily correlated with basic chromaticity definitions if the boundaries of these definitions are lines of constant hue and saturation.

The practical problem of inspection is greatly simplified if there is a requirement in the specification that the chromaticity characteristics be similar to those of a standard filter. This type of requirement is discussed in sec. 8.

6.4. Laws and Regulations

A somewhat distinct problem in the control of signal-light colors arises in connection with laws and regulations. These are usually written in terms of

²⁴ See Association of American Railroads [1953].

²⁵ For the status of national standards see McNish 1958.

color designations that are defined in technical Standards. To be suitable for such legal citation the Standard must define the signal as observed by the user, in order to be impartial, and this definition must be stated with sufficient precision to avoid any significant uncertainty of interpretation.

As has been mentioned in sec. 2.5, the application of laws and regulations to the performance of signal-light units involves the same procedures as the inspection of ware on procurement. Whether the enforcement is by an inspector on the staff of the regulatory authorities, or by an inspector in the employ of the manufacturer, his basis for acceptance or rejection will be similar to that in a

procurement transaction, that is, it will be a comparison of light from a test piece with light of known chromaticity. For this reason the same principles apply to the choice of boundaries for chromaticity definitions underlying laws and regulations as in setting up procurement specifications. In this case, however, there is more likelihood that no standard having chromaticity characteristics²⁶ similar with respect to the test item will be available, and the visual comparison may have to be made with a greater chromaticity difference than is commonly encountered in the inspection of purchased filter ware. This makes it even more necessary that the type of judgment required be one of hue or saturation.

7. Chromaticity Boundaries

7.1. Selection of Boundaries for Chromaticity Definitions

Chromaticity boundaries are involved in two ways in the control of signal-light colors, in the basic chromaticity definitions and in the tolerances for the duplication of standard filters. The problems are essentially different in these two applications. The basic definitions will be considered first.

As in the case of signal color specifications as a whole, it is essential that the chromaticity definitions in particular be so designed that they not only describe chromaticities that can be dependably recognized and can be practicably produced, but also that the boundaries differentiate between acceptable and unacceptable chromaticities in ways that afford the most prospect of positive differentiation in practice. These definitions are used mainly for three purposes, namely: (1) the control of color standards; (2) the inspection of filter ware; and (3) the application of regulations.

For the selection of standards, any positively defined lines in a coordinate system that can be computed from spectral transmittances will suffice because standard filters are regularly tested, either by spectrophotometry or differential colorimetry, both of which methods of measurement provide the chromaticity coordinates of the filter source combination.

The inspection of ware and the enforcement of regulations, on the other hand, usually involve visual comparisons and the only such chromaticity judgments that can be directly made are those of hue and saturation. It follows that reliable judgments with reference to whether a difference indicates a chromaticity inside, or outside, of the chromaticity definitions can only be made if the definition boundary is a line of constant hue or constant saturation.

7.2. Constant Hue Boundaries

It was stated in sec. 3.4 that the C.I.E. and RUCS diagrams are color-mixture diagrams with pure spectrum colors situated along the curved outside boundary and white located somewhere in the center. The selection of the white point, which is technically known as the heterogeneous stimulus, should properly be determined by the conditions of observation. In developing the RUCS diagram, the equal-energy point, that is, the point representing a mixture of all visible wavelengths in equal parts, was selected as the white point because it is centrally located among the points usually advocated as desirable white points and also because it is easily defined. For signal-lighting purposes it might be preferable to use a point corresponding more closely to the color temperature of the lights which are most commonly seen at night, but the difference between the equal energy point and the point representing 2854 °K, for example, produces no difficulties that are serious so far as present requirements are concerned.

It was pointed out in sec. 3.4 that in any mixture diagram all the points lying between two points on a straight line through them represent chromaticities that may be produced by adding light of the chromaticities represented by the two selected points. It follows, therefore, that all the points on a line from the origin in the RUCS diagram through any point on the spectrum locus represent chromaticities which may be produced by adding white light to the spectrum color represented by the point on the spectrum locus. Such a line is a line of constant dominant wavelength and one would expect it to be a line of constant hue. This is approximately true.

²⁶ See NBS Handbook 95, p. 2, sec. 2.11 [1964] and chapter 8 of this Monograph.

Newhall, Nickerson, and Judd [1943] have determined lines of constant hue from the spacing of elements in the Munsell uniform color space and D. L. MacAdam [1950, 51] has made a direct determination. While both of these determinations show some curvature, the departure from straight lines is not greater than the uncertainty of applying determinations made with surface colors to the case of signal lights. In view of this and the lack of standard definitions for constant-hue lines, there would seem to be little advantage in complicating a specification to allow for the difference between the straight and the curved lines.

7.3. Constant Saturation Boundaries

On a uniform chromaticity diagram which faithfully represented chromaticity differences for given conditions of observation, the loci of constant saturation would be circles having the white point, or heterogeneous stimulus, as center. If circular arcs on the RUCS diagram were adopted as saturation boundaries for the basic chromaticity definitions, the corresponding boundaries on the C.I.E. system would be ellipses. Since the C.I.E. system is the only one recognized as standard at the present time, and ellipses are too difficult to apply as criteria in practical problems, it is necessary to use the chords of elliptical arcs to express boundaries that approximate boundaries of constant saturation on the C.I.E. diagram. Transformed to the RUCS, these lines become chords of circular arcs concentric with the origin.

7.4. Comparison of Rational and Adopted Boundaries

Figure 7-1 shows a comparison of the boundaries of signal-light colors as defined in the C.I.E. Recommendations [1955] and the U.S. Standard [NBS 1964] with a system of radial and chord boundaries on the RUCS diagram. The radii and chords of this figure may be considered as lines approximating constant hue and constant saturation, as described in the discussions of secs. 7.2 and 7.3. In addition, four of these radii have been evenly spaced, since it is plausible to assume that optimum boundaries for signal colors can be constructed from such lines if the RUCS diagram is uniform in its chromaticity spacing. To construct this diagram, line (A, A') was drawn from the red terminus of the spectrum locus at $x'' = +0.075$, $y'' = -0.500$ through the origin and prolonged to intersect the spectrum locus in the upper left quadrant. This is a significant line on the RUCS diagram because it is both a line of constant dominant wavelength and close to a locus of confusion for the protanopic type of colorblindness. The angle between (AA') and the $-X''$ -axis has been bisected to obtain (BB') , and this line has been reflected about AA' to obtain

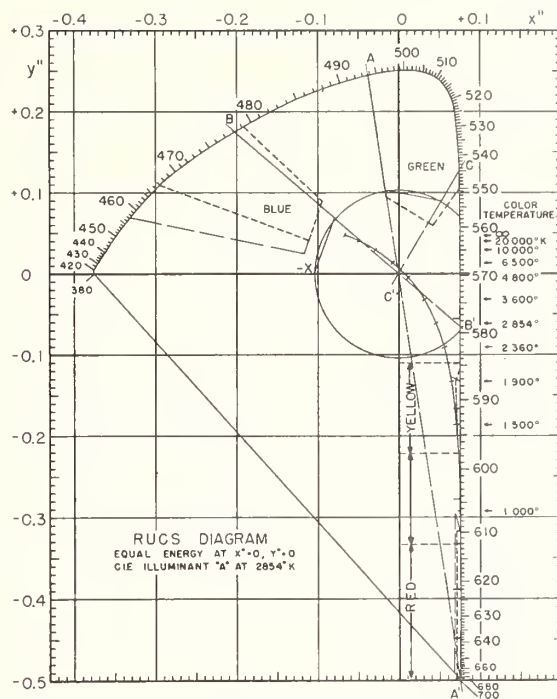


FIGURE 7-1. Comparison of rational boundaries with those of the U.S. Standard.

Figure 7-1 shows a comparison of boundaries for signal light colors as defined in the C.I.E. Recommendations [1955] and the U.S. Standard, N.B.S. [1964] with a system of radial and chord boundaries on the RUCS diagram. The construction of the diagram is explained in the text. It is intended to show that there is a general approximation between the spacing of chromaticities in the RUCS diagram and that to be expected if the limits of the C.I.E. Recommendations and U.S. Standard are accepted as representing optimal signal colors. The absence of signal colors in the lower left hand quadrant is explained by the lack of filters giving such chromaticities.

(CC') . Designating the origin as O , this makes $\angle X''OB = \angle BOA = \angle AOC$. The chords were drawn through the points in which an arc centered at the origin intersected the hue boundaries $(-X''O, BO, AO, \text{ and } CO)$. The radius of this arc was made equal to the mean distance from the origin to the four inner corners of the C.I.E. blue and green regions.

7.5. Boundaries for Blue and Green

A comparison of these rationally constructed boundaries (sec. 7.4 and figure 7-1) with those which have been adopted on the basis of research and experience shows that in the blue and green regions there is a considerable degree of correspondence between the rational and the recommended boundaries. The coincidence of the blue boundary recommended for green with the line through the red terminus may be accidental. The blue boundary of green was, however, intentionally displaced towards the blue to make it possible to use green signals that can be differentiated from red and yellow signals by red-green colorblind observers. The similarity of the rational divisions to the chromaticity boundaries of the C.I.E. Recom-

mendations and the U.S. standard in the blue and green regions favors the use of a uniform-chromaticity-scale diagram, such as the RUCS diagram, for studying the chromaticities of such signals.

7.6. Boundaries for Red and Yellow

Returning to figure 7-1, it is seen that the boundaries for yellow are displaced clockwise and the boundaries for red counterclockwise from the sectors opposite the blue and green. This places both the red and yellow in the sector opposite the gap between blue and green. The displacement of the yellow is reasonable in view of the fact that the spectrum locus in the region opposite the blue, which should have been selected on the basis of equal arcs, is too close to the white point to be acceptable as a color distinct from white. The sector opposite the green, on the other hand, is characterized by much lower transmittances and saturations than are obtainable in the long wavelength region.

From the fact that both recommended red and recommended yellow lie in the same sector, it appears that the discrimination is not made on the basis of hue angle alone, and that the linear distance of separation on the RUCS diagram rather than the angle may be the basis for the recognition of these colors. Since the acceptable regions here lie so close to the spectrum locus, we may take distance along the line $x'' = +0.075$ as a possible basis for assigning boundaries to red and yellow. In figure 7-1 the rational boundaries in the yellow-red quadrant have been drawn by dividing the spectrum locus in this quadrant into nine equal parts. Two-ninths have been allotted to yellow and two-ninths to each of the unused sectors on both sides of yellow, leaving three-ninths for red. This extra allotment to red may be, to some extent, justified by the fact that the longest wavelength portion of the red allotment is not serviceable for some red signals because it cannot be produced with a transmittance sufficient for use where long-range visibility is important. As in the blue and green regions, the similarity of the rational divisions of the RUCS diagram to the chromaticity boundaries of the C.I.E. 1955 Recommendations is sufficient to indicate that the RUCS diagram is an advantageous one for studying the chromaticities of light signals.

The unused sector along the spectrum locus from the yellow limit of green to the X'' -axis is approximately equal to the unused sector between the X'' -axis and the green limit of yellow. This makes the total gap between green and yellow twice that between yellow and red. As we have just observed, however, this long gap is required because the spectrum locus in this region is so close to the Planckian locus that both can not be used to represent distinct colors, and the chromaticities of the Planckian locus are those of the lights regularly used as white signals.

Having considered the hue limits of red and yellow on the basis of their extent along the spectrum

locus, it remains to complete the boundaries of these colors. To extend the paler hue limits towards the white point is to include chromaticities that are paler than some chromaticities these limits exclude at the spectrum locus. It seems desirable, therefore, that the boundaries from these points should follow loci of constant saturation. A possible way of completing each of these areas in a manner somewhat analogous in form to that of the green and blue areas would be to follow arcs having their centers at the origin until these arcs intersect radii from the origin through the second hue limits. This would cause the yellow area to include the chromaticities of lamps incandescing at low voltage. From the standpoint of producibility there is no need to include these chromaticities in the yellow region. For any system which includes a white signal, it is advantageous to limit the yellow to a narrow band along the spectrum locus. In the case of red lights, an area defined by an arc of minimum saturation through the pale hue limit and a radius through the reddest chromaticity visible would include chromaticities which, according to Hill's results shown in figure 4-9, are much less dependable than those closer to the spectrum locus. On the other hand, it is quite practical to close the red and yellow areas with boundaries approximately parallel to the spectrum locus which is the direction of the natural chromaticity variations of the glasses used. The C.I.E. green boundary of yellow and yellow boundary of red do extend somewhat inward on lines directed towards the white area but since this is not necessary to secure adequate transmittances, the U.S. Standard uses lines approximating constant saturation for these boundaries.

7.7. Boundaries for White

White lights are commonly lights without filters, so that the chromaticity definitions for white lights usually follow the Planckian, or blackbody, locus. This suggests a definition for white in the form of a maximum permissible departure from the blackbody locus. It appears feasible to accomplish this in the C.I.E. system by a tolerance in y alone. When such a tolerance is transformed into RUCS coordinates, however, it is evident that the boundaries are far from uniform tolerances. The U.S. Standard [NBS 1964] has tolerances expressed as functions of x and y , and it is a region of fairly uniform width in RUCS which does not actually overlap the yellow region.

There is another type of white light which requires pale blue filters if the source is an incandescent lamp. These are called "lunar white" or "blue white" lights. Since they require filters, it is frequently desired to compute their chromaticities from spectrophotometric data. Curved boundaries are unsatisfactory in this case because of the undue amount of computing which may be required to find out whether the chromaticity is

inside or outside the chromaticity definition. In this region, also, the Planckian locus is much less curved than in the region of the unfiltered white lights. For these reasons, it is both desirable and feasible in this region to use a narrow quadrilateral as a boundary.

The U.S. Standard for the Colors of Signal Lights [NBS 1964] uses straight boundaries for lunar white and blue white since these require filters and curved boundaries based upon the blackbody locus for beacon white and variable white. The International Commission on Illumination [1955] and the International Civil Aviation Organization [1964] solve the problem by using complex straight-sided figures allowing rather large departures from blackbody chromaticities in some regions in order not to limit the departure too severely in other regions. The problems encountered in distinguishing yellow from white will be discussed in sec. 10, "Use of Signal Light Colors."

7.8. Mathematical Simplicity

In the past, specification writers have shown a preference for limits that were easily expressed in the C.I.E. system, as for example, $x = \text{constant}$. This is quite reasonable as long as no reason for preferring other limits is known, but the consideration of mathematical simplicity seems entitled to little weight in comparison with the importance of having the limits follow lines representing sequences of colors which an inspector can recognize; that is, the chromaticities on one side of the boundary should differ from those on the other in some common, recognizable respect.

In another sense mathematical simplicity is a sound principle. The direction and location of a boundary having been determined in accordance with the general principles discussed above, the equation used to define the boundary should not be expressed with more significant figures than are warranted by the precision with which the facts are known. On the other hand the use of equations that are not reduced to mathematically simplest terms may be warranted if by their use it is possible to make it clear that the equation passes through a particular point of interest.

7.9. Relation of Chromaticity Boundaries to Primary Limit Standards

In approving primary standards, the laboratory concerned must consider the use to be made of these standards. If they are to be used to test duplicates by direct colorimetric comparison with a visual instrument, the primary standards should be far enough within the boundaries as to insure that the duplicates in turn can be used by direct comparison to keep the ware within those

boundaries. Color standards may, of course, be located even further inside the basic chromaticity definitions if it is deemed advisable to restrict the chromaticity of the signals controlled by these standards to something less than the entire range permitted by the definition.

7.10. Boundaries for Duplicates

Boundaries for tolerances are quite a different problem from those for basic chromaticity definitions. Although duplicate filters are frequently tested visually, this testing is done with colorimeters designed to give precise values for the color differences between the filters being compared. For this reason, it is not important that the boundaries of these tolerances follow lines of constant hue and saturation.

It would be logical and quite simple to assign tolerances for duplicate filters by requiring that the chromaticity coordinates for the duplicates, expressed in the RUCS system, fall within a fixed radius of the point representing the chromaticity of the primary standard. It is not at present practicable to do this, however. The calibration of the existing colorimeters used for this work is in terms of C.I.E. coordinates and their immediate recalibration in RUCS would be a considerable task. More than this, the duplicates must sometimes be ground a number of times to bring them within the tolerances, and the increase in the complexity of the calculations involved in checking glasses for compliance with circular tolerance areas would not be offset by any advantage to be gained from excluding the corners of rectangular tolerance areas.

A. J. Werner²⁷ has developed a rational method of constructing tolerances for duplicates. When the glass for duplicates is ground to meet a tolerance, the chromaticities of all the ground filters will be found to lie on a curve which is approximately parallel to that generated by the chromaticity of the primary calculated for various thicknesses (see fig. 5-4). Werner proposed to make the slope of two sides of the tolerance parallelogram parallel to the tangent to the locus for the primary filter at the point representing the primary itself. The spread between the curves for glasses from different melts, all of which represent good practice in the reproduction of the original melt, determines the separation of the long sides of the tolerance. If the tolerance area lies close to a boundary for a basic chromaticity definition, its other two sides may be made parallel to that boundary. Making these sides parallel to the boundary may facilitate the inclusion of the necessary area within the tolerance without encroaching too close to the boundary.

²⁷ Unpublished memorandum prepared in connection with the revision of Specification 59-39 (now 59-61), Signal Section, Association of American Railroads. For further information consult Mr. A. J. Werner, Corning Glass Works, Corning, New York.

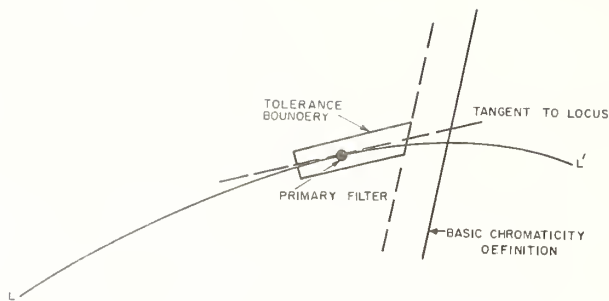


FIGURE 7-2. *Illustration of filter tolerance.*

The diagram shows the relationship of tolerance boundaries to the chromaticity of the primary filter and the locus of chromaticities for filters made from the same type of material but differing in thickness, LL' .

Such a tolerance area is illustrated in figure 7-2. In this figure LL' represents the locus of a primary standard filter. The straight lines represent a tolerance area constructed in accordance with Werner's proposal as used for specifications of the Association of American Railroads and subsequently adopted for

tolerances in the U.S. National Standard [NBS 1964].

Another construction, preferred by the author, is to make the slope of the ends approximately perpendicular to the sides when this is feasible. This construction gives the most compact area practicable. The distance between the ends of the parallelogram is determined by the accuracy within which the optimum thickness may be approximated, which, in turn, is determined by the difficulties of computing and grinding. The size of the tolerance required varies from primary to primary according to the closeness with which melts can be duplicated and the precision with which thickness may be adjusted.

Since chromaticity differences are more easily measured to the accuracy required in duplicating tolerances than chromaticity coordinates are, it is preferable that the tolerances be expressed in terms of the differences between the chromaticity of the primary and that of the duplicate rather than as equations in the coordinates themselves. This, however, need not prevent the use of tolerances in accordance with the principles outlined above.

8. Requirement for Chromaticity Characteristics Similar to Standard

8.1. Purpose of Requirement

It is evident from the purpose of standard filters that their selection is primarily controlled by the basic chromaticity definitions, but since it has been pointed out in sec. 5 that both the chromaticity and the intensity of a light depend not only on the colorant in its filter but also on the filter's thickness, the color temperature of a light source and, in some cases, the physical temperature of the filter, the variation of these factors must also be taken into account. In the case of green filters especially, and to a lesser extent in the case of blue filters, the chromaticity variations caused by changes in the source are at substantial angles on the diagram with those caused by changes in thickness, or in colorant concentration, with the result that the total variation caused by both covers an area on a chromaticity diagram. This was illustrated in figures 5-5 to 5-8.

If a change is made in the colorant, the area of utilized chromaticities will be changed, and even though the original standards had been carefully checked to make certain that ware of the same colorant would stay within the basic chromaticity definitions for all the anticipated conditions of use, the ware with the changed colorant might produce signals outside of these definitions. A requirement to prevent the acceptance of improperly colored ware is an essential part of a dependable specification.

A change of colorant may also make a satisfactory inspection impossible. So long as the chromaticity differences in ware are such as can result from changes in the thickness of the ware, the variations

in chromaticity will be such that they may be represented on a chromaticity diagram by a smooth curve such as the broken curves in figures 8-1 and 8-2. It is a simple task for an inspector with normal eyesight to compare the chromaticities produced by the members of a group of filters which vary in this manner with a pair of standard chromaticities located on the same curve and determine that the tested chromaticities are within, or outside of, the acceptable segment of chromaticities. If the colorant used in the inspected ware is identical with that used for the standard filters, or if it is a satisfactory equivalent, the curve of chromaticity variation will pass through the points which represent the chromaticities of the color standard. If the colorant used in the inspected ware gives rise to a curve of chromaticity variation slightly displaced but parallel to the curve for the color standard, the inspector is still able to make satisfactory judgment and this material should be considered a satisfactory equivalent for that used in the standard filter. If, on the other hand, the curve of chromaticity variation is considerably displaced from that of the standard filter, for some thicknesses of the ware, it may be impossible for the inspector to arrive at any satisfactory judgment. A requirement to assure that the material submitted can be inspected with the standard is therefore an essential part of a dependable specification.

In addition to these two reasons for restricting the chromaticities more strictly than the basic chromaticity definitions alone would restrict them, a third reason arises from the fact that these defini-

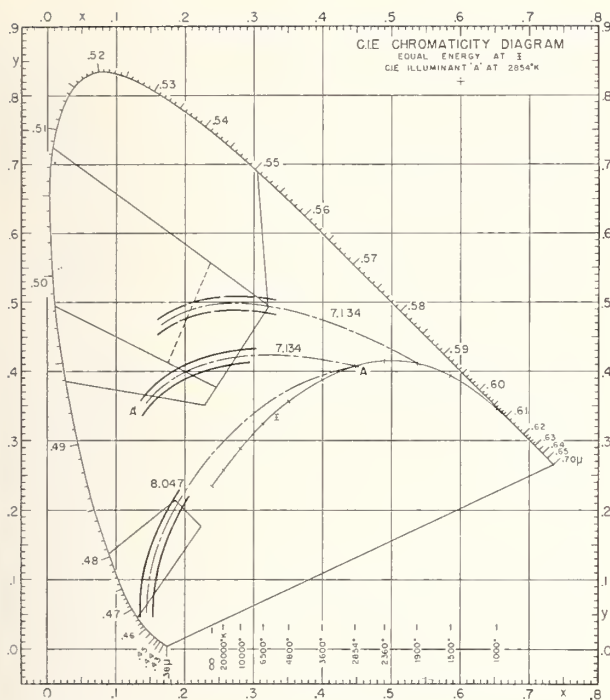


FIGURE 8-1. Similar chromaticity characteristics (C.I.E.).

The figure shows the limits allowed in the U.S. Standard, N.B.S. [1964] for similar chromaticity characteristics in the case of national green standard No. 7.134 and national blue standard No. 8.047. The green standard is shown for light sources of two color temperatures, 1900 °K and 2854 °K, but in the case of the blue only the limits for 2854 °K are shown because the curves would overlap each other so much as to produce a confused pattern.

tions which are intended only to restrict the colors used to those that may be relied upon under usual circumstances, necessarily accept a considerable range of chromaticities. A purchaser who has been accustomed to receive material of one type might well feel he has been provided with an inadequate specification if, upon receiving filters of a noticeably different type, he finds he has no grounds for rejecting the new material. If a producer is able to improve his product, or finds that efficient production requires a change in his colorant, he should inform his customers and arrange for the change of product upon the basis of new standard filters.

The requirement that we have similar chromaticity characteristics to those of the standard filter cited in the specification was developed for the three reasons outlined above. It is designed to limit the chromaticity characteristics of the product furnished without introducing unnecessary restrictions on the physical characteristics, or chemical constitution, that might add to the cost by restricting competition or that might retard improvements which did not change the color characteristics.

8.2. Statement of Requirement

In the U.S. Standard for the Colors of Signal Lights [NBS 1964] the similarity requirement is

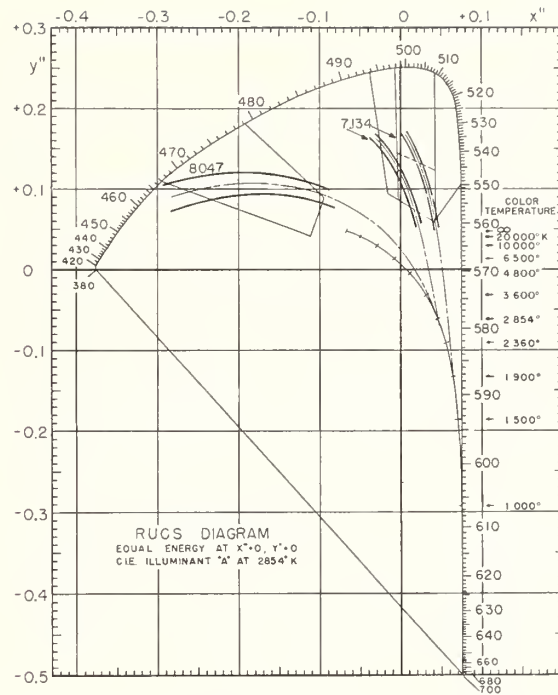


FIGURE 8-2. Similar chromaticity characteristics (RUCS).

This diagram shows the same data as figure 8-1, transformed into the RUCS system. It is evident from this figure that the blue is less closely controlled than the green notwithstanding that in the C.I.E. diagram the blue and green tolerances appear about equal.

stated as follows:

Similar Chromaticity Characteristics

A light-transmitting material has chromaticity characteristics similar to a standard material for a given light source over a stated chromaticity range if the chromaticity coordinates, within that range, of the light from the given source transmitted by any thickness of the standard material can be duplicated within stated tolerances by the chromaticity coordinates of the light from the same source transmitted by some thickness of the subject material. As used in this Standard and in specifications based upon it, the phrase "similar chromaticity characteristics" will be understood to include the range of illuminants to be used with the light-transmitting material in service, and the chromaticity range extends from the chromaticity of the applicable pale limit to that of the second hue limit, or to the chromaticity of such thickness of the subject material as has the minimum acceptable transmittance, whichever gives the lesser range. In this Standard the range of illuminants is that specified in Part II, Table II-1; the minimum transmittance is that stated in Table II-2; and the tolerance is that given in Table II-3.

8.3. Effect of Requirement

Figures 8-1 and 8-2 illustrate this requirement by showing the area within which the chromaticity must remain to qualify green signal ware as conforming to the definition for similar chromaticity characteristics with respect to the pale limits indicated. The figures show the application of the definition with respect to a green pale limit for illuminants at 1900 °K and 2854 °K and for a blue pale limit for the illuminant at 2854 °K. As stated

in the definition, the test is applied only for the range of illuminants and chromaticities which may be involved in the use of the ware to be delivered in fulfilling the order or orders which are being inspected.

As already pointed out, the advantages of this requirement are so considerable from the standpoint of both inspection and service as to warrant the drafting of purchase specifications for signal ware to require ware having chromaticity characteristics similar to those of the pale-limit and transmittance-standard filters adopted to control the purchases. If a purchasing agency finds that such a restriction limits the competition unnecessarily, then it is possible to adopt two or more alternative sets of standard filters. Experience to date suggests that this will rarely, if ever, be necessary. The additional standard filters, if required, should be obtained from prospective bidders who are not satisfied with the existing standards. The requirement for similar chromaticity characteristics places no burden on manufacturers who have produced acceptable material so long as such manufacturers do not change the chromaticity characteristics of their ware. When they do desire to make such a change, and when a new bidder offers ware, an analysis of the chromaticity characteristics of the new colorant is necessary to allow the effects of the change to be evaluated. Moreover, since the manufacturers who have established the characteristics of their ware have presumably already furnished standard filters to customers who regularly purchase from them, or to inspection agencies which inspect the ware for them, it is equitable to require

new bidders to furnish at least a pair of filters for each color. Preferably a bidder should submit duplicate filters, one of which would be returned for his guidance and the other held by the purchaser for inspection purposes. That no more difficulty has been experienced in the past in checking the chromaticities of glassware must be attributed to the fact that manufacturers have adhered quite closely to material that was similar in its chromaticity characteristics to existing pale-limit standards or to samples submitted to customers.

8.4. Application to Plasticware

Plasticware should be required to meet the same specifications for similarity of chromaticity characteristics as glassware although the requirements may be based upon different standards. Thickness in this case is likely to offer little difficulty because it usually varies much less than in the case of glassware. Changing the color temperature of the source can cause the chromaticity of a sample to depart from that of the standard by more than the allowable amount, but since the variation in thickness is likely to be less than with glass, variations from changes in source color temperature should be easier to keep within the basic chromaticity definition. This is because the chromaticity limits can be more favorably located within the area allowed by the chromaticity definitions. Information on the variability of dyes from mix to mix is unfortunately not available. It would be of considerable interest.

9. Instrumentation

9.1. Chromaticity Measurements

The measurement of signal-light chromaticities is carried out, for the most part, on the same principles as other color measurements. When spectrophotometric measurements are to be made, the measurements are made with photoelectric spectrophotometers and the results are evaluated with the use of accepted mathematical functions. Such measurements are regarded as free from subjective estimates. On the other hand, nearly all direct colorimetry of signal lights is carried out with visual instruments which depend upon visual estimations. In general, the photoelectric colorimeters which have been developed for the measurement of surface colors are not satisfactory for use in measuring the colors of the light transmitted by signal ware. The lack of suitable photoelectric instrumentation is partly due to the difficulties inherent in the problem of making such an instrument and partly the result of the small number of laboratories which would be justified in purchasing such instruments. A new

approach to the problem of objective instrumentation is discussed in sec. 9.3, Photoelectric Measurements.

Whenever new standards are to be adopted, the spectral transmittances of the filters must be measured. This is essential because it is necessary to compute the chromaticities to be obtained with the type of colorant represented in different densities (thicknesses or concentrations) and also with sources of different color temperatures. The procedures of spectrophotometry have been described in NBS Circular 484 [Gibson, 1949] to which reference may be made for details of procedure and the probable accuracy to be expected.^{28, 29}

²⁸ The fact that chromaticity differences are more easily and more accurately measured than the chromaticity itself suggests the desirability of standardizing a set of precision standards judiciously spaced over the regions of interest and making all other chromaticity measurements by chromaticity difference. Even if it is important to know the spectral characteristics of a filter the results of the spectrophotometry should be checked against the chromaticity as determined by difference with respect to the precision standards. W. D. Wright [1959] has advocated this procedure for surface color measurements and it seems even more advantageous in the case of filters.

²⁹ To assist in keeping the errors of spectrophotometry to a minimum, the National Bureau of Standards has made sets of standard filters designed for this purpose available. See Keegan, Schleter, and Judd [1962].

In sec. 6.2 it was stated that the U.S. Standard makes provision for duplicates of the national and organization standard filters by requiring that they conform to definite chromaticity tolerances. Such duplicates are polished squares and may be measured with a spectrophotometer. More frequently, however, they are tested by visual methods with colorimeters which measure the chromaticity differences between the duplicates and the originals. This method is generally preferable when there are satisfactory reasons for assuming that the duplicates are similar in chromaticity characteristics to the standard filters they represent. The chromaticity-difference measurements are more accurate than the ordinary spectrophotometric measurements and take less time. The instruments and procedures used for these chromaticity-difference measurements are described in NBS Circular 478 [Judd, 1950].

In principle the procedures for inspecting ware are those of colorimetry, and from the colorimetric standpoint they are relatively simple. They are, however, unusual in having to be made on ware that has optical properties which make it desirable that the chromaticity measurements be made at some distance from the test specimen in order that the measurements represent an average chromaticity such as is seen in service. For this reason the instrumentation is likely to resemble that used for the visual photometry of signal units rather than the usual colorimeters. The instrument may, in fact, be an ordinary Lummer-Brodhun photometer with one side illuminated with light of a known chromaticity.

The variations of chromaticity which have to be checked in an inspection of signal ware will usually constitute a one-dimensional system, that is, they all lie on a smooth curve. The problem is to discriminate between ware which transmits light falling within an acceptable interval from ware which transmits light of a chromaticity outside of that interval. If the similarity of chromaticity requirement has been met, all differences of chromaticity are of a clearly recognizable nature although small differences may require more careful observations than large ones. If such a requirement is applicable and the inspector experiences difficulty in determining whether units are inside or outside of the hue limits, he should refer samples to a colorimetric laboratory equipped to determine whether or not the similarity requirement has been met. If there is no similarity requirement, the inspector should likewise feel free to seek the assistance of a well-equipped colorimetric laboratory since the discriminations required may be such that they can not be made by comparison with one or two hue limits.

9.2. Transmittance Measurements

Most specifications for signal colors contain requirements with reference to the transmittance³⁰ or transmittance ratio³¹ of the ware since this is closely associated with the chromaticity in that both result from the nature of the colorant and the amount of it in the path of the light. The transmittance of signal ware is usually determined as the ratio of two intensity measurements, one with test ware in the optical path and one with a standard filter in the path. If the test ware has no optical effect on the beam, that is, if there is no designed convergence or divergence of the light, nor any appreciable scattering at the surface, the simple interchange of the test ware and standard filter suffices to give the ratio of the transmittances from which we may compute true transmittance values. But most signal ware does not meet these conditions and it is necessary to insert a piece of colorless ware of the same optical characteristics into the path with the standard filter when the light transmitted by the filter is being measured. This results in a nonsymmetrical comparison with two extra glass-air surfaces in the light path when the standard is being measured. The symmetry may be restored by inserting a colorless (neutral) filter into the path when the test ware is being measured. Numerically, the same result is obtained by introducing the equivalent of the colorless filter mathematically.

The formulae for transmittance and transmittance ratio, with and without the colorless flat filter, may be derived as follows: Let us assume that a seasoned lamp (figure 9-1), standardized for the color temperature for which the measurements are to be made, is mounted at a fixed distance from an illuminometer (I) which gives readings in proportion to the illuminance at the instrument. Between the lamp and illuminometer, two lenses are placed successively, the first a standard lens (L_s) which is colorless but of the same design as the lenses to be tested and the second a test lens (L_t). The lenses must be properly focused with respect to the lamp and the illuminometer must be far enough from the lens so that the inverse-square law holds [Waldran, 1951, 1952]. When the colorless standard lens is in place, a standard filter (F_s) of the color of the test lens is placed between it and the illuminometer. When the test lens is in place a colorless filter (F_c) may be substituted for the standard filter, or may be omitted as indicated by the instructions.

³⁰ The transmittance values of all filters are ultimately derived from the spectrophotometric measurements on the filter or some other filter of similar spectral transmittance. In signal lighting most transmittance values can be traced back to the work of Gibson, Haupt and Keegan [1945] and [1946].

³¹ See p. 41, eq (4) for the definition and discussion following.

Let the pertinent quantities be represented as follows:

E = illuminance at illuminometer from standard lamp, without lens or filter.

L_s = optical effect of standard lens, including transmittance.

L_t = optical effect of test lens, including transmittance.

T_s = transmittance of standard filter.

T_c = transmittance of colorless filter, assumed to be 0.92.

R_s = illuminometer reading for standard lens with standard filter in place.

R_t = illuminometer reading for test lens with colorless filter in place.

R'_t = illuminometer reading for test lens without colorless filter in place.

K = calibration constant of illuminometer which is assumed to give a linear response and to approximate the spectral response of the C.I.E. luminosity function.

From which:

$$R_s K = E L_s T_s \quad (1)$$

$$R_t K = E L_t T_c \quad (2)$$

$$R'_t K = E L_t \quad (3)$$

$$(2)/(1) \text{ gives } L_t/L_s = (R_t/R_s)(T_s/T_c) \\ = \text{transmittance ratio of test lens.} \quad (4)$$

$$\text{or } (L_t/L_s)T_c = (R_t/R_s)T_s \\ = \text{"transmittance" of test lens.} \quad (5)$$

$$(3)/(1) \text{ gives } L_t/L_s = (R'_t/R_s)T_s \\ = \text{transmittance ratio of test lens.} \quad (6)$$

$$\text{or } (L_t/L_s)T_c = (R'_t/R_s)(T_s \times 0.92) \\ = \text{"transmittance" of test lens.} \quad (7)$$

Quotation marks are used around the word "transmittance" in eqs (5) and (7) because the ratio obtained in this measurement does not, in general, conform to the usually accepted definition of "transmittance", namely: "The ratio of the luminous flux transmitted by the body to that which it received" [CIE 1957]. The purpose of the measurement in this case is to obtain an index of the loss of light due to the colorant in the specimen. There may be much light wasted by a lens or diffusing cover which does not in principle affect the determined ratio, it being the purpose of the standard lens to cancel this out of the measurement. In actual practice the measured ratio may be influenced in either direc-

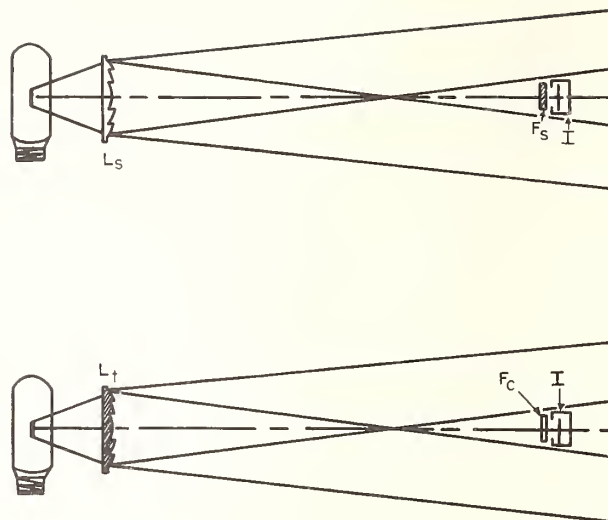


FIGURE 9-1. Arrangement of parts for transmittance measurements.

L_s = Standard lens (colorless) L_t = Test lens (colored)
 F_s = Standard filter (colored) F_c = Colorless filter
 I = Receptor of measuring instrument. See text for procedure.

tion by this wasted light according to the relative amount of light lost by the test and standard pieces. The factor T_c , or 0.92, allows only for the surface reflection which may be small as compared with the other wasted light. It is for this reason that the term "transmittance ratio" was introduced as a term referring to the relative efficiency of a lens. It matters little to the user of a light signal whether the light was lost in a poor colorant, or a poorly molded surface, or an optical design that is less efficient than that of the standard lens.

Another procedure may be used which makes it unnecessary to measure either the "transmittance" or transmittance ratio. The specification writer may multiply the candlepower distribution expected for a clear lens by the desired minimum transmittance ratio and specify a performance curve in the colored light. This requires a candlepower distribution curve to be made on enough pieces to insure that all those accepted meet the specification.

9.3. Photoelectric Measurements

There has been, over a period of years, an increasing interest in the use of photoelectric instruments for the measurement of the chromaticity and transmittance of signal ware. Efforts to produce a suitable instrument seem thus far to have been directed to the development of an instrument that will read chromaticity coordinates and the transmittance of the test specimen directly. This is a difficult objective. A much more simple approach would be to develop instruments that could be substituted for the human eye in the operations which are performed by an inspector. The simplest approach might be a combination of photoelectric cell, filter, and meter, which

would give readings that are a simple, single-value function of the position of a test chromaticity with respect to that of a standard chromaticity when both lie on the path of the normal variation of chromaticity with thickness for the specimen being measured. A change of photocell, filter, or instrument scale in changing from one color to another would not be a prohibitive complication.

The difficulty in applying this simple approach lies in the variation in optical efficiency from piece to piece. Suppose, for example, some small areas of the test piece are blanked out so that they contribute nothing to the illuminance on the photocell but do not change the chromaticity at all. This sort of effect has to be distinguished from those changes that result from changes in colorant density which do affect the chromaticity. This can probably be done by means of a preliminary filter which transmits only a narrow band of wavelengths in a region that is relatively well transmitted by the colorant. This should enable one to interpret the readings taken with the chromaticity filter and determine whether or not the chromaticity is paler than is allowed by the specifications. If there is a second hue limit it might be necessary to have a second chromaticity filter to check the test piece for this possible deficiency.

To inspect ware for compliance with transmittance requirements, a photocell corrected to the spectral luminous efficiency of the C.I.E. standard observer [CIE 1924] can be used and the measure-

ments interpreted in accordance with the appropriate formula of sec. 9.2.

9.4. Sampling

Measurements are of no value unless the things measured represent something significant. This requires a comment on sampling. The variability of color in most glass filterware is so great that it is not practicable to apply to its inspection the ordinary principles for sampling material. Instead of random samples, samples deliberately chosen to contain the palest and densest specimens in the lot are required to assure a satisfactory product. This selection can be based upon a visual inter-comparison of the items submitted. An instrument test is then made on the few specimens so chosen. If all of these specimens are satisfactory, that is good evidence that the whole lot is within the limits. If some of these are not satisfactory, that does not show that the lot is all unacceptable but rather that the lot has not had an adequate inspection by the contractor. When the inspector is satisfied that the contractor has re-inspected the lot and replaced those pieces found outside of the limits, there need be no bar to another inspection.

In the case of plastic filters and perhaps some types of glass filters, it is possible to keep the production well inside of the limits. In such cases it is possible to use such statistical sampling procedures as are approved for other materials that are normally well within safe limits.

10. Use of Signal Light Colors

10.1. General Principles

In considering the recognition of signal light colors in sec. 4, it was pointed out that such recognitions depend upon the conditions under which the light is seen, and nine elements which affect the certainty of recognition were listed. In designing or redesigning a system of light signals, it is important to take these elements into consideration and determine how they will affect the certainty of recognition under the conditions of service. Unfortunately, there are almost no research results available for correlating these conditions with service applications. For this reason it is possible to give only a general discussion of this problem. The following simple principles indicate ways in which the most common sources of signal-color confusion can be reduced to a minimum.

10.2. Number of Colors

The first condition affecting the recognition of signal-light colors is the number of colors in the signal system. The number of colors should be kept to the minimum that will serve the needs

of the signaling since every color that is added increases the risk of mistaken identification.

10.3 The Primary Signal Colors

The most elementary signal-light system is a one color system. Its use in the form of a beacon fire on a tower serving as a landfall light can be traced to archeological times and there is no reason to doubt that signal fires were used by primitive tribes long before the towers were built. From the standpoint of color, it might be thought that such a system would be completely safe since it does not lose its identity at the limit of range, or even when it is used by a colorblind person. There may be, however, the possibility that some stray light may be mistaken for the signal.

If more colors are required, the next step is a two-color system. If it is required that a single signal color be distinguishable from the general aggregate of ordinary lights, then such a system must be regarded as a two-color system since the ordinary lights are in effect being given a negative significance. It might be thought that for maximum dependability the colors of a two-color

signal-light system should be as far apart on the mixture diagram as is practicable. According to the RUCS diagram these would be the longest wavelength red and the most saturated bluish green available. This theoretical solution, however, has to be modified in practice since these highly saturated colors are available only with extremely low luminance.

With a fixed amount of light available from the lamp, the distance at which the signal color may be recognized is at first increased by the greater illuminance that may be obtained by transmitting more light even though the resultant signal is less saturated.³² After an optimum mixture has been reached, further additions of less saturated light reduce the recognition distance as the saturation becomes too low for long range recognition. In practice a two-color system is likely to consist of a red signal and a yellow-white signal, that is, a signal of whatever color the light source may be. Such a system should have dependable color recognition almost to the threshold of the red, for just above threshold all lights except red ones appear white or bluish white, but red lights retain some redness almost, if not quite, to their threshold. If a two-color system is to be used by red-green confusers, a bluish white light should be substituted for the yellow-white light.

The next step is a three-color system in which red, green, and yellow-white or yellow, lights are all used. This is the most common of signal systems. It is used for the long-range lights of aviation, for the navigation lights of ships and for aids for marine navigation with nominally white lights. With yellow lights it is used for highway traffic signals, and for the primary signaling system of most railroads. It is doubtful if any signal system should contain colors other than these three unless the additional colors are to be used solely under conditions that are favorable to color recognition. These will be discussed in connection with the four-, five-, and six-color signal systems.

It has already been pointed out in connection with the two-color system that there is an optimum color density for a color filter to be used on a light intended for recognition at maximum range, and this, of course, is equally applicable to a three-color system.

J. G. Holmes included a determination of the optimum color density for two green filters in the report of his investigation of "The Recognition of Coloured Light Signals" [1941]. For signal green glass he found that 70 percent correct recognition

was obtained at maximum range with the following transmittances:³³

Light source at 1904 °K, optimum transmittance 0.125
Light source at 2365 °K, optimum transmittance 0.16
Light source at 2854 °K, optimum transmittance 0.12

For 80 and 90 percent correct recognition, the transmittances of the optimum densities for light of color temperature 1904 °K were the same within the accuracy of the determination, the greater reliability corresponding to a decreased range. Since both increasing and decreasing the density of the filter tend to reduce its effectiveness as a signal, the transmittances given above presumably should apply to the middle of the range of transmittances to be furnished under the specification. The values given above may not be strictly applicable to a three-color signal-light system since they are based upon tests in which the observers were allowed six color names, but the location of signal green on the chromaticity diagram is such as to indicate much more probability that unsaturated signals would be called white than that they would be called blue. It is also probable that the green glass made by Messrs. Chance Brothers and Co. Ltd., of England is somewhat different from the corresponding green glass made in the United States, but a comparison of the above values for signal green glass with those Holmes found for emerald green glass indicates that the presumably smaller difference between glasses of the same color made in the two countries is likely to have less effect than that resulting from the changes of light source color temperature shown above (see footnote³³).

The practice in the United States is to use green glass of higher transmittance than the optimum found by Holmes. This may be justified on the ground that the additional range or conspicuity is of more value than the greater certainty. A light must be noticed before it can be identified, and in the short interval between discovery and identification the range is usually reduced, which increases the certainty of identification. On the other hand green signals are also easily confused with white at excessive luminances.

10.4. Systems with Yellow and White Signals

For aviation, both yellow and white signals are used for beacons and for runway-lighting systems. In accordance with the principles set forth in the last section these are questionable practices. So far as they are permissible, it is because of the circumstances of their use, for in both cases the two colors are associated in the same application in such a way that they are subject to intercomparisons which enable the observer to distinguish them. Seadrome beacons flash yellow and white alternately at intervals of 2 to 5 seconds, and this permits comparison of a present

³² The thresholds of visibility and color recognition have been investigated by Hill [1947]. More recently, Middleton and Wyszecki have determined peripheral thresholds for red, green and white lights [1951].

³³ These values are from table 20 of the Holmes report. Holmes notes a risk of confusion with blue if the Chance Bros. signal green glass is used with a light source at 2850 °K and recommends against the practice. The signal green glasses used in this country with sources at 2850 °K are designed for this use and have given no difficulty, although it should be noted that the blue lights in this case are secondary signals for short range identification.

flash with the memory image of its predecessor. In the case of some runway-light systems, yellow lights have been used to differentiate the last section of a runway from the rest. In this case the intercomparison is of two groups of lights in the same field of view. Both cases therefore involve only the distinguishing of chromaticity differences rather than the recognition of two chromaticities independently. A much smaller chromaticity difference is sufficient for such distinctions than is required to assure the correct recognition of two chromaticities independently.

The justification of the use of both yellow and white lights on runways has to be qualified. In clear weather when the whole length of the runway lighting system is visible at once, the contrast between the two segments is easily detected and a pilot can keep the transition point in view until he is on the ground and nearly abreast of the point where the color changes. In fog, however, the interval during which both types of lights are visible may be limited to 2 or 3 seconds and under the circumstances this is not long enough to insure that the distinction will be observed. This method of marking the last segment of a runway has never been universally used in this country and it has been used very little, if at all, in any other country. The requirements for use of yellow both in seadrome beacons and on runways are still in effect but seadrome beacons are becoming less common and there is a growing recognition that the runway use is not dependable. These uses, therefore, do not constitute safe precedents for any similar usage in any other application.

The importance of exercising caution in the use of both yellow and white as primary signal colors in the same system was appreciated by the International Civil Aviation Organization at the time when it approved its Standards and Recommended Practices for "Aeronautical Ground Light and Surface Marking Colours" [ICAO 1964]. This is shown by the following quotation from Annex 14 to the Convention on International Civil Aviation (p. 52). These paragraphs are quoted as being the best summary available of the precautions which should be exercised if circumstances seem to make it unavoidable to use both yellow and white lights as primary signal colors.

2.2—Discrimination Between Yellow and White Lights

2.2.1 Recommendation.—If yellow and white are to be discriminated from each other, they should be displayed in close proximity of time or space as, for example, by being flashed successively from the same beacon.

Note.—The limits of yellow and white have been based on the assumption that they will be used in situations in which the characteristics (colour temperature) of the light source will be substantially constant.

2.2.2 Recommendation.—The colours variable-yellow and variable-white are intended to be used only for lights that are to be varied in intensity, e.g., to avoid dazzling. If these colours are to be discriminated from each other, the lights should be so designed and operated that:

a) the chromaticity of the yellow lights will be represented by coordinates such that y is not greater than $x - 0.160$ at any time when the chromaticity of the white lights is represented by coordinates of which x is greater than 0.470; and

b) the disposition of the lights will be such that the yellow lights are displayed simultaneously and in close proximity to the white lights.

Highway traffic signal systems do not include white lights, but the assumption that the yellow traffic lights are recognizably different from ordinary lights commonly seen along streets and highways involves the same difficulties as using yellow and white in the same system. The warning light of the traffic signal system is generally recognizable for reasons other than color. In some installations it does not appear except in association with the green light and in this use any color could serve the purpose. In other installations where this is not true, the warning light is seen following a green one or preceding a red one, and this change of colors, if seen, is an indication that both signals are traffic lights. In still other installations the yellow light is flashed and this differentiates it from street lights and miscellaneous lights which may be within the view of the driver. At short range it is possible for drivers to recognize that the warning light is coming from a traffic light fixture. All of these considerations contribute to making the use of yellow traffic signals possible notwithstanding that many other yellowish lights are frequently visible to drivers.

The realization that the circumstances associated with the use of yellow as a traffic signal have been important to the satisfactory experience with this color makes it clear that it is not safe to conclude that such yellow lights may be used as independent signals. In particular, it is not safe to depend upon the exposure of a fixed yellow light as a warning signal to automobile drivers unless the situation has been examined to make certain that drivers cannot mistake such a light for a stray light. This becomes an increasingly important consideration with the growing use of traffic lights on open highways where the higher speeds of traffic, and the longer distances from which lights must be observed, make it more difficult for drivers to distinguish between traffic lights and stray lights than is the case in city driving.

Another use for yellow light signals on highways is in turn signals on vehicles. In this use yellow and white have been used somewhat interchangeably on the front end of vehicles without difficulty. This would seem to be primarily because the location of the signals on the vehicles has indicated their purpose and the location has been recognizable from the outline of the car in the day time and from its relation to headlights or tail-lights at night. The Society of Automotive Engineers has now adopted a standard which requires all turn signals to be yellow signals and this should in time simplify the situation constructively.

10.5. The Secondary Signal Colors

In some signaling systems one set of colors is needed for operations at long range and additional colors are needed for purposes which permit observing the lights at shorter range and with more deliberation. For these secondary applications it must be possible to design the fixtures so that by the time a decision must be made the lights will either be observed as small surfaces, rather than as point sources, or they will be seen at luminances which are neither very high nor very low. Under these conditions it is possible to use as many as three additional colors. The colors usually adopted for such purposes are lunar white (which could equally well be called lunar), blue, and purple. The general policy in such systems should be to define the primaries for optimum recognition as if the secondaries were not used because the primaries perform the more important functions and do it under the more severe conditions. The blue and lunar secondaries are then fitted in to give the best differentiation possible with the green and yellow. The transmittances obtainable for the lunar glasses are about the same as those for the green glasses, but the transmittances of the blue glasses are about one tenth of those feasible in the green. Blue signal ware should be required to be free from an excessive transmission of red light.

Purple signals have been used in the United States to differentiate certain fixed signals from blue hand lanterns used in railroad yards, but the present trend is away from their use for this purpose. Purple signals fall into a class by themselves in that they are recognized by their dichroic character. The red and blue components of the light transmitted by purple glasses are separated by the chromatic aberration of the eye and are seen separately, usually as a reddish signal surrounded by a blue halo. For this reason the specifications for purple signal ware must contain a requirement to insure that the proportion of red light transmitted is not too low. Otherwise the signal is liable to be mistaken for a blue signal. On the other hand, too high a proportion of red in the transmitted light may cause a risk that the signal will be mistaken for a red signal. If the system uses both blue and purple signals, the limitations for the transmittance of red by the purple ware have to be held within narrow limits and the complementary requirement that the blue ware not transmit too much red light becomes essential.

Requirements relating to the proportion of red light transmitted by a blue or purple filter are represented as minimum or maximum limits for T_r/T_w . T_w , the transmittance for "white" light, is the total transmittance for light of the specified color temperature, and T_r is the transmittance of red light which has been defined for this purpose as light of wavelengths longer than 0.650μ . T_r and T_w are not difficult to compute from spectrophotometric measurements in cases where spectro-

photometric measurements are available, but the inspection of signal ware has to be carried out without spectrophotometry. As a practical equivalent, the blue or purple light may be measured through a deep red, sharp-cutoff filter, but it is important that the use of this procedure be clearly defined in the specification in order to avoid possible misunderstanding and litigations based upon the difference between the actual cutoff of the filter and the abrupt cut implied by the reference to a wavelength.

Table 3 shows the defining red filters, color temperatures for testing, and the limits prescribed for blue ware in the specifications and standards named.

TABLE 3—Maximum transmittance of red light allowed for blue ware

Specification	Test filter	Color temperature	Maximum T_r/T_w
Federal Standard No. 3 Colors Aeronautical Lighting	3113A	2854 °K	0.015
Military Specification, MIL-C-25050 Colors Aeronautical Lights and Lighting Equipment (p. 9)	3055A	2854 °K	0.015
A.A.R. Specification 59-61 Glasses for Kerosene Hand Lanterns (p. 4, 8)	A.A.R.* 211	1900 °K	0.029
A.A.R. Specification 69-59 Signal Glasses (p. 11)	A.A.R. 86	2854 °K	0.006

*Expressed as light of wavelength greater than 0.650μ but tested with filter.

10.6. Signals for Colorblind Observers

Signal light systems are generally planned for persons having normal color vision. As pointed out in section 3.1, for such persons the world of color is three dimensional. In looking at surface colors, luminances are relative and are governed by the reflectances of the surfaces. Such differences are recognized as true color differences and in normal surroundings a baby blue ribbon is easily distinguished from one of navy blue by any person with normal color vision even though the hue and saturation may be the same. In signal lighting the transmittance of the signal ware, which is the analog of reflectance, merely affects the intensity, and while this is technically part of the color, it is not available for distinguishing signals since the apparent intensity is dependent upon the distance to the signal and the state of the intervening atmosphere. In the interpretation of signal lights, even the normal observer, or trichromat, is limited to chromaticity differences which constitute a two dimensional space.

In contrast to the normal trichromats, the completely colorblind monochromats appraise what they see with reference to a single primary. These persons have no chromatic world and no signal light colors can be of any use to them for they see only differences in brightness or brilliance. Intermediate between these groups are the partially colorblind dichromats who live in one dimensional chromaticity worlds.³⁴

Since very few persons are totally colorblind, whereas a substantial fraction are dichromats, it seems reasonable to consider the possibility of designing a system of signal colors that can be used by at least a part of the dichromats. A dichromat, however, may lack any one of the three normal processes and hence there are three kinds of dichromats, known as protanopes, deuteranopes, and tritanopes. The differences between these three groups can be appreciated from figures 10-1, 10-2, and 10-3. In each of these figures there is a family of lines radiating from a focus outside the spectrum locus, that is, outside the region of visible colors. These are chromaticity confusion lines and each set corresponds to the vision of one of the three types of dichromats. All the chromaticities along any one line seem the same to a colorblind observer of the type to which the locus applies.

D. B. Judd has designed a three-color specification for use in control panels for electronic equipment which should be satisfactory for protanopes and deuteranopes as well as for normal observers. This system consists of aviation red, aviation blue, and aviation green with its yellow boundary moved to the line

$$y = 0.667(1 - x).$$

These areas have been indicated in figures 10-1, 10-2, and 10-3 by the boundaries of the aviation colors, the allowable portion of the green area being distinguished by shading. It appears from figures 10-1 and 10-2 that the system should be useful to protanopes and deuteranopes but figure 10-3 indicates that tritanopes will easily confuse the blues and greens. The Judd proposals have been tested by Sloan and Habel [1955] in a research with 22 deuteranopes and 18 protanopes using signals subtending 1° at the observer's eye. Normal observers, all the deuteranopes, and 13 protanopes recognized the colors correctly. Five protanopes experienced difficulty with some of the test colors, making 16 errors out of 120 identifications. The results confirm the principle but indicate that further study is desirable before depending upon such a system in any situation involving appreciable risks. The test must also be considered inconclusive for the more common applications of signal light colors in which the subtended angle is a few minutes more or less.

³⁴ For discussions of the response functions of protanopes and deuteranopes see Judd [1944] and [1949].

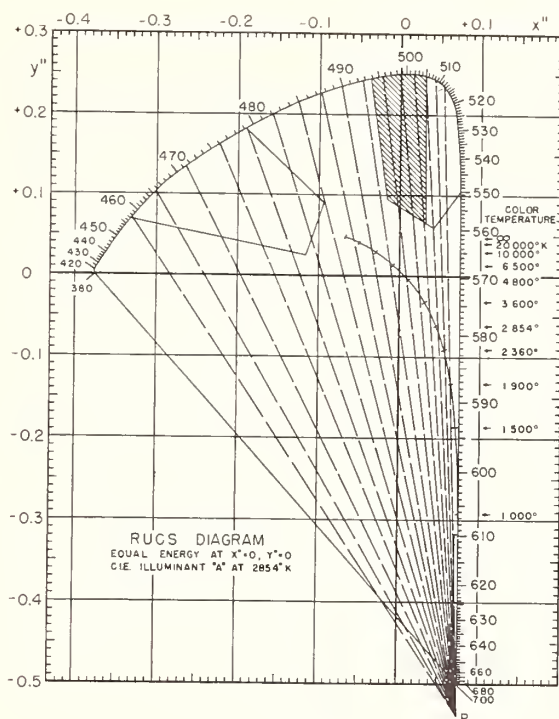


FIGURE 10-1. *Confusion loci for protanopes.*

All points lying on any one of the radii through P will appear to a person having partial color blindness of the type called protanopic as having the same chromaticness. This diagram is a transformation into RUCS coordinates of the diagram given in the original paper by Pitt [1935].

The boundaries for signal red, green and blue and intermediate green (shaded) as defined in the U.S. Standard have been added showing that protanopes may be expected to have difficulty distinguishing between signal green and red.

The possibility of a system designed to be recognizable by all three types of dichromats deserves consideration. Three small areas have been cross-hatched on figure 10-3 to make a modification of the Judd system for this purpose. This proposal has not been tested because the most important application for such a system would seem to be in connection with the control of traffic. For this use, however, the proposal has a serious handicap in the very low transmittance (1 or 2 percent) which must be accepted in order to obtain sufficiently saturated green and blue signals.

10.7. Backgrounds

The background of a signal light can have a considerable effect on its conspicuity and its recognition. The conspicuity of a signal is greatly reduced if it must be seen among a number of other lights from which the observer receives an equal or greater illuminance. A yellow traffic light, for example, hanging in the center of a number of highway lights installed to illuminate a crossing may be difficult to find even though it is known to be there. A driver who has noticed a green light must transfer his attention to the roadway surface to check his alignment and make sure his path is still clear, and he may see only a cluster of yellowish lights when he looks back to check the traffic signal. If

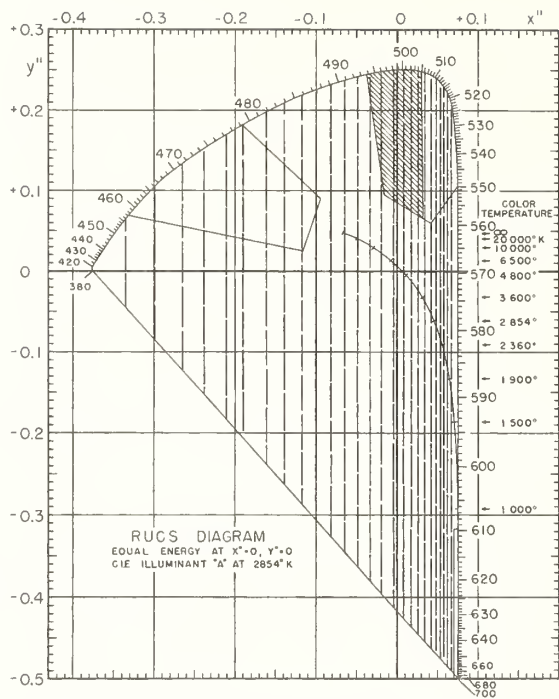


FIGURE 10-2. *Confusion loci for deuteranopes.*

All points lying on any one of the broken vertical lines will appear to have the same chromaticity to a person of deuteranopic vision. The spacing of the lines near the spectrum locus is based upon the results obtained by Pitt [1935]. The lines are drawn vertical because different investigators disagree as to the location of the conversion point in the case of deuteranopes. The conversion point found by some researches would lie at a fairly large negative value, whereas another research would place the conversion point at a positive value. In view of this uncertainty, it is felt that the vertical lines represent the facts as far as known adequately. The other lines have the same significance as in figure 10-1.

the driver is traveling at a good speed, he cannot devote much attention to playing hide and seek with lights, and so he reverts to his relation to the roadway surface. When he has reassured himself that all is well here, he looks again at the lights to find a red one among them. Unless he has reduced his speed as a response to a realization that he is dealing with a situation that he may not completely understand, he must now make an emergency stop, if he can, before he reaches the crossing.

Green lights given a high intensity to insure their visibility by daylight look quite pale to the dark-adapted eye at night. This may cause a green traffic light to become lost among highway lights with mercury lamps, which are frequently quite greenish as seen from a few hundred yards away. The obscuration of red traffic lights by neon signs is a common source of concern.

The loss of conspicuity because of competing lights is not as serious a problem with city traffic lights as it would otherwise be because drivers become accustomed to the exact location of traffic lights with reference to the driving lane in their own city and city driving speeds allow more attention to be given to lights. A study, however, of the whole problem of conspicuity is needed. The mere increasing of the intensities may increase the problem of recognition and do little to solve the problem of conspicuity.

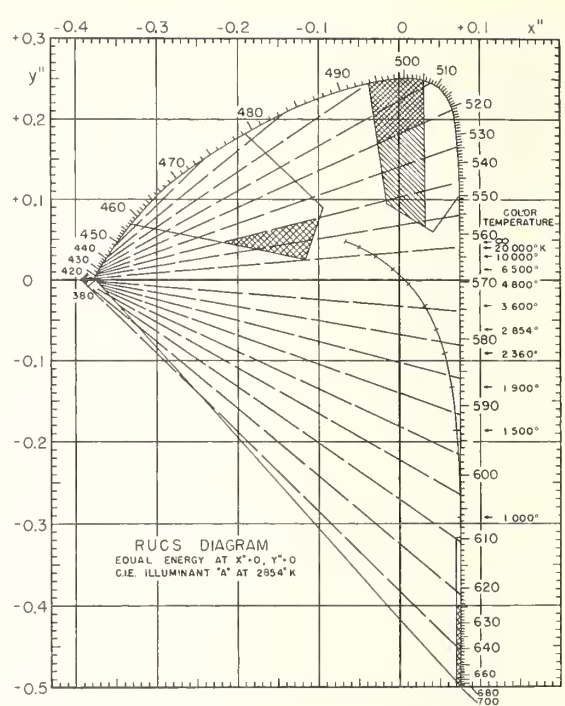


FIGURE 10-3. *Confusion loci for tritanopes.*

In this case all of the points on any one of the converging lines will have the same chromaticity for a tritanope. The conversion point represents a mean of the locations found by Thomson and Wright [1953], Judd [1944], and one computed from the Muller theory [Judd, 1949]. The other lines have the same significance as in figure 10-1.

Closely related to the problem of conspicuity is the problem of visibility. At night the presence of many bright lights will destroy the dark adaptation and make it impossible for observers to see distant signals if they are faint. This is a situation which has to be avoided in marine and aeronautical navigation, and it has received some attention in these fields, but it has not been thought a serious problem in land transportation where signals are closer to the observer. The increasing use of highway lights and the higher intensities used for headlamps and brakelights is making it a problem on the highway.

Both visibility and conspicuity may be reduced by too high a background brightness. This is particularly true of traffic lights hung over the center of an intersection which by day are seen against the sky. This problem has long been recognized in railroad signaling, and high intensity railroad signals used in the daytime are regularly provided with artificial backgrounds by surrounding the light with a black shield.

A promising field for the improvement of the color recognition of traffic lights is the possibility of improving their backgrounds. While it is probably impracticable to use as large background shields as are used for some of the railroad signals, it should be possible to redesign the traffic light fixtures so as to present the largest practicable black cross section surrounding the signals. An effort might be made to have special lighting units designed for use in locations near traffic signals

in order to eliminate the competition between street and highway lights and traffic lights. The location of traffic signals should be considered with reference to competing lights and means sought to eliminate lights from the background if they are interfering with the conspicuity of a needed traffic light. A first approach might be to seek the cooperation of landowners in order to educate the public with reference to the problem. If this proves inadequate legal means should be considered.³⁵

These are only the obvious ways in which the situation might be improved. It is quite probable that a thorough study of the problem would develop other and possibly more important steps that could be taken to improve the recognition of traffic signals. The same principles apply to other types of signal lights when such lights are used in close proximity to stray lights.

10.8. Naming of Signal Colors

The naming of signal colors might not be expected to have much importance but one problem has arisen that merits some attention. It is common practice to think of an ordinary incandescent lamp seen through colorless signal ware as white. This has been the accepted practice in connection with lighthouses and aviation beacons and it is probable that people in general seldom ascribe any color to the lights used for illumination. If such a color, at moderate brightness, is shown an observer

³⁵ From an engineering point of view the projection of hazardous photons would not seem to be essentially different from the projection of hazardous pebbles.

whose eyes are adapted to ordinary daylight, he will generally classify it as yellow. In the case of airport approach and runway lights the situation is aggravated by the necessity of dimming the lights to a color temperature close to 1500 °K which is redder than candlelight and almost at the red limit of some signal yellow lights although not as saturated. At full intensity these lamps may operate at more than twice this color temperature. It is necessary to have a single name for this entire range of colors. The International Civil Aviation Organization selected the term "aviation variable white." The Recommendations of the International Commission on Illumination for colors of signal lights refer to these lights as "white/yellow lights." The U.S. Standard for the Colors of Signal Lights calls it "variable-source white," and this may be a satisfactory name provided all concerned can be made to realize the scope of the term. The limitations which should be placed on the use of both yellow and white in the same system have already been discussed in sec. 10.4.

A minor difficulty encountered in the effort to bring the several specifications for signal-light colors used in the United States into a more consistent relationship has arisen from the practice of naming signal colors with the names of the services in which they are used. It may seem quite inappropriate to railroad signal engineerings, for example, to use an aviation green. For this reason the names used in the U.S. Standard [NBS 1964] have been designed to avoid reference to any applications.

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- Photometry, Walsh, J. W. T., Constable & Company Ltd., 10 Orange Street, London, WC2 (1953).
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- I. E. S. Lighting Handbook, 1959, Address: Illuminating Engineering Society, 345 East 47th Street, New York, N.Y.
- Fifty Years of Signal Lighting, F. C. Breckenridge, *Ill. Eng.* **53**, 311 (1958).

Appendix 3. List of Specifications and Standards

Commission Internationale de l'Eclairage, Colors of Light Signals, Publication C.I.E. No. 2 (W-1.3.3) (1959). Address: Central Bureau of C.I.E., Assistant Secretary, M. J. J. Chap-pat, 57 Rue Cuvier, Paris 5, France, or Secretary, U.S. National Committee, L. E. Barbrow, National Bureau of Stand-ards, Washington, D.C.

International Civil Aviation Organization, International Stand-ards and Recommended Practices, Aerodromes, Annex 14, Int. Civ. Av. Org., Fourth Edition (1964). Address: 1080 University Street, Montreal 3 (Quebec), Canada.

U.S. Standard for the Colors of Signal Lights, National Bureau of Standards Handbook 95 (1964). Address: Superintendent of Documents, Government Printing Office, Washington, D.C., 20402.

Federal Standard No. 3, Colors, Aeronautical Lighting (21 March, 1951). Address: Superintendent of Documents, U.S. Govern-ment Printing Office, Washington, D.C., 20402.

Military Specification MIL-C-25050 (ASG), Colors, Aero-nautical Lights and Lighting Equipment, General Require-

ments for (2 December, 1963). Address: Superintendent of Documents, U.S. Government Printing Office, Washington, D.C., 20402.

Association of American Railroads, Signal Section, Part 136, Specification 69-59, Signal Glasses (Exclusive of Kerosene Lantern Globes) (1960). Address: Secretary AAR, Signal Section, 30 Veasey Street, New York, N.Y.

Association of American Railroads, Signal Manual Part 66, Specification 59-61, Glasses for Kerosene Hand Lanterns (Exclusive of Electric Lantern Glassware) (1962). Address: Secretary AAR, Signal Section 30 Veasey St., New York, N.Y.

American Standards Association, Adjustable Face Traffic Control Signal Head Standards, ASA D10.1-1958, UDC-656.057 (1958). Address: American Standard Association, 70 East 45th Street, New York, N.Y.

Society of Automotive Engineers, Inc., SAE Standards, TR-34, Lighting Equipment and Photometric Tests. Address: Soci-ety of Automotive Engineers, Inc., 485 Lexington Ave., New York 17, N.Y.

Other Countries

Australia

Standard in preparation.

Germany

DIN 6163 Blatt 1: Farben und Farbgrenzen für Signallichter, Allgemeines

DIN 6163 sheet 1: Colors and color limits for signal lights; General.

DIN 6163 Blatt 2: Farben und Farbgrenzen für Signallichter, Ortsfeste Signallichter an See-und Binnenschiffahrtstrassen

DIN 6163 sheet 2: Colors and color limits for signal lights; Fixed signal lights at maritime and inland waterways.

DIN 6163 Blatt 3: Farben und Farbgrenzen für Signallichter, Signallichter an Strassenfahrzeugen und Strassenbahn

DIN 6163 sheet 3: Colors and color limits for signal lights; Signal lights on road vehicles and tramways.

DIN 6163 Blatt 4: Farben und Farbgrenzen für Signallichter, Signallichter der Eisenbahn

DIN 6163 sheet 4: Colors and color limits for signal lights; Signal lights of the railways.

DIN 6163 Blatt 5: Farben und Farbgrenzen für Signallichter, Ortsfeste Signallichter im Strassen-und Strassenbahnverkehr

DIN 6163 sheet 5: Colors and color limits for signal lights; Fixed signal lights in road and tramway traffic.

DIN 6163 Blatt 6: Farben und Farbgrenzen für Signallichter, Signallichter an Wasserverkehrsmitteln

DIN 6163 sheet 6: Colors and color limits for signal lights; Signal lights on vessels.

DIN 6163 Blatt 7: Farben und Farbgrenzen für Signallichter, Luftfahrtfeuer und Signallichter zur Luftverkehrsregelung

DIN 6163 sheet 7: Colors and color limits for signal lights; Air navigation lights and signal lights for air traffic regulation.

DIN 6163 Blatt 8: Farben und Farbgrenzen für Signallichter, Signallichter an Luftfahrzeugen

DIN 6163 sheet 8: Colors and color limits for signal lights; Signal lights on aircraft.

DIN 6171: Aufsichtfarben für Verkehrszeichen, Farben und Farbgrenzen

DIN 6171: Control colors for traffic signals; Colors and color limits.

Deutschen Normenausschusses, Berlin W 15.

United Kingdom

British Standards Institution, British Standard 1376:1953, Colours of Light Signals (1953). Address: British Standards Institution, British Standards House, 2 Park Street, London W1, England.

Appendixes 4 and 5

The tables of 4 and 5 show the equations used as boundaries for the signal-light colors in the U.S. Standard [N.B.S. 1964], together with the corresponding equations used in the Standards of the International Civil Aviation Organization [1964] and the Recommendations of the International Commission on Illumination [1955]. All of the equations of App. 4 are expressed in

the coordinates of the C.I.E. system, which is the system used in all the specifications, but since the RUCS equivalents of the C.I.E. recommendations are more significant for color differen-tiation, App. 5 has been added to give these also. For the titles of the Standards and the addresses of the issuing authorities, see App. 3.

Appendix 4. Corresponding Boundary Equations CIE Coordinates

Colors and Boundaries ¹	International Commission on Illumination ²	International Civil Aviation Organization ³	U.S. Standard for the Colors of Signal Lights ⁴
Signal Red			
Purple boundary	$x = 0.980 - y$	$y = 0.980 - x$	$x + y = 0.992$
Yellow boundary	$y = 0.335$	$y = 0.335$	$x - y = 0.330$
Intermediate Signal Red			
Purple boundary	No equivalent	No equivalent	$x + y = 0.992$
Yellow boundary			$x - y = 0.380$

Appendix 4. Corresponding Boundary Equations CIE Coordinates—Continued

Colors and Boundaries ¹	International Commission on Illumination ²	International Civil Aviation Organization ³	U.S. Standard for the Colors of Signal Lights ⁴
Restricted Signal Red Purple boundary Yellow boundary	$y = 0.985 - x$ $y = 0.300$	No equivalent	$x + y = 0.997$ $x - y = 0.405$
Signal Yellow Red boundary White boundary Green boundary	$y = 0.382$ $y = 0.790 - 0.667x$ $y = x - 0.120$	$y = 0.382$ $y = 0.790 - 0.667x$ $x = 0.550$	$y = 0.383$ $x + y = 0.872 + 0.200x$ $x = 0.550$
Intermediate Signal Yellow Red boundary White boundary Green boundary	No equivalent	No equivalent	$y = 0.383$ $x + y = 0.872 + 0.200x$ $x - y = 0.124$
Restricted Signal Yellow Red boundary White boundary Green boundary	No equivalent	$y = 0.400$ $y = 0.790 - 0.667x$ $x = 0.560$	$y = 0.400$ $x + y = 0.872 + 0.200x$ $x - y = 0.124$
Signal Green Yellow boundary White boundary Blue boundary	$x = 0.360 - 0.080y$ $x = 0.650y$ $y = 0.390 - 0.171x$	$x = 0.333$ $x = 0.650y$ $y = 0.390 - 0.171x$	$x = 0.360 - 0.080y$ $x = 0.650y$ $y = 0.390 - 0.171x$
Intermediate Signal Green Yellow boundary White boundary Blue boundary	No equivalent	No equivalent	$y = 0.730 (1 - x)$ $x = 0.650y$ $y = 0.500 - 0.500x$
Restricted Signal Green Yellow boundary White boundary Blue boundary	No equivalent	$y = 0.850 - x$ $x = 0.650y - 0.030$ $y = 0.390 - 0.171x$	$y = 0.730 (1 - x)$ $x = 0.425y$ $y = 0.500 - 0.500x$
Signal Blue Green boundary White boundary Purple boundary	$y = 0.065 + 0.805x$ $x = 0.400 - y$ $x = 0.133 + 0.600y$	$y = 0.805x + 0.065$ $y = 0.480 - x$ $x = 0.600y + 0.133$	$y = 0.065 + 0.805x$ $y = 0.400 - x$ $x = 0.100 + 0.700y$
Signal White	Variable source white is nearest equivalent	Variable source white is nearest equivalent	Signal white includes all chromaticities which qualify as one of the following four.
Variable Source White Green boundary Purple boundary	$\{ y = 0.150 + 0.640x$ $\{ y = 0.440$ $\{ y = 0.382$ $\{ y = 0.050 + 0.750x$	$y = 0.640 - 0.400x$ $y = 0.150 + 0.640x$ $y = 0.050 + 0.750x$ $y = 0.390$	$\} y - y_0 = 0.003$ $\} y_0 - y = 0.003$
Yellow boundary Blue boundary	$\{ x = 0.255 + 0.750y$ $\{ y = 0.790 - 0.667x$ $x = 0.310$	$\} x = 0.560$ $x = 0.310$	$x = 0.555 + 0.750(y - 0.405)$ $x = 0.325$
Beacon White Green boundary Purple boundary Yellow boundary Blue boundary	$\{ y = 0.150 + 0.640x$ $\{ y = 0.440$ $\{ y = 0.382$ $\{ y = 0.050 + 0.750x$ $x = 0.500$ $x = 0.310$	$y = 0.640 + 0.400x$ $y = 0.150 + 0.640x$ $y = 0.050 + 0.750x$ $y = 0.390$ $x = 0.500$ $x = 0.310$	$\} y - y_0 = 0.003$ $\} y_0 - y = 0.003$ $x = 0.490 + 0.500(y - 0.410)$ $x = 0.325$
Lunar White Green boundary Purple boundary Yellow boundary Blue boundary	$y = 0.150 + 0.640x$ $y = 0.050 + 0.750x$ $x = 0.440$ $x = 0.310$	No equivalent	$y = 0.195 + 0.500x$ $y = 0.170 + 0.500x$ $x = 0.445 + 0.333(y - 0.405)$ $x = 0.325$

Appendix 4. Corresponding Boundary Equations CIE Coordinates—Continued

Colors and Boundaries ¹	International Commission on Illumination ²	International Civil Aviation Organization ³	U.S. Standard for the Colors of Signal Lights ⁴
Blue White Green boundary For $x < 0.330$ For $x > 0.330$ Purple boundary For $x < 0.330$ For $x > 0.330$ Yellow boundary Blue boundary	No equivalent	No equivalent	$y = 0.030 + x$ and $y = 0.195 + 0.500x$ $y = 0.005 + x$ or $y = 0.170 + 0.500x$ $x = 0.445 + 0.333(y - 0.405)$ $x = 0.265 - 0.250(y - 0.280)$

¹Boundary names are from the U.S. Standard. In the case of the I.C.A.O. the correspondence is readily evident with these possible exceptions: "variable yellow" is classified as signal yellow because it is the most inclusive; restricted green is an I.C.A.O. recommended practice; the use of the C.I.E. white boundary for blue is recommended in I.C.A.O. The correspondence in the case of the C.I.E. is clear except for white lights. In this case the correspondence has been determined from the uses indicated in the two documents.

²Recommendations, Publication No. 2.

³International Standards and Recommended Practices, Aerodromes, Annex 14.

⁴NBS Handbook 95.

Appendix 5. Corresponding Boundary Equations RUCS Coordinates

Colors and Boundaries ¹	International Commission on Illumination ²	International Civil Aviation Organization ³	U.S. Standard for the Colors of Signal Lights ⁴
Signal Red Purple boundary Yellow boundary Intermediate Signal Red Purple boundary Yellow boundary Restricted Signal Red Purple boundary Yellow boundary Signal Yellow Red boundary White boundary Green boundary Intermediate Signal Yellow Red boundary White boundary Green boundary Restricted Signal Yellow Red boundary White boundary Green boundary Signal Green Yellow boundary White boundary Blue boundary Intermediate Signal Green Yellow boundary White boundary Blue boundary Restricted Signal Green Yellow boundary White boundary Blue boundary	$\Delta x'' = x'' - 0.075$ $x'' = +0.070$ $y'' = -0.317 - 4\Delta x''$ No equivalent $x'' = +0.071$ $y'' = -0.400 - 4\Delta x''$ $y'' = -0.225 - 4\Delta x''$ $x'' = +0.063 - 0.050y''$ $y'' = -0.134 - 0.50\Delta x''$ No equivalent $x'' = +0.0715y''$ $y'' = +0.085 - 0.600x''$ $x'' = -0.150y''$ No equivalent No equivalent	$\Delta x'' = x'' - 0.075$ $x'' = +0.070$ $y'' = -0.317 - 4\Delta x''$ No equivalent No equivalent $y'' = -0.194 - 4\Delta x''$ $x'' = +0.063 - 0.050y''$ $y'' = -0.134 + 3\Delta x''$ $x'' = +0.875y''$ $y'' = +0.085 - 0.600x''$ $x'' = -0.150y''$ No equivalent $x'' = +0.047 + 0.050y''$ $y'' = +0.104 - 0.650x''$ $x'' = -0.150y''$	$\Delta x'' = x'' - 0.075$ $x'' = +0.0725$ $y'' = -0.317 + 0.16\Delta x''$ $x'' = +0.0725$ $y'' = -0.375 + 0.4\Delta x''$ $x'' = +0.0741$ $y'' = -0.407 + 0.5\Delta x''$ $y'' = -0.225 - 4\Delta x''$ $x'' = +0.068 - 0.026y''$ $y'' = -0.120 + 3\Delta x''$ $y'' = -0.225 - 4\Delta x''$ $x'' = +0.068 - 0.026y''$ $y'' = -0.136 + 0.5\Delta x''$ $y'' = -0.194 - 4\Delta x''$ $x'' = +0.068 - 0.026y''$ $y'' = -0.136 + 0.5\Delta x''$ $x'' = +0.715y''$ $y'' = +0.085 - 0.600x''$ $x'' = -0.150y''$ $x'' = +0.045 - 0.0075y''$ $y'' = +0.085 - 0.600x''$ $x'' = -0.020y''$ $x'' = +0.045 - 0.0075y''$ $y'' = +0.142 - 0.450x''$ $x'' = -0.020y''$

Appendix 5. Corresponding Boundary Equations RUCS Coordinates—Continued

Colors and Boundaries ¹	International Commission on Illumination ²	International Civil Aviation Organization ³	U.S. Standard for the Colors of Signal Lights ⁴
Signal Blue Green boundary White boundary Purple boundary	$x'' = -0.950y''$ $x'' = -0.125 + 0.350y''$ $y'' = -0.205x''$	$x'' = -0.950y''$ $x'' = -0.075 + 0.250y''$ $y'' = -0.205x''$	$x'' = -0.950y''$ $x'' = -0.125 + 0.350y''$ $y'' = -0.370x''$
Signal White	Variable source white is nearest equivalent	Variable source white is nearest equivalent	Signal White includes all chromaticities which qualify as one of the following four
Variable Source White Green boundary Purple boundary Yellow boundary Blue boundary	$\{x'' = +0.020 - 0.700y''$ $\{x'' = +0.042 - 0.247y''$ $\{x'' = +0.021 - 0.240y''$ $\{x'' = -0.026 - 0.900y''$ $\{y'' = -0.161 + 0.086x''$ $\{x'' = +0.063 - 0.050y''$ $x'' = -0.025 + 0.984y''$	$x'' = +0.055 - 0.100y''$ $x'' = +0.020 - 0.700y''$ $x'' = -0.026 - 0.900y''$ $x'' = +0.024 - 0.240y''$ $x'' = +0.115 + 0.300y''$ $x'' = -0.025 + 1.000y''$	$\left. \begin{array}{l} \text{Curved boundaries, no simple} \\ \text{transformation} \end{array} \right\}$ $y'' = -0.1515$ $x'' = -0.000 + 0.900(y'' - 0.009)$
Beacon White Green boundary Purple boundary Yellow boundary Blue boundary	$\{x'' = +0.020 - 0.700y''$ $\{x'' = +0.042 - 0.247y''$ $\{x'' = +0.021 - 0.240y''$ $\{x'' = -0.026 - 0.900y''$ $x'' = +0.100 + 0.420y''$ $x'' = -0.025 + 0.984y''$	$x'' = +0.055 - 0.100y''$ $x'' = +0.020 - 0.700y''$ $x'' = -0.026 - 0.900y''$ $x'' = +0.024 - 0.240y''$ $x'' = +0.100 + 0.420y''$ $x'' = -0.025 + 1.000y''$	$\left. \begin{array}{l} \text{Curved boundaries, no simple} \\ \text{transformation} \end{array} \right\}$ $y'' = -0.094 + 0.400(x'' - 0.056)$ $x'' = -0.000 + 0.900(y'' - 0.009)$
Lunar White Green boundary Purple boundary Yellow boundary Blue boundary	$x'' = +0.020 - 0.700y''$ $x'' = -0.026 - 0.900y''$ $x'' = +0.074 + 0.536y''$ $x'' = -0.025 + 0.984y''$	No equivalent	$x'' = +0.016 - 0.571y''$ $x'' = -0.002 - 0.586y''$ $y'' = -0.060 + 0.700(x'' - 0.045)$ $x'' = +0.000 + 0.900(y'' - 0.009)$
Blue White Green boundary For $x < 0.330$ For $x > 0.330$ Purple boundary For $x < 0.330$ For $x > 0.330$ Yellow boundary Blue boundary	No equivalent	No equivalent	$y'' = +0.021 - 0.844x''$ $x'' = +0.016 - 0.571y''$ $y'' = +0.004 - 0.794x''$ $x'' = -0.002 - 0.586y''$ $y'' = -0.060 + 0.700(x'' - 0.045)$ $x'' = -0.037 + 0.700(y'' - 0.040)$

^{1, 2, 3, 4} See corresponding notes to Appendix 4.

Appendix 6. Comparative Diagrams of Signal Color Boundaries

Figures A-1, A-1A, and A-1B, and figures A-2, A-2A and A-2B show a comparison of the chromaticity boundaries of the three standards for signal light colors listed in App. 3 of the Appendix: the U.S. Standard, the C.I.E. Recommendations, and the I.C.A.O. Standards and Recommended Practice. To avoid confusion, some boundary lines in the yellow-white region and one in the red region have been omitted from figure A-2. The RUCS boundaries for restricted red in the C.I.E. Recommendations may be fairly well located by noting the location of these boundaries on the C.I.E. diagram in comparison with other boundaries that are shown on both the C.I.E. and RUCS diagrams. Space limitations have also dictated the use of different types of lines to distinguish the boundaries of the three standards. When the same equation has been used as a boundary in more than one standard, a combination of the different types of lines has been used to represent the common boundary. The significances of the different types of lines have been shown in the figures. Labels have been used to distinguish boundaries of the same standard which are specified for different uses and again it has been found necessary to conserve space by abbreviating the terms. The following code has been used:

U.S. Standard:

SIG = Signal Color (broadest category)
INT = Intermediate Color
RES = Restricted Color (highest certainty category)
BW = Blue white (special category)
LW = Lunar white (distinguishable from yellow)

C.I.E. Recommendations:

RES = Restricted (red only)

I.C.A.O. Standard and Recommended Practices:

VAR = Variable (color of signal varies with source as intensity is changed for different conditions)
CON = Constant (signals not in previous category)
REC = Recommended practice.

In the case of the C.I.E. white boundaries, the selection of the boundary depends upon the number of colors in the system and this seemed to make it impracticable to label the C.I.E. boundaries. Reference should be had to the Standard itself for the interpretation of the boundary lines.

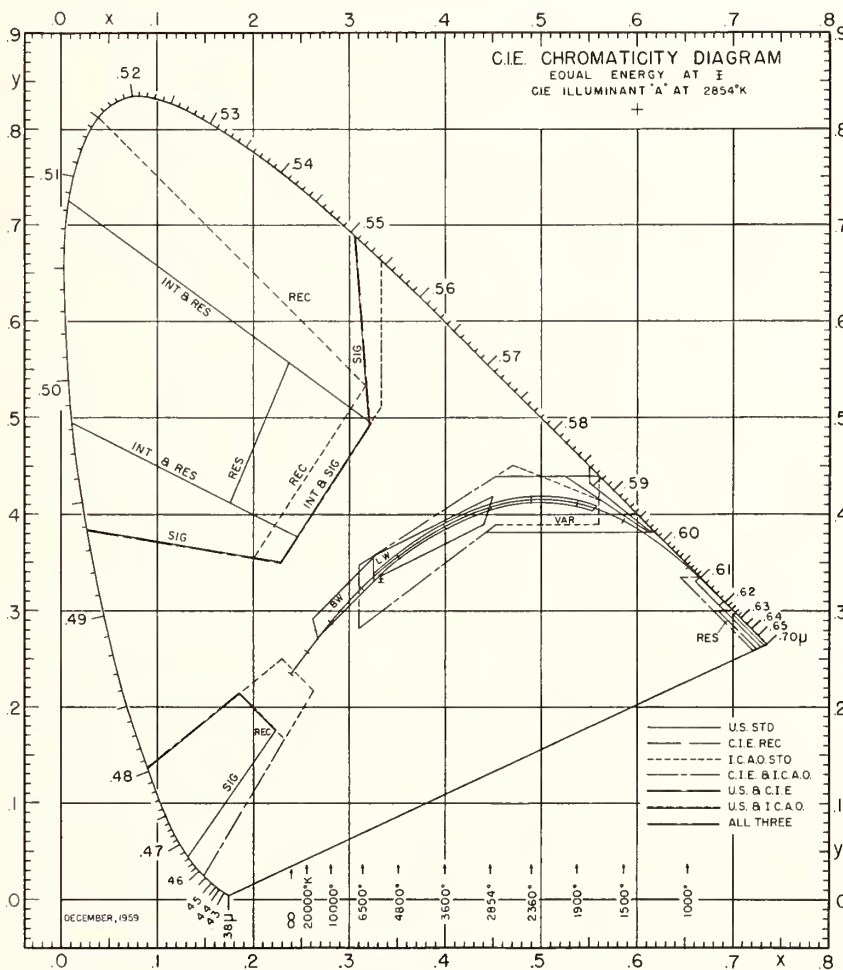
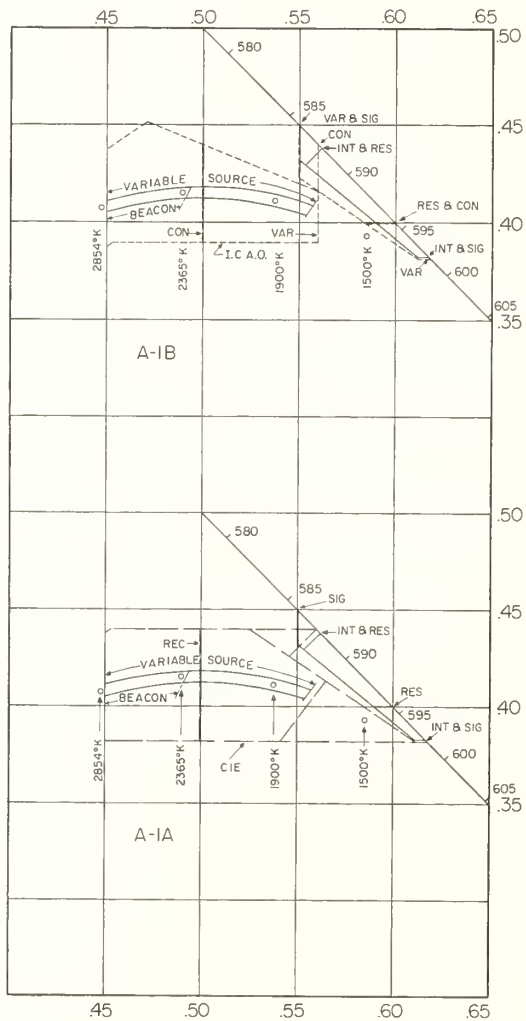


FIGURE A-1.



FIGURES A-1A and A-1B.

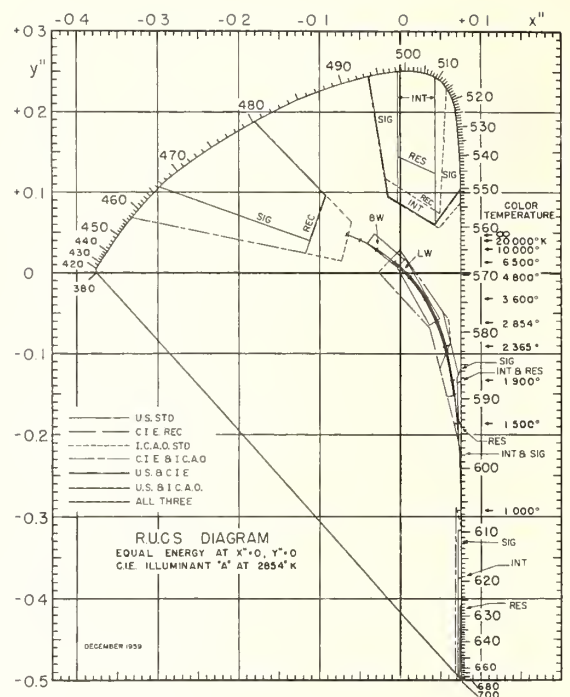
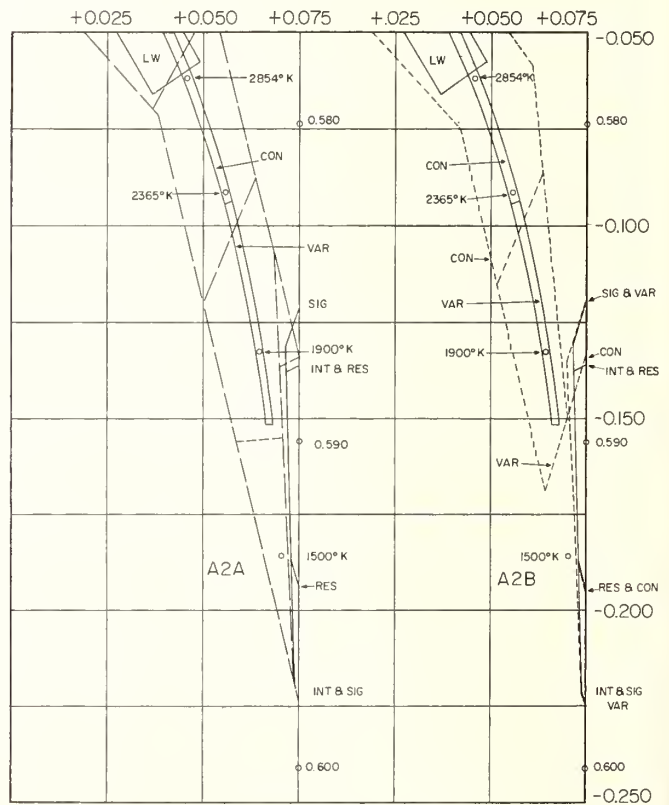


FIGURE A-2.



FIGURES A-2A and A-2B.

4. Light Sources and Color Instrumentation

	Page
Foreword	286
Papers	
4.1. Blue-glass filters to approximate the blackbody at 6500 K. Judd, Deane B., <i>Die Farbe</i> 10, 31 (1961). Key words: Blackbody at 6520 K; color-temperature- altering filters; daylight	287
4.2. Lines of constant correlated color temperature based on Mac- Adam's (<i>u.v.</i>) uniform chromaticity transformation of the CIE diagram. Kelly, Kenneth L., <i>J. Opt. Soc. Am.</i> 53, 999 (1963). Key words: Correlated color temperature; CIE chroma- ticity diagram; Planckian locus of chromaticity; uniform chromaticity	293
4.3. Spectral distribution of typical daylight as a function of cor- related color temperature. Judd, Deane B., MacAdam, David L., and Wyszecki, Gunter, <i>J. Opt. Soc. Am.</i> 54, 1031 (1964). Key words: Characteristics vector method; correlated color temperature; daylight chromaticities; daylight spec- tral distribution	297
4.4. A colorimeter for pyrotechnic smokes. Nimeroff, Isadore and Wilson, S. W., <i>J. Res. NBS</i> , 52, 195 (1954). Key words: Filter design; mosaic optical filter; photo- electric tristimulus colorimeters; pyrotechnic smoke colors	307
4.5. Development of filters for a thermoelectric colorimeter. Emara, Sayeda H. and Teele, Ray P., <i>J. Res. NBS—C. Eng. and Instr.</i> 67C, 319 (1963). Key words: Filter design: thermoelectric colorimeter ..	312
4.6. Mechanized conversion of colorimetric data to Munsell renota- tions. Rheinboldt, Werner C. and Menard, John P., <i>J. Opt. Soc. Am.</i> 50, 802 (1960). Key words: Intersystem chromaticity-coordinate conver- sion; high-speed digital computer; Munsell renotations ..	319

Foreword

Colorimetry has been recognized as a collection of measurement techniques that stem from the interaction of radiant power from sources, the optical characteristics of intervening media and the response of the photo-receptions of the eye. The most common sources and illuminants under which colors are seen, judged, and matched are daylight, sunlight, incandescent tungsten lamps, and fluorescent lamps. Correlated color temperature is an attempt to characterize the colorimetric properties of sources and illuminants in terms of the temperature of a blackbody radiator that has the same color (chromaticity) of those sources or illuminants.

The first three papers in this section treat the subject of color temperature. In the first paper, Judd describes filters that alter, on transmission, the light from a tungsten lamp at 2854 K to appear as light from a blackbody at 6500 K. The second paper, by Kelly, describes the lines of constant correlated color temperature on a uniform chromaticity diagram as perpendicular to the locus of a Planckian radiator. The third, by Judd, MacAdam and Wyszecki, gives the spectral distributions of typical daylight as related to correlated color temperature. The spectral irradiances of daylight under a variety of conditions are given in terms of characteristic vectors. These vectors are useful in deriving spectral distributions for any preselected correlated color temperature in the range 4,000 to 40,000 K.

Photoelectric tristimulus colorimeters are devices that avoid the computation of color coordinates from spectral data by means of three source-filter-photocell combinations to approximate the three standard colorimetric observer functions. The next two papers, one by Nimeroff and Wilson and one by Emara and Teele, describe such colorimeters developed at the National Bureau of Standards. In the first of these papers, multiplier phototubes were used, while in the second, a thermopile is used.

The last paper in this section, by Rheinboldt and Menard, describes the conversion of colorimetric data to Munsell notations by means of a program for high-speed digital electronic computers. It demonstrates that the arduous tasks of computing color coordinates by means of desk calculators, tasks that are approximately solved by photoelectric tristimulus colorimeters, can be completed speedily and accurately by means of digital computers. All such spectrophotometric colorimetry work is now being done at NBS by means of specially prepared high-speed computer programs.

Sektion 1: Farbmetrik
Gruppe 1a: Farbmeßtechnik

Deane B. Judd, WASHINGTON D. C.*:

**Blue-Glass Filters to Approximate the Blackbody
at 6500° K**

DK 535.65.084.852
666.242

Im Jahre 1956 haben Rautian-Lobanova-Znamenskaja eine Lichtart beschrieben, die für exakte Farbmessungen an Körperfarben gedacht ist und die spektrale Strahlungsverteilung des Planckschen Strahlers bei 6500°K zwischen 400 und 700 nm ergeben soll. Sie wird durch Normlichtart A (2854°K) und ein davorgesetztes Dreischichten-Glasfilter nach Ronis realisiert. Die Filter sind jedoch bis jetzt nicht im Handel erhältlich. Auf Anregung des National Bureau of Standards (Washington, D. C.) haben die Corning Glaswerke (Corning, N. Y.) unter Leitung von A. J. Werner die Entwicklung und Herstellung solcher Filter aufgegriffen und im Dezember 1960 einen Satz geliefert. Die Messungen des N. B. S. zeigen, daß die Anpassung an die Strahlungsverteilung 6500°K mit dem dreifachen Corning-Filter noch ein wenig besser als mit den Filtern nach Ronis gelungen ist.

En 1956 les auteurs Rautian-Lobanova-Znamenskaja ont décrit un illuminant pour la colorimétrie précise des couleurs des objets; les auteurs avaient en vue que cet illuminant imite la répartition spectrale du rayonnement du corps noir à 6500°K dans la région de 400 à 700 nm. L'illuminant peut être réalisé par l'illuminant A (2854°K) en combinaison avec un triple filtre de verre d'après Ronis. Mais on ne peut pas l'acheter. C'est pourquoi le National Bureau of Standards (Washington, D. C.) a proposé aux usines Corning Glass Works (Corning, N. Y.) de développer et produire de tels filtres; sous la direction de A. J. Werner ces usines ont commencé ce travail, et en décembre 1960 les premiers filtres sont délivrés. Les mesures du N. B. S. ont montré que l'on a réussi à imiter la répartition spectrale du rayonnement noir à 6500°K par la combinaison triple de Corning un peu mieux que par le filtre Ronis.

In 1956, Rautian, Lobanova, and Znamenskaja described a light source for precise color measurement of non-self-luminous objects with an energy distribution between 400 and 700 nm closely duplicating that of a black body at 6,500°K. This light source took the form of an incandescent lamp at a color temperature of 2,854°K (CIE source A) combined with a three-layer, blue-glass filter of the Ronis type. An attempt in February 1959 to purchase such Ronis filters through Stanko-import indicated that they were not yet commercially available. At the request of the National Bureau of Standards (Washington, D. C.), Corning Glass Works (Corning, N. Y.) undertook the design and production of such a filter under the direction of their A. J. Werner, and in December 1960 delivered a supply of them. Measurements at the N. B. S. show that the degree of duplication of the black-body energy distribution for 6,500°K available from the Corning three-layer filter is slightly better than that reported for Ronis-type filters.

* National Bureau of Standards, Photometry and Colorimetry Section

In 1956, RAUTIAN, LOBANOVA, and ZNAMENSKAYA [1] described a light source for precise color measurement of non-self-luminous objects. This light source took the form of an incandescent lamp at a color temperature of 2854°K (CIE source A) combined with a three-layer filter made from glasses developed in the USSR by D. RONIS. The spectral energy distribution of the light transmitted by the filter so illuminated closely duplicates that of a blackbody at 6500°K .

At the first meeting of *CIE Committee E-1.3.1, Colorimetry*, in 1957, a proposal was considered to redefine CIE standard sources B and C as Planckian blackbodies and to realize them by glass filters instead of liquid filters [2]. This proposal was made by the *Permanent Commission on Colorimetry* (G. N. RAUTIAN, Vice-Chairman), *State Institute of Metrology, Leningrad*. It was pointed out that progress in the glass industry since the adoption of liquid filters in 1931 makes it feasible to substitute glass for liquid filters at the present time. The advantages of glass filters in convenience and stability are obvious, and it was pointed out that by using three kinds of colored glass developed by D. RONIS the spectral distribution of energy of the blackbody at 4800 and 6500°K can be duplicated to a quite satisfactory degree. As a result of considering this proposal, CIE Committee E-1.3.1, agreed "*that the Russian proposal to define standard sources solely as blackbody radiators and to substitute glass filters for liquid filters in their realization is worthy of study. We accept with thanks the Russian offer to demonstrate the adequacy of duplication of blackbody energy distributions by means of the glass filters developed by D. Ronis*".

In September 1958, four RONIS-type filters to approximate CIE source B were circulated for study to the experts of CIE Committee E-1.3.1, and in June 1959, four more such filters together with four to duplicate CIE source C, were circulated for study. These studies substantiated in every detail the claims made for these filters.

At the second meeting of *CIE Committee E-1.3.1* in 1959, this Russian proposal was further discussed briefly, and three points were brought out, two in favor, one against. First, the merit of color-temperature-altering filters made of RONIS glasses was granted, and second, the use of blue-glass filters instead of liquid filters was favored because such filters are already used to a considerable extent in Great Britain, Australia, and the United States of America for color grading and measurement¹, but it was con-

¹ One of the blue filters most used in the USA is the *Corning Daylite* glass filter at such a thickness as to produce a color temperature of 7500°K , the resulting source being sometimes known as „source D“. Another frequently used type of blue-glass filter is that known as *Corning Whiterlite* [3; 4; 5; 6].

sidered premature to recommend any change in the definition of CIE sources for color measurement until blue filters to realize a good approximation of sources defined in terms of ideal blackbody radiators should become generally available. In comment on this view, RAUTIAN (letter of 17 March 1959 to D. B. JUDD) stated, "...I can inform you that Soviet Vneshtorg intends to turn out for sale a tentative set of 'B' and 'C' glass filters." It was agreed to postpone decision on the Russian proposal until suitable blue-glass filters should become readily available. In order to ensure

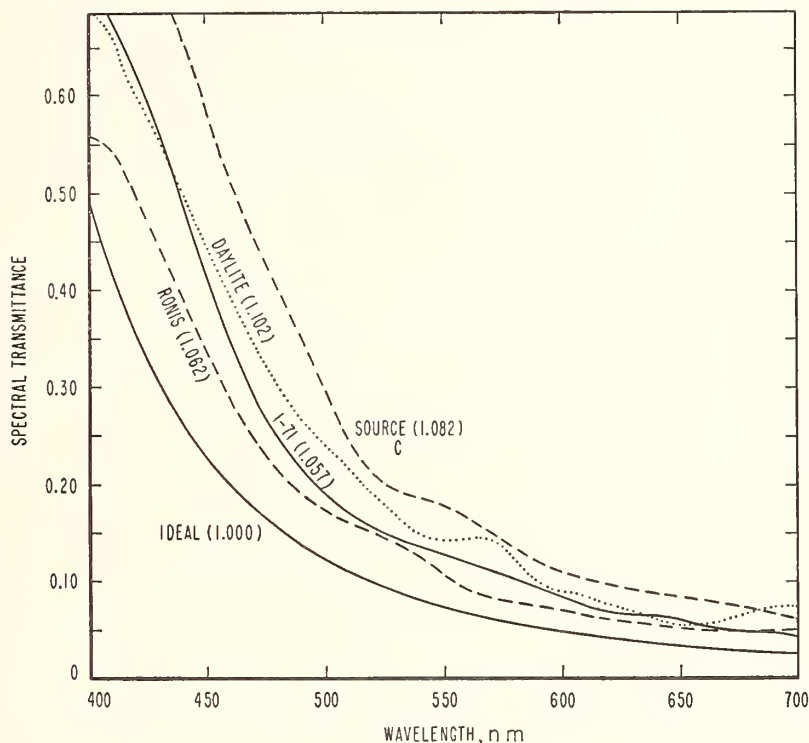


Fig. 1. Spectral transmittances of an ideal filter to convert CIE source A to the relative spectral energy distribution of a blackbody at 6500°K compared to those of four actual blue filters yielding more or less close approximations. The numbers in parentheses following the identification of each filter are the average transmittance ratios, actual to ideal (taken from table 1).

prompt consideration of this proposal, the Chairman requested Mr. A. J. WERNER, *Corning Glass Works*, to undertake development of a blue-glass filter which should convert CIE source A to as close a duplication as possible of the energy distribution of the blackbody at 6500°K. The lower curve of fig. 1 shows a spectral-transmittance function between 400 and

700 nm meeting this requirement perfectly. It is simply a plot proportional to the ratio of the spectral emittance of the blackbody at 6500° K to that of a blackbody at 2854° K, the constant of proportionality being such as to yield a transmittance value of 50 percent at 400 nm. The shape of this curve is thus the target shape for the spectral-transmittance functions of the blue-glass filters to be developed. The other solid curve shows the actual spectral-transmittance function of a trial filter produced by *Corning Glass Works*, now identified by Glass Code No. 1—71. It may be seen that the shape of this curve approximates to a rather high degree the target shape. The three dotted curves of Fig. 1 show the spectral transmittances of previously developed blue filters. One is that of the CIE source C filter [7] which may be made up from the prescribed concentrations of the two solutions comprising this two-component liquid filter, another is that of Corning Daylite glass, commercially available for many years. The third is that of one of the filters made of RONIS-type glass circulated within *CIE Committee E-1.3.1* for study, but which are not yet available commercially.

Table 1: Average departure between 400 and 700 nm from the blackbody at 6500°K of the spectral energy distributions available from various blue filters illuminated by CIE source A (2854°K)

Blue Filter	Average Departure in Optical Density (log 1/T)	Average Transmittance Ratio
Ideal filter (target)	0.000	1.000
Corning 1-71 (trial filter)	.024	1.057
(delivered filter)	.025	1.059
Ronis C.9.5	.026	1.062
Davis-Gibson Source C	.034	1.082
Corning Daylite	.042	1.102
Corning Whiterlite	.098	1.1253

Table 1 serves to appraise the degree to which these four filters, and a fifth filter of the Corning Whiterlite type, succeed in duplicating the target shape. The criterion used is the average departure, between 400 and 700 nm, of the actual spectral optical density (logarithm to the base 10 of the reciprocal of the spectral transmittance) from that corresponding to the target shape where the constant of proportionality has been adjusted to reduce the average departure to a minimum. This is the criterion used by RAUTIAN [1]. Table 1 also shows the average factor by which the spectral transmittance of each filter departs from that of the target.

Table 2: CIE chromaticity coordinates, x , y , and luminous transmittance, Y , of various blue filters illuminated by CIE source A

Blue Filter	Chromaticity Coordinates		Luminous Transmittance,
	x	y	Y
Ideal filter (target)	0.313	0.324	12.2°
Corning 1-71 (trial filter)	.315	.317	11.2
(delivered filter)	.320	.327	13.1
Ronis C.9.5	.312	.325	9.5
Davis-Gibson Source C	.310	.316	15.5
Corning Daylite	.313	.330	13.0
Corning Whiterlite	.317	.325	10.7

* Ideal filter with transmittance at 400 nm set at 92%; computed as the luminous transmittance for the ideal curve shown on Fig. 1 multiplied by the ratio: 92/50

Table 2 compares the chromaticity coordinates, x , y , of the light transmitted by each of the blue filters of Table 1 to those of the ideal filter, and lists the luminous transmittance of the filters for CIE source A.

Table 1 indicates that Corning filter 1-71 duplicates the energy distribution of the blackbody at 6500°K with the smallest average departure, but the filter made up of RONIS-type glasses is a good second. Not too much should be made of this slight superiority of two filters of Corning Glass Code No. 1-71 over one filter made of RONIS-type glasses. The delivered filter is a poorer chromaticity match. The important conclusion to be drawn from Table 1 is that the approximations to the ideal color-temperature-altering filter now available in glass are much better than those previously available in glass, and even somewhat better than that available in the liquid filter defining CIE source C. This corroborates the statement made in 1957 by RAUTIAN to CIE Committee E-1.3.1 „that progress in the glass industry . . . makes it feasible to substitute glass for liquid filters at the present time“.

Since filters of Corning Glass Code No. 1—71 are now available by purchase to anyone from *Corning Glass Works, Corning, New York, U.S.A.*, there is no longer any reason for postponing serious consideration of the advantages and disadvantages of redefining CIE standard sources for color measurement as suggested in 1957 by the *Permanent Commission on Colorimetry, State Institute of Metrology, Leningrad*.

At the third meeting of the experts of CIE Committee E-1.3.1, *Colorimetry*, held in London, 19 May 1961, a tabulation of these advantages and disadvantages was made for circulation to the various national committees of the CIE with a view to obtaining a decision at the next meeting planned in conjunction with the CIE Congress scheduled for Vienna in 1963.

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Lines of Constant Correlated Color Temperature Based on MacAdam's (u,v) Uniform Chromaticity Transformation of the CIE Diagram

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The chromaticities of hot bodies (such as the filaments of incandescent lamps) and the various phases of daylight are often expressed on the color-temperature scale. Such chromaticities do not usually coincide precisely with those of the blackbody at any temperature, but are specified by giving the temperature of the blackbody yielding the nearest chromaticity match. Thus each chromaticity point near the Planckian locus is correlated with a chromaticity point on it, and the color temperature corresponding to the Planckian chromaticity is called the correlated color temperature. If the decision as to the nearest Planckian chromaticity is based on a uniform-chromaticity-scale (UCS) diagram, the chromaticity points correlated with each Planckian chromaticity fall on a straight line (isotemperature line) on the chromaticity diagram normal to the Planckian locus at that Planckian chromaticity. The UCS diagram proposed by MacAdam in 1937 and recommended in 1959 by the International Commission on Illumination (CIE) has been used here to define the isotemperature lines. Graphs showing the Planckian locus and the isotemperature lines both in the CIE (x,y) -diagram and in the MacAdam CIE (u,v) -diagram are shown at every 10 microreciprocal degrees for all color temperatures higher than 1515°K.

TABLE I of a paper by Judd¹ gives the chromaticity coordinates of the intersections of a series of lines of constant correlated color temperature^{2,3} (isotemperature lines) with the Planckian locus for $C_2 = 14\,380\,\mu$ degrees along with the reciprocals of their slopes as a function of reciprocal color temperature. The isotemperature lines were established as the normals to the Planckian locus when plotted on the uniform chromaticity scale (UCS) diagram represented by the equilateral Maxwell triangle of a linear transformation of the CIE system.⁴ When these normals are transformed to the CIE diagram, they indicate the nearest color temperature estimated according to the UCS system read approximately from specifications in the CIE system. The expression of color temperature (θ) in reciprocal form ($10^6/\theta$) with the microreciprocal degree ("μrd" or "mired") as the unit is used in these graphs

because as Priest,⁵ who suggested its use, pointed out, "a difference of one micro-reciprocal degree is fairly representative of the doubtfully perceptible difference in chromaticity under the most favorable conditions of observation." Thus the stepping between successive isotemperature lines is nearly uniform on the UCS diagram and as the color temperature scale approaches infinity at the blue end, the reciprocal-temperature scale approaches zero. It has also been possible in one application⁶ to extend this locus to values less than zero in order to interpret experimental data.

MacAdam in 1937⁷ proposed a transformation of the CIE system which would have the desirable properties that the resultant diagram would closely resemble Judd's uniform chromaticity scale diagram in that colors represented as at the ends of lines of equal length throughout the diagram would appear about equally different. It has the additional advantages that the luminosity coefficients of the corresponding color-

¹ D. B. Judd, *J. Opt. Soc. Am.* **26**, 421 (1936).

² R. Davis, *J. Res. Natl. Bur. Std.* **7**, 659 (1931).

³ O.S.A. Committee on Colorimetry, *The Science of Color* (The T. Y. Crowell Company, New York, 1953), p. 303.

⁴ D. B. Judd, *J. Res. Natl. Bur. Std.* **14**, 41 (1935); also *J. Opt. Soc. Am.* **25**, 24 (1935).

⁵ I. G. Priest, *J. Opt. Soc. Am.* **23**, 41 (1933).

⁶ K. L. Kelly, *J. Res. Natl. Bur. Std.* **60**, 97 (1958).

⁷ D. L. MacAdam, *J. Opt. Soc. Am.* **27**, 294 (1937).

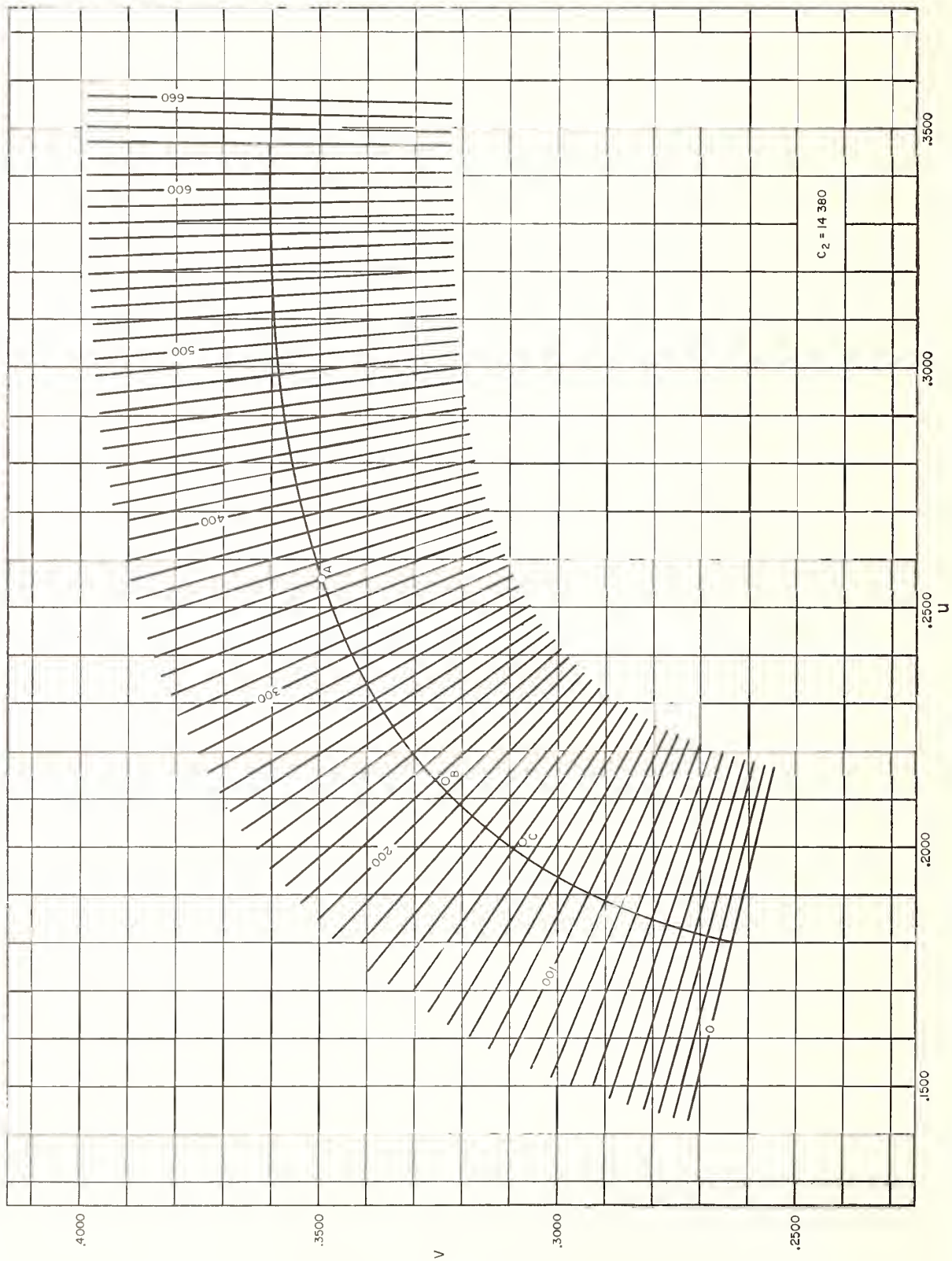


FIG. 1. The Planckian locus with the normals to it (lines of constant correlated color temperature) at each 10 microreciprocal degrees plotted on the MacAdam CIE (u, v) -diagram. Sources A, B, and C are indicated by a circle and A, B, or C.

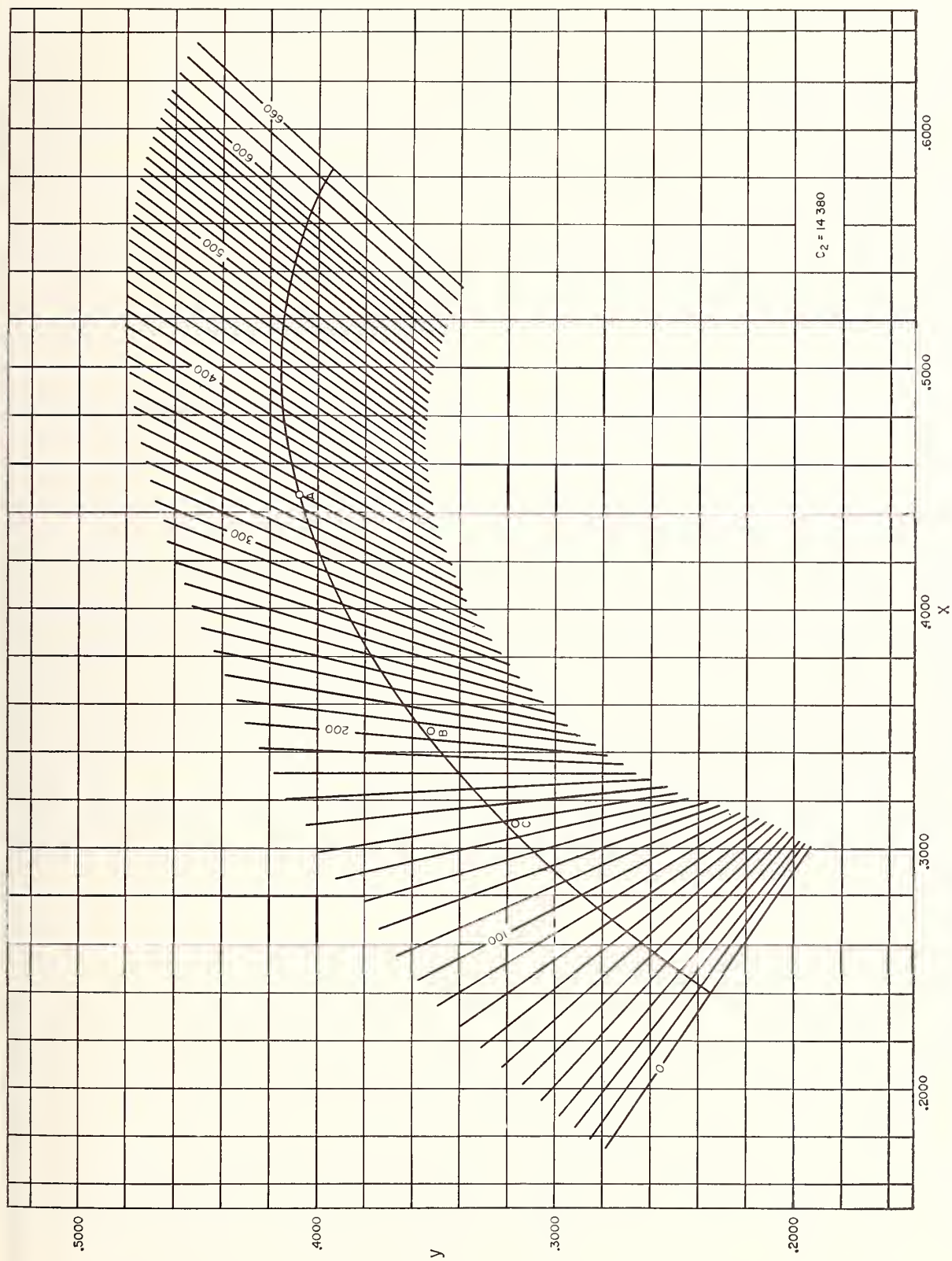


FIG. 2. The Planckian locus with the normals to it from the (u, v) -diagram transformed into the (x, y) -diagram. The lines of constant correlated color temperature are again given at every 10 microreciprocal degrees and the standard sources are indicated by a circle and A, B, or C.

matching functions are 0-1-0, and that the transformation equations from the 1931 CIE system are fairly simple. The CIE, in its 1959 meeting in Brussels,⁸ officially recommended the use of the MacAdam 1937 diagram "whenever a diagram yielding colour spacing perceptually more nearly uniform than the (x,y) -diagram is desired." The chromaticity diagram is produced by plotting $4X/(X+15Y+3Z)$ as abscissa and $6Y/(X+15Y+3Z)$ as ordinate, in which X, Y, Z are the tristimulus values corresponding to the 1931 CIE standard observer and coordinate system.⁹ An equivalent form of the recommendation is to plot: $4x/(-2x+12y+3)$ as abscissa and $6y/(-2x+12y+3)$ as ordinate, where x, y are chromaticity coordinates according to the 1931 CIE standard observer and coordinate system.

Normals to the Planckian locus have been computed for the MacAdam CIE (u,v) -diagram by resort to the data supplied by Harding and Vickers¹⁰ giving the chromaticity coordinates in the 1931 CIE (x,y) -diagram of points on the Planckian locus for each microreciprocal degree for $C_2=14\,380\,\mu$ degrees. A code was written for transforming these data into (u,v) -coordinates, for determining the normals to the locus at each microreciprocal degree, and for determining the coordinates of points on each of these normals for four increments of Δv (0.002, 0.004, 0.006, and 0.008) from the locus both above and below it. The coordinates of these points were found by automatic digital computation (IBM 7090) by means of this code.

The (u,v) -coordinates of the Planckian locus and the points on the normals to it are plotted at each 10 micro-

reciprocal degrees in Fig. 1. Each set of points was connected by a straight line of arbitrary length, this length selected to be useably long but not so long as to invite the criticism that they represent notably non-Planckian colors. These (u,v) -data were transformed by a similar code back into (x,y) -coordinates and plotted on a similar diagram in Fig. 2, which likewise shows the Planckian locus and points on the transformed normals at each 10 microreciprocal degrees.¹¹ Each set of these points was connected by a straight line. Each graph therefore represents this information for a color-temperature range from 1515.15°K to infinity.

It is interesting to note that if the isothermperature lines are extended downward they all pass fairly near to the point $x=0.325, y=0.154$; that is, there is a chromaticity point in the purple region which, by the spacing of the (u,v) -diagram, is about equally distinguishable from each local section of the Planckian locus in turn within the range evaluated. If it had turned out that all isothermperature lines passed through a single point, this point would have to be equally distant on the (u,v) -diagram from every point on this part of the Planckian locus, and this part of the Planckian locus would have to be precisely circular. The point on the (u,v) -diagram corresponding to $x=0.325, y=0.154$ is $u=0.31$ and $v=0.22$. It is seen by reference to Fig. 1 that the point (0.31, 0.22) is indeed separated from the points on the Planckian locus by nearly the same distance.

The assistance of J. C. Schleter in writing the IBM codes and of Ray W. Gates in preparing the two graphs is gratefully acknowledged.

⁸ Compt. Rend. CIE 14e Brussels 1959, 107, 108.

⁹ D. B. Judd, J. Opt. Soc. Am. 23, 359 (1933).

¹⁰ H. G. W. Harding and T. Vickers, "Colours of Planckian Radiators Expressed on the CIE Trichromatic System for the Temperature Ranges 1500-10-9000°K and 0-1-660 Mired ($C_2=14\,380$)," National Physical Laboratory, Teddington, Middlesex, England.

¹¹ A limited number of Xerox copies of Figs. 1 and 2 of this paper as well as of the 14 large-scale working graphs in x, y , are available on request. The large-scale graphs cover the same range of the Planckian locus as Figs. 1 and 2 but show a line of correlated color temperature for each microreciprocal degree.

Spectral Distribution of Typical Daylight as a Function of Correlated Color Temperature

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(Received 29 April 1964)

Spectral distributions of 622 samples of daylight (skylight, and sunlight plus skylight) have been subjected to characteristic vector analysis, as composite data and in three subgroups (99 distributions measured by Budde; 249, by Condit; and 274, by Henderson and Hodgkiss). The chromaticity coordinates (x, y) computed from these distributions have been compared with direct visual determinations of chromaticity coordinates of daylight by Nayatani and Wyszecki, and by Chamberlin, Lawrence, and Belbin. It was found that the chromaticities indicated by the spectral distributions and by direct visual colorimetry cluster about the curve: $y = 2.870x - 3.000x^2 - 0.275$. This curve of typical daylight chromaticities falls slightly on the green side of the Planckian locus. From the mean and the first two characteristic vectors of the composite data, spectral distribution curves have been reconstituted by choice of scalar multiples of the vectors such that the chromaticity points fall on the curve of typical daylight chromaticities at places corresponding to correlated color temperatures of 4800°, 5500°, 6500°, 7500°, and 10 000°K. The representative character of these reconstituted spectral-distribution curves has been established by comparison with the measured curves from each subgroup yielding the closest approximation to the same chromaticities. The agreement so found suggests that this family of curves is more representative of the various phases of daylight between correlated color temperatures 4800° and 10 000°K than any previously derived distributions.

INTRODUCTION

EVER since light sources B and C were recommended in 1931 by the International Commission on Illumination at its meeting in Cambridge, England, they have served as standard sources for the colorimetry of materials.¹ These sources are combinations of a gas-filled lamp at a color temperature of 2854°K with Davis-Gibson² liquid filters intended to approximate within the visible spectrum the spectral distributions of the complete radiator (blackbody) at temperatures of 4800° and 6500°K, respectively. Source B (4800°K) has served in the colorimetric laboratory as a representative of noon sunlight, and source C (6500°K) has served as a representative of average daylight. This laboratory use of these standard sources has steadily diminished in recent years because of the increasing dependence placed on indirect colorimetry by means of the spectrophotometer; but the spectral energy distributions of sources B and C have been widely and

increasingly used in the reduction of spectrophotometric data to the colorimetric variables (\bar{Y}, x, y) of the 1931 CIE standard-observer system.¹

The development of the Davis-Gibson liquid filters, and their use to realize standard sources for colorimetry, was an important step toward an internationally recognized general method of colorimetry, but standard sources so realized have limitations that prevent them from leading to a completely general method. For example, the ultraviolet content³ is too low to permit these sources to be an adequate representation of sunlight or daylight in the colorimetry of fluorescent materials. Furthermore, the standard sources cannot yield the high luminous flux required for the most precise inspection of industrial products because the liquids in the filters would become heated above the boiling point. Because of these limitations, because of minor inconveniences arising from the use of liquid filters (difficulties of preparation according to the careful prescription given, need to replace the filters every few months because of drift in their spectral character, temperature coefficient of spectral transmittance), and because improved glass filters could be made available,⁴

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¹ *Proceedings of the International Commission on Illumination, 8th Session, Cambridge, 1931* (Cambridge University Press, Cambridge, England, 1932) p. 19.

² R. Davis and K. S. Gibson, "Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature," *Natl. Bur. Std. (U. S.), Misc. Pub. 114* (1931).

³ R. Davis, K. S. Gibson, and G. W. Haupt, *J. Res. Natl. Bur. Std.* **50**, 31 (1953); *J. Opt. Soc. Am.* **43**, 172 (1953).

⁴ G. N. Rautian, N. V. Lobanova, and M. A. Znamenskaya, *Z. Tech. Fiziki* **26**, 193 (1956). D. B. Judd, *Farbe* **10**, 31 (1961).

a proposal was made by the Russian delegate to the CIE Committee on Colorimetry in 1957 to define standard sources solely as blackbody radiators and to substitute glass filters for liquid filters in their realization.⁵

As a part of the studies of this Russian proposal, undertaken at the request of the CIE Committee on Colorimetry, a questionnaire devised by one of us (GW) was circulated by the Canadian Committee on Colorimetry, and a similar questionnaire devised by D. Nickerson was circulated by the U. S. Technical Committee on Colorimetry.

The replies to the Canadian questionnaire have been summarized⁶ as follows:

"Question 4.—Do you suggest that the present CIE standard sources should be changed? If so, which sources do you recommend? Please give references and reasons for your choice."

The replies received can be separated into three groups, each of which is about equal in size:

"(i) No comments; leaves it up to the CIE to investigate the problem and come forward with recommendations."

"(ii) No change recommended; emphasizes the possible confusion a change may bring about and the considerable effect it may have on existing tabulations and instrumentation."

"(iii) New sources or changes are desirable; it would be desirable to have one or more standard daylight sources which are of more practical use than the present Source C and thus encourage the use of the standard source in inspection work. A new standard source should provide a high level of illuminance over a large area of the visual field. The ultraviolet content should be comparable with that of natural daylight both in amount and spectral distribution."

Nickerson⁶ summarized the replies to the questionnaire as follows: "No change should be made in CIE standard light sources until a 300–800 nm definition for a target standard for daylight can be agreed upon, and a sufficiently close match to this can be obtained in an actual lamp, or in a lamp-filter combination, that can be agreed upon for use as a standard source for colorimetric and visual use. Further, that any series adopted for light-source standards should include one that in color temperature is at least as high as 7400°K. This is for use as a daylight substitute in colorimetry and for visual use, to be adopted either in replacement for, or in addition to, the present source C."

⁵ *Proceedings of the International Commission on Illumination, 14th Session, Brussels, 1959* (Bureau Central de la Commission Internationale de l'Eclairage, 57, rue Cuvier, Paris, 5^e, France, 1960), Vol. A, p. 93.

⁶ *Proceedings of the International Commission on Illumination, 15th Session, Vienna, 1963* (Bureau Central de la Commission Internationale de l'Eclairage) (to be published).

A meeting of U. S. Technical Committee on Colorimetry was convened in Rochester in October 1962 to consider the results of these questionnaires and to explore what next steps ought to be taken in the development of standard sources for colorimetry. All members of the committee were present (I. A. Balinkin, University of Cincinnati; D. B. Judd, National Bureau of Standards; D. L. MacAdam, Eastman Kodak Company; D. Nickerson, U. S. Department of Agriculture; and W. T. Wintringham, Bell Telephone Laboratories). Present also as guests at one time or another were W. E. K. Middleton and G. Wyszecki, National Research Council of Canada; Norman Macbeth, Macbeth Corporation; and H. R. Condit, O. E. Miller, and J. L. Simonds, Eastman Kodak Company. Macbeth criticized existing standards of daylight, including Abbot-Gibson daylight⁷ and Middleton's computed distributions⁸ for the overcast sky with an urban terrain suggested by the committee in 1955 as a target standard,⁹ as well as CIE sources B and C, as being too pink to be acceptable representations of typical daylight. These recommended spectral distributions correspond to chromaticities on the pink side of the Planckian locus on the chromaticity diagram. He called attention to the fact that over a period of several decades his company has sold thousands of artificial daylighting units not only with a color temperature range, but also with a pink-green range. Many units yielding colors on the pink side of the Planckian locus, and even those falling very slightly on the green side, have been returned by customers with the complaint that they were too pink to give color renditions characterizing natural daylight.¹⁰

Examination of a chromaticity diagram supplied by Nickerson comparing the chromaticities corresponding to the spectral distributions of sunlight and daylight by Abbot¹¹ as summarized by Moon,¹² by Taylor and Kerr,¹³ and by Henderson and Hodgkiss supplied in advance of publication¹⁴ was found to support Macbeth's indirect finding that typical daylight is more greenish than existing standards of daylight.

MacAdam reported that extensive spectral distribution data for various phases of daylight had been obtained in Rochester in recent years independently in

⁷ K. S. Gibson, *J. Opt. Soc. Am.* **30**, 88 (1940).

⁸ W. E. K. Middleton, *J. Opt. Soc. Am.* **44**, 793 (1954).

⁹ *Proceedings of the International Commission on Illumination, 13th Session, Zürich, 1955* (Bureau Central de la Commission Internationale de l'Eclairage, 57, rue Cuvier, Paris 5^e, France, 1955), Vol. I, p. 1.3.1 U-9.

¹⁰ Norman Macbeth and W. B. Reese, "Some practical notes on standard illumination practices for color matching in the U. S. A.—past, present, and future," 7th International Conference on Color; Florence, Prato, and Padua, Italy; 2–7 May 1963.

¹¹ C. G. Abbot, F. E. Fowle, and L. B. Aldrich, "The distribution of energy in the spectra of the sun and stars," *Smithsonian Miscellaneous Collections* **74**, No. 7, Publ. No. 2714 (1923).

¹² P. Moon, *J. Franklin Inst.* **230**, 583 (1940).

¹³ A. H. Taylor and G. P. Kerr, *J. Opt. Soc. Am.* **31**, 3 (1941).

¹⁴ S. T. Henderson and D. Hodgkiss, *Brit. J. Appl. Phys.* **14**, 125 (1963).

two projects of the Eastman Kodak Company, one (Condit)¹⁵ by spectroradiometer, the other (Miller) by interference filters, and that some of the data gathered by Condit had been subjected to characteristic vector analysis by Simonds.¹⁶ Wysecki said that a project of the National Research Council was about to get under way to measure the spectral distribution of north sky-light in Ottawa simultaneously with visual determinations of chromaticity as a check. In view of the fact that data recently taken, or about to be taken, were far more numerous, and sampled much more completely the range of spectral distributions of daylight than previously published data, it was decided that the technical colorimetry committees of Canada and the United States would jointly undertake to evaluate from these recent data (Ottawa, Rochester, and Enfield, England), a series of related spectral distributions of typical daylight extending over a considerable range of correlated color temperatures. It is the purpose of this paper to describe the method of evaluation, to present the series of spectral distributions, and to present evidence supporting the view that each individual spectral distribution of the series is typical of daylight of a particular correlated color temperature.

COLLABORATION OBTAINED

Condit, Henderson, and Budde supplied spectral distribution data relative to 560 nm for various samples of daylight in the form of relative spectral irradiance for each 10-nm interval between 330 and 700 nm in advance of publication. Condit supplied distributions for 249 samples of daylight; Henderson, 274; and Budde, 99.

Simonds supervised derivation of the first four characteristic vectors not only for the composite data (622 distributions), but also for the three subgroups (Condit, Henderson, Budde) over the same spectral range, and for the Henderson data over the range 330 to 780 nm. The program and computer time for computing the chromaticity coordinates for each input distribution, for evaluation of the characteristic vectors, for obtaining reconstituted distributions from the means and the first two characteristic vectors, and for checking these against the corresponding individual input distributions were supplied by the Eastman Kodak Company.

CHARACTERISTIC VECTOR METHOD

A mathematical statement of the method, adapted from Simonds,¹⁶ might be given as follows:

Response data E_λ (spectral irradiance) are available for r (38) levels of the variable λ (wavelength). For each experimental condition, then, the r values of E_λ constitute a one-row r -column vector of response data.

¹⁵ H. R. Condit and F. Grum, J. Opt. Soc. Am. 53, 1340 (1963); 54, 937 (1964).

¹⁶ R. H. Morris and J. H. Morrissey, J. Opt. Soc. Am. 44, 530 (1954); J. L. Simonds, *ibid.* 53, 968 (1963).

For n (622, 249, 274, or 99) sample sets of data, the response vectors can be arrayed to form a data matrix of n rows and r columns.

It is possible to find a set of characteristic vectors, which, when added in the proper amounts to the mean response vector, will adequately approximate any of the original family of n response vectors. Details of the computational procedure are given by Simonds.¹⁶ The procedure involves the computation of the variance-covariance matrix S from the original n -by- r data matrix of response vectors. Characteristic vectors of this matrix are obtained corresponding to the latent roots of the determinantal equation

$$|S - LI| = 0, \quad (1)$$

where I is an r -by- r unit matrix and L is a diagonal matrix (r -by- r) of the latent roots. The characteristic vectors, like the response vectors themselves, are sets of r numbers. Mathematically stated, the sample responses at each value of wavelength are given by Eq. (2):

$$\left. \begin{aligned} E_1 &= \bar{E}_1 + M_1 V_{1,1} + M_2 V_{2,1} \\ &\quad + M_3 V_{3,1} + \cdots + M_p V_{p,1}, \\ E_2 &= \bar{E}_2 + M_1 V_{1,2} + M_2 V_{2,2} \\ &\quad + M_3 V_{3,2} + \cdots + M_p V_{p,2}, \\ E_r &= \bar{E}_r + M_1 V_{1,r} + M_2 V_{2,r} \\ &\quad + M_3 V_{3,r} + \cdots + M_p V_{p,r}. \end{aligned} \right\} p \leq r. \quad (2)$$

The M 's are the amounts of the characteristic vectors which must be added to the mean response vector in order to produce the sample response vector. The characteristic vectors V_p are uniquely determined for a given family of response curves. The same characteristic vectors apply to all response vectors belonging to the original family from which the vectors are derived. Only the values of M_1, M_2, \dots, M_p vary from one response curve to another. The M 's, therefore, are a complete specification of the response vector to which they apply. Together with the uniquely determined characteristic vectors and the mean response vector, the M 's are sufficient information with which to reconstruct the entire response vector from which they were derived.

The number p of characteristic vectors required to explain all the differences among a family of curves, each represented by r responses, will be equal to, or less than r . For a perfect fit to all the response vectors, r characteristic vectors may be required. The power of the tool, however, comes from the fact that a large percentage of the variability among the family of homologous response vectors may be explained by using only a few characteristic vectors.

The p vectors, when graphically presented, are basis curves or vectors. The following statements are given without proof:

(1) The p characteristic vectors are orthogonal; that is, they represent independent types of response

variability. Mathematically stated,

$$\sum_{i=1}^r V_{a,i} V_{b,i} = 0, \quad a \neq b.$$

(2) The M 's, called scalar multiples, can be determined for each sample response vector as a simple linear combination of the response data at the r values of wavelength. The weighting coefficients are uniquely determined for a particular set of vectors.

(3) The derivation of the vectors ensures that the first vector accounts for the largest amount of the total response variability; the second vector accounts for the second largest amount of variability; and so forth.

EXPERIMENTAL DATA

The 249 spectral distributions of Rochester daylight supplied by Condit were obtained by comparing the light reflected from a magnesium oxide test plate illuminated by incandescent lamp light of known color temperature with that reflected from a barium sulfate test plate tilted 15° off the vertical and illuminated either by light from the sky and light from the sun at various altitudes in the plane perpendicular to the test plate, or simply by light from the sky with the sun being back of the plate or obscured by clouds. The instrument used was the Beckman DK spectrophotometer whose slitwidths are automatically adjusted to maintain the output of the photodetector constant. The spectral bands transmitted varied from about 1 to 3 nm at half-height. The scanning time was about 1 min. The measured spectral distribution curves extended from 330 to 700 nm, and were read at somewhat irregular intervals dictated by an attempt to obtain values of relative spectral irradiance at the middle of each major absorption or transmission band detected. The input data were derived at intervals of 10 nm by interpolation of these data. Small corrections were applied for the difference in spectral reflectance between magnesium oxide and barium sulfate. The measurements refer to sky conditions not yielding any visual evidence of industrial contamination.

The 274 spectral distributions of daylight in Enfield, England, supplied by Henderson, were obtained partly by light reflected from a horizontal diffusing white plate of known spectral reflectance relative to magnesium oxide, consisting of mat-finish Vitrolite receiving light from nearly the whole hemisphere including direct sunlight if it was not obscured by clouds. The remainder of the observations refer to the north sky at 45° with an acceptance angle of about 6° . The Hilger and Watts D290 grating monochromator was used, and the scan was divided between two photocells (Mazda 27M3 from 300 to 580 nm, RCA 1P22 from 540 to 780 nm). The resulting spectroradiometer was calibrated by light from an incandescent lamp at a color temperature of 2854°K reflected from a magne-

sium oxide screen.¹⁴ Narrow equal slits gave a spectral bandwidth of 1.5 nm. "With the instrument's acceptance angle of about 6° , a comparatively small area was viewed and consequently the signal was sensitive to changes in this small region of the sky. . . . With readings taken every 100 Å . . . a run could be completed in about 10 min, . . . Attempts to compensate for the variation of intensity with time by using a second monochromator set at a fixed wavelength throughout the scan gave some improvement but this could not be carried out regularly. It was decided that a program involving many measurements over an extended period would compensate for the random variations during any one run. With this direct-view arrangement it was desirable to make the measurements at times of apparently steady sky conditions. On a good day the variation over a period of 10 min would be within 3% of the mean value at any given wavelength, but often it was worse than this, up to 10% variation being common. . . . The majority of the observations were made on the laboratory roof and the white plate was used to give an average distribution for most of the hemisphere. The measurement procedure was as before, with some reduction in variability during a run on account of the integrating effect of the white plate."

The 99 spectral distributions of Ottawa daylight were obtained by Budde by measuring the light received at wavelength λ either from the total sky, or from the north sky, relative to that at wavelength 560 nm. This measurement relative to the irradiance at 560 nm eliminated fluctuations of the total irradiance during scanning of the spectrum. The apparatus consisted basically of an integrating sphere, a Hilger quartz prism double monochromator (D191) and a 1P28 RCA photomultiplier tube. The spectrum was scanned with a bandwidth ranging from 1 nm in the ultraviolet to approximately 7 nm in the red. The spectral range covered was from 300 to 720 nm. Simultaneous measurements of the same daylight by means of a Donaldson six-primary colorimeter¹⁷ were made by two observers as a closing check on the accuracy of the measurements of spectral distribution. The agreement in chromaticity coordinates, calculated from the visual and the spectroradiometric measurements, was better than 0.006 in x and y . The data supplied for analysis were those derived from averaging the original distribution curves over bands of 10 nm width.

The composite input data consisted of 622 spectral distributions E_λ (Condit 249, Henderson 274, Budde 99) weighted in inverse proportion to the residual variances $V(E_\lambda/E_{560})$ when the data from each source were separately analyzed and reconstituted with four vectors. The variance for each set of data was computed in the usual way as the sum of the squares of the differences between each input data and the corre-

¹⁷ R. Donaldson, Proc. Phys. Soc. (London) **59**, 554 (1947).

TABLE I. Mean and first four characteristic vectors for composite data on spectral irradiance of daylight.

Wavelength (nm)	Mean	V_1	V_2	V_3	V_4
330	553	420	85	91	12
340	573	406	78	78	21
350	618	416	67	66	19
360	615	380	53	52	19
370	688	424	61	43	11
380	634	385	30	4	-41
390	658	350	12	-2	-40
400	948	434	-11	-22	-32
410	1048	463	-5	-35	-28
420	1059	439	-7	-50	-28
430	968	371	-12	-51	-11
440	1139	367	-26	-50	2
450	1256	359	-29	-60	15
460	1255	326	-28	-63	18
470	1213	279	-26	-65	21
480	1213	243	-26	-59	20
490	1135	201	-18	-58	20
500	1131	162	-15	-47	23
510	1108	132	-13	-45	21
520	1065	86	-12	-33	22
530	1088	61	-10	-20	17
540	1053	42	-5	-15	13
550	1044	19	-3	-9	8
560	1000	0	0	0	0
570	960	-16	2	11	-7
580	951	-35	5	-10	-5
590	891	-35	21	-3	-7
600	905	-58	32	-5	-7
610	903	-72	41	-10	-7
620	884	-86	47	-20	-9
630	840	-95	51	-22	-8
640	851	-109	67	-36	-5
650	819	-107	73	-48	-3
660	826	-120	86	-55	-2
670	849	-140	98	-61	2
680	813	-136	102	-65	2
690	719	-120	83	-57	3
700	743	-133	96	-64	5

sponding value reconstituted from the mean and the first four characteristic vectors by Eq. (2) divided by the number of input data. On this basis, each of the monitored distributions supplied by Budde received full weight, the unmonitored distributions supplied by Condit and Henderson receiving, as expected, somewhat lower weights.

MEANS AND CHARACTERISTIC VECTORS

Table I gives, for the composite data, the means and the first four characteristic vectors. The analogous means and vectors derived from the three subsets of data (Condit, Henderson, Budde) for the same spectral range are available from the authors on request as are also means and characteristic vectors derived from the Henderson data for the range 330 to 780 nm.

It will be noted from Table I that, at the normalizing wavelength 560 nm, the mean of the spectral distributions is set at 1000, and the values of the characteristic vectors are all identically zero there. Reconstitutions of spectral distributions of daylight obtained by Eq. 2 will thus also have values of 1000 at the normalizing wavelength 560 nm.

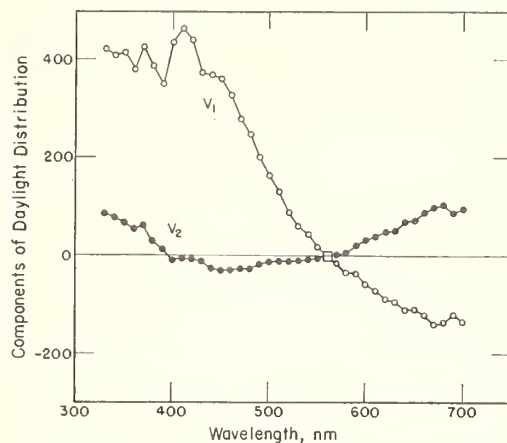


FIG. 1. Vectors V_1 (open circles) and V_2 (solid circles) derived from the composite data (622 spectral distributions).

It will also be noted that the values of the first characteristic vector are greater than zero for wavelengths less than 560 nm, and less than zero for wavelengths greater than 560 nm. The most common variation from one measured curve of daylight spectral distribution to another is thus such as to correspond chiefly to a change in correlated color temperature of the daylight. This yellow-to-blue variation corresponds to presence or absence of clouds in the sky, and to the inclusion or exclusion of direct sunlight.

Note also that the second vector has low values from 380 to 560 nm and higher values in the longwave (red) part of the visible spectrum. This second most common variation thus corresponds to a pink-green variation such as might be caused by the presence of little or much water in the form of vapor and haze.

Figure 1 is a plot of the first and second vectors V_1 and V_2 derived from the composite data. By adding various scalar multiples M_1 and M_2 of these vectors to the mean curve, it is evident that variations either in the yellow-blue sense or the pink-green sense may be introduced into the reconstituted curves. Thus, by adjustment of the scalar multiples M_1 and M_2 , reconstituted daylight distribution curves would be generated to correspond to any chromaticity within the daylight range; in particular, values of these scalar multiples exist which make the chromaticity point corresponding to the reconstituted curve fall at any desired point on a locus of typical daylight chromaticities. Any two linear combinations of characteristic vectors would similarly yield an unique distribution curve for each chromaticity point. It was found, however, that reconstituted curves satisfactorily reproducing the measured curves were generated by using only the first two vectors. The remainder of this paper describes the derivation of such a locus on the chromaticity diagram, the derivation of the corresponding scalar multiples for correlated color temperatures 4800°, 5500°, 6500°, 7500°, and 10 000°K, and demonstrations of the degree to which curves reconstituted in

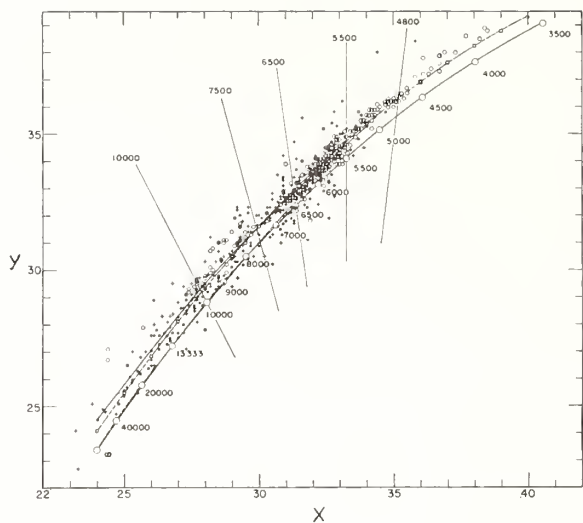


FIG. 2. Chromaticities of daylight compared to the locus of chromaticities implied by the Planck radiation law (open circles connected by solid lines). The temperatures of the complete radiator corresponding to the points at the centers of these open circles are indicated in degrees Kelvin. The straight lines intersecting this locus at 4800°, 5500°, 6500°, 7500°, and 10 000°K correspond to lines of constant correlated color temperature computed by Kelly from the 1960 CIE-UCS diagram. The center of the chromaticity range found for daylight by Nayatani and Wyszecki is shown by solid circles connected with a solid line; that found by Chamberlin, Lawrence, and Belbin, by a dotted line. Chromaticity points computed from the measured spectral distribution curves are indicated by open circles (Condit), by crosses (Henderson and Hodgkiss), and by solid circles (Budde). The locus of chromaticities taken in the present paper to be typical of daylight conforms to the relation: $y = 2.870x - 3.000x^2 - 0.275$, and is shown by squares connected by a dotted line.

accord with these scalar multiples may be considered reliable representatives of typical daylight spectral distributions.

CHROMATICITIES OF VARIOUS PHASES OF DAYLIGHT

To derive a curve of typical daylight chromaticities, use has been made of recent visual measurements by Chamberlin, Lawrence, and Belbin¹⁸ on the north sky in southern England, and by Nayatani and Wyszecki¹⁹ on a near-north sky at Ottawa, Canada, and of the chromaticities computed from each of the 622 spectral distributions subjected to characteristic vector analysis. Figure 2 is a central portion of (x,y)-chromaticity diagram of the 1931 CIE coordinate system. It shows as solid circles connected by a solid line the locus derived by Nayatani and Wyszecki, as a dotted line that derived by Chamberlin, Lawrence, and Belbin, as open circles the chromaticity points computed from the 249 spectral distributions obtained by Condit, as vertical crosses the chromaticity points computed from the 274 spectral distributions supplied by Henderson,

and as solid circles the chromaticity points computed from the 99 spectral distributions supplied by Budde. In general, the plotted points derived from the spectral distributions agree well with the two lines representing the visual observations, and they cluster closely about the line defined by: $y = 2.870x - 3.000x^2 - 0.275$, and shown as squares connected by a solid line on Fig. 2. The Nayatani-Wyszecki line diverges from the Chamberlin-Lawrence-Belbin line for correlated color temperatures greater than 10 000°K. Since there are too few chromaticity points computed from measured spectral distributions to decide between these two lines above 40 000°K, the adopted locus has been drawn about half-way between the two lines. This adopted locus below 40 000°K is drawn closer to the Nayatani-Wyszecki locus because the computed chromaticity points indicate that course. The adopted locus follows the Planckian locus (shown as open circles connected by a solid line on Fig. 2) fairly closely, but stays slightly on the green side. Figure 2 also shows lines of correlated color temperature computed by Kelly²⁰ from the 1960 CIE-UCS diagram for correlated color temperatures of 4800°, 5500°, 6500°, 7500°, and 10 000°K. Table II gives the chromaticity coordinates at which these lines of correlated color temperature intersect the locus adopted as typical of daylight chromaticities defined by $y = 2.870x - 3.000x^2 - 0.275$.

RELATION BETWEEN SCALAR MULTIPLES AND CHROMATICITY COORDINATES

Any spectral distribution curve E_λ reconstituted from a mean and two characteristic vectors derived from a family of measured spectral distributions is computed by Eq. (2) as:

$$E_\lambda = \bar{E}_\lambda + M_1 V_{1,\lambda} + M_2 V_{2,\lambda}, \quad (2a)$$

and the tristimulus value X of the irradiance specified by this distribution may be written:

$$X = \sum E_\lambda \bar{x}_\lambda \Delta\lambda = \sum (\bar{E}_\lambda + M_1 V_{1,\lambda} + M_2 V_{2,\lambda}) \bar{x}_\lambda \Delta\lambda,$$

where the weighting function \bar{x}_λ is one of the color matching functions of the 1931 CIE standard observer for colorimetry.¹ From the fact that the scalar multiples M_1 and M_2 are constants independent of wavelength for any single reconstituted curve, the expression for

TABLE II. Chromaticity coordinates (x,y) of typical daylight for various correlated color temperatures.

Correlated color temperature (°K)	Chromaticity coordinates	
	x	y
4800	0.3518	0.3634
5500	0.3324	0.3475
6500	0.3127	0.3291
7500	0.2991	0.3150
10 000	0.2787	0.2919

¹⁸ G. J. Chamberlin, A. Lawrence, and A. A. Belbin, *Light and Lighting* 56, 73 (1963).

¹⁹ Y. Nayatani and G. Wyszecki, *J. Opt. Soc. Am.* 53, 626 (1963).

²⁰ K. L. Kelly, *J. Opt. Soc. Am.* 53, 999 (1963).

TABLE III. Scalar multiples of the first two characteristic vectors of the composite data required to reconstitute spectral distribution curves of typical daylight of five correlated color temperatures.

Correlated color temperature (°K)	Scalar multiples of first characteristic vector	Second characteristic vector
4800	-1.140	0.677
5500	-0.784	-0.195
6500	-0.293	-0.689
7500	0.145	-0.752
10 000	1.005	-0.378

X may be written:

$$X = X_0 + M_1 X_1 + M_2 X_2, \quad (3)$$

where X_0 is the X -tristimulus value of the mean distribution, and X_1 and X_2 are the X -tristimulus values of the two characteristic vectors; for example, $X = \sum V_{1,\lambda} \bar{x}_\lambda \Delta\lambda$. By analogous argument for Y and Z we may write:

$$Y = Y_0 + M_1 Y_1 + M_2 Y_2, \quad (3)$$

$$Z = Z_0 + M_1 Z_1 + M_2 Z_2.$$

Note that values of X_0 , Y_0 , Z_0 are nowhere less than zero, but the tristimulus values of the characteristic vectors may be less than zero because the vectors themselves (see Table I and Fig. 1) are adjusted to zero at 560 nm with both positive and negative values for other parts of the spectrum.

The connection between scalar multiples M_1 and M_2 and chromaticity coordinates x , y follows directly from Eq. 3 and from definitions: $x = X/(X+Y+Z)$, $y = Y/(X+Y+Z)$. A convenient form of the connection, for $X+Y+Z$ abbreviated as S , is:

$$x = \frac{X_0/S_0 + M_1 X_1/S_0 + M_2 X_2/S_0}{1 + M_1 S_1/S_0 + M_2 S_2/S_0}, \quad (4)$$

$$y = \frac{Y_0/S_0 + M_1 Y_1/S_0 + M_2 Y_2/S_0}{1 + M_1 S_1/S_0 + M_2 S_2/S_0}.$$

To find the scalar multiples M_1 and M_2 required to yield a reconstituted spectral distribution of irradiance having any arbitrarily chosen values x and y of chromaticity coordinates, it is necessary to solve for M_1 and M_2 from Eq. 4. A convenient form of solution is:

$$M_1 = \frac{X_0 Y_2 - X_2 Y_0 + (Y_0 S_2 - Y_2 S_0)x + (X_2 S_0 - X_0 S_2)y}{X_2 Y_1 - X_1 Y_2 + (Y_2 S_1 - Y_1 S_2)x + (X_1 S_2 - X_2 S_1)y}, \quad (5)$$

$$M_2 = \frac{X_1 Y_0 - X_0 Y_1 + (Y_1 S_0 - Y_0 S_1)x + (X_0 S_1 - X_1 S_0)y}{X_2 Y_1 - X_1 Y_2 + (Y_2 S_1 - Y_1 S_2)x + (X_1 S_2 - X_2 S_1)y}.$$

The tristimulus values of the mean of the composite data and of their first two composite vectors have been

calculated and found to be as follows: $X_0 = 102434$, $Y_0 = 106769$, $Z_0 = 123630$; $X_1 = 1866$, $Y_1 = 1914$, $Z_1 = 34810$; $X_2 = 2133$, $Y_2 = 762$, $Z_2 = -2355$. Substitution of these tristimulus values into Eqs. 4 and 5 yielded both the explicit direct connection between the scalar multiples for the composite data and the chromaticity coordinates of the reconstituted spectral distributions of irradiance, and the reverse connection as follows:

$$x = \frac{0.30776 + 0.00561 M_1 + 0.00641 M_2}{1.00000 + 0.11594 M_1 + 0.00162 M_2}, \quad (4a)$$

$$y = \frac{0.32079 + 0.00575 M_1 + 0.00229 M_2}{1.00000 + 0.11594 M_1 + 0.00162 M_2},$$

$$M_1 = \frac{-1.3515 - 1.7703x + 5.9114y}{0.0241 + 0.2562x - 0.7341y}, \quad (5a)$$

$$M_2 = \frac{0.0300 - 31.4424x + 30.0717y}{0.0241 + 0.2562x - 0.7341y}.$$

SPECTRAL DISTRIBUTIONS OF DAYLIGHT FOR VARIOUS CORRELATED COLOR TEMPERATURES

To derive a series of related spectral distributions of typical daylight over a considerable range of correlated color temperatures from the composite data, values of scalar multiples M_1 and M_2 were computed by inserting into Eq. (5a) the values of chromaticity coordinates x and y given in Table II for typical daylight chromaticities of correlated color temperatures 4800°, 5500°, 6500°, 7500°, and 10 000°K. Table III shows

TABLE IV. Extension of the mean and the first two characteristic vectors of the composite data given in Table I to the spectral ranges 300 to 330 nm and 700 to 830 nm from Moon's compilation of the spectral absorbance of the earth's atmosphere due to ozone and water vapor.

Wavelength (nm)	Mean	V_1	V_2
300	0.4	0.2	0.0
310	60	45	20
320	296	224	40
330	553	420	85
700	743	-133	96
710	764	-129	85
720	633	-106	70
730	717	-116	76
740	770	-122	80
750	652	-102	67
760	477	-78	52
770	686	-112	74
780	650	-104	68
790	660	-106	70
800	610	-97	64
810	533	-83	55
820	589	-93	61
830	619	-98	65

TABLE V. Relative spectral irradiance of typical daylight reconstituted from mean and characteristic vectors of the composite data (Tables I and IV) by the scalar multiples of Table III.

Wavelength (nm)	Correlated color temperature (°K)				
	4800	5500	6500	7500	10 000
300	0.2	0.2	0.3	0.4	0.6
310	23	21	33	52	97
320	68	112	202	298	506
330	132	207	371	550	943
340	163	240	400	573	952
350	190	279	450	627	1011
360	218	307	467	630	977
370	246	344	522	703	1091
380	215	326	500	668	1010
390	267	382	547	700	1006
400	446	610	828	1019	1388
410	516	686	916	1119	1515
420	554	716	935	1128	1503
430	537	679	868	1033	1346
440	704	856	1049	1211	1518
450	827	981	1171	1330	1628
460	864	1004	1178	1323	1594
470	878	999	1149	1272	1503
480	916	1026	1159	1269	1469
490	894	980	1088	1177	1344
500	936	1007	1094	1165	1300
510	949	1008	1078	1137	1246
520	959	1000	1049	1086	1156
530	1011	1042	1077	1105	1153
540	1002	1021	1044	1063	1097
550	1020	1030	1040	1049	1064
560	1000	1000	1000	1000	1000
570	979	973	964	956	943
580	995	977	957	942	914
590	945	914	886	870	848
600	993	944	900	873	835
610	1012	951	896	862	816
620	1014	942	876	836	780
630	983	904	833	787	726
640	1020	923	837	785	716
650	990	889	800	748	683
660	1021	903	802	745	673
670	1075	940	822	755	671
680	1037	900	783	717	638
690	912	797	697	640	567
700	960	829	716	652	573
710	969	849	743	681	602
720	801	702	616	565	500
730	901	793	699	643	572
740	963	850	751	692	617
750	814	719	636	587	524
760	601	528	464	427	379
770	864	759	668	614	545
780	815	718	634	584	520
790	828	729	643	592	527
800	764	674	594	548	488
810	665	587	519	480	429
820	736	650	574	530	472
830	775	683	603	556	496

the values of the scalar multiples so obtained. Reconstituted spectral distributions were then obtained for the wavelength range 330 to 700 nm by applying these scalar multiples to the mean and first two character-

istic vectors of the composite data given in Table I in accord with Eq. (2a). To extend these spectral distribution curves to cover the wavelength ranges 300 to 330, and 700 to 830 nm, resort was had to Moon's¹² compilation of data on the spectral absorptance of the earth's atmosphere due to ozone and water vapor. Extensions of the mean and first two characteristic vectors of the composite data in this way are shown in Table IV. Table V shows the reconstituted spectral distributions of typical daylight over the extended range 300 to 830 nm, and Fig. 3 shows a plot of these distributions.

The spectral distributions of typical daylight were also evaluated in an analogous way from the means and first two characteristic vectors derived from the four subsets of data (Condit 249, Henderson 274, from 330 to 700 nm; Henderson 274, from 330 to 780 nm; and Budde 99) as a check on the distributions derived from the composite data comprising all 622 of the measured distributions. The agreement within the spectral range 400 to 700 nm was found to be very satisfactory, but the amount of the ultraviolet component (330 to 390 nm) indicated by the four subsets of data showed significantly different dependences on correlated color temperature. One might be tempted to ascribe these indicated different dependences of ultraviolet content on correlated color temperature to real differences between the atmospheric conditions at Rochester, Ottawa, and Enfield, were it not for the fact that the two Henderson subsets of data (330 to 700 nm and 330 to 780 nm, identical from 330 to 700 nm) also showed this different dependence. It is our view that ultraviolet content is poorly correlated with correlated color temperature, and that the amount of ultraviolet indicated by the composite data should be taken as typical.

In the opinion of the authors, the spectral distributions of irradiance produced by daylight at the earth's surface, shown in Fig. 3, are the most typical that can

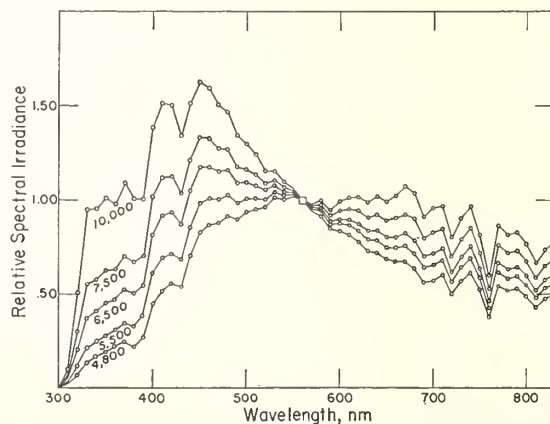


FIG. 3. Spectral distribution of typical daylight for correlated color temperatures: 4800°, 5500°, 6500°, 7500°, and 10 000°K reconstituted from the mean and the first two characteristic vectors of the composite data (622 measured distributions).

TABLE VI. Chromaticity coordinates (x, y) corresponding to the measured spectral distributions shown in Figs. 4-6.

Correlated color temperature (°K) (and Fig. number)	Spectral distribution	Chromaticity coordinates corresponding to the distribution	
		x	y
5500 (Fig. 4)	From Table V	0.3324	0.3475
	Condit 51	0.3318	0.3465
	Henderson 107	0.3324	0.3473
	Budde 84	0.3256	0.3334
6500 (Fig. 5)	From Table V	0.3127	0.3291
	Condit 120	0.3124	0.3285
	Henderson 67	0.3116	0.3280
	Budde 12	0.3134	0.3295
7500 (Fig. 6)	From Table V	0.2991	0.3150
	Condit 241	0.2982	0.3157
	Henderson 247	0.2990	0.3146
	Budde 44	0.2968	0.3100

be derived from the experimental data gathered to date. Although the initial experimental data employed in this analysis refer to different spectral bandwidths of 10 nm and less, the spectral distributions proposed here should be taken as the average of the true values of spectral irradiance over wavelength intervals of 10 nm throughout the spectrum relative to that for the interval 555 to 565 nm with its central wavelength at 560 nm. They are recommended as guides in the development of sources of artificial daylight that might be proposed as standard sources for colorimetry; in particular, the spectral distributions found for correlated color temperatures 5500°, 6500°, and 7500°K are proposed for the role of target curves to CIE Committee E-1.3.1 which has already indicated⁶ its intention to develop standard sources of these correlated color temperatures. Should phases of typical daylight of other correlated color temperatures between 4000° and 40 000°K be desired, derivation of other curves of this family can be accomplished by means of Eq. 5a by substitution of other chromaticity coordinates satisfying the relation: $y = 2.870x - 3.000x^2 - 0.275$. Reliable curves of relative spectral irradiance of the pinker or greener phases of daylight may also be derived by means of Eq. 5a provided that, for $0.25 \leq x \leq 0.38$, the value of the y -coordinate departs from the typical value by not more than 0.008. This range in y -coordinates includes about 90% of the 622 computed chromaticities.

COMPARISONS OF A SAMPLING OF THE MEASURED SPECTRAL DISTRIBUTIONS WITH RECONSTITUTIONS FROM THE MEAN AND FIRST TWO CHARACTERISTIC VECTORS OF THE COMPOSITE DATA

Measured curves were selected from each subset of data by taking those whose corresponding chromaticity points on the 1960 CIE-UCS diagram are closest to the chromaticity points given in Table II for correlated color temperature 5500°, 6500°, and 7500°K. Table VI compares these chromaticity coordinates with those

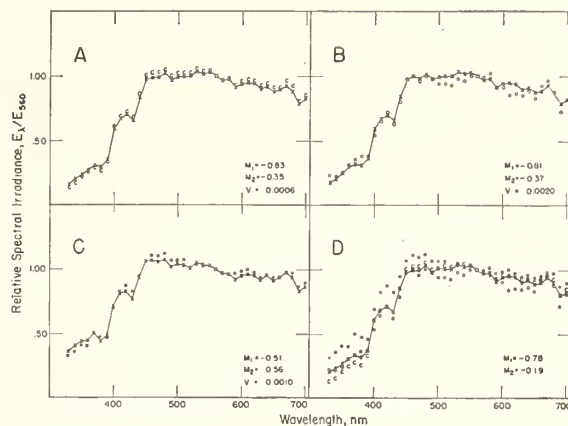


FIG. 4. Comparison of measured spectral distributions of daylight, whose chromaticities are near to that of typical daylight of correlated color temperature 5500°K, with distributions reconstituted from the mean and the first two characteristic vectors derived from the composite data (622 measured distributions). The measured distribution chosen from each subset of data is that whose chromaticity point is nearest on the 1960 CIE-UCS diagram to that for typical daylight of correlated color temperature 5500°K; see Table VI. Quadrants A, B, and C compare measured distributions (A—Condit 51; B—Henderson 107; C—Budde 84) with the reconstitution giving the best least-squares fit. The values of the scalar multiples (M_1 and M_2) so found are indicated together with the variance of the measured distribution from the reconstitution. Quadrant D compares all three of these measured distributions with the reconstituted distribution shown in Fig. 3 for 5500°K.

corresponding to the nine measured curves selected. Figures 4-6 compare the measured relative spectral irradiances with the corresponding curves reconstituted from the mean and first two characteristic vectors by applying the scalar multiples. These scalar multiples give the best least-squares fit and are shown on the plots. Figures 4-6 also compare (lower right quadrant) these measured distributions of spectral irradiance with the reconstituted curves of Fig. 3 and Table V for correlated color temperatures 5500°, 6500°, and 7500°K. The agreement is seen to be satisfactory. There is no indication that spectral distributions reconstituted from the mean and the first two characteristic vectors of the

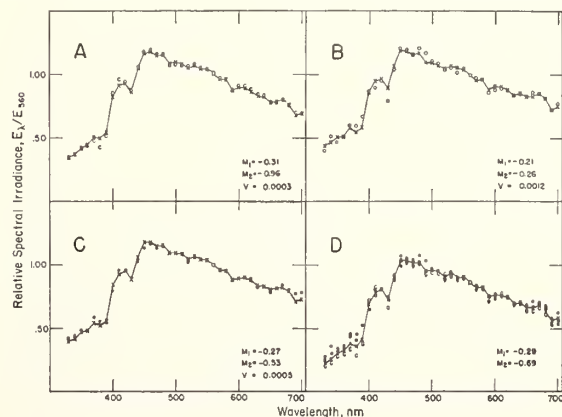


FIG. 5. Same as Fig. 4 except that the correlated color temperature is 6500°K. A—Condit 120; B—Henderson 67; C—Budde 12.

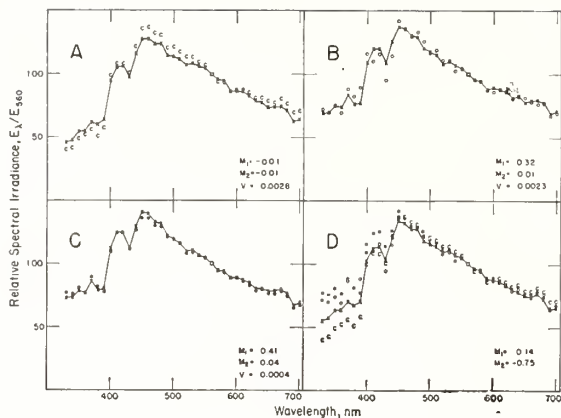


FIG. 6. Same as Fig. 4 except that the correlated color temperature is 7500°K. A—Condit 241; B—Henderson 247; C—Budde 44.

composite data fail importantly to agree with measured spectral distributions corresponding to the same chromaticity. The variance $V(E_\lambda/E_{560})$ for each curve is shown on the plot. The average variance for all nine curves is only 0.0011 which corresponds to a standard deviation of about 3 to 4 on the scale of 100 at the normalizing wavelength 560 nm. These curves show somewhat better than average agreement. The average variance of the 99 Budde curves is 0.0018; for the 249 Condit curves, 0.0020; and for the 274 Henderson curves, 0.0053. Perhaps a more striking indication of the degree to which the individual measured curves agree with the curves reconstituted from the mean and the first two characteristic vectors is to count the number of curves having variances no greater than that ($V=0.0028$) for Condit curve 241 shown on Fig. 6. Of the 99 Budde curves, 89 have variances no greater than 0.0028; of the 249 Condit curves, 222; and of the

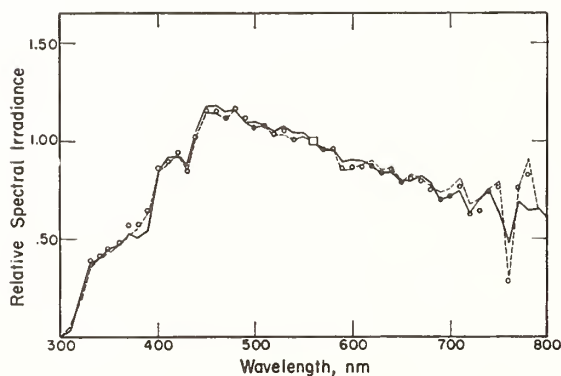


FIG. 7. Comparison of three spectral distributions intended to represent typical daylight at a correlated color temperature of 6500°K. The distribution reconstituted from the mean and first two characteristic vectors of the composite data is shown as a solid line; this distribution is also shown in Fig. 3. The distribution reconstituted from the Henderson data for 330 to 780 nm is shown by open circles, and the distribution derived for the British Standards Institution by taking an average of a selection of measured spectral distributions from the same data is shown by a dotted line.

274 Henderson curves, 113; so of the 622 curves comprising the composite data, 424 agree with the reconstituted curves more closely than the agreement shown on Fig. 6 for Condit curve 241. The finding that the Henderson measured curves agree less well with their reconstitutions than do the curves measured by Budde and Condit is to be expected from the longer time taken by Henderson to scan the spectrum; but this finding should not be construed to mean that averages of many Henderson curves corresponding to nearly the same chromaticity are in error. The assumption by Henderson and Hodgkiss¹⁴ "that a programme involving many measurements over an extended period would compensate for the random variations during any one run" seems to have been amply justified by the good agreement already noted between curves reconstituted from the Henderson subsets of data and the Condit and Budde subsets.

COMPARISON WITH SPECTRAL DISTRIBUTION CURVE DERIVED FOR THE BRITISH STANDARDS INSTITUTION FOR COLOR MATCHING AND APPRAISAL

A Draft British Standard Specification circulated 13 June 1963 for consideration as a revision of British Standard 950:1941 for artificial daylight for color matching and color appraisal includes a definition of the "spectral power distribution of artificial daylight." This distribution was derived by taking an average of a group of spectral distributions measured by Henderson and Hodgkiss.¹⁴ These measured spectral distributions all had chromaticities in the neighborhood of CIE source C but on the green side of the Planckian locus. The average curve corresponds to the chromaticity coordinates: $x=0.314$, $y=0.329$. Note that these chromaticity coordinates agree well with those derived by us ($x=0.313$, $y=0.329$; see Table VI) to characterize typical daylight of correlated color temperature 6500°K.

Figure 7 compares our recommendation (solid curve) for this correlated color temperature with the spectral distribution (dotted curve) derived for the British Standards Institution and with the spectral distribution (shown by open circles) derived from the mean and first two characteristic vectors of the Henderson 330–780 nm data. The agreement shown is very encouraging. It demonstrates that, for the spectral range 330 to 700 nm, average daylight agrees well with British daylight of the same correlated color temperature. The discrepancies between 700 and 780 nm may be at least partially ascribed to the smaller bandwidth (1.5 compared to 10 nm) used by Henderson and Hodgkiss, and to the presence of more water vapor in the Enfield atmosphere than in Rochester and Ottawa atmospheres. The good agreement affords a basis for the hope that a single set of related spectral distributions, such as those presented in this paper, may be found suitable for international use.

A Colorimeter for Pyrotechnic Smokes*

Isadore Nimeroff and Samuel W. Wilson

A tristimulus photoelectric colorimeter has been designed to measure the chromaticities of pyrotechnic smokes having highly saturated colors. To minimize edge effects and rear-wall reflection, the smoke chamber was designed as a 36-inch cube. Three filters were designed to approximate the daylight CIE standard observer functions for the source-filter-phototube combinations of the instrument. As the CIE \bar{y} - and \bar{z} -functions are unimodal, little difficulty was encountered in designing filters to approximate these functions. The CIE \bar{x} -function, which is bimodal, required a divided filter to approximate its two lobes. The filter constructed was composed of two sectors, one approximating the "blue lobe", the other approximating the "red lobe" of the \bar{x} -function. Satisfactory results have been obtained with this colorimeter, using Munsell papers as standards.

1. Introduction

In 1931 the International Commission on Illumination [1]¹ (CIE) adopted a system for reducing spectrophotometric data to color coordinates in three-space. The reduction of such data for a single specimen entails a considerable amount of computation. To avoid this computation, Twyman and Perry [2] suggested in 1930 that three filters duplicating the then proposed standard-observer functions could be used in a photoelectric instrument. Several such instruments, now called photoelectric colorimeters [3], have been designed since then. The photosensitive elements in colorimeters have been barrier photocells [4, 5], vacuum phototubes [6], and multiplier phototubes [7, 8].

This paper describes a colorimeter, using multiplier phototubes, designed at the request of representatives of the Army Chemical Center to be used for evaluating the colors of pyrotechnic smokes. This instrument was needed to determine progress in the development of saturated smoke colors and to check on reproduction of specified smoke colors.

2. Filter Design

The colorimetric spectral functions approximated in this colorimeter by combined source-filter-phototube response are those established by the CIE. The theory of the design of filters is based on two propositions (1) that the three CIE observer functions represent the colorimetric system of the average normal observer, and (2) that each of these functions can be approximated by the proper selection of optical filters in combination with a light source and a phototube. As the instrument is intended to evaluate the color of nonfluorescent smokes as if they were viewed in daylight (6,500° K), even though in the colorimeter they are illuminated by incandescent-lamp light (2,780° K), the proper correction was incorporated in the filter design. Had the colorimeter been intended to evaluate the color of fluores-

cent, as well as nonfluorescent, smokes as viewed under daylight, the approximation of the CIE function would be accomplished by phototube and filter combination alone, but the illuminator would then be required to yield artificial daylight.

The CIE spectral-response functions, \bar{x} , \bar{y} , and \bar{z} , under daylight spectral conditions, E_v , are respectively, $\bar{x}E_v$, $\bar{y}E_v$, and $\bar{z}E_v$. If the illuminator yields spectral distribution, E_i , the spectral responses, S_x , S_y , and S_z , of the phototubes for each of these functions must be altered by filters having spectral transmittances T_x , T_y , and T_z , respectively, in order to refer to viewing by daylight. This condition may be stated mathematically:

$$\left. \begin{aligned} E_i T_x S_x K_x &= \bar{x} E_v \\ E_i T_y S_y K_y &= \bar{y} E_v \\ E_i T_z S_z K_z &= \bar{z} E_v \end{aligned} \right\} \quad (1)$$

where K_x , K_y , and K_z are proportionality constants, independent of wavelength for their respective functions. The desired relative spectral transmittances then are determined by solving eq (1) for $K_x T_x$, $K_y T_y$, and $K_z T_z$.

The filter design is accomplished by determining the proper combination of colored glasses. Where the transmittance, T_0 , of a glass of stock thickness d_0 , is to be changed to a transmittance, T_1 , the following equation must be solved for the new thickness, d_1 :

$$T_1 = (T_0)^{d_1/d_0} (k^2)^{1 - (d_1/d_0)} \quad (2)$$

where k is unity minus the Fresnel reflectance of the glass. The approximate value of k for glass is 0.96. To minimize the work entailed in this computation a thickness-transmittance nomograph was used [9].

Figure 1 shows the spectral-response curves of several multiplier phototubes (RCA 5819) as calibrated by the NBS Radiometry Laboratory. The first phototube purchased had the response shown by curve A. Because of the relatively high response of this phototube in the region about 580 m μ the spectral transmittances of required filters for CIE functions must have absorption bands in this region.

*This project was done under contract with the Chemical Corps, Department of the Army.

¹ Figures in brackets indicate the literature references at the end of this paper.

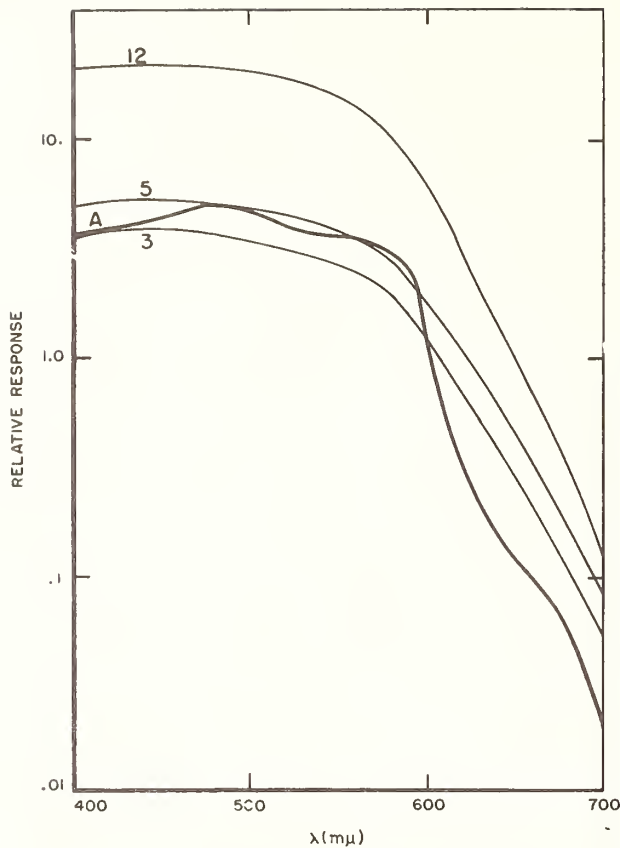


FIGURE 1. Spectral-response curves of several multiplier phototubes

To duplicate these complex transmittance curves the technique of divided-disk filters was devised. Fortunately, the phototubes purchased later had normal responses, as shown by curves 3, 5, and 12, of figure 1, thereby simplifying the filter-design problem. The phototubes having these curves are the ones used in the colorimeter for functions y , x , and z , respectively.

As no combination of glasses has ever achieved duplication of the bimodal \bar{x} -function in a single filter, the divided-disk technique devised for the complex transmittances required for phototube A was applied to the design of the bimodal \bar{x} -function. This division requires that one part of the single filter approximate the "blue lobe" and the other part approximate the "red lobe" of the \bar{x} -function. The transmittance of the ideal filter, T_x , for the \bar{x} -function may be divided into its two lobes, T_{xb} and T_{xr} , thus $T_x = T_{xb} + T_{xr}$. In terms of relative transmittance, this becomes

$$K_x T_x = K_x (T_{xb} + T_{xr}). \quad (3)$$

The actual approximation consists of a disk, area A , with one fraction, a/A , of the total area having spectral transmittance T'_{xb} , and the remaining fraction $1 - (a/A)$, having spectral transmittance

T'_{xr} . Then the transmittance, T'_x , of the actual filter is

$$T'_x = \frac{a}{A} T'_{xb} + \frac{A-a}{A} T'_{xr}. \quad (4)$$

The conditions that the spectral lobes of the approximate filter be properly proportional to those of the ideal filter are: The summation under each lobe of the approximation must equal the corresponding summation under the ideal filter, thus

$$\begin{aligned} \sum K_{xb} T'_{xb} &= \sum K_x T_{xb} \\ \sum K_{xr} T'_{xr} &= \sum K_x T_{xr}. \end{aligned} \quad (5)$$

The ratio of the summation under the two lobes in the approximate filter must equal the ratio of the summation under the two lobes of the ideal filter, thus

$$\frac{\sum \frac{a}{A} T'_{xb}}{\sum \frac{A-a}{A} T'_{xr}} = \frac{\sum T_{xb}}{\sum T_{xr}}. \quad (6)$$

But substituting the values of T'_{xb} and T'_{xr} from eq (5) into eq (6) gives

$$\frac{a}{A} = \frac{K_{xb}}{K_{xr} + K_{xb}}. \quad (7)$$

The area of the disk of radius r may be divided into either segments or sectors. Consider the segment division. The total area, A , is πr^2 , and the area of the segment, a , is $(r^2/2)(\theta - \sin \theta)$, where θ , in radians, is the central angle subtended by the segment, as shown in figure 2,a. Then

$$\frac{a}{A} = \frac{1}{2\pi} (\theta - \sin \theta). \quad (8)$$

After the value of θ is computed, the segment is determined by computing, $x = r \cos(\theta/2)$.

Although relatively easy to construct, a filter with this type of division of the disk has two major difficulties. The disk, once constructed, is useful only with a circular opening of radius r . As the segment is displaced from the center of the disk, one must assume that the spectral response of the photosensitive surface of the phototube is completely uniform.

Consider now the sector division of a disk filter. The area of the sector (see fig. 2,b) is equal to $r^2\theta/2$, where θ , the sector angle, is in radians. Then $a/A = \theta/2\pi$. If the sector angle, expressed in degrees, is ρ , then

$$\frac{a}{A} = \frac{\rho}{360^\circ}. \quad (9)$$

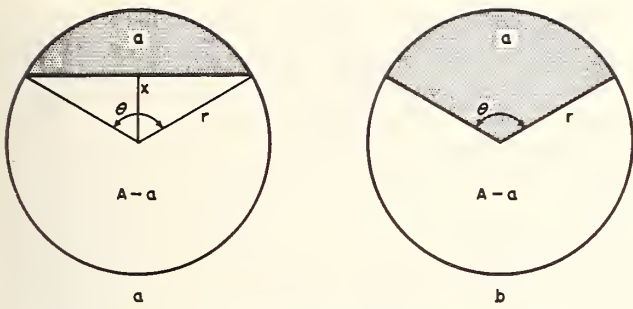


FIGURE 2. Divided disks for \bar{x} -function filter
a, Segment division; b, sector division.

This type of division, although more difficult to construct, does overcome the major difficulties of the segment division. The spectral character of the transmitted light is independent of the radius of the disk. One must assume only that the spectral response of the photosensitive surface of the phototube is radially symmetrical. This is a reasonable assumption for the head-on-type phototube used, as the photosensitive surface is evaporated onto the center of the cathode, thereby yielding a close approach to radially symmetrical sensitivity. This radial symmetry can readily be tested, furthermore, by rotating the sectored filter in its own plane and observing if the output of the phototube changes. For either type of division the center of the disk must be carefully positioned over the center of the photosensitive surface.

Because of the advantages outlined above, the sector division of the filter was used. On the basis of the application of eq (2) to the published spectral transmittance of stock glass, the combinations yielding the best expected approximation to the two lobes of the \bar{x} -function were chosen. The design data are

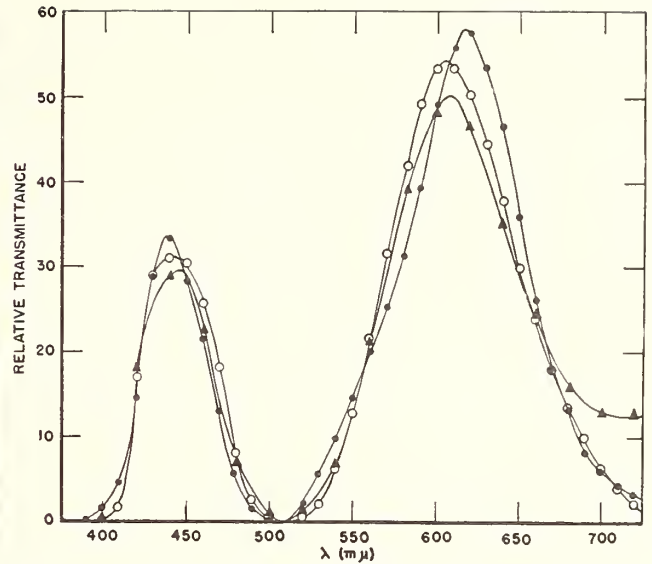


FIGURE 3. Spectral transmittance of the desired (●), expected (▲), and actual (○) filters for CIE \bar{x} -function.

shown in table 1. The sector angle was determined by substituting eq (7) into eq (9), giving

$$\rho = 360^\circ K_{xb} / (K_{xr} + K_{xb}). \quad (10)$$

for the \bar{x} -function filter designed for this colorimeter, $K_{xb} = 0.915$, $K_{xr} = 5.483$, and $\rho = 51.5^\circ$. The expected relative transmittance, $K_x''T_x$, for this filter is shown in figure 3 compared with the desired relative transmittance, $K_x'T_x$, and the achieved relative transmittance, $K_x'T_x'$, of the actual filter. The actual filter proved to be a better approximation to the desired filter than had been expected from the design data.

TABLE 1. x -function filter design data

λ	$K_x T_x$	$K_x T_{xb}$ (blue lobe)	$K_x T_{xr}$ (red lobe)	T_{xb}'' 5433 ^a (7.5 mm) 3389 ^a (1 mm)	T_{xr}'' 9788 ^a (3 mm) 3304 ^a (6 mm)	$T_{xb}'' K_{xb}$ $K_{xb} = 0.915$	$T_{xr}'' K_{xr}$ $K_{xr} = 5.483$	$K_x T_x'$
				%	%			
380	0.1	0.1	-----	0	-----	0	-----	0
400	1.6	1.6	-----	0	-----	0	-----	0
20	14.7	14.7	-----	19.8	-----	18.1	-----	18.1
40	33.4	33.4	-----	31.8	-----	29.1	-----	29.1
60	21.5	21.5	-----	24.6	-----	22.5	-----	22.5
80	5.8	5.8	-----	7.5	0	6.9	0	6.9
500	0.2	0.1	0.1	0.7	0.09	0.6	0.5	1.1
20	2.2	-----	2.2	0	.29	0	1.6	1.6
40	9.9	-----	9.9	-----	1.23	-----	6.7	6.7
60	20.1	-----	20.1	-----	3.86	-----	21.2	21.2
80	31.5	-----	31.5	-----	7.17	-----	39.3	39.3
600	49.2	-----	49.2	-----	8.85	-----	48.5	48.5
20	57.5	-----	57.5	-----	8.49	-----	46.6	46.6
40	46.6	-----	46.6	-----	6.43	-----	35.3	35.3
60	26.1	-----	26.1	-----	4.43	-----	24.3	24.3
80	13.0	-----	13.0	-----	2.92	-----	16.0	16.0
700	6.1	-----	6.1	-----	2.36	-----	12.9	12.9
20	3.5	-----	3.5	-----	2.36	-----	12.9	12.9
	343.0	77.2	265.8	84.4	48.48	77.2	265.8	343.0

^a Corning glass code numbers.

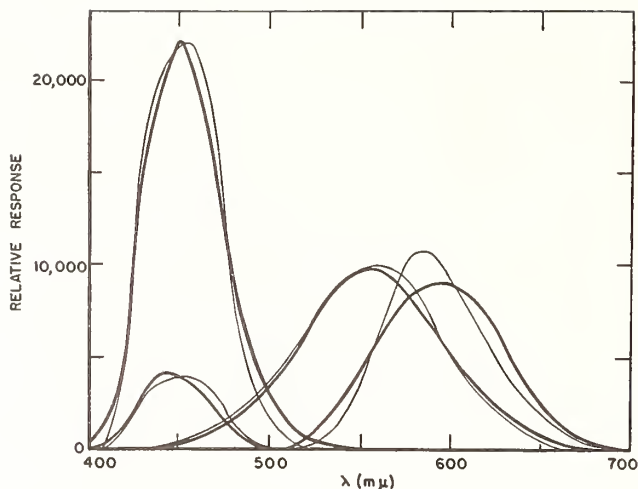


FIGURE 4. CIE tristimulus functions (heavy lines) and the approximate functions of the colorimeter.

As the CIE \bar{y} - and \bar{z} -functions are unimodal little difficulty was encountered in designing filters to approximate these functions. For the source-filter-phototube combinations of this colorimeter, the approximations (light lines) to the CIE tristimulus functions (heavy lines) are shown in figure 4.

3. Description of Apparatus

3.1. Chamber

Most of the colorimeters heretofore designed have been used for measurement of colors of opaque or transparent media. For these media, where edge effect is negligible, the size of the illuminated and viewed area of the medium was determined by the sensitivity of the instrument. The instrument here described is being used to measure the color of translucent media, pyrotechnic smokes. As the smokes are translucent, that is, light scattering as well as transmitting, and reflecting, a rather large chamber having a uniformly illuminated window is required to minimize edge effects.

Figure 5 shows a general view of the apparatus. The chamber is a 1-yd cube made of Dural. One side of the cube has a $\frac{1}{4}$ -in. plate-glass window 32 in. sq. To determine if the window was sufficiently large, circular openings of increasing size were placed over the window. The aperture effect decreased to zero when the opening was still well within the window size. To determine if reflection from the chamber walls would affect the results obtained when the chamber is filled with smoke, measurements were made on a smoke by using alternately a black and a white rear wall. As the wall reflection and window size had little effect upon the measurements, the conclusion was reached that the chamber had been made sufficiently large.

A blower is installed either to stir the smoke in order to obtain uniform distribution in the chamber or to exhaust the smoke after colorimetric measurements are made. A door serves as access to the

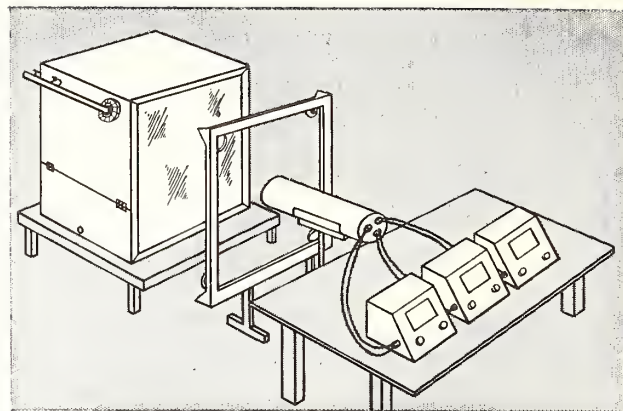


FIGURE 5. General view of apparatus, showing smoke chamber, illuminators, optical-assembly tube, and photometers.

chamber for purposes of placing a smoke grenade in the chamber and firing it. To facilitate entering the chamber in order to clean the window, an additional door was constructed on the side of the chamber (not shown in fig. 5).

3.2. Optical Arrangement

The smoke is illuminated by four 150-w incandescent floodlamps, mounted on a frame. The uniformity of illumination on the window was checked by means of a foot-candle meter.

The smoke is viewed perpendicularly through a lens that focuses upon a central region of the window. The lens is mounted at one end of a brass tube equipped with baffles to minimize stray light. The tristimulus filters are mounted in front of the phototubes. To achieve compactness of the viewing assembly, head-on-type phototubes were used.

3.3. Photometers

To measure accurately the chromaticity coordinates of saturated colors, a colorimeter is required to evaluate low reflectance in one spectral region and high reflectance in another spectral region with equal precision. As the colors of the pyrotechnic smokes are highly saturated and further development will improve their saturation, this colorimeter requires highly sensitive photometers and phototubes. To satisfy these requirements and for expediency, commercially available multiplier photometers and phototubes were used. The Photovolt Multiplier Photometer, model 520-M, known to be reasonably sensitive and stable, was chosen for use in the colorimeter. One minor adaptation of the photometers, required to permit the use of 5819 phototubes, instead of 1P21 phototubes for which the photometer was designed, was the replacement of the 10-prong socket with 9 dynode resistors by a 14-prong socket with 10 dynode resistors. To obtain direct readings of tristimulus values, X , Y , and Z , on the corresponding photometers, modification of the microammeter circuit of each photometer was anticipated, but

only the Z-scale photometer required an increase in meter sensitivity.

4. Results With Colorimeter

As the source-filter-phototube combination does not completely duplicate the CIE functions, best results are obtained by calibrating the colorimeter with standards having spectral selectivities similar to those of the smokes. But as no spectral data are available for any smokes, standards of Munsell charts having colors similar to those of the smokes investigated were chosen and gave satisfactory results. Table 2 shows the Munsell notations for three differ-

TABLE 2.

Smoke color	Munsell notations		Time
	Instrumental	Visual	
Red.....	{ 3R3.5/15 3.5R 3/13	5R4/10 4R4/10	t_0 t_{10}
Do.....	{ 4.5R3.5/14 5R 2/10	5P4/12 5P2/7	t_0 t_{10}
Yellow.....	{ 5Y6.5/10 5Y 4/6	5Y6/8 4.5Y5/6	t_0 t_{10}

ent smokes reduced from data obtained with the colorimeter at the time of firing (t_0) and within 10 min (t_{10}) of that time, compared with notations obtained simultaneously by visual estimates through a daylight filter. The instrumental colorimetric data were converted to Munsell book notation, so that comparisons could be made with the visual estimates obtained by comparison with the color scales in a Munsell book of color. The colorimeter yields hues and values in substantial agreement with visual estimates, but yields generally higher chromas than visually obtained estimates. The uncertainty in a visual estimate of hue and value is approximately one half-step. As the Munsell book does not contain chips with chromas as high as those of these smokes, the uncertainty in visual estimates of chroma is of the order of two steps. One possible reason for the

discrepancy between the instrumentally and visually obtained chromas is that the approximation to the \bar{z} -function is deficient in the region between 500 and 550 m μ , thereby yielding relatively low Z-values for yellow and red smokes.

Table 3 shows Munsell notations of three different smoke colors taken on smokes fired under outdoor daylight (clear, sunny day) compared with notations obtained by reduction of instrumental data for duplicate smokes. The satisfactory correspondence between the sets of data indicates that the smokes were not strongly fluorescent and that the CIE functions had been adequately approximated.

TABLE 3.

Smoke color	Munsell notations	
	Instrumental	Visual
Red.....	5R3/14	5P5/14
Yellow.....	7Y8/12	7.5Y7/10
Green.....	4G2/3	3G3/3

5. References

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Development of Filters for a Thermoelectric Colorimeter

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(May 21, 1963)

Filters have been designed for use with photocells and phototubes in tristimulus colorimeters, but because of the variation in spectral response of these detectors from one to the other, it has not been worthwhile to attempt to achieve the best possible duplication of the CIE tristimulus functions. In the case of a thermoelectric detector, however, the response is so nonselective that the same filter designs can be used with any thermopile.

The new filters permit CIE tristimulus functions and chromaticity coordinates to be obtained directly with reasonable accuracy. Results obtained on a thermoelectric colorimeter for five standard filters are compared with tristimulus values obtained by computation from spectral transmittance data.

1. Introduction

Experience has shown that the results obtained in visual color measurements by a single observer are likely to differ from those of some other observer. For this reason, a number of observers must be used in order to obtain representative results. Furthermore, an observer using the equality-of-brightness method of photometry will change his criterion for photometric balance over a period of several days; some observers exhibit variation during a single day. If the properties of the normal average eye (CIE standard observer) can be reproduced in a completely physical instrument, these problems can be avoided. However, it is to be emphasized that physical color measurements must be made with high precision if they are to compare favorably with visual judgments of color, especially of small color difference. A colorimeter comparable to the eye of the standard observer in discrimination sensitivity must be capable of measuring tristimulus values, X , Y , Z , to at least one part in a thousand. For many years, the Bureau has been examining glasses and chemicals for possible use in physical colorimeters. It is more desirable to design tristimulus filters for use with a nonselective detector rather than a selective detector because such filters are not dependent on the useful life of an individual detector nor limited to a particular one.

No physical detector has been constructed which has the same relative response curve as the average human eye for radiant energy of different wavelengths. It is possible, however, to make a nonselective receiver, which indicates the amount of received energy per unit time independent of its wavelength throughout the visible spectrum. If we interpose between the source and such a receiver filters which duplicate the CIE tristimulus functions, we can then measure the tristimulus values and by

computation obtain the chromaticity coordinates without reference to human observers. Such filters would permit CIE tristimulus functions and chromaticity coordinates to be obtained directly. The object of the present work was to determine if such filters could be realized closely enough to determine these quantities with reasonable accuracy.

Numerous instruments have been proposed and many constructed for direct color measurement over the years. Ives (1915) [1][†] described a thermopile tristimulus colorimeter in which he made use of a spectrum selectively projected through a series of carefully cut templates in place of the filters employed in the more recent devices. Twyman and Perry (1930) [2] suggested that three filters duplicating the then proposed standard observer could be used in a photoelectric instrument. Guild (1934) [3] described an experimental model of a photoelectric tristimulus colorimeter. A few years later Winch and Palmer (1937) [4], Dresler and Frühling (1938) [5], and Barnes (1939) [6] described devices for the same purpose. Gibson (1936) [7] and Van den Akker (1937) [8] have discussed the problem of obtaining source-filter-photocell combinations giving the closest equivalence to the standard observer. Hunter (1942) [9] described a three-filter method for photoelectric tristimulus colorimetry which was successfully applied to the Hunter multipurpose reflectometer [10] to measure surface color. Several new instruments have been designed in recent years. However, the photosensitive elements in these colorimeters have been barrier-layer photocells as in the instruments of Barnes (1939) [6] and Hunter (1942) [9], phototubes in the devices described by Glasser and Troy (1952) [11] and Hunter (1958) [12], and multiplier phototubes in the instruments designed by Sziklai (1951) [13], Bentley (1951) [14], and Nimeroff and Wilson (1954) [15]. Only in the instrument described by Ives (1915) [1] was a thermopile used.

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[†] Figures in brackets indicate the literature references at the end of this paper.

2. Filter Design

The only parts of a colorimeter over which the designer can exercise much control are the filters used to modify the detector response to obtain the \bar{x} , \bar{y} , and \bar{z} functions of the CIE standard observer. The \bar{x} function is bimodal and therefore two filters are used; one designated \bar{x} -short represents the \bar{x} function from 380 to 500 nm and the other, \bar{x} -long, the \bar{x} function from 510 to 760 nm. The components of these filters are usually absorbing solutions and glasses. Information on characteristics of various solutions and glass cells tested for photometric use is given in papers published by Ives and Kingsbury [16], [17], [18]. In this investigation, spectrophotometric curves of various solutions and glasses were made on a General Electric recording spectrophotometer. The data obtained on these filter components were used to compute the concentrations and thicknesses required to obtain a good fit for the spectral tristimulus values, \bar{x}_λ , \bar{y}_λ , \bar{z}_λ . Each of the filters designed contains as its components a solution, a cell, and two or more glasses.

2.1. Solutions

Chemicals used for preparation of solutions were labeled "analytical reagent quality" or "reagent quality." To prevent contamination from cork stoppers, glass-stoppered bottles were used as containers for all solutions.

The compositions of solutions used are as follows:

a. Solution I (\bar{x} -short and \bar{z})

Cobalt ammonium sulphate ($\text{CoSO}_4 \cdot (\text{NH}_4)_2\text{SO}_4 \cdot 6\text{H}_2\text{O}$)	13.0 g
Nickelous sulphate ($\text{NiSO}_4 \cdot 6\text{H}_2\text{O}$)	19.5 g
Glacial acetic acid	60 ml
Distilled water to make one liter of solution.	

b. Solution II (\bar{x} -long)

Copper sulphate ($\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$)	11.0 g
Ferric sulphate ($\text{Fe}_2(\text{SO}_4)_3 \cdot 9\text{H}_2\text{O}$)	11.5 g
Glacial acetic acid	60 ml
Distilled water to make one liter of solution.	

c. Solution III (\bar{y})

Potassium dichromate ($\text{K}_2\text{Cr}_2\text{O}_7$)	0.140 g
Copper sulphate ($\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$)	17.0 g
Cobalt ammonium sulphate ($\text{CoSO}_4 \cdot (\text{NH}_4)_2\text{SO}_4 \cdot 6\text{H}_2\text{O}$)	2.10 g
Sulphuric acid (1.835 sp gr)	10 ml
Distilled water to make one liter of solution.	

2.2. Cells

The filter accuracy obtained with solutions in cells depends in part on the spectral transmittance and uniformity of the cell. Cells having nonselective windows, closely parallel, and with close tolerances

on window spacing are commercially available. Three identical cells with 1-mm thick windows and 50-mm spacing were selected. Spectrophotometric measurements indicated that the windows were nonselective in transmittance from 380 to 780 nm except for variations less than 1.5 percent. The solutions I, II, and III prepared as described above were placed in cells I, II, and III, respectively.

2.3. Glasses

The specifications of the glasses which were selected are shown in table 1.

TABLE 1. *Glass components of the four tristimulus filters*

No.	Mfg.	Mfg. No.	Name	Color spec. No.	Thickness
					<i>mm</i>
1	Corning	5433	Heat resistant projector blue	5-59	4.4
2	Corning	5543	Heat resistant lantern blue	5-60	0.84
3	Corning	9788	Colorimeter blue green light	4-97	.615
4	Corning	3780	Uranium yellow	3-80	.67
5	Kopp		Heat absorbing	K-48	^a 5

^a(This thickness is about 5 mm as furnished by the manufacturer. This glass may no longer be obtainable from Kopp Glass Co., but it is similar to Corning filter No. 4600, color specification No. 1-69. The spectral transmittance of these heat-absorbing glasses from 560 to 780 nm is very similar to that of copper sulphate solution. Therefore, adjustment of concentration could be used to compensate for individual differences in spectral transmittance of the glass; however, glass samples obtained from three manufacturers were found to be sufficiently similar that no change in solution concentration was required.

2.4. Tristimulus

Solution-cells and the glasses listed in table 1 are combined to provide tristimulus filters as follows:

a. \bar{x} -short Filter

Solution-cell I and glasses numbered 1, 2, and 5

b. \bar{x} -long Filter

Solution-cell II and glasses numbered 3 and 5.

c. \bar{y} Filter

Solution-cell III and glasses numbered 4 and 5. This filter design is a slight modification of the luminosity filter design by Gibson, Teele, and Keegan (1939) [19]. The use of heat-absorbing glass necessitated a reduction in copper sulphate and potassium dichromate and an increase in cobalt ammonium sulphate.

d. \bar{z} Filter

Solution-cell I and glasses numbered 1 and 5.

3. Filter Characteristics

Spectral transmittance data for each of the four filters were obtained on a recording spectrophotometer [20], [21], [22]. Routine corrections for 100 percent, zero, and wave length scale were made [23] but additional corrections for inertia, slit width, and back reflectance were not made [24] since their total for these filters is less than the uncertainty of a single run on the spectrophotometer. The results

obtained are summarized in table 2. For each filter, the first column gives the spectral transmittance, the second column gives the relative transmittance adjusted to have the same area as the CIE curve, and the third column gives the CIE data. For ease of comparison the data from columns two and three for each filter are shown in figures 1, 2, 3, and 4. The transmittances less than 0.001 for the z colorimeter filter from 590 to 830 nm were found by remeasuring the three individual components of the filter on a wider range spectrophotometer after unexplained discrepancies turned up in the check of the performance of the colorimeter to be described presently.

For each colorimeter filter the sum of column three divided by the sum of column one gives the factor used to obtain the values in column two. These factors are also used to obtain the relative tristimulus values of any light source directly from the instrument readings, I .

$$\begin{aligned} X &= 2.560 & I_{\bar{x}\text{-short}} & + 5.596 & I_{\bar{x}\text{-long}}, \\ Y &= 2.033 & I_{\bar{y}}, & & \\ Z &= 12.166 & I_{\bar{z}}, & & \end{aligned}$$

Where the subscript for I corresponds with the particular colorimeter filter being used.

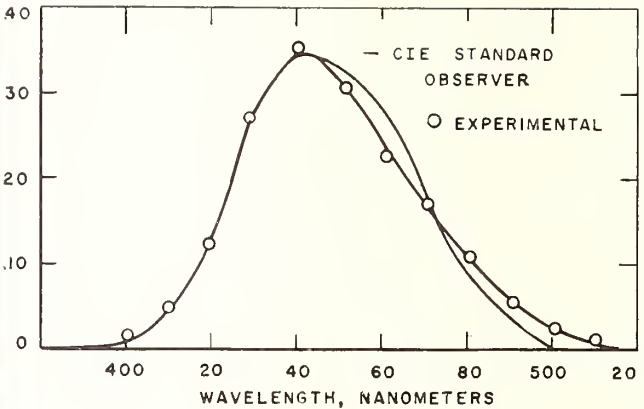


FIGURE 1. Adjusted transmittance of x-short filter compared to CIE tristimulus values \bar{x}_λ .

TABLE 2. Measured spectral transmittances, T , and adjusted spectral transmittances for each of the four filters compared to CIE spectral tristimulus values

Wave-length	\bar{x} -short			\bar{x} -long			\bar{y}			\bar{z}		
	T	$2.5605T$	CIE	T	$5.5963T$	CIE	T	$2.0327T$	CIE	T	$12.166T$	CIE
<i>nm</i>												
380	0.0005	0.0013	0.0014				0.0000	0.0000	0.0000	0.0006	0.0073	0.0065
390	.0028	.0072	.0042				.0007	.0014	.0001	.0034	.0414	.0201
400	.0063	.0161	.0143				.0007	.0014	.0004	.0074	.0900	.0679
410	.0173	.0443	.0435				.0007	.0014	.0012	.0199	.2421	.2074
420	.0528	.1352	.1344				.0011	.0022	.0040	.0608	.7397	.6456
430	.1079	.2763	.2839				.0030	.0061	.0116	.1242	1.5111	1.3856
440	.1399	.3582	.3483				.0092	.0187	.0230	.1623	1.9746	1.7471
450	.1196	.3062	.3362				.0187	.0380	.0380	.1414	1.7203	1.7721
460	.0900	.2304	.2908				.0297	.0604	.0600	.1094	1.3310	1.6692
470	.0679	.1739	.1954				.0523	.1063	.0910	.0872	1.0609	1.2876
480	.0443	.1134	.0956	0.0000	0.0000		.0731	.1486	.1390	.0629	0.7652	0.8130
490	.0244	.0625	.0320	.0016	.0090		.1020	.2073	.2080	.0394	.4793	.4652
500	.0113	.0289	.0049	.0046	.0257		.1530	.3110	.3230	.0209	.2543	.2720
510	.0048	.0123		.0077	.0431	0.0093	.2437	.4954	.5030	.0106	.1289	.1582
520	.0019	.0049		.0121	.0677	.0633	.3545	.7206	.7100	.0053	.0645	.0782
530	.0007	.0018		.0212	.1186	.1655	.4250	.8639	.8620	.0026	.0316	.0422
540	.0005	.0013		.0414	.2317	.2904	.4671	.9495	.9540	.0021	.0255	.0203
550	.0010	.0026		.0716	.4007	.4334	.4925	1.0011	.9950	.0036	.0438	.0087
560	.0018	.0046		.1080	.6044	.5945	.4938	1.0037	.9950	.0058	.0706	.0039
570	.0013	.0033		.1493	.8355	.7621	.4692	0.9537	.9520	.0049	.0596	.0021
580	.0001	.0003		.1772	.9917	.9163	.4263	.8665	.8700	.00113	.0138	.0017
590	.0000	.0000		.1894	1.0599	1.0263	.3716	.7554	.7570	.00021	.0026	.0011
600				.1840	1.0297	1.0622	.3077	.6255	.6310	.00013	.0016	.0008
610				.1656	0.9267	1.0026	.2443	.4966	.5030	.00010	.0012	.0003
620				.1374	.7689	0.8544	.1832	.3724	.3810	.00007	.0008	.0002
630				.1078	.6033	.6424	.1297	.2636	.2650	.00003	.0004	.0000
640				.0778	.4354	.4479	.0853	.1734	.1750	.00001	.0001	
650				.0531	.2972	.2835	.0544	.1106	.1070	.00001	.0001	
660				.0338	.1892	.1649	.0316	.0642	.0610	.00001	.0001	
670				.0212	.1186	.0874	.0171	.0348	.0320	.00002	.0003	
680				.0121	.0677	.0468	.0090	.0183	.0170	.00005	.0006	
690				.0067	.0375	.0227	.0043	.0087	.0042	.00012	.0015	
700				.0035	.0196	.0114	.0013	.0026	.0041	.00019	.0023	
710				.0019	.0106	.0058	.0006	.0012	.0021	.00016	.0020	
720				.0010	.0056	.0029	.0003	.0006	.0010	.00013	.0016	
730				.0001	.0006	.0014	.0001	.0002	.0005	.00010	.0012	
740				.0000	.0000	.0007	.0001	.0002	.0003	.00008	.0010	
750						.0003	.0000	.0000	.0001	.00007	.0009	
760						.0002	.0000	.0000	.0001	.00006	.0007	
770-830						.0001				.00020	.0025	
Sums.....	0.6971	1.7850	1.7849	1.5901	8.8986	8.8987	5.2569	10.6855	10.6857	0.8751	10.6770	10.6770

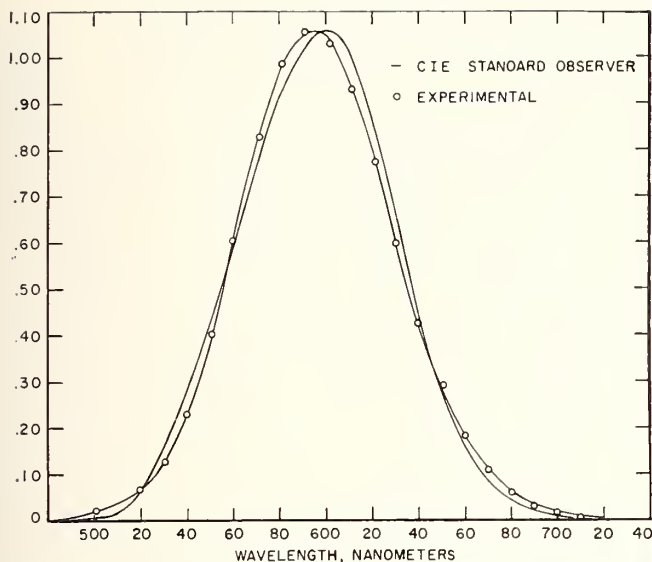


FIGURE 2. Adjusted transmittance of \bar{x} -long filter compared to CIE tristimulus values \bar{x}_λ .

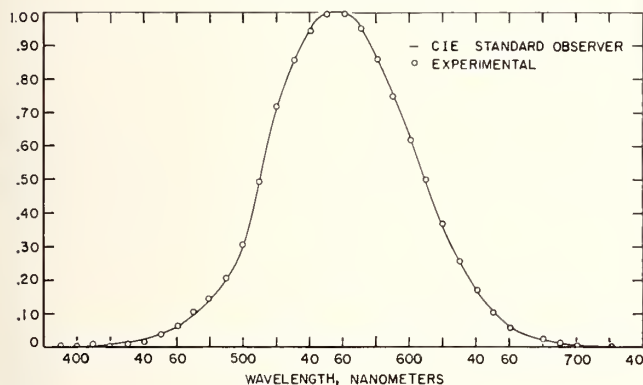


FIGURE 3. Adjusted transmittance of \bar{y} filter compared to CIE tristimulus values \bar{y}_λ .

4. Performance

To check the performance of these four colorimeter filters, use was made of the instrument described by Teele [24] which consists essentially of a thermopile, potentiometer, and recorder.

The first check was a measurement of the chromaticity coordinates of an incandescent lamp at a color temperature of 2,854 °K (CIE source A). The tristimulus values obtained by reducing these data by the factors given above were divided by their sum in the usual way to obtain values of chromaticity coordinates as follows: $x=0.457$, $y=0.403$, $z=0.140$. These chromaticity coordinates differ from those obtained by computation from the spectral distribution of a Planckian radiator at a temperature of 2,854 °K by $+0.009$, -0.004 , and -0.005 , respectively. These differences correspond to a combination of errors from possible failure of the voltage assigned to the lamp to operate it at a

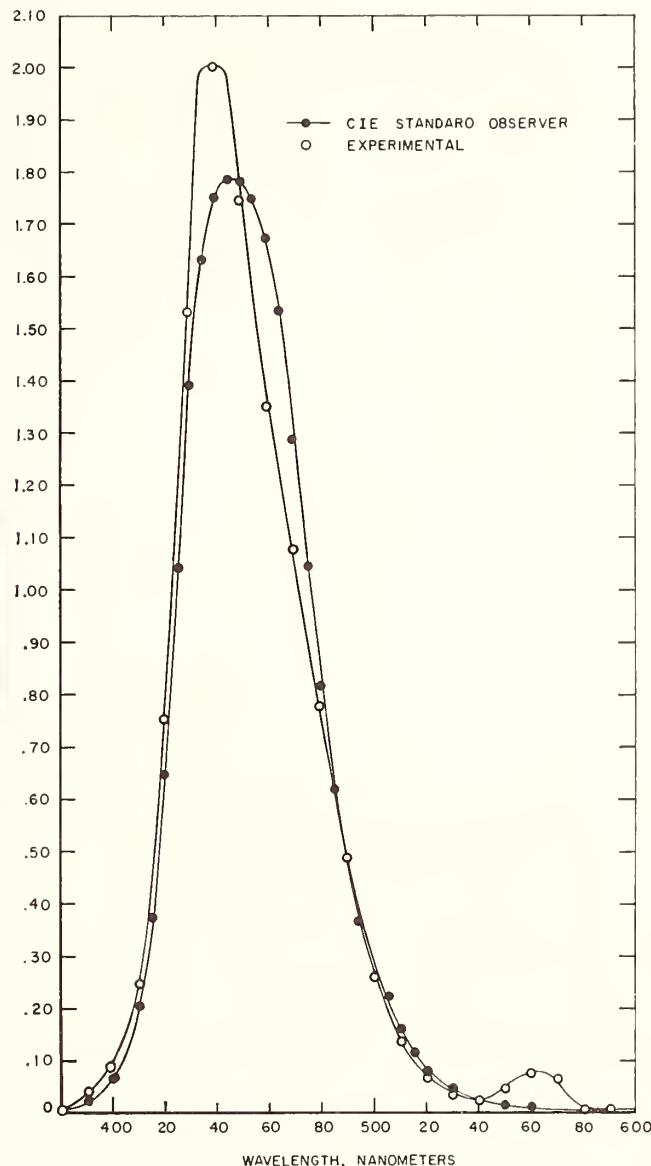


FIGURE 4. Adjusted transmittance of \bar{z} filter compared to CIE tristimulus values \bar{z}_λ .

color temperature of 2,854 °K, from failure of the lamp to yield spectral distributions characteristic of the Planckian radiator, from failure of the colorimeter filters to duplicate the CIE standard observer, and from the experimental errors of reading the thermopile.

A more detailed check of the performance of the colorimeter filters was obtained by measuring five NBS standard spectrophotometer-integrator filters, NBS 2101 through 2105, master set number 1. These are glass filters described as selenium reddish orange, carbon yellow, sextant green, cobalt blue, and selective neutral.

They were originally calibrated to check the performance of spectrophotometers equipped with tristimulus integrators. [25] The five filters were measured on two spectrophotometers (Cary 14 and

GE); appropriate corrections (such as wavelength, zero, 100%, slit width, inertia, and back reflectance) were applied; spectral transmittances were adopted as weighted means; and uncertainties were estimated. The spectral transmittances of these filters are plotted in figure 5.

To provide a more detailed test of the performance of the colorimeter filters use was made of a device already devised by Teele, which consists essentially of a thermopile (nonselective receiver for radiant energy), potentiometer, and recorder, the incandescent lamp was operated at a color temperature of 2,854 °K as before. To eliminate back reflection effects, a concave-convex lens was positioned before and another after the filter to be measured as shown in figure 6. The colorimeter filter, say \bar{x} -short, was then introduced between the light source and thermopile, and the reading was recorded. The filter to be measured was placed in the beam between the concave-convex lenses, and a second reading was recorded. This procedure was repeated with each of the four colorimeter filters. The ratios, source through filter to source alone, of such readings through the four colorimeter filters, \bar{x} -short, \bar{x} -long, \bar{y} , and \bar{z} , gives the values: $R_{\bar{x}\text{-short}}$, $R_{\bar{x}\text{-long}}$, $R_{\bar{y}}$ and $R_{\bar{z}}$ respectively, which may in themselves serve as a colorimetric specification of the filter to be measured relative to the reference source. The CIE tristimulus values of the test filter with source A, adjusted to make Y_A equal to the luminous transmittance of the filter in percent, are computed as follows:

$$X_A = 5.37 R_{\bar{x}\text{-short}} + 104.45 R_{\bar{x}\text{-long}},$$

$$Y_A = 100.00 R_{\bar{y}},$$

$$Z_A = 35.55 R_{\bar{z}},$$

where the coefficients 5.37, 104.45, 100.00, and 35.55 are related to the source-A tristimulus values. The coefficients of $R_{\bar{x}}$ and $R_{\bar{z}}$ are the Y and Z tristimulus values for source A, respectively. The coefficients of $R_{\bar{x}\text{-short}}$ and $R_{\bar{x}\text{-long}}$ are the summations of the spectral tristimulus values for the \bar{x} function and source A from 380 to 500 nm and from 510 to 770 nm, respectively.

The chromaticity coordinates, x , y , z , of the test filters are calculated from the tristimulus values, X , Y , Z , in the usual manner:

$$x = X/(X+Y+Z), y = Y/(X+Y+Z), z = Z/(X+Y+Z)$$

Table 3 compares the measured values of the five standard filters with their assigned values. Each measured value was computed from 10 independent determinations of the ratio R . The uncertainties of the assigned values shown in the table are those estimated by Keegan, Schleter, and Judd as the ranges of the values found by three different spectrophotometers [25]; see their table 11. The uncertainties shown for the measured values are based upon computations of huge error (4.9 times the probable error) of the mean of the ten settings of the ratios R . These computations showed that the huge error (uncer-

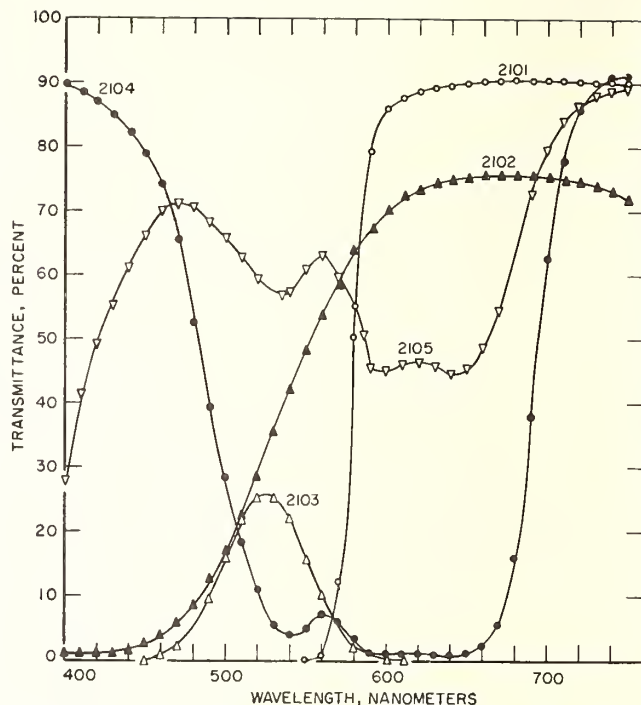


FIGURE 5. Spectral transmittances of the five test filters.

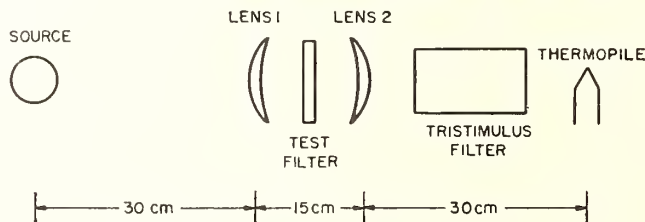


FIGURE 6. Diagram of the optical arrangement.

tainty at the one-tenth of one percent level) of values of the ratio R was on the average equal to ± 0.0025 , and these estimates of uncertainty are given directly for the values of Y in table 3. Propagation of these errors to chromaticity coordinates, x , y , z , leads to the estimates of uncertainties entered in table 3.

It will be noted from table 3 that the sum of the uncertainties listed fails in 11 cases out of 20 to exceed the difference between the measured and assigned values. Of course, since the colorimeter filters do not succeed precisely in duplicating the distribution functions of the 1931 CIE standard observer (see table 2), some discrepancy between measured and assigned values is to be expected. Table 4 gives in column 2 the differences ascribable to failure of the spectral transmittances of the colorimeter filters to have precisely the wavelength variation striven for. In column 3 are given the observed differences copied from the last column of table 3, and in column 4 are given the sums of the uncertainties listed in columns 2 and 3 of table 3. If the expected difference departs from the observed difference by less than the combined uncertainty, we may say that a reasonable basis for explaining the discrepancy has been found. Column 5 gives the amounts of the unexplained discrepancies. It is computed as the excess of the abso-

lute value of the difference between columns 2 and 3 over the sum of the uncertainties in column 4. If there is no excess, no entry is inserted.

TABLE 3. Comparison of measured and assigned values of chromaticity coordinates, x , y , z , and luminous transmittance, Y , for CIE source A for the five NBS standard filters

Filter No. and coordinate	CIE colorimetric coordinates for standard filters with source A		Difference
	Measured	Assigned	
2101			
x	0.6364 ± 0.0018	0.6488 ± 0.0009	-0.0124
y	$.3577 \pm .0018$	$.3509 \pm .0009$	$+.0068$
z	$.0059 \pm .0018$	$.0003 \pm .0000$	$+.0056$
Y	$38.0 \pm .25$	$37.6 \pm .5$	$+.4$
2102			
x	0.5443 ± 0.0015	0.5497 ± 0.0002	-0.0054
y	$.4298 \pm .0014$	$.4324 \pm .0003$	$-.0026$
z	$.0260 \pm .0007$	$.0179 \pm .0001$	$+.0081$
Y	$55.1 \pm .25$	$55.3 \pm .02$	$-.2$
2103			
x	0.253 ± 0.016	0.236 ± 0.005	$+0.017$
y	$.654 \pm .015$	$.668 \pm .004$	$-.014$
z	$.093 \pm .003$	$.096 \pm .004$	$-.003$
Y	$8.56 \pm .25$	$9.07 \pm .13$	$-.51$
2104			
x	0.1992 ± 0.0060	0.1871 ± 0.0019	$+0.0121$
y	$.1609 \pm .0059$	$.1630 \pm .0034$	$-.0021$
z	$.6399 \pm .0064$	$.6499 \pm .0057$	$-.0100$
Y	$5.8 \pm .25$	$5.9 \pm .23$	$-.1$
2105			
x	0.4211 ± 0.0014	0.4163 ± 0.0019	$+0.0048$
y	$.4076 \pm .0014$	$.4087 \pm .0020$	$-.0011$
z	$.1712 \pm .0005$	$.1750 \pm .0016$	$-.0038$
Y	$53.3 \pm .25$	$53.9 \pm .2$	$-.6$

TABLE 4. Analysis of the discrepancies between the differences listed in table 3 and those to be expected from the actual colorimeter filters used.

Filter No. and coordinate	Differences to be expected between measured and assigned values	Differences actually found (taken from last column of table 3)	Sum of the uncertainties in the assigned and measured values (from columns 2 and 3 of table 3)	Unexplained discrepancies. (If the absolute value of the difference between columns 2 and 3 exceeds the sum of the uncertainties in column 4, the amount of this excess is recorded below.)
(1)	(2)	(3)	(4)	(5)
2101				
x	-0.0023	-0.0124	0.0027	0.0074
y	$-.0015$	$+.0068$	$.0027$	$.0056$
z	$+.0038$	$+.0056$	$.0008$	$.0010$
Y	$-.2$	$+.4$	$.75$	
2102				
x	-0.0032	-0.0054	0.0017	0.0005
y	$-.0042$	$-.0026$	$.0017$	
z	$+.0073$	$+.0081$	$.0008$	
Y	$-.1$	$-.2$	$.27$	
2103				
x	-0.0009	$+0.017$	0.021	
y	$-.0010$	$-.014$	$.019$	
z	$+.0020$	$-.003$	$.007$	
Y	$+.02$	$-.51$	$.38$	0.15
2104				
x	$+0.0096$	$+0.0121$	0.0079	
y	$+.0003$	$-.0021$	$.0093$	
z	$+.0099$	$-.0100$	$.0121$	
Y	$+.01$	$-.1$	$.48$	
2105				
x	$+0.0025$	$+0.0048$	0.0033	
y	$-.0017$	$-.0011$	$.0034$	
z	$-.0007$	$-.0038$	$.0021$	0.0010
Y	$.0$	$-.6$	$.45$	$.15$

Table 4 shows that nearly all discrepancies are accounted for by the imperfection of the fit of the

four filters to the respective CIE curves and that improvement in performance depends upon finding better constituents for the solutions and different glasses (or different thicknesses of the same glasses that are now used).

It will be noted from table 4 that for four of the five filters the unexplained discrepancies (column 5) are few and small. For filter 2101, however, the discrepancies in chromaticity coordinates are large. Perhaps it is significant that this filter (selenium reddish orange) is the only one of the five which has a large temperature coefficient of spectral transmittance.

By comparing the uncertainties in the measured values (column 3, table 3) ascribable to the uncertainty of 2.5 on a scale of a thousand in the measured ratios with the differences (column 2, table 4) to be expected because the colorimeter filters do not have precisely the ideal spectral character striven for, it may be noted that those two sources of error are about equally important. The performance of the colorimeter could be significantly improved only by developing better \bar{x} and \bar{z} colorimeter filters and at the same time reducing the noise from the thermopile and amplifier. Significant improvement of the colorimeter filters might be accomplished by combining the components in parallel as well as in series as was originally suggested by Dresler [26], and used among others by Nimeroff and Wilson [15], Frühling and Krempel [27], Geutler [28], and Davies and Wyszecki [29].

The authors express their appreciation to the members of the staff who ran the numerous spectrophotometric curves required to develop the filters.

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(Paper 67C4-143)

Mechanized Conversion of Colorimetric Data to Munsell Renotations

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A program for a high-speed digital electronic computer is described for performing the computation of the Munsell renotations H , V , C corresponding to given CIE chromaticity coordinates x , y and daylight reflectance Y . Mathematically, this is equivalent to a three-dimensional coordinate transformation where two of the three transformation functions are given only numerically for a grid of discrete points. Since this grid consists of approximately 5000 points which are nonuniformly spaced, the major problem was to devise an economic scanning routine in order to find the point used in the interpolation. This was accomplished by consistent use of vector algebra and the help of an interpretive routine for vector operations.

1. INTRODUCTION

FOR the past two years, the Computation Laboratory and the Photometry and Colorimetry Section of the National Bureau of Standards have been engaged jointly in the development of methods for the computation of spectrophotometric and colorimetric data on automatic electronic computers. During this time, computer programs have been written for (1) the calculation of the CIE chromaticity coordinates from given spectrophotometric data, and (2) the conversion of the chromaticity coordinates and daylight reflectance (or transmittance) for CIE source C into terms of the Munsell renotation system.

The first of these programs, while of little interest mathematically, represents—as far as we know—one of the most extensive programs of its kind for the standardization of color from spectrophotometric data with the help of an electronic computer.

In the following we will present an account of the mathematical and programming details of the second program. To our knowledge this program represents the first routine ever written for an electronic computer which performs the conversion of the CIE specifications of a given color sample into terms of the Munsell renotation system.

2. STATEMENT OF THE PROBLEM

In their report,¹ Newhall *et al.* present the modified and enlarged Munsell color solid in terms of the CIE standard observer and coordinate system for CIE source C (representative of average north-sky daylight). The charts and tables of that report constitute the definition of the new Munsell renotation system. Together with these data and charts a graphical method is outlined that enables one to find the Munsell renotation, that is Munsell hue H , Munsell value V , and Munsell chroma C from the given CIE-specification of a color sample in terms of chromaticity coordinates (x, y) and daylight reflectance (or transmittance) Y for CIE source C .

Mathematically, the connection between H , V , C and x , y , Y is a three-dimensional coordinate transformation of the form

$$\begin{aligned} Y &= f_1(V) \\ x &= f_2(H, V, C) \\ y &= f_3(H, V, C). \end{aligned} \quad (1)$$

The functions f_1 , f_2 , f_3 were obtained numerically by experimental comparison of spectrophotometric measurements with estimates of color spacing by human observers.² For the function f_1 the following polynomial approximation was suggested and adopted by the OSA Subcommittee on the spacing of Munsell colors¹:

$$Y = f_1(V) = 1.2219 \times 10^{-2} V - 2.3111 \times 10^{-3} V^2 + 2.3951 \times 10^{-3} V^3 - 2.1009 \times 10^{-4} V^4 + 8.404 \times 10^{-6} V^5. \quad (2)$$

No useful mathematical formulas are known that provide a sufficiently good approximation of the function f_2 and f_3 . Hence, these two functions are known only by the points of a three-dimensional grid in the (H, V, C) -space. These grid points are spaced according to the following specifications:

$$\begin{aligned} V &= 0.2(0.2)1; \quad 1(1)10 \\ C &= 0(2)C_{\max} \quad (2 \leq C_{\max} \leq 50) \\ H &= 0(2.5)97.5 \end{aligned} \quad (3)$$

Altogether, this grid consists of 4996 points. The values of the functions f_2 , f_3 at 3089 of these points are to be found in Newhall *et al.*¹ and the paper³ by Judd and Wyszecki. The values at the remaining 1907 points were obtained by numerical and graphical extrapolation of the data of Newhall *et al.*¹ and Judd and Wyszecki.³ This extension is described in the paper⁴ by Schleter *et al.* The reasons for this extension will be explained later.

² Proceedings, Eighth Session, Commission Internationale de l'Eclairage, Cambridge, England, September, 1931, pp. 19–29.

³ D. B. Judd and G. Wyszecki, *J. Opt. Soc. Am.* **46**, 281 (1956).

⁴ J. C. Schleter, D. B. Judd, and H. J. Keegan, *J. Opt. Soc. Am.* **48**, 863(A) (1958).

⁵ The vector notation of boldface letters in the text corresponds to the notation of letters with superior arrows in the figures.

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¹ S. M. Newhall, D. Nickerson, and D. B. Judd, *J. Opt. Soc. Am.* **33**, 385 (1943).

One possible way of finding the Munsell renotation H , V , C corresponding to specific x , y , V would be to determine once and for all the inverse of the transformation (1).

This inverse transformation has a form similar to that of (1):

$$\begin{aligned} V &= g_1(Y) \\ H &= g_2(x, y, V) \\ C &= g_3(x, y, V). \end{aligned} \quad (4)$$

Starting from what is known about f_1 , f_2 , f_3 , the functions g_2 and g_3 will only be known numerically by the points of a discrete grid in the (x, y, V) -space, while $g_1(Y)$ assumes the form of a power series in Y . At this point it would then be best to approximate the functions g_2 and g_3 in some suitable way—possibly by polynomials—and to replace also the power series of g_1 by a polynomial, e.g., with the help of an economization procedure. Once this is accomplished one can find H , V , C corresponding to given x , y , V by simply computing the values of three analytically given functions.

Such a method of solving this problem—though elegant from a purely mathematical point of view—is not very desirable from a numerical standpoint. The data defining the functions f_2 and f_3 were obtained experimentally and are known only with an accuracy of at most four decimal digits. Hence, the inverse system (4) obtained from these data after the lengthy computations just described could scarcely be guaranteed to give results compatible with the original experimental data. In other words, this inverse system (4) would constitute the creation of a second Munsell renotation system which then would have to be checked out and confirmed experimentally.

Such a procedure was not desired and we therefore proceeded to use in our computation of H , V , C for given x , y , V on an electronic computer a method that is similar to the graphical method used for normal hand computation. This method was established—as we mentioned previously—by the OSA Subcommittee on the spacing of Munsell colors,¹ and is essentially a three-dimensional interpolation procedure which—when accomplished graphically—does not present any major difficulties. By using an automatic computer, however, this interpolation becomes considerably involved due to the large number of grid points to be handled.

FIG. 1. Munsell value (solid line) as a function of reflectance, approximating curve $V = V^2$ (dashed line).

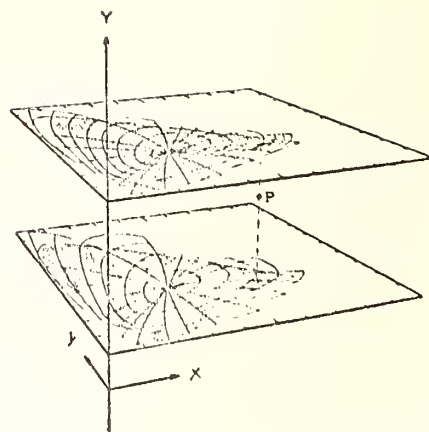
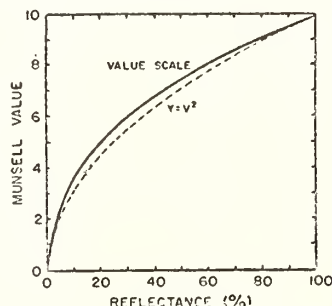


FIG. 2. The point $P(x, y, V)$ situated between two data planes.

3. EVALUATION OF MUNSELL VALUE

We will assume in the following that the CIE specifications \bar{x} , \bar{y} , \bar{Y} of a specific color sample are given.

The special form of the transformation (1) permits the evaluation of V independent of that of H and C . This evaluation of V is equivalent with finding a specific root $V = \bar{V}$ of the fifth-degree polynomial (2) when Y is given. As can be seen from Fig. 1 the polynomial $f_1(V) - Y = 0$ possesses for every Y with $0 \leq Y \leq 102.56$ exactly one root V with $0 \leq V \leq 10$. This root is to be computed.

The convenient behavior of the curve $Y = f_1(V)$ in $0 \leq V \leq 10$ suggests the straightforward application of Newton-Raphson's method.

Thus, starting with a suitable initial value V_0 we employ the iteration scheme

$$V_{n+1} = V_n - \{[f_1(V_n) - Y]/f_1'(V_n)\}, \quad (5)$$

where

$$\begin{aligned} f_1'(V) &= 1.2219 \times 10^{-2} - 4.6222 \times 10^{-3}V + 7.0853 \\ &\quad \times 10^{-3}V^2 - 8.4036 \times 10^{-4}V^3 + 4.202 \times 10^{-5}V^4 \end{aligned} \quad (6)$$

denotes the derivative of $f_1(V)$. It is well known and can also be seen from Fig. 1 that the function $Y = f_1(V)$ can be approximated by $Y = (1/100)V^2$. This suggests the initial approximation

$$V_0 = 10\sqrt{Y}. \quad (7)$$

The iteration (5) and (7) has proved to be very satisfactorily convergent.

4. EVALUATION OF MUNSELL HUE AND CHROMA

After the Munsell value $V = \bar{V}$ of the given color sample \bar{x} , \bar{y} , \bar{Y} has been found, we can determine those values V_1 and V_2 among the data (3), that is, among the numbers

$$0.2, 0.4, \dots, 1.0, 2.0, \dots, 9.0, 10.0 \quad (8)$$

which fulfill the inequality $V_1 \leq \bar{V} \leq V_2$ and which are closest to \bar{V} .

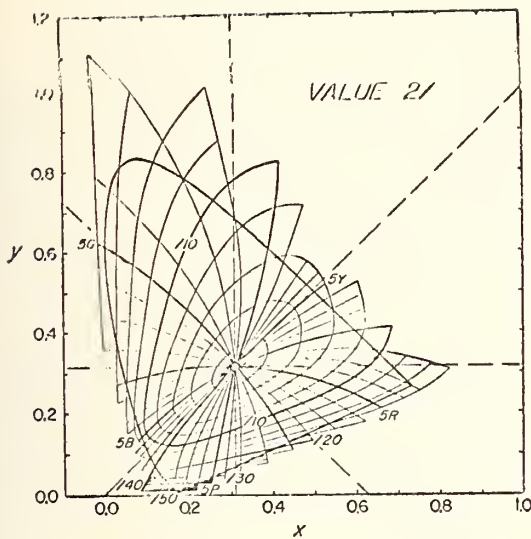


FIG. 3. Example (Munsell value 2/) of the division of the CIE chromaticity diagram into 8 sectors.

Then we have a geometrical configuration as shown in Fig. 2. The given point $P: (\bar{x}, \bar{y}, \bar{Y})$ in the (x, y, Y) -space is situated between the two data planes $Y_1 = f_1(V_1)$ and $Y_2 = f_2(V_2)$ which are parallel to the (x, y) plane.

We project the point P onto these value levels $Y = Y_1$ and $Y = Y_2$, the direction of projection being parallel to the Y axis. Let the footpoints of P which we obtain in the two value planes under this projection be denoted by P_1 and P_2 , respectively.

Assume that the Munsell rennotations (H_1, V_1, C_1) and (H_2, V_2, C_2) of the two footpoints P_1 and P_2 , respectively, are known. Then one can find the Munsell rennotations $(\bar{H}, \bar{V}, \bar{C})$ of P by interpolation:

$$\begin{aligned} \bar{H} &= H_1 + \frac{\bar{V} - V_1}{V_2 - V_1} (H_2 - H_1); \\ \bar{C} &= C_1 + \frac{\bar{V} - V_1}{V_2 - V_1} (C_2 - C_1). \end{aligned} \quad (9)$$

The use of linear interpolation was already established in the original report¹ of the OSA subcommittee on the spacing of Munsell colors. Linear interpolation is also mathematically well justified, since the data changes between the value levels are sufficiently small and since, furthermore, the data are only given with an accuracy of at most 3 to 4 decimal places.

Thus our problem reduces to that of finding the Munsell rennotations (H_1, C_1) and (H_2, C_2) of the footpoints P_1 and P_2 , respectively. Mathematically, this is equivalent to solving the two systems of equations

$$\begin{aligned} \bar{x} &= f_2(H, V_n, C) \\ \bar{y} &= f_3(H, V_n, C) \end{aligned} \quad (n=1, 2), \quad (10)$$

where \bar{x}, \bar{y} are given.

Figure 3 shows—for the case $V_n=2.0$ as a typical example—the location of the grid points where f_2 and f_3 are known. The point where the hue lines originate is the neutral point with the coordinates $x_0=0.3101$, $y_0=0.3163$.

Consider the four points at which two adjacent hue lines intersect with two adjacent chroma lines. Form a quadrilateral by connecting these four points by straight lines. The point $P_n = \{\bar{x}, \bar{y}\}$ will be contained in one of these small quadrilaterals. The geometrically simple problem of finding the four corner points of this quadrilateral becomes a rather involved task when it is to be done on an automatic computer where all the coordinates of the data points are stored consecutively.

In this connection one should recall that there are 4996 data points, that is, a minimum of 9992 words of machine information. The average number of points per value level is approximately 700 which is equivalent to about the same number of quadrilaterals.

Since the use of approximations for the hue and chroma lines is not desirable, we are left with the task of devising a fast technique which enables us to search through the quadrilaterals in order to find that one which contains the given point.

After investigating a number of possibilities we decided on a method which uses only simple vector algebra. This search method is based on the following principle:

Let $\mathbf{a} = \{\alpha_x, \alpha_y\}$ be any vector in the plane and g the directed straight line defined by \mathbf{a} . Let $\mathbf{b} = \{\beta_x, \beta_y\}$ be the vector from any fixed point on g to a given point P . We compute the scalar product

$$S = \mathbf{a}_1 \mathbf{b} = \alpha_x \beta_y - \alpha_y \beta_x$$

of \mathbf{b} with the vector $\mathbf{a}_1 = \{-\alpha_y, \alpha_x\}$ which is orthogonal to \mathbf{a} . This number S is positive for all points P on the "left-hand" side of g and negative for all points on the other side. Hence, by simply checking the sign of S we can determine on which side of g the point P is situated.

Now, let Q_j be a (nondegenerated) quadrilateral with the corners given by the vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4$ from the origin. The numbering may be as shown in Fig. 4. We want to know whether a given point $\mathbf{x} = \{x, y\}$ is inside Q_j or not. To answer this question we compute the four numbers

$$\begin{aligned} S_1 &= (\mathbf{a}_2 - \mathbf{a}_1)_1 (\mathbf{x} - \mathbf{a}_1) \\ S_2 &= (\mathbf{a}_3 - \mathbf{a}_2)_1 (\mathbf{x} - \mathbf{a}_2) \\ S_3 &= (\mathbf{a}_4 - \mathbf{a}_3)_1 (\mathbf{x} - \mathbf{a}_3) \\ S_4 &= (\mathbf{a}_1 - \mathbf{a}_4)_1 (\mathbf{x} - \mathbf{a}_4). \end{aligned}$$

The point \mathbf{x} is contained in the quadrilateral if and only if

$$S_k \geq 0 \quad \text{for } k=1, 2, 3, 4.$$

Some of the quadrilaterals are degenerated into triangles as Fig. 3 shows. In these cases the method

applies similarly. One computes S_1 , S_2 , and

$$S_3^* = (\mathbf{a}_1 - \mathbf{a}_3)_\perp (\mathbf{x} - \mathbf{a}_3).$$

Then, if and only if S_1 , S_2 , and S_3 are nonnegative, the point is inside the triangle.

With the help of an interpretive routine for vector algebra the programming of the process just described proved to be very simple.

By applying this search technique successively to all closed quadrilaterals in the given value level $V = V_n$ one will finally arrive at that one which contains P_n —provided that P_n is at all located inside a closed quadrilateral. Then the problem of solving the system (10) reduces to an interpolation problem which we will describe later.

However, in the original publications,^{1,3} by far not all the quadrilaterals are closed, and hence it is possible that for certain samples $P = \{\bar{x}, \bar{y}, \bar{Y}\}$ the mentioned interpolation problem becomes an extrapolation problem.

In order to avoid this, it was found necessary to extend the Munsell renotation system once and for all in such a way that at least all quadrilaterals become closed. Since we have to solve the system (10) in two adjacent value charts, this extension has even to go so far that for a given point P the projections are always either both contained in closed quadrilaterals or both outside the entire system.

For the method that was applied to perform this extension of the Munsell renotation system and for other related questions we refer to the paper⁴ of Schleter *et al.*

Despite the fact that the search routine just described

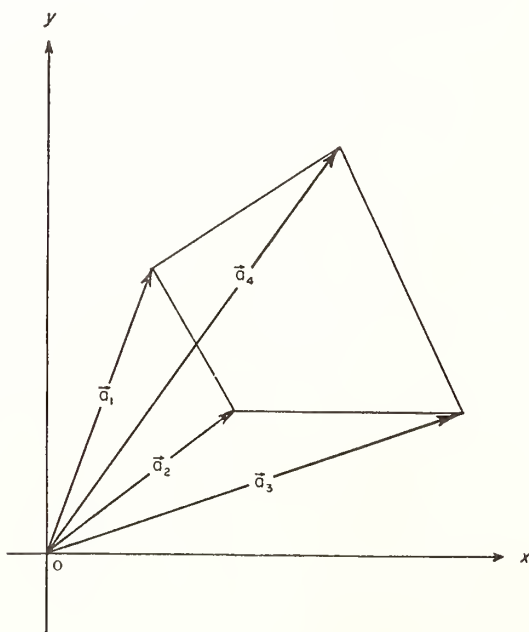


FIG. 4. Quadrilateral as defined by the vectors \mathbf{a}_1 , \mathbf{a}_2 , \mathbf{a}_3 , and \mathbf{a}_4 .

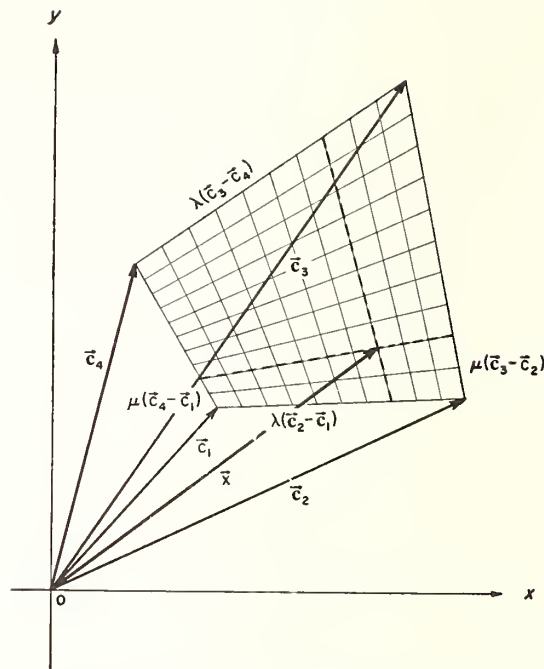


FIG. 5. Linear interpolation using vectors. $\mathbf{x} - \mathbf{c}_1 = \lambda(\mathbf{c}_2 - \mathbf{c}_1) + \mu(\mathbf{c}_4 - \mathbf{c}_1) + \mu(\mathbf{c}_1 - \mathbf{c}_2 + \mathbf{c}_3 - \mathbf{c}_4)$.

is relatively fast, we found it desirable to reduce the number of quadrilaterals which are to be searched. This can be done by performing the following preselection:

We subdivide each value level into eight equal sectors having their vertex at the neutral point. In other words, we introduce in the (x, y) plane the sectors

$$m\left(\frac{1}{4}\pi + \alpha_0\right) \leq \arctan \frac{y - y_0}{x - x_0} < (m+1)\left(\frac{1}{4}\pi + \alpha_0\right) \quad (m=0, 1, \dots, 7), \quad (11)$$

where α_0 is a given initial angle.

The quadrilaterals which have a nonempty intersection with a given sector define a certain block of data. Assume that in the computer all these data blocks are separately addressable. Then—after determining the sector which contains the given point P_n —one will have to search only through the quadrilaterals of the block of data that belongs to this sector.

In the first version of the code the data had to be stored on magnetic tape and it was only possible to read into the core-memory one data block at a time. This required that each of these data blocks had to be self-contained. In other words, the data corresponding to quadrilaterals which belong to more than one sector had to be stored several times, namely, in the data block of each sector involved.

When the second version of the code was to be written, the size of the core-memory of the NBS IBM 704 computer had been increased to 8000 words

and we found it possible to store all data within the core memory. Thus it was no longer necessary to store color space coordinates several times. The only overlap which proved to be convenient (though not absolutely necessary) was the one between the last and first sector in each value level. In order to minimize this overlap we selected the initial angle α_0 in Eq. (11) in such a way that the first sector in each value level has its one side close to the almost straight "10 yellow" hue line. This is the case for

$$\alpha_0 = \arctan 1.55.$$

In the first version of the code, where the last-mentioned consideration was of no importance, we chose $\alpha_0 = 0$.

The selection of the sector which contains the given point $P_n = \{\bar{x}, \bar{y}\}$ can easily be accomplished by another application of our vector algebra routine:

$$\text{Let } y = y_0 + m(x - x_0)$$

be the straight line through the neutral point which forms the angle α_0 with the axis. Then the eight vectors

$$\begin{aligned} a_1 &= a = \{-m, 1\} \\ a_2 &= b = \{-m-1, -m+1\} \\ a_3 &= a_1 = \{-1, -m\} \\ a_4 &= b_1 = \{m-1, -m-1\} \\ a_5 &= -a_1 = -a \\ a_6 &= -a_2 = -b \\ a_7 &= -a_3 = -a_1 \\ a_8 &= -a_4 = -b_1 \end{aligned}$$

have the direction of the dividing lines of the eight sectors (11). With

$$x = \{x - x_0, y - y_0\}$$

we compute the four scalar products

$$\alpha_1 = x \cdot a, \quad \alpha_2 = x \cdot b, \quad \alpha_3 = x \cdot a_1, \quad \alpha_4 = x \cdot b_1.$$

Then it is easy to check that the sector containing $P_n = (x, y)$ is determined according to the following table of relations:

Sector	α_1	α_2	α_3	α_4	Sector	α_1	α_2	α_3	α_4
1	>0	<0	<0	<0	5	<0	>0	>0	>0
2	>0	>0	<0	<0	6	<0	<0	>0	>0
3	>0	>0	>0	<0	7	<0	<0	<0	>0
4	>0	>0	>0	>0	8	<0	<0	<0	<0

After we have now determined the quadrilateral (or triangle) in value chart $V = V_n$ which contains the given point P_n the problem of solving the system (10) reduces to a linear interpolation. However, since the quadrilaterals are in general rather irregularly shaped there is no general interpolation formula available. We found it therefore best to apply our vector algebra routine again to accomplish the interpolation in the following way:

From Fig. 5 it can easily be deduced that

$$x = c_1 - \lambda(c_2 - c_1) + \mu(c_4 - c_1) + \lambda\mu(c_1 - c_2 + c_3 - c_4). \quad (12)$$

With the abbreviations

$$\begin{aligned} d_1 &= c_2 - c_1 \\ d_2 &= c_4 - c_1 \\ d_3 &= c_1 - c_2 + c_3 - c_4 \\ z &= x - c_1, \end{aligned}$$

Eq. (12) assumes the form

$$d_1\lambda + d_2\mu + d_3\lambda\mu = z.$$

This is a system of two quadratic equations for λ and μ . By a simple vector algebraic computation this system can be brought into the form

$$\begin{aligned} (d_1d_3)\lambda^2 + (d_1d_2 - zd_{31})\lambda &= (zd_{21}) \\ (d_2d_3)\mu^2 + (d_{11}d_2 - zd_{31})\mu &= (zd_{11}). \end{aligned} \quad (13)$$

These are two separate quadratic equations for λ and μ . In the program the coefficients are easily obtained by the vector algebra routine.

The solution of these equations proceeds in the usual way and presents no difficulties.

When the quadrilateral degenerates into a triangle, that is, when $d_1 = 0$, the first equation degenerates into a linear equation. It is easy to prove that both equations always have exactly one root between 0 and 1, which we will denote by λ_0, μ_0 .

In order to find the final hue and chroma values of the given point, let us assume that the Munsell representation of the four corner points of the quadrilateral are

$$\begin{aligned} c_1 &= \{H, C\} \\ c_2 &= \{H + \Delta H, C\} \\ c_3 &= \{H + \Delta H, C + \Delta C\} \\ c_4 &= \{H, C + \Delta C\}. \end{aligned}$$

In our case $\Delta H = \pm 2.5$ and $\Delta C = \pm 2$.

Then we finally obtain as solution of the system (10)

$$\begin{aligned} H_k &= H + \lambda\Delta H \\ C_k &= C + \mu\Delta C. \end{aligned}$$

These values are to be used in Eq. (9).

5. CODING OF THE PROBLEM

The general outline of the program may be seen from the block diagram in Fig. 6. In general the block diagram is self-explanatory. However, some of the boxes are of sufficient interest to warrant further comment.

Box 2 refers to the iteration method outlined in Sec. 3. In box 3 a sequence of comparisons is performed in order to determine the interval I_k which contains \bar{V} . This value of k corresponds to the number of value charts below the point in question. The vector sub-

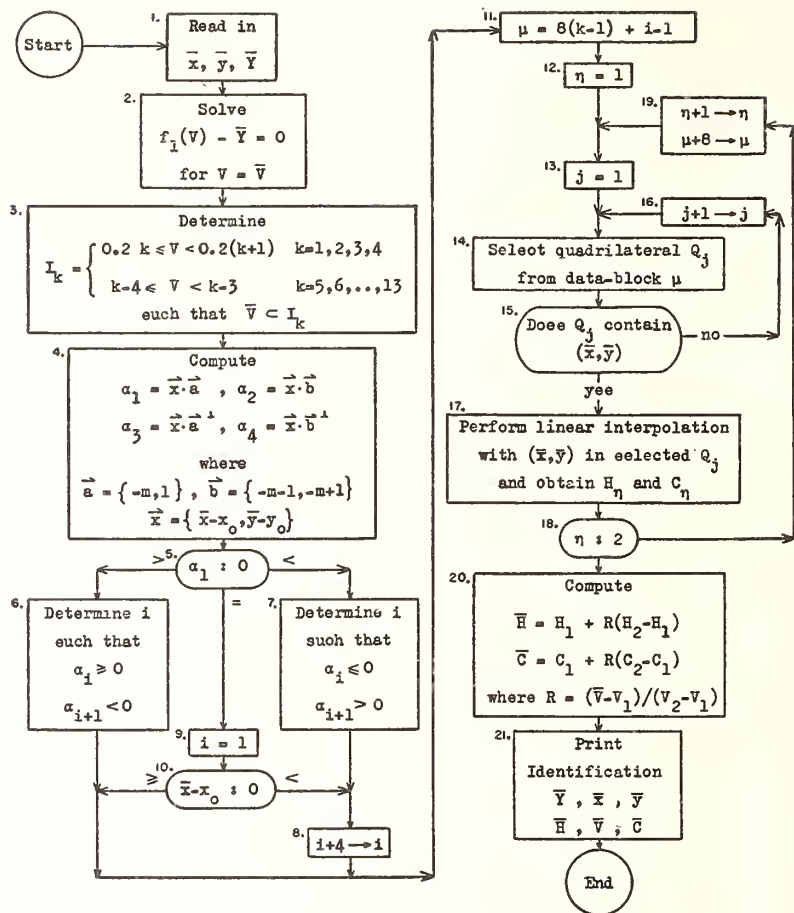


FIG. 6. Block diagram of the conversion of CIE chromaticity coordinates for CIE source C to Munsell renotations.

routine is then utilized to perform the computations indicated in box 4. Another sequence of comparisons as indicated in boxes 5 through 10 is performed to evaluate i , the number of the sector containing the point (\bar{x}, \bar{y}) . The value of μ may now be computed as shown in box 11.

The next eight boxes of the diagram relate to the searching of the data for the quadrilaterals containing P_1 and P_2 , and the computation of H_1, C_1, H_2, C_2 as discussed in Sec. 4. The data are stored in memory according to hue lines, and each hue line according to increasing chroma, starting with $C=2$. Since the neutral point (x_0, y_0) is common to all the hue lines, Q_1 becomes a triangle formed by the neutral point and the first points of the first and second hue lines in a given sector. If the point (\bar{x}, \bar{y}) is not contained in Q_1 , Q_2 is formed using the first and second point of the first and second hue lines. This process is continued until either a Q_j is found containing the point (\bar{x}, \bar{y}) or the end of one of the hue lines has been reached. If

the end of one of the hue lines has been reached, the process is repeated using the second and third hue lines, the third and fourth hue lines, and so forth, until a Q_j is found containing the point (\bar{x}, \bar{y}) . Once the corner points of Q_j are known, a simple linear interpolation is performed to obtain H_η and C_η (box 17).

After the hue and chroma of the footpoints P_1 and P_2 have been found, \bar{H} and \bar{C} may then be computed according to the formula in box 20. The program then prints identification for the sample, the input data $\bar{x}, \bar{y}, \bar{Y}$, and the Munsell renotation $\bar{H}, \bar{V}, \bar{C}$ of the sample.

ACKNOWLEDGMENTS

The authors wish to thank John C. Schleter for preparing the data for the codes, for his extensive help in the checkout, for planning Figs. 2 and 3, and for executing the drawings for all illustrations. We are furthermore indebted to the late Dr. Milton Abramowitz whose initiative helped to start the work.

5. Color Spacing and Metamerism

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5.2. Interval scales, ratio scales, and additive scales for the sizes of differences perceived between members of a geodesic series of colors. Judd, Deane, B., J. Opt. Soc. Am. 57, 380 (1967). Key words: Color; color difference; colorimetry; uniform color scales; vision	330
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Foreword

While section 3 deals with color specification so that exact color control can be maintained, this section treats color differences or color spacing. Although the 1931 CIE standard observer system for colorimetry is adequate for color specification, the desire on the part of many workers in colorimetry for a system that yields equal spacing between adjacent colors, no matter what the color, has led to the search for such a space. The first paper in this section gives the spectral tristimulus values for the small-field and the large-field CIE observer systems. Before this paper was written by Nimeroff, all use of this uniform-space system devised by MacAdam, entailed first computation in the non-uniform space and then transformation into the uniform space by equations provided for this purpose. The data in this paper permit direct computation, with spectral data, into the uniform space.

The second paper, by Judd, discusses various scales for the sizes of perceived differences between members of a geodesic series of colors. This paper concludes that there is a possibility, for engineering applications, of developing an approximately uniformly-spaced color solid.

Another type of color difference exists because of the integrative nature of color vision. It has been found that two photometric fields will color match if simultaneously each of the three integrated tristimulus values of one field is equal to the corresponding integrated value of the other field. This can occur in either of two ways: (1) There is a complete spectral match between the two fields; or, (2) There is no such complete spectral match. In the first of these ways the match is said to be spectral; in the second the match is said to be metameric. The last paper in this section, written by Nimeroff, discusses several attempts to develop indexes to represent the degree of metamerism (spectral mismatch) in metameric pairs of colors. The next to the last paper, written by Nimeroff and Yurow, describes a general index of metamerism that has proven to be quite successful in evaluating degree of metamerism.

Spectral Tristimulus Values for the CIE (u, v, w) Uniform Spacing System

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The spectral tristimulus values, \bar{u}_λ , \bar{v}_λ , \bar{w}_λ , of the uniform space adopted by the CIE in 1959 have two bimodal functions, \bar{u}_λ and \bar{w}_λ . Because in 1963 the CIE recommended data for a 10° matching field as well as the 1931 2° data, spectral tristimulus values are given for both systems. The chromaticity coordinates of several standard sources, the locus of Planckian radiators, and the spectrum locus are also reported.

IN 1959, the International Commission on Illumination (CIE)¹ recommended the use of the MacAdam u, v chromaticity diagram² for representing chromaticity in a spacing that is perceptually more uniform than the x, y chromaticity diagram. The values of the coordinates u, v are usually obtained by use of the chromaticity transformation equations:

$$\begin{aligned} u &= 4x/(12y-2x+3), \quad v = 6y/(12y-2x+3), \\ w &= 1 - (u+v). \end{aligned} \quad (1)$$

This set of transformation equations facilitates conversion of data obtained with existing tristimulus integrators. However, integrators and other methods for obtaining u, v values directly from spectrophotometric data of a sample will probably come into use, now that the CIE has recommended the u, v diagram. For such integrators and methods, spectral tristimulus values \bar{u}_λ ,

\bar{v}_λ , and \bar{w}_λ are needed. In order to facilitate the use of the CIE 1959 u, v diagram, tables of the spectral tristimulus values \bar{u}_λ , \bar{v}_λ , and \bar{w}_λ have been prepared. From these, the spectral transmittance (or reflectance) T_λ , and spectral flux distribution of the source P_λ , the tri-

TABLE I. CIE 1959 u, v, w , coordinates of the 2° observer system.

λ (nm)	Spectral tristimulus values			Chromaticity coordinates		
	\bar{u}_λ	\bar{v}_λ	\bar{w}_λ	u_2	v_2	w_2
380	0.0009	0.	0.0025	0.2647	0.0000	0.7353
390	0.0028	0.0001	0.0081	0.2545	0.0091	0.7364
400	0.0095	0.0004	0.0274	0.2547	0.0107	0.7346
410	0.0290	0.0012	0.0837	0.2546	0.0105	0.7349
420	0.0896	0.0040	0.2616	0.2523	0.0113	0.7365
430	0.1894	0.0116	0.5682	0.2462	0.0151	0.7387
440	0.2323	0.0230	0.7339	0.2348	0.0233	0.7419
450	0.2242	0.0380	0.7749	0.2162	0.0366	0.7472
460	0.1940	0.0600	0.7792	0.1878	0.0581	0.7542
470	0.1303	0.0910	0.6826	0.1442	0.1007	0.7552
480	0.0638	0.1390	0.5672	0.0829	0.1805	0.7366
490	0.0213	0.2080	0.5286	0.0281	0.2744	0.6975
500	0.0033	0.3230	0.6180	0.0035	0.3421	0.6545
510	0.0062	0.5030	0.8289	0.0046	0.3759	0.6195
520	0.0422	0.7100	1.0724	0.0231	0.3891	0.5877
530	0.1104	0.8620	1.2313	0.0501	0.3912	0.5587
540	0.1937	0.9540	1.2959	0.0793	0.3904	0.5303
550	0.2891	0.9950	1.2801	0.1127	0.3880	0.4992
560	0.3965	0.9950	1.1972	0.1532	0.3844	0.4625
570	0.5083	0.9520	1.0480	0.2026	0.3795	0.4178
580	0.6112	0.8700	0.8477	0.2624	0.3736	0.3640
590	0.6845	0.7570	0.6229	0.3316	0.3667	0.3017
600	0.7085	0.6310	0.4158	0.4036	0.3595	0.2369
610	0.6687	0.5030	0.2533	0.4693	0.3530	0.1778
620	0.5699	0.3810	0.1444	0.5203	0.3478	0.1318
630	0.4285	0.2650	0.0763	0.5566	0.3442	0.0991
640	0.2987	0.1750	0.0385	0.5832	0.3417	0.0752
650	0.1891	0.1070	0.0187	0.6007	0.3399	0.0594
660	0.1100	0.0610	0.0090	0.6111	0.3389	0.0500
670	0.0583	0.0320	0.0043	0.6163	0.3383	0.0455
680	0.0312	0.0170	0.0021	0.6203	0.3380	0.0417
690	0.0151	0.0082	0.0009	0.6240	0.3388	0.0372
700	0.0076	0.0041	0.0004	0.6281	0.3388	0.0331
710	0.0039	0.0021	0.0002	0.6290	0.3387	0.0323
720	0.0019	0.0010	0.0000	0.6552	0.3448	...
730	0.0009	0.0005	0.0000	0.6429	0.3571	...
740	0.0005	0.0003	0.0000	0.6250	0.3750	...
750	0.0002	0.0001	0.0000	0.6667	0.3333	...
760	0.0001	0.0001	0.0000	0.5000	0.5000	...
770	0.0001	0.	0.0000	1.0000

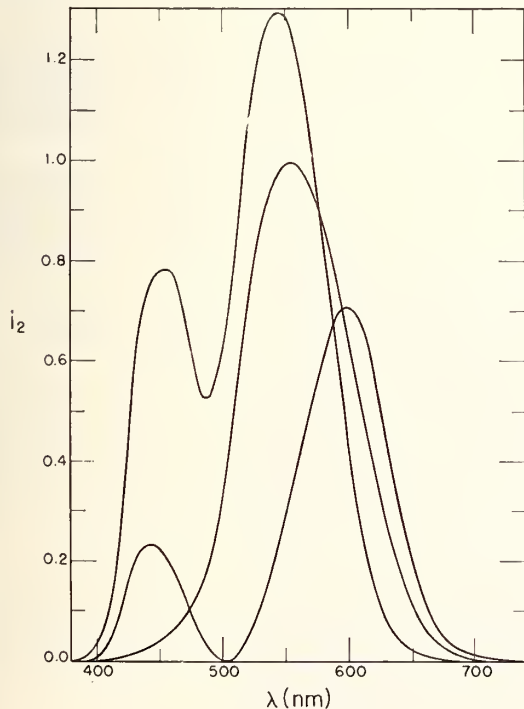


FIG. 1. Spectral tristimulus values i_2 for the 2° observer system.

¹ Proc. Intern. Comm. Illum., 14th Session, Brussels, 1959.

² D. L. MacAdam, J. Opt. Soc. Am. 27, 294 (1937).

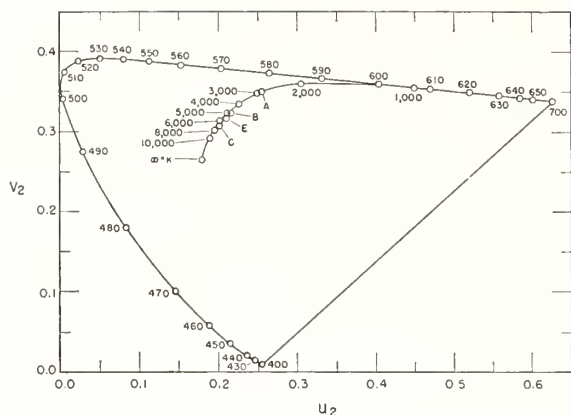


FIG. 2. Chromaticity diagram (u_2, v_2) showing the loci of the spectrum and of the Planckian radiators and the points for other sources for the 2° observer system.

stimulus values may be computed as:

$$U = \int_0^\infty \bar{u}_\lambda P_\lambda T_\lambda d\lambda, \quad V = \int_0^\infty \bar{v}_\lambda P_\lambda T_\lambda d\lambda, \quad (2)$$

$$W = \int_0^\infty \bar{w}_\lambda P_\lambda T_\lambda d\lambda,$$

and the chromaticity coordinates may be computed as:

$$u = U/(U+V+W), \quad v = V/(U+V+W), \quad (3)$$

$$w = W/(U+V+W).$$

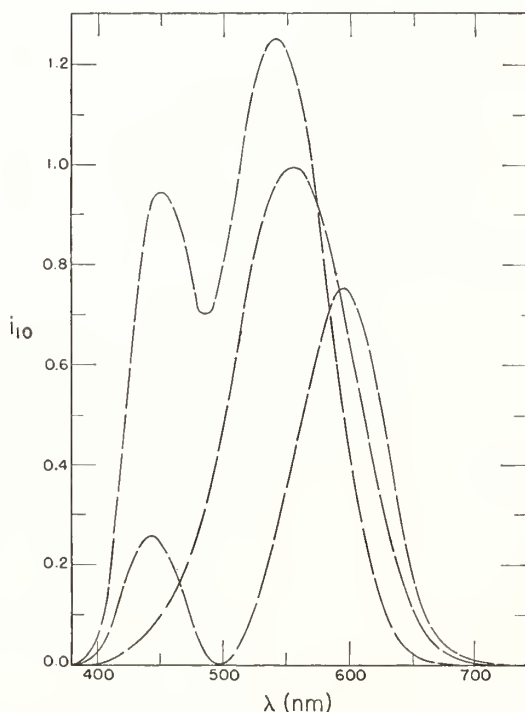


FIG. 3. Spectral tristimulus values i_{10} for the 10° observer system. The spectral tristimulus values and the chromaticity

TABLE II. CIE 1959 u, v, w , coordinates of the 10° observer system.

λ (nm)	Spectral tristimulus values			Chromaticity coordinates		
	\bar{u}_λ	\bar{v}_λ	\bar{w}_λ	u_{10}	v_{10}	w_{10}
380	0.0001	0.	0.0002	0.3333	0.0000	0.6667
390	0.0016	0.0003	0.0045	0.2500	0.0469	0.7031
400	0.0127	0.0020	0.0364	0.2485	0.0391	0.7123
410	0.0565	0.0088	0.1655	0.2448	0.0381	0.7171
420	0.1364	0.0214	0.4161	0.2377	0.0373	0.7250
430	0.2099	0.0387	0.6774	0.2267	0.0418	0.7315
440	0.2559	0.0621	0.8847	0.2127	0.0516	0.7356
450	0.2473	0.0895	0.9463	0.1927	0.0698	0.7375
460	0.2016	0.1282	0.9138	0.1621	0.1031	0.7348
470	0.1305	0.1852	0.8388	0.1130	0.1604	0.7265
480	0.0537	0.2536	0.7262	0.0520	0.2454	0.7027
490	0.0108	0.3391	0.7082	0.0102	0.3205	0.6693
500	0.0025	0.4608	0.7985	0.0020	0.3652	0.6328
510	0.0250	0.6067	0.9473	0.0158	0.3842	0.5999
520	0.0785	0.7618	0.1142	0.0402	0.3898	0.5701
530	0.1577	0.8752	0.2098	0.0703	0.3902	0.5394
540	0.2513	0.9620	1.2614	0.1015	0.3887	0.5097
550	0.3534	0.9918	1.2248	0.1375	0.3859	0.4766
560	0.4704	0.9973	1.1433	0.1809	0.3834	0.4396
570	0.5861	0.9555	0.9939	0.2312	0.3768	0.3920
580	0.6765	0.8689	0.7962	0.2889	0.4711	0.3400
590	0.7460	0.7774	0.6068	0.3502	0.3649	0.2849
600	0.7497	0.6583	0.4254	0.4089	0.3591	0.2320
610	0.6873	0.5280	0.2767	0.4607	0.3539	0.1854
620	0.5712	0.3981	0.1690	0.5018	0.3497	0.1485
630	0.4319	0.2835	0.1015	0.5287	0.3470	0.1243
640	0.2879	0.1798	0.0539	0.5520	0.3447	0.1033
650	0.1790	0.1076	0.0272	0.5704	0.3429	0.0867
660	0.1018	0.0603	0.0141	0.5777	0.3422	0.0800
670	0.0542	0.0318	0.0071	0.5822	0.3416	0.0763
680	0.0273	0.0159	0.0034	0.5858	0.3412	0.0730
690	0.0133	0.0077	0.0016	0.5885	0.3407	0.0708
700	0.0064	0.0037	0.0007	0.5926	0.3426	0.0648
710	0.0031	0.0018	0.0004	0.5849	0.3396	0.0755
720	0.0015	0.0008	0.0001	0.6250	0.3333	0.0417
730	0.0007	0.0004	0.0001	0.5833	0.3333	0.0833
740	0.0003	0.0002	0.	0.6000	0.4000	0.0000
750	0.0002	0.0001	0.	0.6667	0.3333	...
760	0.0001	0.	0.	1.0000	0.0000	...
770	0.0001	0.	0.	1.0000

Since the tristimulus values U, V , and W , of the CIE 1959 u, v, w , system are related to the tristimulus values, X, Y , and Z , by the equations:

$$U = 2X/3, \quad V = Y, \quad W = (3Y - X + Z)/2, \quad (4)$$

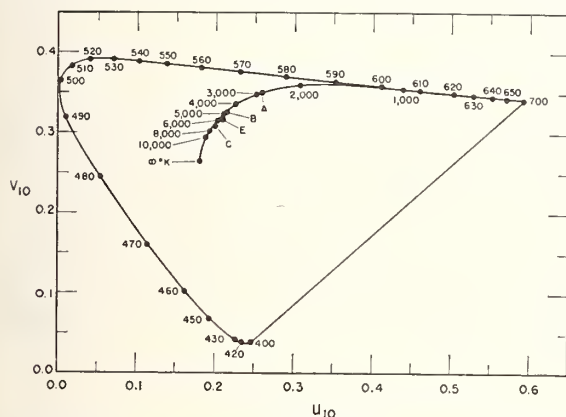
the spectral tristimulus values $\bar{u}_\lambda, \bar{v}_\lambda$, and \bar{w}_λ may be transformed from the spectral tristimulus values $\bar{x}_\lambda, \bar{y}_\lambda$, and \bar{z}_λ by the equations:

$$\bar{u}_\lambda = 2\bar{x}_\lambda/3, \quad \bar{v}_\lambda = \bar{y}_\lambda, \quad (5)$$

$$\bar{w}_\lambda = (3\bar{y}_\lambda - \bar{x}_\lambda + \bar{z}_\lambda)/2.$$

TABLE III. Chromaticity coordinates of Planckian radiators ($c_2=14\,380$) and standard sources.

Source (°K)	2° observer system				10° observer system			
	x_2	y_2	u_2	v_2	x_{10}	y_{10}	u_{10}	v_{10}
1000	0.6526	0.3447	0.4482	0.3547	0.6474	0.3504	0.4383	0.3555
2000	0.5266	0.4133	0.3050	0.3590	0.5300	0.4122	0.3078	0.3591
3000	0.4368	0.4041	0.2506	0.3476	0.4403	0.4026	0.2532	0.3476
4000	0.3804	0.3767	0.2251	0.3344	0.3827	0.3759	0.2266	0.3345
5000	0.3450	0.3516	0.2115	0.3231	0.3464	0.3515	0.2119	0.3234
6000	0.3220	0.3317	0.2034	0.3141	0.3227	0.3323	0.2030	0.3146
7000	0.3063	0.3165	0.1982	0.3070	0.3066	0.3176	0.1973	0.3078
8000	0.2952	0.3048	0.1947	0.3014	0.2951	0.3063	0.1934	0.3022
9000	0.2869	0.2956	0.1922	0.2970	0.2866	0.2975	0.1905	0.2979
10 000	0.2806	0.2883	0.1904	0.2933	0.2802	0.2905	0.1884	0.2944
∞	0.2399	0.2342	0.1800	0.2636	0.2394	0.2366	0.1786	0.2648
A	0.4476	0.4075	0.2560	0.3495	0.4512	0.4059	0.2588	0.3496
B	0.3485	0.3517	0.2137	0.3235	0.3498	0.3527	0.2138	0.3241
C	0.3101	0.3163	0.2009	0.3073	0.3104	0.3191	0.1994	0.3086
E	0.3333	0.3333	0.2105	0.3158	0.3333	0.3333	0.2105	0.3158

FIG. 4. Chromaticity diagram (u_{10}, v_{10}) showing the loci of the spectrum and of the Planckian radiators and points for other sources for the 10° observer system.

coordinates of the spectrum locus for the 2° observer system are given in Table I. Figure 1 shows these spectral tristimulus values $i_2 = \bar{u}_\lambda$, \bar{v}_λ , or \bar{w}_λ for the 2° observer system. These reveal the bimodality of both the \bar{u}_λ and the \bar{w}_λ functions. Figure 2 shows the spectrum locus on the 2° chromaticity diagram (u_2, v_2).

The spectral tristimulus values and the chromaticity coordinates of the spectrum locus for the 10° observer system are given in Table II. Figure 3 shows, for the 10° observer system, the bimodality of both the \bar{u}_λ and the \bar{w}_λ functions. Figure 4 shows the chromaticity diagram (u_{10}, v_{10}) for colorimetry in large fields and its spectrum locus. The chromaticity coordinates of Planckian radiators at different temperatures, as well as for some standard sources, are listed in Table III and are shown also in the chromaticity diagram of Fig. 2 for the 2° and in Fig. 4 for the 10° observer system.



Isay Balinkin, University of Cincinnati. Banquet lecturer at Chicago, 1963. Photograph by H. R. Condit, Eastman Kodak Company.

Interval Scales, Ratio Scales, and Additive Scales for the Sizes of Differences Perceived between Members of a Geodesic Series of Colors

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From larger-smaller judgments of color differences, compared visually two at a time, the perceived sizes may be evaluated on an interval scale. Given numbers B so evaluated, and such that B is linearly connected to some power p , of the physical measure D (such as distance on any chromaticity diagram) of the differences, the additive constant K_{br} such that the numbers $B + K_{br}$ are expressed on a ratio scale may be found from judgments of the ratio of sizes of pairs of differences. To evaluate p , it is sufficient to observe the three differences, 12, 23, and 13 between the pairs of three colors, 1, 2, 3, forming a geodesic series, and chosen so that B_{12} is not much different from B_{23} . The scale formed by the numbers $(B + K_{br})^{1/p}$ is additive if the D scale is additive. If the largest of the color differences judged exceeds the smallest by a factor not greater than 3, a close approximation to the $(B + K_{br})^{1/p}$ scale may be found without evaluating K_{br} by ratio judgments. This approximation is based on the empirical discovery that scales based on the additivity condition: $(B_{12} + K_{bd})^{1/p} + (B_{13} + K_{bd})^{1/p} = (B_{13} + K_{bd})^{1/p}$, though it implies that K_{bd} depends strongly on p , are essentially identical regardless of the choice of p between 1 and $\frac{1}{3}$. It is sufficient therefore, to derive the additive scale by setting $p=1$, and computing K_{bd} as $B_{13} - B_{12} - B_{23}$.

INDEX HEADINGS: Color; Vision; Colorimetry.

A GUIDING principle in the work of the OSA Committee on Uniform Color Scales from its very beginning in 1948 has been that there exists a color space each point of which represents one and only one color, the spacing of the points being uniform in the sense that to a degree of approximation suited to engineering applications the distance between any two points is proportional to the perceived size of the difference between the colors represented by the two points. Equally spaced points along any path in this color space would thus yield a uniform color scale. The committee has been engaged since 1955 in experiments intended to result in the production of several hundreds of painted color chips that would sample this color space by the regular rhombohedral lattice-sampling method.¹ The first experiment (1956) determined a locus of colors of chromaticity points on the CIE (x, y)-diagram for colors of Munsell value 6/ perceived as having the same saturation (departure from gray). The second experiment (1957) determined 40 chromaticity points on this locus equally spaced in hue. The third experiment (1961) determined chromaticness spacing near grays of Munsell value 3/, 6/, and 8/. The latest experiment has determined chromaticness spacing of the entire mat-finish paint gamut at Munsell value 6/. In the course of these studies, the committee evaluated scales of three sorts: interval scales, ratio scales, and additive scales. It is the purpose of this paper to define these three types of scales, to indicate the methods by which the experimental data must be reduced to achieve them, and to restate the guiding principle in more precise terms. In the course of following the above-stated guiding principle, the committee

has shown that color space cannot have all the properties included in the statement of this principle.

INTERVAL SCALES AND RATIO SCALES

Let us consider a series of stimuli, S_1, S_2, \dots, S_n , evaluated physically by numbers, D_1, D_2, \dots, D_n , and producing responses, R_1, R_2, \dots, R_n , evaluated by numbers, B_1, B_2, \dots, B_n .

Interval Scale

If for all pairs of samples such that the difference between the members of the pair, S_1 and S_2 , is perceived as equal to the difference between some two other samples, S_3 and S_4 (that is, if $R_1 - R_2 = R_3 - R_4$), then the B scale is defined as an interval scale provided that $B_1 - B_2 = B_3 - B_4$, and provided that the converse is true (that is, provided that any two pairs of stimuli, S_5, S_6 , and S_7, S_8 for which $B_5 - B_6 = B_7 - B_8$, yield responses such that $R_5 - R_6 = R_7 - R_8$). Stated another way, the B scale is defined as an interval scale if and only if

$$B = m_{br}R - K_{br}. \quad (1)$$

The subscript br , by which the constants m and K are identified, means that these constants apply to the relation between the B scale and the R scale.

Ratio Scale

If the response R_1, R_2, R_3 , and R_4 with R_2 and $R_4 \neq 0$, are such that R_1 is the same multiple (or fraction) of R_2 as R_3 is of R_4 (that is, if $R_1/R_2 = R_3/R_4$), then the B scale is defined as a ratio scale provided that $B_1/B_2 = B_3/B_4$, and provided that the converse is true. Stated another way: the B scale is defined as a ratio scale if and only if

$$B = m_{br}R. \quad (2)$$

* Chairman, OSA Committee on Uniform Color Scales.

¹ G. Wyszecki, J. Opt. Soc. Am. 44, 725 (1954); D. B. Judd, J. Opt. Soc. Am. 45, 673 (1955).

Any B scale derived from a model of the form

$$\sum [B_i - B_j - F(f_i - f_j)]^2 = \text{a minimum} \quad (3)$$

by the method of least squares is presumed to approximate an interval scale. Here f_i is the fraction of the total times, S_i is judged to be larger than S_j , and f_j is the fraction of the times S_j is judged to be larger than S_i . The function $F(x)$ may be chosen in many ways including, for example, "normal scores" and also simply $F(x) = x^2$. When the responses R_i are not very different, as is the case in the committee's experiments, the choice of the function F has very little effect on the B -scale values derived.

If the scores, $f_i - f_j$, are analyzed by a method, such as the Morrissey-Gulliksen matrix method,³ in which the average of the responses is assigned arbitrarily a B -scale value of zero (that is: $\sum B_n/n = 0$), then this scale obviously could not be a ratio scale. An interval scale for positive quantities like color differences can be converted into a ratio scale by adding a constant K_{br} to each B -scale value. That is, if B_1, B_2, \dots, B_n form an interval scale, then $B_1 + K_{br}, \dots, B_n + K_{br}$ form a ratio scale. The value of the additive constant K_{br} may be determined by making ratio judgments. If the ratio judgments indicate that

$$(B_i + K_{br}) / (B_j + K_{br}) = r_{ij},$$

then

$$K_{br} = (r_{ij}B_j - B_i) / (1 - r_{ij}). \quad (4)$$

In the work of the OSA Committee on Uniform Color Scales, the stimuli S_i have been pairs of simultaneously presented colors of closely constant luminance. They are evaluated physically by distance D_i on a chromaticity diagram between the chromaticity points of the colors forming each pair. The responses R_i are the subjective impressions of the sizes of the differences perceived between the pairs of colors, and the numbers B_i are the interpoint distances derived by the Morrissey-Gulliksen matrix method from larger-smaller judgments. From its derivation, the B scale is presumed to approximate an interval scale; if ratio judgments for a suitable sampling of pairs of these color differences were available, the B scale could be converted to a ratio scale by determining K_{br} in accord with Eq. (4).

GEODESIC SERIES OF COLORS

Colors C_1, C_2, \dots, C_n are said to form a geodesic series of colors if all of the intermediate colors, C_2, C_3, \dots, C_{n-1} , lie on the geodesic in color space connecting the extreme colors, C_1 and C_n . Color space is any three-dimensional space each point of which repre-

sents one and only one color, and in which each color is represented by one and only one point. Given two separate points in color space representing two separate colors, there exist infinitely many paths starting with the one point and terminating in the other. For any of these infinitely many paths it is conceptually possible for an observer to determine the number of thresholds of color difference required to traverse this path from one end to the other. That path yielding the smallest number of thresholds is the geodesic path. Examples of color spaces are those formed by plotting the tristimulus values, X, Y, Z , of the colors along orthogonal axes, by plotting the chromaticity coordinates x, y , and the luminance Y greater than zero along orthogonal axes, or by plotting Munsell hue H , value V , and chroma C in cylindrical coordinates, that is, by plotting H according to angle, C as the distance from the black-white axis, and V as distance parallel to this axis.

ADDITIVE SCALES

If the stimuli S_{gh} are differences among a geodesic series of colors, and if color 2 is an intermediate along the geodesic between any two colors 1 and 3, then the physical measures D_{gh} of the stimuli S_{gh} form an additive scale if and only if the sum of D_{12} and D_{23} equals D_{13} . Most physical measures are expressed on essentially additive scales; in particular, distance D on any chromaticity diagram computed from the usual formula

$$D = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{\frac{1}{2}}$$

corresponds to the length of the straight line on the chromaticity diagram between the points (x_1, y_1) and (x_2, y_2) ; it would be nonadditive only for such geodesic series of colors as depart significantly from this straight line. Distance measured along the geodesic between any two points on the chromaticity diagram conforms to the additivity condition except for such geodesics as leave the plane of constant luminance. That is, if the geodesic between points 1 and 3 corresponding to colors of the same luminance has such a course in color space that the intermediate color represented by point 2 has closely the same luminance as colors 1 and 3, then distance measured along the geodesic in the chromaticity diagram is additive. Only if color 2 has a significantly different luminance than that of colors 1 and 3 would this distance scale be nonadditive. It has been presumed in the work of the committee that restriction of the difference between the terminal colors of the same luminance to amounts less than about 24 NBS units (4 Munsell chroma steps) is sufficient to make distance on any chromaticity diagram additive.

Similarly, the numbers B_i that evaluate the responses R_i to the stimuli S_i formed from a geodesic series of colors are on an additive scale if and only if the sum of B_{12} and B_{23} equals B_{13} , where colors 1, 2, and 3 are

² H. A. David, *The Method of Paired Comparisons* (Hafner Publishing Co., New York, 1963), Sec. 1.3 and Ch. 4, especially p. 55.

³ J. E. Jackson and M. Fleckenstein, *Biometrics* 13, 51 (1957).

as defined above. The condition for an additive scale is

$$B_{12} + B_{23} = B_{13}. \quad (5)$$

If the geodesic series does not depart significantly from a straight line on the chromaticity diagram, then the D scale is known to be an additive scale and if the B scale is linearly connected to the D scale, then the numbers, B_1, B_2, \dots, B_n , may be converted into an additive scale by adding a constant K_{bd} to each number, that is, $B_1 + K_{bd}, B_2 + K_{bd}, \dots, B_n + K_{bd}$ evaluate the responses R_1, R_2, \dots, R_n on an additive scale. The required constant K_{bd} may be computed⁴ as

$$K_{bd} = B_{13} - B_{12} - B_{23}. \quad (6)$$

Proof: If the B -scale is linearly connected to the D scale it must be that $B = m_{bd}D - K_{bd}$, or $D = (B + K_{bd})/m_{bd}$. If the D scale is additive, then $D_{12} + D_{23} = D_{13}$, or

$$(B_{12} + K_{bd})/m_{bd} + (B_{23} + K_{bd})/m_{bd} = (B_{13} + K_{bd})/m_{bd},$$

whence it is seen that the $(B + K_{bd})$ scale is additive, and it is also seen that

$$B + K_{bd} = m_{bd}D.$$

That is, the D scale and the $(B + K_{bd})$ scale differ only by some constant factor m_{bd} , relating the sizes of the units of the two scales. The subscript bd by which the constants m and K are identified means that these constants apply to the relation between the B and D scales.

IMPLICATIONS OF AN ADDITIVE $(B + K_{bd})$ SCALE FOR THE RESPONSES R

If by observing several geodesic series of colors it is established within experimental uncertainty that the $(B + K_{bd})$ scale is linearly connected to an additive D scale, what does this imply about the connection between the responses R_n and the physical measures D_n ? If the numbers B_i have been derived by applying the criterion of Eq. (3), it is presumed that the relation between B_i and R_i is that given by Eq. (1). From the additivity of the $(B + K_{bd})$ scale we have

$$B_{12} + K_{bd} + B_{23} + K_{bd} = B_{13} + K_{bd},$$

and from Eq. (1) may be written

$$m_{br}R_{12} - K_{br} + K_{bd} + m_{br}R_{23} - K_{br} + K_{bd} = m_{br}R_{13} - K_{br} + K_{bd},$$

whence it is seen that only if $K_{br} = K_{bd}$, does R satisfy the requirement of additivity: $R_{12} + R_{23} = R_{13}$. If the B and D scales are linearly connected (that is, if $B = m_{bd}D - K_{bd}$), we have already seen that

$$B + K_{bd} = m_{bd}D.$$

That is, the $(B + K_{bd})$ scale differs from the D scale only by a constant of proportionality. If the responses R are themselves proportional to the measures D of the stimuli, this would satisfy the condition: $K_{br} = K_{bd}$.

IMPLICATIONS OF THE HYPOTHESIS THAT THE $(B + K_{br})$ SCALE IS PROPORTIONAL TO SOME POWER p , LESS THAN ONE, OF D

We have seen that the assumption that B is linearly connected to D implies that the $(B + K_{bd})$ scale differs from the D scale only by a constant of proportionality, where K_{bd} is given by Eq. (6). MacAdam⁵ has proposed the hypothesis that the psychometric scale values $B + K_{br}$, where K_{br} is determined by ratio judgments in accord with Eq. (4), are really proportional to chromaticity differences D raised to a power p , thus

$$B + K_{br} = m_{bd}D^p. \quad (7)$$

He has found evidence from the work of the committee on three seven-color clusters each centering on a gray, one for each of Munsell values 3/, 6/, and 8/, that the hypothesis is valid, and that $0.37 < p < 0.80$. From some very preliminary experiments, Judd and Howett⁶ reported ratio judgments from two geodesic series of approximately equally spaced colors indicating that the double difference was perceived as only 1.5 to 1.7 times as large as the single difference. The implication of this preliminary result is shown as follows:

p	Single difference ($B_{12} + K_{br}$)	Implied perceived size of the double difference ($B_{13} + K_{br}$)/($B_{12} + K_{br}$) = 2
1	1	2.00
0.50	1	1.41
0.333...	1	1.26

The values of $(B_{13} + K_{br})/(B_{12} + K_{br})$ found by Howett and Judd (1.5 to 1.7) are seen to correspond to values of p between 1 and 0.50. From the law that the perceived size of the double difference D_{13} is equal to 2^p times the perceived size of the single differences $R_{12} = R_{23} = 1$, of which the double difference is composed, that is,

$$(B_{13} + K_{br})/(B_{12} + K_{br}) = 2^p,$$

we may solve explicitly for p

$$p = \log \left[\frac{B_{13} + K_{br}}{B_{12} + K_{br}} \right] / [\log 2 = 0.301]. \quad (8)$$

Or, to generalize, if we have n colors along a geodesic producing $n-1$ equal differences, that is, differences

⁴ W. S. Torgerson, *Theory and Methods of Scaling* (John Wiley & Sons, New York, 1958), p. 271.

⁵ D. L. MacAdam, *J. Opt. Soc. Am.* **53**, 754 (1963).

⁶ G. L. Howett, *Report of Eighth Meeting of OSA Committee on Uniform Color Scales*, 20 November 1964, p. 4, unpublished.

TABLE I. Demonstration of the degree to which $[(B_{13}+K_{bd})/(B_{12}+K_{bd})]^{1/p}$ approaches 2 for K_{bd} approximated from Eq. (11), B_{12} taken equal to $B_{23}=-0.5$, and $B_{13}=0.7$.

p	K_{bd} approximated from Eq. (11)	$B_{12}+K_{bd}$	$B_{13}+K_{bd}$	$\frac{B_{13}+K_{bd}}{B_{12}+K_{bd}}$ implied ratio of perceived sizes	$(B_{12}+K_{bd})^{1/p}$	$(B_{13}+K_{bd})^{1/p}$	$\left(\frac{B_{13}+K_{bd}}{B_{12}+K_{bd}}\right)^{1/p}$ implied ratio D_{13}/D_{12} of the physical measures
1	1.700	1.200	2.400	2.00	1.200	2.400	2.000
1.5	2.266	1.766	2.966	1.68	2.134	4.261	1.997
2	2.548	2.048	3.248	1.59	2.931	5.853	1.996
2.5	3.397	2.897	4.097	1.41	8.393	16.785	2.000
3	5.094	4.594	5.794	1.26	6.956	194.507	2.006
4	6.791	6.291	7.491	1.19	1566.30	3148.89	2.010
5	13.579	13.079	14.279	1.09	856241589.	1728153633.	2.018

such that

$$R_{12}=R_{23}=R_{34}\cdots=R_{n-1,n}$$

$$\frac{R_{1n}}{R_{12}} = \frac{B_{1n}+K_{br}}{B_{12}+K_{br}} = (n-1)^p, \quad (9)$$

then

$$p = \log \left[\frac{B_{1n}+K_{br}}{B_{12}+K_{br}} \right] / \log(n-1).$$

If we have a D scale such that also $D_{12}=D_{23}=\cdots=D_{n-1,n}$, then the law may be stated as MacAdam did:

$$B+K_{br}=m_{bd}D^p \quad \text{or} \quad R_{1n}/R_{12}=(D_{1n}/D_{12})^p.$$

Since these judgments involved three colors from Eq. (8) ($n=3$), the values of p implied by the ratios 1.5 and 1.7 may be computed as follows:

$\frac{B_{13}+K_{br}}{B_{12}+K_{br}}$	\log_{10}	$p = \log \left[\frac{B_{13}+K_{br}}{B_{12}+K_{br}} \right] / 0.301$
1.5	0.176	0.58
1.6	0.204	0.68
1.7	0.230	0.76

Since the implied values of p (0.58 to 0.76) found by Howett and Judd fall within the range of 0.37 to 0.80 found by MacAdam, the Howett-Judd preliminary determination corroborates the MacAdam values found from the three seven-color clusters in which K_{br} was evaluated from ratio judgments by many observers.

DETERMINATION OF K_{bd} FROM OBSERVATIONS OF 43 VALUE 6/ COLORS

For the linear hypothesis, $(B+K_{bd})=m_{bd}D$, the value of K_{bd} for the interpoint distances among the 43 value 6/ colors used in the latest experiment of the committee, was intended to be found from observations of three double differences among triads of colors along three separate near-geodesic series. The following table identifies the three triads of differences, gives the corresponding values of B , and the values of K_{bd}

computed from Eq. (6):

Geodesic series	B_{12}	B_{23}	B_{13}	K_{bd}
2-6-12	-0.335	-0.767	0.615	1.717
37-38-39	-0.581	-0.605	0.614	1.800
41-42-43	-0.130	-0.741	0.695	1.566
			Average	1.694

If the linear hypothesis is assumed, we would therefore expect to compare the numbers $B+K_{bd}=B+1.694$ with the corresponding physical measures D of the color differences to determine the applicability of each particular D scale [distance on (x,y) diagram, distance on (u,v) diagram, distance on Munsell hue-chroma diagram, and so on] for the prediction of the perceived size of the corresponding differences. The three series were chosen by taking the smallest pairs of single differences along a near-geodesic. It is to be expected that B_{12} and B_{23} should be less than average for such a choice, and B_{13} greater than average; and it may be noted from the above table that the single differences have B values less than zero, and the double differences greater than zero, as expected.

For the MacAdam nonlinear hypothesis given in Eq. (7), $(B+K_{bd})=m_{bd}D^p$, we would expect to compare the values of $(B+K_{bd})^{1/p}$ with the corresponding physical measures D of the color differences, because by this hypothesis $(B+K_{bd})^{1/p}$ is proportional to D . Since the D scale is assumed to be additive, K_{bd} must be adjusted so that $(B+K_{bd})^{1/p}$ is additive; that is, the condition to be satisfied by K_{bd} is that

$$(B_{12}+K_{bd})^{1/p}+(B_{23}+K_{bd})^{1/p}=(B_{13}+K_{bd})^{1/p}.$$

For $p=\frac{1}{2}$, an explicit solution for K_{bd} can be found readily:

$$K_{bd}=B_{13}-B_{12}-B_{23}+[2(B_{13}-B_{12})(B_{13}-B_{23})]^{\frac{1}{2}}. \quad (10)$$

By comparing Eq. (6) with Eq. (10), we see immediately that K_{bd} depends upon the value of p which, as indicated in Eq. (8), can be evaluated by ratio judgments. Note that the first three terms of Eq. (10) are identical to Eq. (6), and further note that the last term in Eq. (10) can never be zero unless one of the single differ-

TABLE II. Dependence of the values of $2[(B+K_{bd})/K_{bd}]^{1/p}$ on the value of p over the range $p=1$ to $p=\frac{1}{3}$.

i, j	B_{ij}	$2[(B_{ij}+K_{bd})/K_{bd}]^{1/p}$				
		$p=1$ $K_{bd}=2.000$	$p=\frac{3}{4}$ $K_{bd}=2.943$	$p=\frac{2}{3}$ $K_{bd}=3.414$	$p=\frac{1}{2}$ $K_{bd}=4.828$	$p=\frac{1}{3}$ $K_{bd}=7.657$
1, 2	-1	1.00	1.18	1.19	1.26	1.31
1, 3	0	2.00	2.00	2.00	2.00	2.00
1, 4	1	3.00	2.95	2.94	2.91	2.89
1, 5	2	4.00	4.00	4.00	4.00	4.00
2, 3	-1	1.00	1.18	1.19	1.26	1.31
2, 4	0	2.00	2.00	2.00	2.00	2.00
2, 5	1	3.00	2.95	2.94	2.91	2.89
3, 4	-1	1.00	1.18	1.19	1.26	1.31
3, 5	0	2.00	2.00	2.00	2.00	2.00
4, 5	-1	1.00	1.18	1.19	1.26	1.31
Average	0	2.00	2.06	2.06	2.09	2.10

ences is judged to be of the same size as the double difference of which it is a part. Explicit solutions for K_{bd} for values of p different from 1 and $\frac{1}{2}$ have not been found, though K_{bd} for any actual values of p , B_{12} , B_{23} , and B_{13} , can readily be found by trial and error.

From a considerable number of such trial-and-error solutions, we discovered that K_{bd} is closely a linear function of $1/p$. This discovery led to the following approximate formula for K_{bd} :

$$K_{bd} \doteq B_{13} - B_{12} - B_{23} - 2^{\frac{1}{p}}(1 - 1/p) \times [(B_{13} - B_{12})(B_{13} - B_{23})]^{\frac{1}{2}}. \quad (11)$$

Table I illustrates how closely the approximate values of K_{bd} found from Eq. (11) conform to the additivity condition for the particular choice of values $B_{12}=B_{23}=-0.5$, $B_{13}=0.7$, which were chosen as representative of the experimental values already given for the three near-geodesic series of colors studied by the committee. This choice of values on the B scale corresponds to three equally spaced colors on a geodesic, and the ratio $[(B_{13}+K_{bd})/(B_{12}+K_{bd})]^{1/p}$ on a truly additive scale yielded by precise values of K_{bd} would be precisely 2. Note from the last column of Table I that the approximate values of K_{bd} found from Eq. (11) yield values of this ratio that depart from 2 by no more than 0.2 of 1% for p ranging between 1 and $\frac{1}{8}$. Note also that this ratio for $p=1$ given in column 5 gives the ratio of the perceived sizes closely equal to 2^p in accord with Eq. (8); that is, whatever value may be found for K_{br} from Eq. (4), there exists a value of p for which Eq. (8) is satisfied by setting $K_{br}=K_{bd}$.

A number of other choices of B_{12} , B_{23} , and B_{13} for which $B_{12}=B_{23}$ were tried with precisely identical results, and this suggested that the degree of approximation to which Eq. (11) gives K_{bd} is independent of the choice of B_{12} and B_{13} . That this is indeed true for $B_{12}=B_{23}$ may be seen by writing out the expression for the ratio $(B_{13}+K_{bd})/(B_{12}+K_{bd})$ obtained by sub-

stituting the value of K_{bd} from Eq. (11), thus:

$$\begin{aligned} & (B_{13}+K_{bd})/(B_{12}+K_{bd}) \\ &= [2 - (1 - 1/p)2^{\frac{1}{p}}]/[1 - (1 - 1/p)2^{\frac{1}{p}}], \quad (12) \\ & B_{12}=B_{23}. \end{aligned}$$

In this expression the term $(B_{13}-B_{12})$ cancels out, yielding the implied ratio of perceived sizes as simply a function of p . The degree to which Eq. (11) gives correct values of K_{bd} thus depends on the degree to which the right-hand expression in Eq. (12) approximates 2^p . Note that for $p=1$, this expression is precisely 2^1 ; for $p=\frac{1}{2}$, it is precisely $2^{\frac{1}{2}}$; and for $p=0$, it is precisely 2^0 .

To explore to what extent the spacing of the $(B+K_{bd})^{1/p}$ scale is dependent on p , take values of B applying for $p=1$ to all differences among five equally spaced colors along a geodesic, and compute values on the $(B+K_{bd})^{1/p}$ scales for other values of p with K_{bd} computed from Eq. (11). Table II shows the 10 values of B_{ij} . Note that they have been chosen so that $\Sigma B_{ij}=0$ in accord with the convention, already mentioned in connection with Eq. (3), of the Morrissey-Gulliksen matrix method. The size of the unit of the $(B+K_{bd})$ scale increases rapidly with $1/p$, the factor of proportionality being approximately $K_{bd}^{1/p}$. The scale formed by $2[(B+K_{bd})/K_{bd}]^{1/p}$ is relatively independent of p both as to spacing and as to size of unit. Table II shows this scale for $p=1$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{3}$, that is, for the range of values of p indicated by MacAdam's studies.⁵ Note that for B_{ij} equal to 0 and 2, the value of $2[(B+K_{bd})/K_{bd}]^{1/p}$ is 2.00 and 4.00, respectively, independent of the value of p ; but there is a slight dependence (less than 0.3 of a unit) for $B_{ij}=-1$ and $+1$. Figure 1 extends this comparison of the scales yielded by $2[(B+K_{bd})/K_{bd}]^{1/p}$ for various values of p in the range $\frac{1}{3}$ to 1 over the range 0 to 7 on the scale for $p=1$. Note that for the range 1.5 to 4.5, it makes little difference which of these scales is chosen, the maximum discrepancy being less than 0.3 unit, or less than 10% of the total range of 1.5 to 4.5. Uncertainties

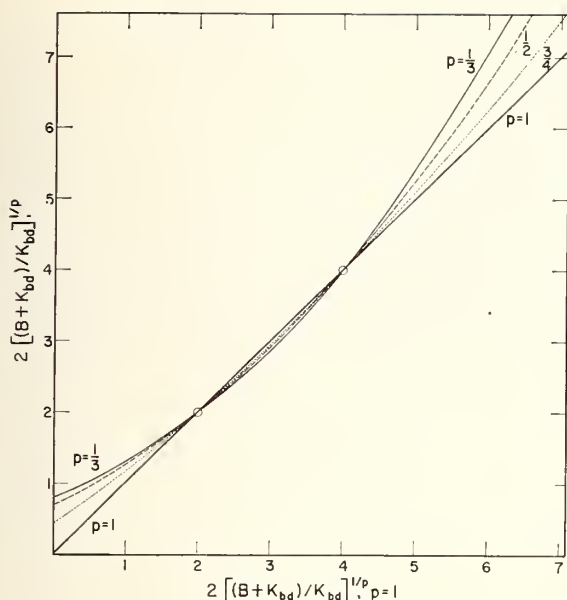


FIG. 1. Scales of $2[(B+K_{bd})/K_{bd}]^{1/p}$ for $p=\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 plotted against the scale for $p=1$. Note that within scale values varying by a factor of 3 (1.5 to 4.5) it makes little difference which of these scales is chosen.

in the values of B_{ij} determined experimentally are somewhat greater than 10% of the range from highest to lowest values of B_{ij} .

REDUCTION OF INTERVAL SCALES OF COLOR DIFFERENCE TO RATIO SCALES AND TO ADDITIVE SCALES

We are now in position to indicate the procedures by which interval scales of color difference can be reduced to ratio scales or to additive scales. Given a set of values of B found experimentally on a B scale by the Morrissey-Gulliksen method such that B is linearly connected to some power p of the physical measure D of the stimuli. Required first to derive a ratio scale from the B scale. Required second to derive an additive scale from the B scale.

The direct way to derive a ratio scale is to obtain judgements of the ratio of sizes of pairs of color differences thus:

$$(B_i + K_{br}) / (B_j + K_{br}) = r_{ij}$$

and to compute K_{br} from Eq. (4), thus:

$$K_{br} = (B_i - r_{ij}B_j) / (r_{ij} - 1).$$

The $(B + K_{br})$ scale is the desired ratio scale, and the linear connection of B to some power of D is given by Eq. (7):

$$B + K_{br} = m_{bd}D^p.$$

To find the value of p implied by K_{br} it is sufficient to observe differences 12, 23, and 13 among colors 1, 2, and 3, forming a geodesic or near-geodesic series, and chosen so that $B_{12} = B_{23}$, and to find p from Eq. (8),

thus:

$$p = (1/0.301) \log[(B_{13} + K_{br}) / (B_{12} + K_{br})].$$

The required additive scale is then given by the basic assumption stated in Eq. (7): $B + K_{br} = m_{bd}D^p$, which implies that $(B + K_{br})^{1/p}$ is proportional to D , the additive physical measure. The $(B + K_{br})^{1/p}$ scale is the desired additive scale.

In the studies so far carried out by the OSA Committee on Uniform Color Scales, the stimuli S_i (color differences) have always been chosen so as to produce responses R_i as nearly identical as possible by using all information available to the Committee either from previously published studies of color spacing or from its own previous work. If the stimuli are chosen so that the ratio of the maximum response R_{max} to the minimum response R_{min} is less than 3, then it is possible to derive an additive scale from the observed values of B_{ij} without making any ratio judgments at all provided it be assumed, as indicated by MacAdam's work,⁵ that the value of p lies somewhere between $\frac{1}{3}$ and 1. The method is to observe differences 12, 23, and 13 among colors 1, 2, and 3, forming a geodesic or near-geodesic series, and chosen so that B_{12} is approximately the same as B_{23} . The value of K_{bd} is then found for $p=1$ from Eq. (11) as

$$K_{bd} = B_{13} - B_{12} - B_{23},$$

in which case the desired additive scale is the $(B + K_{bd})$ scale; that is, a correctly spaced D scale is proportional to the $(B + K_{bd})$ scale well within the experimental uncertainties of determining the values of B if K_{bd} is found for $p=1$; see Table II and Fig. 1. No determination of p is needed.

If a ratio scale is desired, K_{br} must be determined by ratio judgments in accord with Eq. (4), and the scale is the $(B + K_{br})$ scale. This is proportional to the $(B + K_{bd})^p$ scale, where p is found from Eq. (8) and K_{bd} is found for $p=1$ from Eq. (11). The ratio scale depends sharply on the value of p ; the additive scale much less sharply. If $\frac{1}{3} \leq p \leq 1$, and $R_{max} \leq 3 R_{min}$, the dependence on p is not significant.

RESTATEMENT OF GUIDING PRINCIPLE

If the MacAdam finding, corroborated by the preliminary determinations of Howett and Judd, that ratio scales are significantly nonadditive (p significantly less than one in the model: $B + K_{br} = m_{bd}D^p$), is accepted, we must also recognize that there cannot be a color space uniform in the sense that the distance between any two points is proportional to the perceived size of the difference between the colors represented by the two points. Consider n equally spaced colors lying along a geodesic, generating $n-1$ color differences of one unit, $n-2$ differences of two units, . . . and one difference of $n-1$ units. All the one-unit differences must be separated in ratio space by the same unit

distance, all the two-unit differences must be separated by 2^p , all of the three-unit differences by 3^p , and so on. If, as seems likely, p is about $\frac{2}{3}$, the geodesic would be represented in ratio space by a line far from straight, and these theoretical departures of the geodesic from a straight line are far from being within experimental error. For example, the one-unit differences could be represented by one-unit separations; but the two-unit differences would have to be represented by separations of only 1.59 instead of 2; and the three-unit differences by separations of 2.08 instead of 3.

There remains, however, the possibility of develop-

ing a color space that is uniform in the sense that the perceived sizes of the differences between pairs of colors identified by points equally separated in color space will, to a degree of approximation suited to engineering applications, be equal. In such a color space, distance D must be adjusted as closely as possible to be proportional to additive color scales of the form $B + K_{bd}$, where K_{bd} is found from Eq. (6). If it is desired to predict from this color space the ratio judgment for two color differences with different separations D_1 and D_2 in uniform color space, the prediction would be given by D_1^p/D_2^p , where p is evaluated as in Eq. (8).

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Degree of Metamerism

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Colorimetric matches between light from objects and a three-primary visual colorimeter are generally metameric, not spectral. The amount of difference between the spectral energy distributions from the object and the colorimeter determines the degree of metamerism. Three indexes of metamerism are considered that depend on this difference. One of these indexes is shown to correlate well with the chromaticity spreads found among observers' settings for a series of metameric matches. This correlation is independent of luminance. The other indexes do not correlate with the spread of chromaticity of matches by observers.

1. INTRODUCTION

THE basis for colorimetry is that two photometric fields (range 3 to 3000 cd/m²), supplying at the observer's eye spectral irradiances E_λ and E_λ' , will be a color match if simultaneously:

$$\begin{aligned}\int_0^\infty (E_\lambda - E_\lambda') \bar{x}(\lambda) d\lambda &= 0, \\ \int_0^\infty (E_\lambda - E_\lambda') \bar{y}(\lambda) d\lambda &= 0, \\ \int_0^\infty (E_\lambda - E_\lambda') \bar{z}(\lambda) d\lambda &= 0,\end{aligned}\quad (1)$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are the observer's color matching functions. If Eqs. (1) are satisfied because $E_\lambda \equiv E_\lambda'$, the color match may be said to be spectral. If Eqs. (1) are satisfied although $E_\lambda \not\equiv E_\lambda'$, the color match is said to be metameric. Consequently, any attempt to formulate a method for computing the degree of metamerism could reasonably be expected to be based on some function of the difference $E_\lambda - E_\lambda'$.

To test whether a given set of color matching functions actually characterizes the observer's color matching, photometric fields should be chosen to form a metameric match, as has been done in the trials of the 10° observer functions for colorimetry performed by Stiles and Wyszecki¹ and by Nimeroff.² Only with metameric matches will the fields afford a check on the applicability of the given color matching functions to the observer. Otherwise, if these fields form a spectral match, they cannot be distinguished by any observer (normal or color defective) and will not afford a check of the applicability of the functions. Furthermore, the greater the degree of metamerism, the more sensitive is the test of the color matching afforded the observer. On this account alone, if for no other, is it important to have a reliable method for estimating or rating the degree of metamerism between two color matching fields.

There are various methods by which to evaluate the

degree of metamerism of a given pair of photometric fields. Perhaps the most direct method is to have a sufficient number of actual observers inspect the fields predicted to match by a standard observer system and estimate the size of the color difference between the fields. The average of these estimates might be taken as the rating of metamerism.

Another method, proposed by Judd and Wyszecki,³ is to compute this color difference from a set of color matching functions differing from $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, in either of two ways. One way is characteristic of the way one actual observer differs from another; the other way is characteristic of changing from one source to another. For the former way, however, this method is equivalent to the direct method only if observer differences can be expressed in two sets of color matching functions.

A simpler method, mentioned by Judd and Wyszecki,³ is to compute the square root of the sum of the squares of the differences between E_λ and E_λ' . In rejecting this method, they point out that it is unreliable because it overweights the ends of the visible spectrum. They do not, however, make any attempt to develop a technique that will compensate for this overweighting.

It is the purpose of this paper to develop a simple computational method and to validate it by checking against data obtained by the direct method. The validation data used here are the spreads of the chromaticity coordinates of the color matches made by individual observers (observer chromaticity spreads) for each match found on the Donaldson colorimeter by Nimeroff² in the trials of the 10° observer system for colorimetry. Various functions of the differences $E_\lambda - E_\lambda'$ are computed and checked for correlation with the observer chromaticity spreads for corresponding colors.

2. METAMERISM INDEXES

The simple measure of degree of metamerism mentioned by Judd and Wyszecki³ may be computed by

$$M_e = [\sum_\lambda (E_{f\lambda} - E_{p\lambda})^2]^{\frac{1}{2}}, \quad (2)$$

where $E_{f\lambda}$ and $E_{p\lambda}$ are the spectral irradiances at the

¹ W. S. Stiles and G. Wyszecki, *J. Opt. Soc. Am.* **52**, 58 (1962).

² I. Nimeroff, *J. Opt. Soc. Am.* **54**, 696 (1964).

³ D. B. Judd and G. Wyszecki, *Color in Business, Science and Industry* (John Wiley & Sons, Inc., New York, 1963), 2nd ed., p. 152 ff.

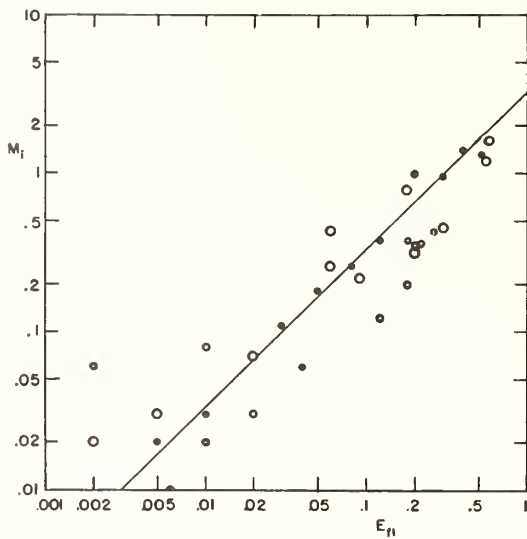


FIG. 1. Spectral irradiance E_{fl} vs trivariate metamerism index, M_i computed by Eq. (3): $i=x$, $l=550$ to 650 nm, \bullet ; $i=y$, $l=500$ to 600 nm, \bullet ; $i=z$, $l=420$ to 460 nm, \circ .

observer's eye from the test filter and from the mixture of the primaries, respectively. This type of formula has no negative values and yields a finite, positive-valued measure for metamerism. Judd and Wyszecki admit, however, that this equation is not completely satisfactory because it gives equal weight at all wavelengths of the visible spectrum, thereby overweighting the extremes of the visible spectrum. It is here suggested that, to overcome this objectionable feature of Eq. (2), the spectral irradiance differences might be weighted according to each of the three standard observer functions, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$. Use of these three functions would yield trivariate metamerism indexes, thus:

$$\begin{aligned} M_x^2 &= \sum \lambda [\bar{x}(\lambda) (E_{f\lambda} - E_{p\lambda})]^2, \\ M_y^2 &= \sum \lambda [\bar{y}(\lambda) (E_{f\lambda} - E_{p\lambda})]^2, \\ M_z^2 &= \sum \lambda [\bar{z}(\lambda) (E_{f\lambda} - E_{p\lambda})]^2. \end{aligned} \quad (3)$$

(The subscripts x , y , and z are used merely to indicate which tristimulus-value function is used in the computation of the metamerism indexes.) Equations (3) may be written in general form as

$$M_i^2 = \sum \lambda [i(\lambda) \Delta E_\lambda]^2, \quad (3a)$$

where $i(\lambda)$ represents spectral tristimulus values as $x(\lambda)$, $y(\lambda)$, or $z(\lambda)$. Each of the trivariate metamerism indexes could be considered as a component of a single-numbered total index obtained from the square root of the weighted sum of the trivariate indexes, M_x^2 , M_y^2 , M_z^2 , thus

$$M_t = [(k_x M_x^2) + (k_y M_y^2) + (k_z M_z^2)]^{1/2}. \quad (4)$$

The weights k_x , k_y , and k_z , may be taken in any desirable manner. In the simplest form of the equation, however, these weights may be taken as unity. Equation (4) would then become

$$M_t = (M_x^2 + M_y^2 + M_z^2)^{1/2}. \quad (5)$$

Equation (3a) yields results in M_i with units that are the same as those of E_λ . The results, therefore, are directly related to the amount of light that passes through the filters. It will be noted from Fig. 1, that the measure of metamerism defined by Eq. (2) depends on the irradiance caused by the flux that leaves the trial filters. Figure 1 shows metamerism indexes M_i computed by Eq. (3) plotted against nominal irradiances for wide spectral bands of range l that correspond to the regions of large values of the spectral tristimulus values $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ of Eqs. (3).

If the colorimetric field were observed through a non-selective filter of transmittance T , the measure would be changed by this factor. As it is generally agreed that metamerism should not be defined in such a manner as to have a dependence on luminance, we must seek an index that will not have this objectionable feature.

The next formula considered for metamerism is suggested by the statistical χ^2 test for goodness of fit. This formula for our problem is of the form

$$M_i^2 = \sum \lambda [(i(\lambda) \Delta E_\lambda)^2 / i(\lambda) E_{f\lambda}], \quad (6)$$

where $E_{f\lambda}$ is the spectral irradiance from the color that is being matched. The units of M_i computed by this equation are the square root of the units of E_λ .

One of the limitations of the χ^2 test for goodness of fit is that the function to which comparisons are being made [that is, the denominator of Eq. (6)] must not become zero; otherwise χ^2 becomes either undefined or boundlessly large. In problems of high metamerism, such as that encountered in the matches obtained on the three-primary Donaldson colorimeter, wherein some of the values of the comparison function may be zero, the χ^2 formula is not valid.

Another equation that suggests itself is the following:

$$M_i^2 = \{ \sum \lambda [i(\lambda) \Delta E_\lambda]^2 \} / [\sum \lambda i(\lambda) E_{f\lambda}]^2. \quad (7)$$

This equation yields metamerism indexes M_i that are without dimensions. Consequently, the resulting total indexes M_t should be independent of specimen-color luminance.

Inherent in Eq. (7) is the problem that if $E_{p\lambda}$, the spectral irradiance distribution of the matching color, is used instead of $E_{f\lambda}$, the spectral irradiance distribution of the test color, the results are generally different. It is true that by Eq. (7)

$$\begin{aligned} \sum \lambda [i(\lambda) \Delta E_\lambda]^2 / [\sum \lambda i(\lambda) E_{f\lambda}]^2 \\ = \sum \lambda [i(\lambda) \Delta E_\lambda]^2 / [\sum \lambda i(\lambda) E_{p\lambda}]^2, \end{aligned} \quad (8)$$

if $i(\lambda)$ represents the spectral tristimulus values of the individual observer who makes the match. In general, however, since the spectral tristimulus values of the individual are not known, these are represented by the spectral tristimulus values of the standard observer, thereby making Eq. (8) hold approximately.

Equation (7) yields meaningful indexes for most metameric pairs because its denominator $[\sum \lambda i(\lambda) E_{m\lambda}]^2$ (where $m=f$ or P) generally is greater than zero so

that the value of the index will rarely become boundlessly large. The denominator may take on the value zero in rare cases, however, such as certain very pure red colors. In such cases, however, the numerator of Eq. (7) will also be zero, so the ratio is not infinite but indeterminate. By arbitrarily assigning in all such cases the value zero to the ratio, the indeterminacy may be avoided. With Eq. (7), little if any discrepancy results when either $\sum \lambda i(\lambda) E_f$ or $\sum \lambda i(\lambda) E_p$ is chosen for the denominator, because the use of the standard observer for the actual observer will make these summations approximately equal. Hence the ratio is a reasonably stable statistic and is not overly sensitive to the use of either E_f or E_p for E_m , except in very unusual cases where both the numerator and denominator are nearly zero and the numerator is much larger than the denominator. These considerations suggest, however, that any function, such as the $\bar{z}(\lambda)$ function, that approaches zero for a considerable fraction of the visible spectrum, is not perfectly suited for a weighting function in an index of degree of metamerism. Weighting functions of the form $k_x \bar{x}(\lambda) + k_y \bar{y}(\lambda) + k_z \bar{z}(\lambda)$, where either k_x or k_y is sufficiently greater than zero, would not be open to this criticism.

3. UNIFORM SPACING

In order to be able to compare the metamerism indexes with observed chromaticity differences for several different colors we must have a system that will have uniform spacing. In this way observer differences

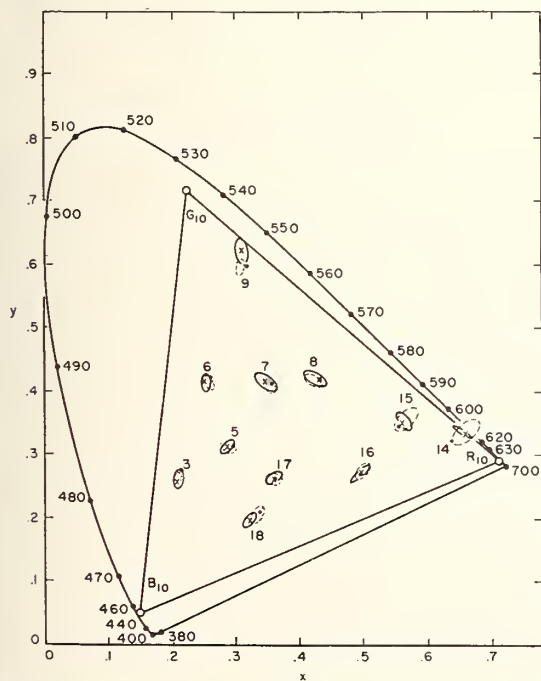


FIG. 2. Predicted chromaticities (\cdot), and observer chromaticity-match boundaries in the NRC (dashed ellipses) and NBS (solid ellipses) field trials of the 10° observer system; NBS observer averages (x).

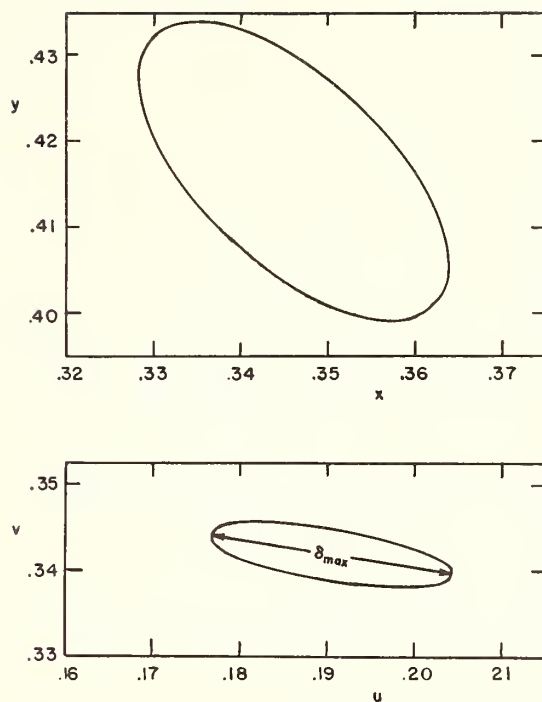


FIG. 3. Ellipses of observers' matches in the (x,y) - and the (u,v) -chromaticity diagrams for trial filter #7.

will be independent of the color and comparisons among colors may be made.

In 1959 the Commission Internationale de l'Eclairage (CIE) recommended⁴ the MacAdam (u,v) chromaticity diagram⁵ for use in making comparisons wherein the uniformity of color spacing is important. This system was chosen because the transformation equations for conversion of the standard (x,y) chromaticity diagram are the simplest of those yielding nearly uniform color spacing. These equations in terms of tristimulus values are:

$$\begin{aligned} U &= 2X/3, \\ V &= Y, \\ W &= (3Y - X + Z)/2. \end{aligned} \quad (9)$$

To obtain the observer differences in the u,v system the ellipses in the x,y system (see Fig. 2) need to be transformed to the u,v system. The transformation equations are

$$\begin{aligned} u &= 4x/(12y - 2x + 3), \\ v &= 6y/(12y - 2x + 3). \end{aligned} \quad (10)$$

Figure 3 shows the ellipses for filter 7 in the chromaticity diagrams for both systems.

It is worth noting that the spectral tristimulus values $\bar{u}(\lambda)$, $\bar{v}(\lambda)$, $\bar{w}(\lambda)$, associated with the chromaticity coordinates u , v , w , promise to be more suitable weighting functions for a metamerism index than those used

⁴ Commission Internationale de l'Eclairage (CIE) Proc. 14th Session A36, Brussels (1959).

⁵ D. L. MacAdam, J. Opt. Soc. Am. 27, 294 (1937).

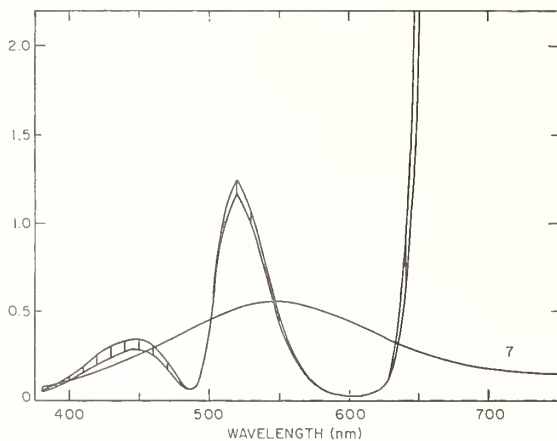


FIG. 4. Spectral irradiance distributions for trial filter #7 and range of observers' matches.

in Eqs. (3). The transformation equations are

$$\begin{aligned}\bar{u}(\lambda) &= 2\bar{x}(\lambda)/3, \\ \bar{v}(\lambda) &= \bar{y}(\lambda), \\ \bar{w}(\lambda) &= [3\bar{y}(\lambda) - \bar{x}(\lambda) + \bar{z}(\lambda)]/2.\end{aligned}\quad (11)$$

Note that $\bar{u}(\lambda)$ and $\bar{v}(\lambda)$ have the same dependence on wavelength as $\bar{x}(\lambda)$ and $\bar{y}(\lambda)$, respectively. The function $\bar{w}(\lambda)$, however, avoids the approach to zero that makes the function $\bar{z}(\lambda)$ not perfectly suited to serve as a weighting function for an index of metamerism. Of course, there exist other transformation equations yielding chromaticity diagrams of the same spacing as the u, v , diagram⁶; these transformation equations yield an infinitely large selection of wavelength functions each one of which might, or might not, be suited to serve as a substitute for the $\bar{z}(\lambda)$ function as a weighting function for a metamerism index. Of this infinite series of possible weighting functions, the suitability of $\bar{u}(\lambda)$, $\bar{v}(\lambda)$, $\bar{w}(\lambda)$ is examined first. To do this we need

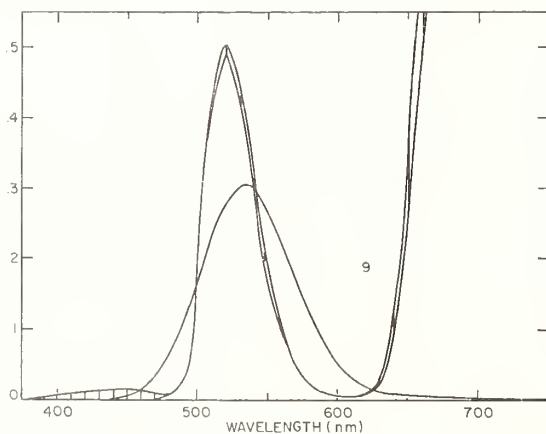


FIG. 5. Spectral irradiance distributions for trial filter #9 and range of observers' matches.

⁶ D. L. MacAdam, J. Opt. Soc. Am. 32, 2 (1942).

only to let the spectral tristimulus values $i(\lambda)$ in Eqs. (3a) and (7) be equal to $\bar{u}(\lambda)$, $\bar{v}(\lambda)$, $\bar{w}(\lambda)$ instead of $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, respectively. The defining formulas for computing metamerism indexes in the x, y, z system, M_x , M_y , M_z , and the corresponding indexes in the MacAdam system, M_u , M_v , M_w , are listed in Table I. The total metamerism index by any method L and in any system of spectral tristimulus values i may be written:

$${}_L M_t^2 = \sum ({}_L M_i^2). \quad (12)$$

The use of Eq. (3a) to compute the total metamerism index of Eq. (12) will be referred to as method A. Total metamerism indexes thus derived may be denoted ${}_A M_t$. The use of Eq. (7) to compute the total metamerism index of Eq. (12) will be referred to as method B. Total metamerism indexes thus derived may be denoted ${}_B M_t$. To distinguish indexes in the uvw system from those in the xyz system, the latter may be written ${}_A M_t'$ and ${}_B M_t'$.

TABLE I. The defining formulas for computing metamerism indexes by Methods A and B in the xyz system and the uvw system.

Method, L	z	Formulas for ${}_L M_t^2$	
		xyz system M_i^2	uvw system M_i^2
A	x	$\sum \lambda [\bar{x}(\lambda) \Delta E_\lambda]^2$	$u \quad \sum \lambda [\bar{u}(\lambda) \Delta E_\lambda]^2$
	y	$\sum \lambda [\bar{y}(\lambda) \Delta E_\lambda]^2$	$v \quad \sum \lambda [\bar{v}(\lambda) \Delta E_\lambda]^2$
	z	$\sum \lambda [\bar{z}(\lambda) \Delta E_\lambda]^2$	$w \quad \sum \lambda [\bar{w}(\lambda) \Delta E_\lambda]^2$
B	x	${}_A M_x^2 / [\sum \lambda \bar{x}(\lambda) E_{m\lambda}]^2$	$u \quad {}_A M_u^2 / [\sum \lambda \bar{u}(\lambda) E_{m\lambda}]^2$
	y	${}_A M_y^2 / [\sum \lambda \bar{y}(\lambda) E_{m\lambda}]^2$	$v \quad {}_A M_v^2 / [\sum \lambda \bar{v}(\lambda) E_{m\lambda}]^2$
	z	${}_A M_z^2 / [\sum \lambda \bar{z}(\lambda) E_{m\lambda}]^2$	$w \quad {}_A M_w^2 / [\sum \lambda \bar{w}(\lambda) E_{m\lambda}]^2$

4. METAMERISM INDEXES AND OBSERVER DIFFERENCES

In the Donaldson colorimeter field trials of the 1959 proposed 10° observer system, performed by Stiles and Wyszecki¹ and by Nimeroff², there was considerable difference between the chromaticity matches obtained by the observers and those predicted by the 10° observer system. There was also considerable difference among the chromaticity matches set by the observers. These differences among observers are shown in Fig. 2 by the dashed-line ellipses for the Stiles-Wyszecki (NRC) observers and by the solid-line ellipses for the Nimeroff (NBS) observers. It is plausible that the degree of metamerism, as represented by any one of the metamerism indexes described above, between the spectral irradiance transmitted by the field trial filters and that from the primaries of the Donaldson colorimeter correlates with the spread of the observer differences.

The definition of degree of metamerism proposed by Judd and Wyszecki,³ while it hints at the relationship between observer differences and metamerism, is arbitrary in the choice of only two observers, or two light sources. The metamerism indexes described here

are more general than the one they proposed. The generality arises from use of weighted spectral irradiance differences in computing the metamerism index instead of using total color differences. Our weighted differences may be written in very general form as

$$\Delta[i_m(\lambda)E_{m\lambda}],$$

in which $i_m(\lambda)$ or $E_{m\lambda}$, individually, or both, simultaneously, may represent data obtained under the different conditions being compared. The different conditions under which differences in $E_{m\lambda}$ occur because of field size, luminance level, viewing time, surround, etc., may be similarly treated. For differences in $i_m(\lambda)$ we may consider, as well, comparisons between any two observers (standard, group average, or individual).

Figures 4, 5, and 6 show the ranges of irradiance distribution for the matches set by the observers in the NBS trials for the field-trial filters 7, 9, and 18. As there were 11 observers in the NBS trial, the average differences were chosen to represent the differences in

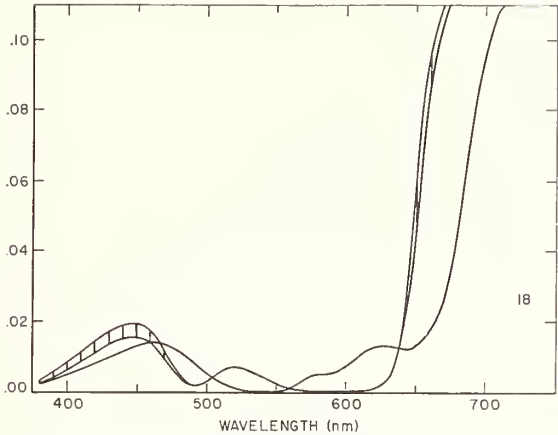


FIG. 6. Spectral irradiance distributions for trial filter #18 and range of observers' matches.

TABLE II. Comparison of observer spreads and metamerism indexes for the field trial filters.

Filter	δ_{\max}	${}_AM_t$	${}_BM_t$
3	0.0136	0.496	0.300
5	0.0196	0.394	0.359
6	0.0129	0.846	0.301
7	0.0277	2.041	0.408
8	0.0252	2.816	0.466
9	0.0160	0.551	0.340
14	0.0392	0.804	0.596
15	0.0363	1.705	0.559
16	0.0373	0.051	0.546
17	0.0227	0.127	0.460
18	0.0248	0.030	0.417

spectral irradiance between the test colors and the matching colors, thus:

$$\Delta E_{\lambda} = \sum_i (E_{f\lambda} - E_{p\lambda i}) / 11.$$

A good measure of the differences among observations on a filter is the maximum dimension of the ellipse that is required to encompass all of the observed chromaticity coordinates for that filter. This observer difference may then be compared with the metamerism index derived from the use of methods A and B.

Table II shows the maximum dimensions of the ellipses for the 11 trial filters δ_{\max} in the (u,v) diagram, and the metamerism indexes ${}_AM_t$ and ${}_BM_t$ for the 11 filters used in the Nimeroff field trial of the 10° observer system.

Figures 7 and 8 show the relationship that exists between δ_{\max} and the metamerism indexes ${}_AM_t$ and ${}_BM_t$, computed by the two methods, A and B, respectively. From Fig. 7, it is evident that there is no overall correlation between the two quantities, observer difference and metamerism index computed by method A. Figure 8 shows that, there is quite good correlation

between δ_{\max} and the metamerism index ${}_BM_t$ computed by method B. The visually estimated curve drawn in this figure indicates that, even with zero metamerism, there may still be a small amount of observer difference, or observer setting error ϵ_0 .

A rough estimate of correlation between two parameters may be obtained by the use of Spearman's rank-order correlation index,⁷

$$\rho = 6 \sum d^2 / n(n^2 - 1),$$

where d is the order difference and n is the number of

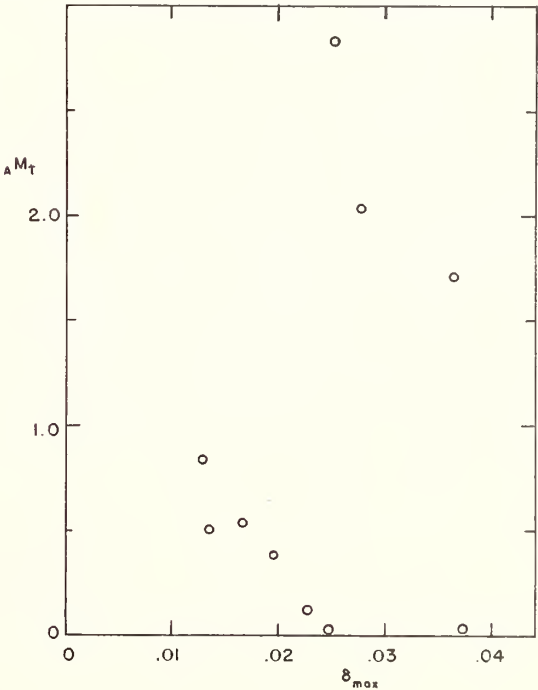


FIG. 7. Observer difference δ_{\max} in the u,v,w , system and metamerism index computed by method A, in the u,v,w , system.

⁷ C. Spearman, Brit. J. Psychol. 2, 89 (1906).

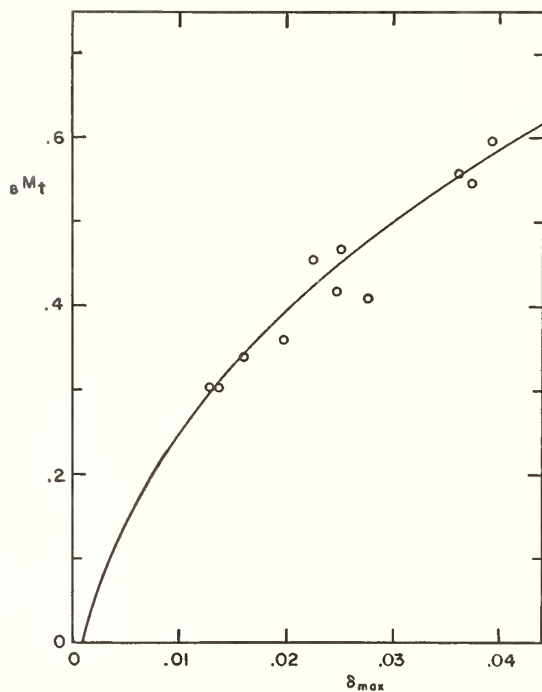


FIG. 8. Observer difference δ_{\max} in the u, v, w system and metamerism index computed by method B, in the v, u, w system. (Curve shown was visually estimated.)

items ordered by the two parameters. The rank-order correlation between observer difference and metamerism indexes obtained by A and B are shown in Table III.

5. DISCUSSION

From Figs. 7 and 8 and the rank-order correlation coefficients listed in Table II we may conclude that:

(1) The metamerism indexes ${}_A M_t$ and ${}_A M'_t$ yield insignificant correlation with observer spread in either system.

(2) The metamerism indexes ${}_B M_t$ and ${}_B M'_t$ yield highly significant correlation with observer spread in the uvw system but not in the xyz system. The correlation is greater with ${}_B M_t$ than with ${}_B M'_t$. The metamerism indexes obtainable from the methods described here are quite general in scope. They depend

TABLE III. Rank-order correlation between observer spreads and metamerism indexes by methods A and B.

Coordinate system of reference δ_{\max}	M	Spearman's rank-order correlation coefficient	
		Method A	Method B
xyz	xyz	0.11	0.21
xyz	uvw	0.14	0.09
uvw	xyz	0.16	0.84
uvw	uvw	0.11	0.93

neither on an arbitrary choice of observers nor on an arbitrary choice of sources as does the Judd-Wyszecki definition of degree of metamerism.

Because of the rather good correlation of metamerism index ${}_B M_t$ with observer spread we may eventually be able to form a reasonable statistical model for estimating anticipated chromaticity coordinates of color matches by a large number of observers. This model may at present be written

$$i_t = i + \epsilon_M + \epsilon_0 + \epsilon_a + \dots,$$

where i_t is the true chromaticity coordinate of a given light, i the chromaticity coordinate observed by metameric match, ϵ_M the metamerism error (measured perhaps by ${}_B M_t$), ϵ_0 the observer error, and ϵ_a is the additivity error. Estimates of the observer error may be made from the within-observer variances reported by Nimeroff, Rosenblatt, and Dannemiller.⁸ Estimates of additivity error (for large viewing fields) may be obtained from a continuance of the work done by Clarke⁹ and Nimeroff¹⁰.

ACKNOWLEDGMENT

The authors are pleased to acknowledge the suggestions and advice given them by D. B. Judd in the writing of this paper.

⁸ I. Nimeroff, J. R. Rosenblatt, and M. C. Dannemiller, J. Res. Natl. Bur. Std. (U. S.) 65A, 475 (1961); J. Opt. Soc. Am. 52, 685 (1962).

⁹ F. J. J. Clarke, PhD. thesis, Imperial College, University of London (1959); Opt. Acta. 7, 355 (1960).

¹⁰ I. Nimeroff, D.I.C. thesis, Imperial College, University of London (1962); J. Opt. Soc. Am. 54, 833 (1964).

A Survey of Papers on Degree of Metamerism

by ISADORE NIMEROFF

Generally, colorimetric matches are metamerism, not spectral. The degree of metamerism between a pair of metameric colors is determined by the differences between the spectral distributions of light from these colors. A survey of work reported on metamerism revealed that colorimetric indexes of metamerism have been devised on the basis of observed color differences that appear with change of illuminating and viewing conditions. These indexes are, however, limited in scope and applicability. A general and fundamental metamerism index should depend directly on the spectral differences of the matched metameric colors. A description is given of an index of metamerism that depends directly on spectral differences, is independent of color lightness, and correlates well with the spread of chromaticity coordinates of metameric matches made by a number of observers for a variety of colors.

At the request of the Chairman of the CIE Colorimetry Committee, a survey of papers dealing with metamerism index computations was conducted. This paper summarizes the three papers found to treat measures of metamerism.

The basis for colorimetry is that two photometric fields, that produce at an

observer's eye spectral irradiances E_λ and E'_λ , will match in color if simultaneously:

$$\begin{aligned}\int_0^\infty (E_\lambda - E'_\lambda) \bar{x}(\lambda) d\lambda &= 0 \\ \int_0^\infty (E_\lambda - E'_\lambda) \bar{y}(\lambda) d\lambda &= 0 \\ \int_0^\infty (E_\lambda - E'_\lambda) \bar{z}(\lambda) d\lambda &= 0\end{aligned}\quad (1)$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are the observer's color-matching functions, or spectral tristimulus values. If these equations are satisfied because all $E_\lambda \equiv E'_\lambda$, that is, the irradiance distributions are spectrally as well as colorimetrically matched, the pair of fields may be said to be an unconditionally matched pair (German: unbedingt-gleich Paar). If, on the other hand, these equations are satisfied although $E_\lambda \neq E'_\lambda$, that is, the distributions are colorimetrically matched but not spectrally matched, the pair of fields may be said to be a conditionally matched pair (German: bedingt-gleich Paar). That both types of matched pairs are possible has been known for a long time. The conditionally matched pair is said to exhibit *meta-*

merism and to form a *metameric match*. Such a match, made on a Donaldson colorimeter by selecting amounts of three primary light colors to match the test light color, is shown in Figure 1. The unconditionally matched pair does not exhibit metamerism and has been called a special match by Nimeroff and Yurov (1).

Index of Differences

The earliest successful attempt (found by this survey) to formulate an index of difference of one kind in the presence of equality of another kind was made by Bodmann (2). This attempt was made in connection with a heterochromatic photometry problem in which a distinction index was required to distinguish between two sources of equal luminance but of different spectral characteristics and different colors. Bodmann derived an equation by which to compute a distinction index d^2_{12} that is based on the spectral luminance differences $_{(1)}b_\lambda - _{(2)}b_\lambda$ of the two sources. The equation, in modern notation, may be written:

$$d_{12}^2 = \frac{\int_0^\infty [(1)b_\lambda - (2)b_\lambda]^2 d\lambda}{\left[\int_0^\infty (1)b_\lambda^2 d\lambda + \int_0^\infty (2)b_\lambda^2 d\lambda \right]^{1/2}}$$

$$\frac{\int_0^\infty [(1)L_\lambda - (2)L_\lambda V(\lambda)]^2 d\lambda}{\left[\int_0^\infty (1)L_\lambda V(\lambda) d\lambda^2 + \int_0^\infty (2)L_\lambda V(\lambda) d\lambda^2 \right]^{1/2}} \quad (2)$$

where $(1)b_\lambda = (1)L_\lambda V(\lambda)$, $(2)b_\lambda = (2)L_\lambda V(\lambda)$, $V(\lambda)$ is the spectral luminous efficiency function, and L_λ is the spectral radiance of the source.

Colorimetric Metamerism Indexes

In connection with the counting of metameric object colors, Stiles and Wyszecki (3) devised a procedure for determining an index of metamerism. They based their index on the empirically observed fact that metameric pairs may be detected if under conditions 1 (say Source A) the pair have the same color and under set of conditions 2 (say Source C) the pair do not have the same color. This index of metamerism D_{12} is defined as the ratio of d_{12} to d_m . The quantity d_{12} is the square root of the sum of squares of the differences in tristimulus values obtained under condition 2 for a metamERICALLY matched pair of colors under condition 1. The quantity d_m is a unit of measure which Stiles and Wyszecki interpret as "the greatest value of separation" of tristimulus values obtainable under condition 2 for a metamERICALLY matched pair under condition 1. They indicate two approaches by which its value may be obtained. The index D_{12} is computed from

$$D_{12} = \frac{d_{12}}{d_m} = \frac{[(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{1/2}}{d_m} \quad (3)$$

where the subscripts of the tristimulus values X, Y, Z refer to the pair of colors. Judd and Wyszecki (4) developed an index of metamerism D based on the same premise as that on which the Stiles-Wyszecki index is based. The only important difference between these two indexes is that the Judd-Wyszecki index uses an estimate of perceived color differences instead of simply tristimulus-value differences. Color difference ΔE involves differences in lightness ΔL and in chromaticity coordinates Δa and Δb .

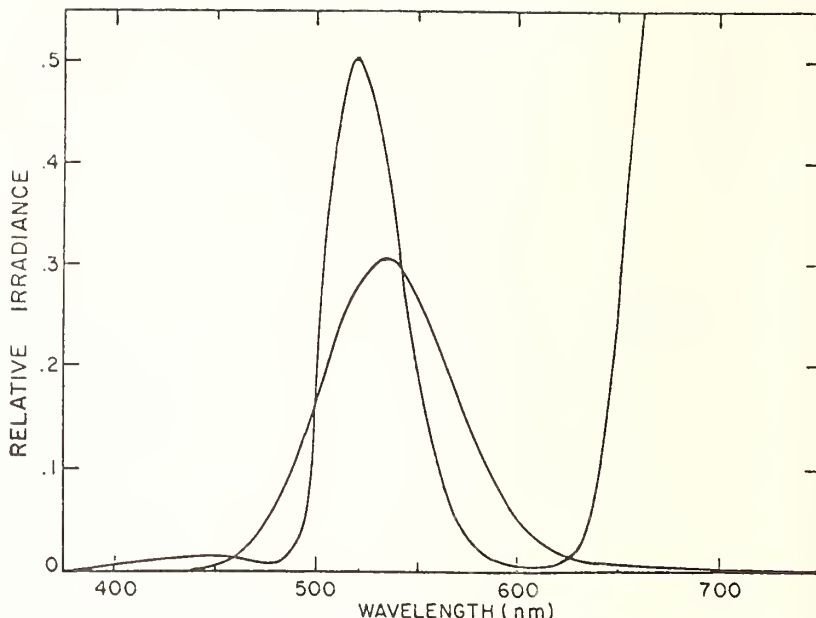


Figure 1—Spectral irradiance distributions of a metameric match.

The equation by which the Judd-Wyszecki index of metamerism is computed may be written:

$$D = \frac{\Delta E_m}{\Delta E_m} = \frac{[(L_1 - L_2)^2 + (a_1 - a_2)^2 + (b_1 - b_2)^2]^{1/2}}{\Delta E_m} \quad (4)$$

where ΔE_m is a just tolerable color difference, taken as one NBS unit of color difference.

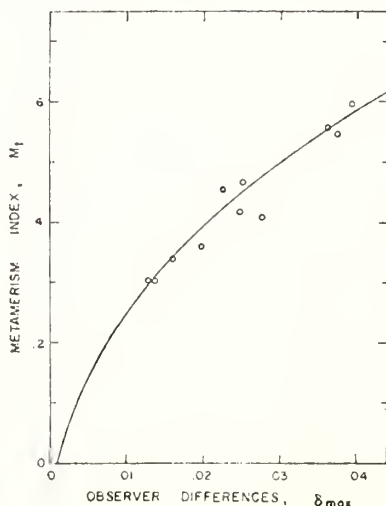


Figure 2—Observers differences δ_{max} and metamerism indexes M_1 for a variety of colors.

The basing of the indexes of metamerism on the fact that metameric pairs are found to match under one set of conditions and to mismatch under another set of conditions, has an inherent weakness. It is possible that there may be no match for a pair of colors under these two conditions, but a match may be found under still another set of conditions. This pair would then be a metameric pair, but the fact of their metamerism would not be revealed by observation under the first two conditions. Also, for this case, even if the existence of a match under the third set of conditions were discovered, the degree of metamerism between the first and third set of conditions may be either larger or smaller than that between the second and third set of conditions. Users of these procedures for computing index of metamerism would then have the problem of deciding which is the measure of the degree of metamerism between the pair of colors.

Spectral Metamerism Index

Nimeroff and Yurow (1) wished to avoid this pitfall in devising their index of metamerism. They accomplished this by basing the index on the differences between the spectral irradiances of one color $(1)E_\lambda$ and of a second color $(2)E_\lambda$ that are found to be a matching pair of metameric colors. These differences $(1)E_\lambda - (2)E_\lambda$ were weighted by a set

of spectral tristimulus values. In order to minimize the effect of the level of the irradiances of the matched metamerie colors, the weighted differences were related to mean values of spectral irradiance ${}_{(m)}E_{\lambda}$. The equation they devised for index of total metamerism M_t is the square root of the sum of squares of the indexes of partial metamerism,

$$M_t = (M_u^2 + M_v^2 + M_w^2)^{1/2}, \quad (5)$$

where

the partial metamerism indexes are

$$\begin{aligned} M_u^2 &= \frac{\Sigma [\bar{u}(\lambda) ({}_{(1)}E_{\lambda} - {}_{(2)}E_{\lambda})]^2}{(\Sigma \bar{u}(\lambda) {}_{(m)}E_{\lambda})^2} \\ M_v^2 &= \frac{\Sigma [\bar{v}(\lambda) ({}_{(1)}E_{\lambda} - {}_{(2)}E_{\lambda})]^2}{(\Sigma \bar{v}(\lambda) {}_{(m)}E_{\lambda})^2} \\ M_w^2 &= \frac{\Sigma [\bar{w}(\lambda) ({}_{(1)}E_{\lambda} - {}_{(2)}E_{\lambda})]^2}{(\Sigma \bar{w}(\lambda) {}_{(m)}E_{\lambda})^2} \end{aligned} \quad (5a)$$

Because observers with normal color vision differ slightly in their selection of metamerie matches, a good correlate of the degree of metamerism is the range of matches made by several such normal observers. This range may be represented by the major axis δ_{\max} of the ellipse that includes the chromaticity coordinates of the matches of a test color by a group of observers. To check on the validity of the metamerism index M_t for color matches made by a group of observers for a variety of colors (5) indexes M_t and maximum chromaticity differences δ_{\max} for the matches of each test color were computed in the CIE uniform chromaticity-spacing system (6) with spectral tristimulus values $\bar{u}(\lambda)$, $\bar{v}(\lambda)$, $\bar{w}(\lambda)$, suggested by MacAdam (7) and published by Nimeroff (8). These two parameters were then intercompared. The relationship between M_t and δ_{\max} is shown qualitatively in Figure 2. The rank-order correlation coefficient between M_t and δ_{\max} was found to be 0.93. It can be seen from Figure 2 that M_t^2 is approximately proportional to δ_{\max}^2 .

Conclusions

Metamerism indexes may be derived either from colorimetric or spectral considerations. Colorimetric metamerism in-

dexes are, however, limited in scope and applicability because, to evaluate metamerism, they require that a condition of color mismatch as well as a condition of color match be known, and therefore cannot be used with data obtained on visual colorimeters that have light sources with only one relative spectral radiance. Spectral metamerism indexes, on the other hand, are generally applicable for they can be evaluated at the condition for which there is a metamerie match without requiring that a condition of color mismatch be found.

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6. Variabilities and Uncertainties in Colorimetry

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Foreword

Colorimetry, as its name implies, is a collection of procedures by which color is measured. The dominant colorimetric technique is spectrophotometric colorimetry. As has been pointed out in section 4, colorimetry stems from the interaction of spectral characteristics of source, intervening media, and the eye. Each of these spectral characteristics, or parameters, are measured quantities and each is subject to measurement uncertainties or variabilities. After having prepared a paper on the propagations of errors in spectrophotometric colorimetry, published in 1953, Nimeroff proceeded to consider the effects of uncertainty in each of the measured parameters in the computation of color coordinates. This treatment is given in the first paper reproduced in this section. The next paper, prepared by Nimeroff, Rosenblatt and Dannemiller derives the variability of the spectral tristimulus values of the large-field colorimetric observer system.

The third paper, by Nimeroff, compares the uncertainty of colorimetric data calculated by one spectrophotometric colorimetry method with those of another method. One of these methods involves the standard observer and its variability, and the other involves actual observers and their variabilities. The overlap of the two sets of variability data was as anticipated.

To summarize the work done at the National Bureau of Standards on the variability and uncertainty in tristimulus colorimetry, Nimeroff prepared the last paper in this section. An attempt was made on this summary paper to present this material in a manner that can be understood by nonstatisticians.

Propagation of Errors in Tristimulus Colorimetry

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(Received February 1, 1957)

The uncertainty ellipse equation is developed for the general case of propagation of errors in tristimulus colorimetry, where spectral tristimulus values as well as spectral radiance factors are deemed uncertain. Two applications are presented as special cases. In the first, uncertainty in chromaticity coordinates for spectrum colors, uncertainties of spectral tristimulus values are based on the CIE standard observer system and the 2°- and 10°-field systems derived from Stiles' data by Judd and Kelly. In the second, uncertainties in chromaticity coordinates of fluorescent light sources, uncertainties in spectral emittances are based on five sets of data for one source. It is important that an international standard-observer system be specified not only by mean tristimulus values of the spectrum but also by their variances; otherwise the significance of the departure of an individual experimental datum from that by the standard observer cannot be assessed.

I. INTRODUCTION

IN an earlier paper, Nimeroff¹ considered a special case of the propagation of errors in tristimulus colorimetry, namely the propagation of errors in spectrophotometric colorimetry. For that special case the spectral tristimulus values for a specified light source, A_λ , B_λ , and C_λ , were deemed free from uncertainty, but the spectral radiance factors, R_λ , of a specimen have associated uncertainties, characterized by their respective variances, $\sigma_{R_\lambda}^2$.

¹ I. Nimeroff, *J. Opt. Soc. Am.* 43, 531 (1953).

Spectrophotometric data may be reduced to tristimulus values by the following defining equations:

$$\begin{aligned} X &\equiv \sum_\lambda R_\lambda A_\lambda \\ Y &\equiv \sum_\lambda R_\lambda B_\lambda \\ Z &\equiv \sum_\lambda R_\lambda C_\lambda. \end{aligned} \tag{1}$$

In these equations R_λ is the spectral radiance factor (spectral transmittance or spectral reflectance) of a specimen and A_λ , B_λ , and C_λ are the spectral tristimulus values for a specified light source, usually

represented $P_\lambda \bar{x}_\lambda$, $P_\lambda \bar{y}_\lambda$, and $P_\lambda \bar{z}_\lambda$, respectively. The defining equations for the chromaticity coordinates x , y , and z are:

$$\begin{aligned}x &\equiv X/S \\ y &\equiv Y/S \\ z &\equiv Z/S\end{aligned}\quad (2)$$

where $S = X + Y + Z$. The computational equations are obtained by combining Eqs. (1) and (2),

$$\begin{aligned}x &= \Sigma_\lambda (R_\lambda A_\lambda) / \Sigma_\lambda [R_\lambda (A_\lambda + B_\lambda + C_\lambda)] \\ y &= \Sigma_\lambda (R_\lambda B_\lambda) / \Sigma_\lambda [R_\lambda (A_\lambda + B_\lambda + C_\lambda)] \\ z &= \Sigma_\lambda (R_\lambda C_\lambda) / \Sigma_\lambda [R_\lambda (A_\lambda + B_\lambda + C_\lambda)].\end{aligned}\quad (2a)$$

The general equation for the uncertainty, characterized by the variance, σ_U^2 , in a computed result, U , where U is a function of the variables, v_n , having respectively associated uncertainties, $\sigma_{v_n}^2$, is*:

$$\sigma_U^2 = \Sigma \left(\frac{\partial U}{\partial v_n} \right)^2 \sigma_{v_n}^2 + 2 \Sigma \left(\frac{\partial U}{\partial v_n} \right) \left(\frac{\partial U}{\partial v_m} \right) \sigma_{v_n v_m}^2 \quad (3)$$

where $\sigma_{v_n v_m}^2$ is the covariance of variables v for $n \neq m$.

The second (covariance) term of Eq. (3) is required in a completely rigorous treatment of this problem, for two reasons, one functional and the other statistical. First, the functions, A_λ , B_λ , and C_λ are functionally related with each other, being linear transformation of individual observer color mixture data, \bar{r}_λ , \bar{g}_λ , and \bar{b}_λ , that is,

$$\begin{aligned}A_\lambda &= P_\lambda (k_1 \bar{r}_\lambda + k_2 \bar{g}_\lambda + k_3 \bar{b}_\lambda) \\ B_\lambda &= P_\lambda (k_4 \bar{r}_\lambda + k_5 \bar{g}_\lambda + k_6 \bar{b}_\lambda) \\ C_\lambda &= P_\lambda (k_7 \bar{r}_\lambda + k_8 \bar{g}_\lambda + k_9 \bar{b}_\lambda)\end{aligned}$$

where the k_i are constants. Second, that once the standard observer system is established and an individual is determined to be a normal observer, his colorimetric evaluations can be predicted within limits.

A treatment involving the functional relationships has been reported by Nimeroff² and is in preparation for publication. Individual observer data, at this writing, are inadequate to estimate the statistical correlation. Consequently, in this present paper the covariance term will be neglected throughout, although its importance is recognized. Equation (3) becomes:

$$\sigma_U^2 = \Sigma \left(\frac{\partial U}{\partial v_n} \right)^2 \sigma_{v_n}^2. \quad (3a)$$

* Where there are repeated observations we may rely on mathematical justification for use of the propagation of error equation. See Cramer, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, 1951), pp. 353-354, and Rao, *Advanced Statistical Methods in Biometric Research* (John Wiley and Sons, Inc., New York, 1952), p. 207. Where there are no repeated observations, use of this formula requires experimental justification.

² I. Nimeroff, J. Opt. Soc. Am. 47, 338(A) (1957).

Where several additional results, V and W , are to be computed and these are correlated with U , the covariances need to be known also. These covariances are:

$$\begin{aligned}\sigma_{UV}^2 &= \Sigma \left(\frac{\partial U}{\partial v_n} \right) \left(\frac{\partial V}{\partial v_n} \right) \sigma_{v_n}^2 \\ \sigma_{UW}^2 &= \Sigma \left(\frac{\partial U}{\partial v_n} \right) \left(\frac{\partial W}{\partial v_n} \right) \sigma_{v_n}^2 \\ \sigma_{VW}^2 &= \Sigma \left(\frac{\partial V}{\partial v_n} \right) \left(\frac{\partial W}{\partial v_n} \right) \sigma_{v_n}^2.\end{aligned}\quad (4)$$

The general case will be developed here. Two special cases and their applications will be fully considered, also.

II. GENERAL CASE

The general case is that in which all the variables, A_λ , B_λ , C_λ , as well as R_λ , have variances, $\sigma_{A_\lambda}^2$, $\sigma_{B_\lambda}^2$, $\sigma_{C_\lambda}^2$, and $\sigma_{R_\lambda}^2$, respectively, associated with them. This is the situation in which a spectrophotometer is used to measure the spectral radiance factors, R_λ , of a specimen and the resulting data are to be treated as though seen by human observers. Both of these, the spectrophotometer and the human observer, are not free from error.

1. Variances

The general equations for the variances in computed chromaticity coordinates, x , y , and z , where the independent variables, A_λ , B_λ , C_λ , and R_λ have variances, $\sigma_{A_\lambda}^2$, $\sigma_{B_\lambda}^2$, $\sigma_{C_\lambda}^2$, and $\sigma_{R_\lambda}^2$, respectively, are:

$$\begin{aligned}\sigma_x^2 &= \Sigma \left(\frac{\partial x}{\partial A_\lambda} \right)^2 \sigma_{A_\lambda}^2 + \Sigma \left(\frac{\partial x}{\partial B_\lambda} \right)^2 \sigma_{B_\lambda}^2 \\ &\quad + \Sigma \left(\frac{\partial x}{\partial C_\lambda} \right)^2 \sigma_{C_\lambda}^2 + \Sigma \left(\frac{\partial x}{\partial R_\lambda} \right)^2 \sigma_{R_\lambda}^2 \\ \sigma_y^2 &= \Sigma \left(\frac{\partial y}{\partial A_\lambda} \right)^2 \sigma_{A_\lambda}^2 + \Sigma \left(\frac{\partial y}{\partial B_\lambda} \right)^2 \sigma_{B_\lambda}^2 \\ &\quad + \Sigma \left(\frac{\partial y}{\partial C_\lambda} \right)^2 \sigma_{C_\lambda}^2 + \Sigma \left(\frac{\partial y}{\partial R_\lambda} \right)^2 \sigma_{R_\lambda}^2 \\ \sigma_z^2 &= \Sigma \left(\frac{\partial z}{\partial A_\lambda} \right)^2 \sigma_{A_\lambda}^2 + \Sigma \left(\frac{\partial z}{\partial B_\lambda} \right)^2 \sigma_{B_\lambda}^2 \\ &\quad + \Sigma \left(\frac{\partial z}{\partial C_\lambda} \right)^2 \sigma_{C_\lambda}^2 + \Sigma \left(\frac{\partial z}{\partial R_\lambda} \right)^2 \sigma_{R_\lambda}^2.\end{aligned}\quad (5)$$

On differentiation for our problem, these variances

become:

$$\begin{aligned}
 \sigma_x^2 &= \left(\frac{S-X}{S^2} \right)^2 \Sigma R_\lambda^2 \sigma_{A_\lambda}^2 + \left(\frac{X}{S} \right)^2 \Sigma R_\lambda^2 (\sigma_{B_\lambda}^2 + \sigma_{C_\lambda}^2) \\
 &\quad + \Sigma \left(\frac{SA_\lambda + X S_\lambda}{S^2} \right)^2 \sigma_{R_\lambda}^2 \\
 \sigma_y^2 &= \left(\frac{S-Y}{S^2} \right)^2 \Sigma R_\lambda^2 \sigma_{B_\lambda}^2 + \left(\frac{Y}{S} \right)^2 \Sigma R_\lambda^2 (\sigma_{A_\lambda}^2 + \sigma_{C_\lambda}^2) \\
 &\quad + \Sigma \left(\frac{SB_\lambda + Y S_\lambda}{S^2} \right)^2 \sigma_{R_\lambda}^2 \\
 \sigma_z^2 &= \left(\frac{S-Z}{S^2} \right)^2 \Sigma R_\lambda^2 \sigma_{C_\lambda}^2 + \left(\frac{Z}{S} \right)^2 \Sigma R_\lambda^2 (\sigma_{A_\lambda}^2 + \sigma_{B_\lambda}^2) \\
 &\quad + \Sigma \left(\frac{SC_\lambda + Z S_\lambda}{S^2} \right)^2 \sigma_{R_\lambda}^2.
 \end{aligned} \tag{5a}$$

As, in practice, CIE colorimetric data are represented on an (x,y) chromaticity diagram, the only variances required for such a representation are σ_x^2 and σ_y^2 .

2. Covariances

The covariances for the general case are σ_{xy} , σ_{xz} , and σ_{yz} . These can be written:

$$\begin{aligned}
 \sigma_{xy} &= \Sigma \left(\frac{\partial x}{\partial A_\lambda} \right) \left(\frac{\partial y}{\partial A_\lambda} \right) \sigma_{A_\lambda}^2 + \Sigma \left(\frac{\partial x}{\partial B_\lambda} \right) \left(\frac{\partial y}{\partial B_\lambda} \right) \sigma_{B_\lambda}^2 \\
 &\quad + \Sigma \left(\frac{\partial x}{\partial C_\lambda} \right) \left(\frac{\partial y}{\partial C_\lambda} \right) \sigma_{C_\lambda}^2 + \Sigma \left(\frac{\partial x}{\partial R_\lambda} \right) \left(\frac{\partial y}{\partial R_\lambda} \right) \sigma_{R_\lambda}^2 \\
 \sigma_{xz} &= \Sigma \left(\frac{\partial x}{\partial A_\lambda} \right) \left(\frac{\partial z}{\partial A_\lambda} \right) \sigma_{A_\lambda}^2 + \Sigma \left(\frac{\partial x}{\partial B_\lambda} \right) \left(\frac{\partial z}{\partial B_\lambda} \right) \sigma_{B_\lambda}^2 \\
 &\quad + \Sigma \left(\frac{\partial x}{\partial C_\lambda} \right) \left(\frac{\partial z}{\partial C_\lambda} \right) \sigma_{C_\lambda}^2 + \Sigma \left(\frac{\partial x}{\partial R_\lambda} \right) \left(\frac{\partial z}{\partial R_\lambda} \right) \sigma_{R_\lambda}^2 \\
 \sigma_{yz} &= \Sigma \left(\frac{\partial y}{\partial A_\lambda} \right) \left(\frac{\partial z}{\partial A_\lambda} \right) \sigma_{A_\lambda}^2 + \Sigma \left(\frac{\partial y}{\partial B_\lambda} \right) \left(\frac{\partial z}{\partial B_\lambda} \right) \sigma_{B_\lambda}^2 \\
 &\quad + \Sigma \left(\frac{\partial y}{\partial C_\lambda} \right) \left(\frac{\partial z}{\partial C_\lambda} \right) \sigma_{C_\lambda}^2 + \Sigma \left(\frac{\partial y}{\partial R_\lambda} \right) \left(\frac{\partial z}{\partial R_\lambda} \right) \sigma_{R_\lambda}^2.
 \end{aligned} \tag{6}$$

For our problem these covariances become, on

differentiation:

$$\begin{aligned}
 \sigma_{xy} &= \left(\frac{Y}{S^2} \right) \left(\frac{X-S}{S^2} \right) \Sigma R_\lambda^2 \sigma_{A_\lambda}^2 + \left(\frac{X}{S^2} \right) \left(\frac{Y-S}{S^2} \right) \\
 &\quad \times \Sigma R_\lambda^2 \sigma_{B_\lambda}^2 + \left(\frac{X}{S^2} \right) \left(\frac{Y}{S^2} \right) \Sigma R_\lambda^2 \sigma_{C_\lambda}^2 \\
 &\quad + \Sigma \frac{(SA_\lambda - X S_\lambda)(SB_\lambda - Y S_\lambda)}{S^2} \sigma_{R_\lambda}^2 \\
 \sigma_{xz} &= \left(\frac{Z}{S^2} \right) \left(\frac{X-S}{S^2} \right) \Sigma R_\lambda^2 \sigma_{A_\lambda}^2 + \left(\frac{X}{S^2} \right) \left(\frac{Z-S}{S^2} \right) \\
 &\quad \times \Sigma R_\lambda^2 \sigma_{C_\lambda}^2 + \left(\frac{X}{S^2} \right) \left(\frac{Z}{S^2} \right) \Sigma R_\lambda^2 \sigma_{B_\lambda}^2 \\
 &\quad + \Sigma \frac{(SA_\lambda - X S_\lambda)(SC_\lambda - Z S_\lambda)}{S^2} \sigma_{R_\lambda}^2 \\
 \sigma_{yz} &= \left(\frac{Z}{S^2} \right) \left(\frac{Y-S}{S^2} \right) \Sigma R_\lambda^2 \sigma_{B_\lambda}^2 + \left(\frac{Y}{S^2} \right) \left(\frac{Z-S}{S^2} \right) \\
 &\quad \times \Sigma R_\lambda^2 \sigma_{C_\lambda}^2 + \left(\frac{Y}{S^2} \right) \left(\frac{Z}{S^2} \right) \Sigma R_\lambda^2 \sigma_{A_\lambda}^2 \\
 &\quad + \Sigma \frac{(SB_\lambda - Y S_\lambda)(SC_\lambda - Z S_\lambda)}{S^4} \sigma_{R_\lambda}^2.
 \end{aligned} \tag{6a}$$

As, in practice, CIE colorimetric data are represented on an (x,y) chromaticity diagram, the only covariance required for such a representation is σ_{xy} .

3. Uncertainty Ellipses

Nimeroff¹ has shown that if we form an ellipse of the general type,

$$T_\epsilon = \frac{(x-x_n)^2 \sigma_y^2 - 2(x-x_n)(y-y_n) \sigma_{xy}^2 + (y-y_n)^2 \sigma_x^2}{\sigma_x^2 \sigma_y^2 - \sigma_{xy}^2}, \tag{7}$$

for each observed specimen mean chromaticity point, x_n, y_n , on the (x,y) chromaticity diagram then, on the average, the true mean chromaticity point x,y will lie inside the ellipses so determined a fraction ϵ of the number of observations. If x_n and y_n have a joint normal distribution, then T_ϵ and ϵ are related exactly by the equation $\epsilon = 1 - e^{-T_\epsilon/2}$; otherwise approximately. For $\epsilon = 0.95$, $T_\epsilon = 6$.

III. APPLICATIONS OF SPECIAL CASES

The first special case to be considered here is that in which A_λ , B_λ , and C_λ alone have associated variances

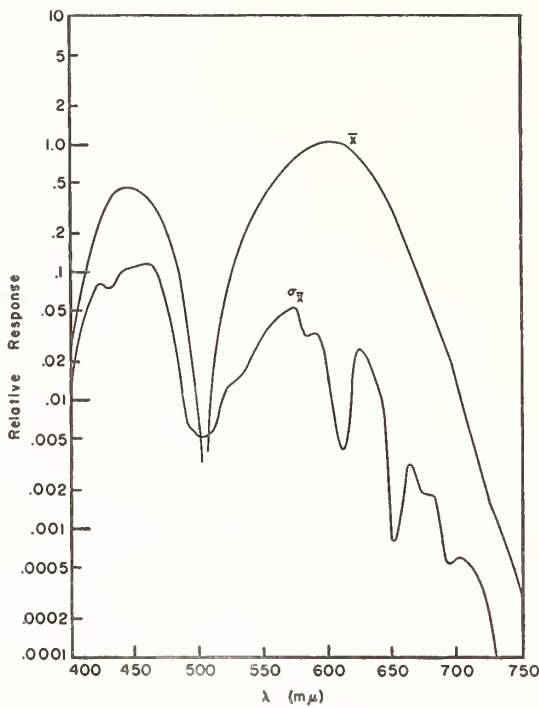


FIG. 1. Mean spectral tristimulus values, \bar{x}_λ , and standard deviations $\sigma_{\bar{x}_\lambda}$ derived from CIE and Judd-Kelly interpretation of Stiles' 2°- and 10°-field color mixture data.

$\sigma_{A_\lambda}^2$, $\sigma_{B_\lambda}^2$, and $\sigma_{C_\lambda}^2$, respectively, resulting from human observer spectral uncertainty. This is the situation in which a specimen is seen by human observers. In this situation the spectral radiance factors, R_λ , of the specimen are deemed certain.

The second special case to be considered here is that in which the variances result from errors in only the evaluation of spectral emittances, P_λ , of a light source. This is the situation in which one wishes to evaluate, as does the average normal human observer, the chromaticity of a source whose measured spectral emittances are uncertain.

To solve these problems we must know the variances in x and y and their covariance when R_λ is free from uncertainty. As by definition

$$A_\lambda \equiv P_\lambda \bar{x}_\lambda, \quad B_\lambda \equiv P_\lambda \bar{y}_\lambda, \quad \text{and} \quad C_\lambda \equiv P_\lambda \bar{z}_\lambda$$

the variances for A_λ , B_λ , and C_λ are:

$$\begin{aligned} \sigma_{A_\lambda}^2 &= \left(\frac{\partial A_\lambda}{\partial P_\lambda} \right)^2 \sigma_{P_\lambda}^2 + \left(\frac{\partial A_\lambda}{\partial \bar{x}_\lambda} \right)^2 \sigma_{\bar{x}_\lambda}^2 = \bar{x}_\lambda^2 \sigma_{P_\lambda}^2 + P_\lambda^2 \sigma_{\bar{x}_\lambda}^2 \\ \sigma_{B_\lambda}^2 &= \left(\frac{\partial B_\lambda}{\partial P_\lambda} \right)^2 \sigma_{P_\lambda}^2 + \left(\frac{\partial B_\lambda}{\partial \bar{y}_\lambda} \right)^2 \sigma_{\bar{y}_\lambda}^2 = \bar{y}_\lambda^2 \sigma_{P_\lambda}^2 + P_\lambda^2 \sigma_{\bar{y}_\lambda}^2 \quad (8) \\ \sigma_{C_\lambda}^2 &= \left(\frac{\partial C_\lambda}{\partial P_\lambda} \right)^2 \sigma_{P_\lambda}^2 + \left(\frac{\partial C_\lambda}{\partial \bar{z}_\lambda} \right)^2 \sigma_{\bar{z}_\lambda}^2 = \bar{z}_\lambda^2 \sigma_{P_\lambda}^2 + P_\lambda^2 \sigma_{\bar{z}_\lambda}^2. \end{aligned}$$

The variances, σ_x^2 and σ_y^2 , and the covariance, σ_{xy} , when $\sigma_{R_\lambda}^2 = 0$, are:

$$\begin{aligned} \sigma_x^2 &= \left(\frac{S-X}{S^2} \right)^2 \Sigma R_\lambda^2 (\bar{x}_\lambda^2 \sigma_{P_\lambda}^2 + P_\lambda^2 \sigma_{\bar{x}_\lambda}^2) + \left(\frac{X}{S^2} \right) \\ &\quad \times \Sigma R_\lambda^2 [(\bar{y}_\lambda^2 + \bar{z}_\lambda^2) \sigma_{P_\lambda}^2 + P_\lambda^2 (\sigma_{\bar{y}_\lambda}^2 + \sigma_{\bar{z}_\lambda}^2)] \\ \sigma_y^2 &= \left(\frac{S-Y}{S^2} \right)^2 \Sigma R_\lambda^2 (\bar{y}_\lambda^2 \sigma_{P_\lambda}^2 + P_\lambda^2 \sigma_{\bar{y}_\lambda}^2) + \left(\frac{Y}{S^2} \right) \\ &\quad \times \Sigma R_\lambda^2 [(\bar{x}_\lambda^2 + \bar{z}_\lambda^2) \sigma_{P_\lambda}^2 + P_\lambda^2 (\sigma_{\bar{x}_\lambda}^2 + \sigma_{\bar{z}_\lambda}^2)] \quad (9) \\ \sigma_{xy} &= \left(\frac{Y}{S^2} \right) \left(\frac{X-S}{S^2} \right) \Sigma R_\lambda^2 (\bar{x}_\lambda^2 \sigma_{P_\lambda}^2 + P_\lambda^2 \sigma_{\bar{x}_\lambda}^2) \\ &\quad + \left(\frac{X}{S^2} \right) \left(\frac{Y-S}{S^2} \right) \Sigma R_\lambda^2 (\bar{y}_\lambda^2 \sigma_{P_\lambda}^2 + P_\lambda^2 \sigma_{\bar{y}_\lambda}^2) \\ &\quad + \left(\frac{X}{S^3} \right) \left(\frac{Y}{S^2} \right) \Sigma R_\lambda (\bar{z}_\lambda^2 \sigma_{P_\lambda}^2 + P_\lambda^2 \sigma_{\bar{z}_\lambda}^2). \end{aligned}$$

1. Spectrum Colors

We will now consider the first special case for the uncertainty ellipses of unit energy spectrum colors in the x, y plane. Here, by definition of these colors, $P_\lambda = 1$ and $\sigma_{P_\lambda}^2 = 0$, then $\sigma_{A_\lambda}^2 = \sigma_{\bar{x}_\lambda}^2$, $\sigma_{B_\lambda}^2 = \sigma_{\bar{y}_\lambda}^2$, and $\sigma_{C_\lambda}^2 = \sigma_{\bar{z}_\lambda}^2$.

The only data conveniently available at this time upon which to estimate the variances in the spectrum

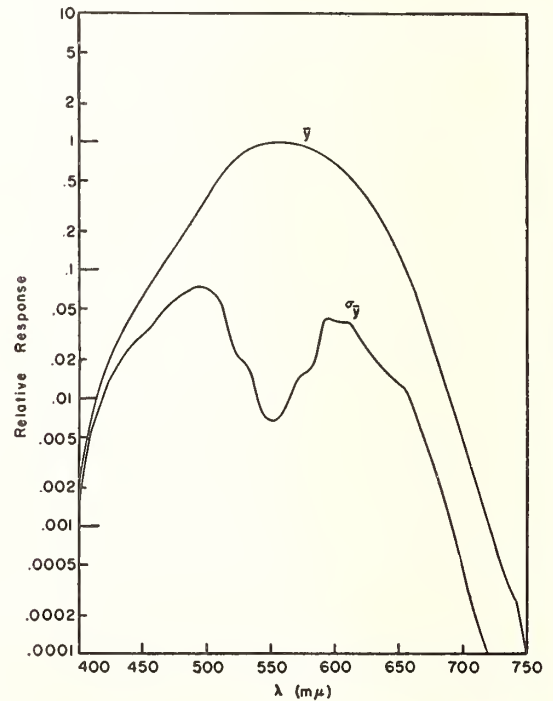


FIG. 2. Mean spectral tristimulus values, \bar{y}_λ , and standard deviations, $\sigma_{\bar{y}_\lambda}$, derived from CIE and Judd-Kelly interpretation of Stiles' 2°- and 10°-field color mixture data.

colors, $\sigma_{\bar{x}\lambda}^2$, $\sigma_{\bar{y}\lambda}^2$, and $\sigma_{\bar{z}\lambda}^2$, are the present CIE and the Judd-Kelly³ interpretation of the Stiles' 2°- and 10°-field color-mixture data. To determine expected observer chromaticity differences, variances for any proposed international system should be based on observer spectral differences for any one set of geometric conditions of observation, for instance, for one field size.

The means, \bar{x}_λ , \bar{y}_λ , and \bar{z}_λ , of these data are computed from:

$$\begin{aligned}\bar{x}_\lambda &= \frac{(\bar{x}_{2^\circ} + \bar{x}_{10^\circ} + \bar{x}_{\text{CIE}})_\lambda}{3} \\ \bar{y}_\lambda &= \frac{(\bar{y}_{2^\circ} + \bar{y}_{10^\circ} + \bar{y}_{\text{CIE}})_\lambda}{3} \\ \bar{z}_\lambda &= \frac{(\bar{z}_{2^\circ} + \bar{z}_{10^\circ} + \bar{z}_{\text{CIE}})_\lambda}{3}\end{aligned}\quad (10)$$

and adjusted so that $\Sigma \bar{x}_\lambda = \Sigma \bar{y}_\lambda = \Sigma \bar{z}_\lambda$. The estimated variances, $\sigma_{\bar{x}\lambda}^2$, $\sigma_{\bar{y}\lambda}^2$, and $\sigma_{\bar{z}\lambda}^2$, are computed from,

$$\begin{aligned}\sigma_{\bar{x}\lambda}^2 &= \frac{\Sigma (\bar{x}_i - \bar{x})_\lambda^2}{n-1}, \quad \sigma_{\bar{y}\lambda}^2 = \frac{\Sigma (\bar{y}_i - \bar{y})_\lambda^2}{n-1}, \\ \text{and} \\ \sigma_{\bar{z}\lambda}^2 &= \frac{\Sigma (\bar{z}_i - \bar{z})_\lambda^2}{n-1}\end{aligned}\quad (11)$$

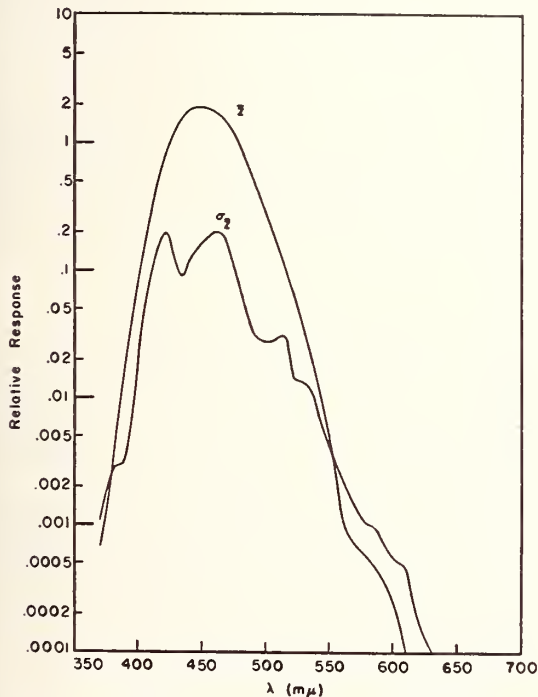


FIG. 3. Mean spectral tristimulus values, \bar{x}_λ , and standard deviations, $\sigma_{\bar{x}\lambda}$, derived from CIE and Judd-Kelly interpretation of Stiles' 2°- and 10°-field color mixture data.

³ D. B. Judd and K. L. Kelly, J. Opt. Soc. Am. 46, 382 (1956).

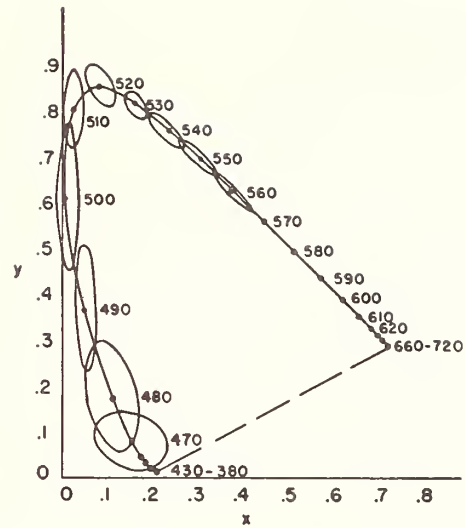


FIG. 4. Uncertainty ellipses of the spectrum colors, based on mean spectral tristimulus values, for the 95% confidence level.

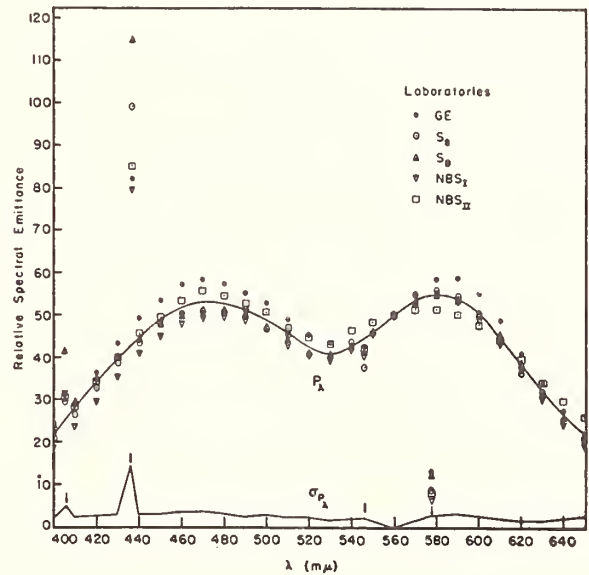


FIG. 5. Spectral emittance data, P_λ , and standard deviation, σ_{P_λ} , for a daylight fluorescent source taken on five spectroradiometers: NBS_I, NBS_{II}, National Bureau of Standards; S_B, S_S, Sylvania Electric Products, Inc.; GE, General Electric Company.

where $n=3$. These means and their respective standard deviations are shown in Figs. 1, 2, and 3, for \bar{x} , \bar{y} , and \bar{z} , respectively.

In computation of the uncertainty for the spectrum colors, we assign to the transmittance of air spectral radiance factors of unity and variances of zero. Thus:

$$R_\lambda = 1 \quad \text{and} \quad \sigma_{R_\lambda}^2 = 0.$$

The variances, $\sigma_{\bar{x}\lambda}^2$ and $\sigma_{\bar{y}\lambda}^2$, and the covariance, $\sigma_{\bar{x}\lambda\bar{y}\lambda}$, for the chromaticity coordinates of the spectrum

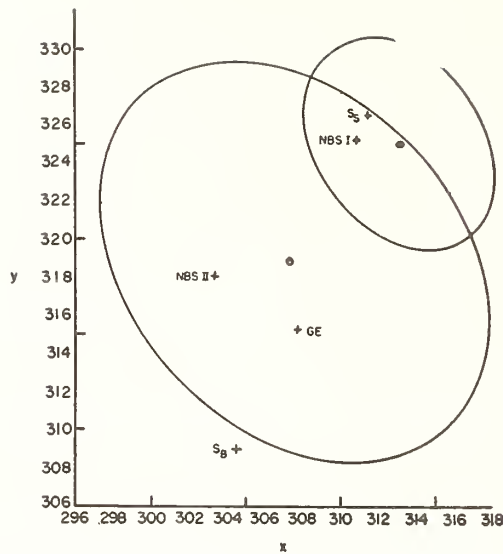


FIG. 6. Uncertainty ellipse plotted about the mean chromaticity point \circ for the daylight fluorescent light source whose spectral emittance data are shown in Fig. 5. Smaller ellipse plotted about the mean chromaticity point \bullet shows the uncertainty for the visual comparison data for this source.

colors are:

$$\begin{aligned}\sigma_{x\lambda}^2 &= \left(\frac{S-X}{S^2}\right)^2 \sigma_{x\lambda}^2 + \left(\frac{X}{S^2}\right)^2 (\sigma_{y\lambda}^2 + \sigma_{z\lambda}^2) \\ \sigma_{y\lambda}^2 &= \left(\frac{S-Y}{S^2}\right)^2 \sigma_{y\lambda}^2 + \left(\frac{Y}{S^2}\right)^2 (\sigma_{x\lambda}^2 + \sigma_{z\lambda}^2) \\ \sigma_{x\lambda}^2 \sigma_{y\lambda} &= \left(\frac{Y}{S^2}\right) \left(\frac{X-S}{S^2}\right) \sigma_{x\lambda}^2 + \left(\frac{X}{S^2}\right) \left(\frac{Y-S}{S^2}\right) \sigma_{y\lambda}^2 \\ &\quad + \left(\frac{X}{S^2}\right) \left(\frac{Y}{S^2}\right) \sigma_{z\lambda}^2.\end{aligned}\quad (12)$$

Figure 4 shows the uncertainty ellipses of the spectrum colors, based on the average values and the spectral uncertainty derived from CIE and Stiles' 2°- and 10°-field data, for the 95% confidence level. The ellipses would indicate the regions where the chromaticity coordinates of the spectrum colors for an observer may be expected to lie for 95% of the observers.

2. Fluorescent Lamp Colors

In determining the colors of fluorescent lamps data are obtained on the spectral emittance, P_λ , of the lamps by using a spectroradiometer. Five sets of spectral emittance data obtained during an ASA cooperative test on a daylight fluorescent lamp indicate a standard deviation, σ_{P_λ} , equal to 2.5. These data are shown in Fig. 5. The variances, $\sigma_{P_\lambda}^2$, are then equal to 6.25.

Here also for the reasons cited above the following conditions hold:

$$R_\lambda = 1 \quad \text{and} \quad \sigma_{R_\lambda} = 0.$$

Strictly, this determination also should be performed for the "totality" of normal observers, that is for means, \bar{x}_λ , \bar{y}_λ , and \bar{z}_λ and variances, $\sigma_{\bar{x}_\lambda}^2$, $\sigma_{\bar{y}_\lambda}^2$, and $\sigma_{\bar{z}_\lambda}^2$, and not for the *average* normal observer alone. But, as the estimated variances of the spectral tristimulus values, here derived for illustration, may be excessively large only the CIE average normal observer has been used. The large variances may result from the equations chosen by Judd and Kelly to transform the Stiles data to a coordinate system approximating the CIE system.

Then, if we consider only the CIE average normal observer, that is $\sigma_{\bar{x}_\lambda}^2 = \sigma_{\bar{y}_\lambda}^2 = \sigma_{\bar{z}_\lambda}^2 = 0$, then Eq. (9) becomes,

$$\begin{aligned}\sigma_{A\lambda}^2 &= \bar{x}_\lambda^2 \sigma_{P_\lambda}^2 \\ \sigma_{B\lambda}^2 &= \bar{y}_\lambda^2 \sigma_{P_\lambda}^2 \\ \sigma_{C\lambda}^2 &= \bar{z}_\lambda^2 \sigma_{P_\lambda}^2\end{aligned}\quad (9a)$$

the variances, σ_x^2 and σ_y^2 , and the covariance, σ_{xy}^2 , are:

$$\begin{aligned}\sigma_x^2 &= \left(\frac{S-X}{S^2}\right)^2 \Sigma \bar{x}_\lambda^2 \sigma_{P_\lambda}^2 + \left(\frac{X}{S^2}\right)^2 \Sigma (\bar{y}_\lambda^2 + \bar{z}_\lambda^2) \sigma_{P_\lambda}^2 \\ \sigma_y^2 &= \left(\frac{S-Y}{S^2}\right)^2 \Sigma \bar{x}_\lambda^2 \sigma_{P_\lambda}^2 + \left(\frac{Y}{S^2}\right)^2 \Sigma (\bar{x}_\lambda^2 + \bar{z}_\lambda^2) \sigma_{P_\lambda}^2 \\ \sigma_{xy}^2 &= \left(\frac{Y}{S^2}\right) \left(\frac{X-S}{S^2}\right) \Sigma \bar{x}_\lambda^2 \sigma_{P_\lambda}^2 + \left(\frac{X}{S^2}\right) \left(\frac{Y-S}{S^2}\right) \\ &\quad \times \Sigma \bar{y}_\lambda^2 \sigma_{P_\lambda}^2 + \left(\frac{X}{S^2}\right) \left(\frac{Y}{S^2}\right) \Sigma \bar{z}_\lambda^2 \sigma_{P_\lambda}^2.\end{aligned}\quad (13)$$

Figure 6 shows the uncertainty ellipse for one daylight fluorescent light source used in a round-robin test conducted during 1951-1952. Also shown is the uncertainty ellipse for visual data on the same source.

IV. CONCLUSION

Application of the theory of propagation of errors in a computed result to computation of chromaticity coordinates permits predictions about the chromaticity regions within which chromaticities may be expected to fall for a specified fraction of observers. Any international standard observer system, if it is to be useful in assessing individual departures, should contain information on the variances in the spectral tristimulus values that are based on individual normal observer differences for any one set of geometric conditions of observation.

It should be clearly understood that, while variances of the spectral tristimulus values may be as large as indicated here, researchers in this field should not consider that they are given freedom to perform inaccurate spectrophotometry or calculations. The utmost care should be taken in these operations to assure that data obtained are the best that is humanly possible.

Variability of Spectral Tristimulus Values

Isadore Nimeroff, Joan R. Rosenblatt, and Mary C. Dannemiller

(July 11, 1961)

As the spectral tristimulus values of the CIE Standard Observer System for Colorimetry are measurable quantities, their variabilities should be known. This paper describes a procedure for deriving "within" and "between" variances and covariances in the spectral tristimulus values, based on color-matching data for individual observers. The "within" variances are based on the replications of color-mixture data by an observer. The "between" variances are based on differences among the color-mixture data of individual observers. A statistical model is given for the system in which the experimental data are obtained. Formulas for expected values (means), variances, and covariances are developed. Variances and covariances belonging to different sources of uncertainties in the experimental data are considered. A procedure is developed for determining the uncertainties in the constants of a linear transformation to a system analogous to the present CIE system. The formulas for variances and covariances after linear transformation are given, for a rigorous empirically-based choice, and also for an arbitrary choice of transformation constants. The complete standard observer system for every 10 mμ consisting of means, variances, and covariances derived from an arbitrary transformation, is listed. The between-observer variabilities are found to be about 10 percent of the averages of the color-mixture data and the average ratio of the between-observer variabilities to the within-observer variabilities is found to be about 5.7.

1. Introduction

Since 1931 the International Commission on Illumination has recommended the use of a Standard Observer System for Colorimetry [1].¹ This system defines the manner in which spectral data for materials are to be reduced to three numbers, called tristimulus values, that describe colors of emitted, reflected, or transmitted lights. The defining equations for these tristimulus values are:

$$X \equiv \int_0^\infty \bar{x}_\lambda N_\lambda T_\lambda d\lambda \doteq \sum_0^\infty \bar{x}_\lambda N_\lambda T_\lambda \Delta_\lambda$$

$$Y \equiv \int_0^\infty \bar{y}_\lambda N_\lambda T_\lambda d\lambda \doteq \sum_0^\infty \bar{y}_\lambda N_\lambda T_\lambda \Delta_\lambda$$

$$Z \equiv \int_0^\infty \bar{z}_\lambda N_\lambda T_\lambda d\lambda \doteq \sum_0^\infty \bar{z}_\lambda N_\lambda T_\lambda \Delta_\lambda$$

The quantities \bar{x}_λ , \bar{y}_λ , and \bar{z}_λ are called spectral tristimulus values and are intended to be descriptive of the spectral-light response of the average human observer with normal color vision. The quantity N_λ describes the spectral emittance of light sources and the quantity T_λ describes the spectral character of the reflecting or transmitting materials.

Tristimulus values are usually reduced to chromaticity coordinates by the equations:

$$x = X/S, y = Y/S, \text{ and } z = Z/S,$$

where S is the sum of the tristimulus values X , Y , and Z . As \bar{x}_λ , \bar{y}_λ , \bar{z}_λ , N_λ , and T_λ are measured

quantities, they are subject to measurement uncertainty. Nimeroff [2,3] has treated, by means of propagation of error theory, the manner in which variabilities in T_λ and in N_λ affect the chromaticity coordinates, x , y , and z .

The general problem and several special cases of propagation of errors in tristimulus colorimetry have been treated by Nimeroff [3]. In that treatment the mean spectral tristimulus values, \bar{x} , \bar{y} , and \bar{z} , were estimated by averaging the mean CIE (17 observers) and mean Stiles' 2°- and 10°-field pilot data (10 observers each). The variances in these values were estimated in the usual manner by using deviations of these three mean data from the estimated overall mean values; the covariances were ignored. The variances as well as the covariances should, however, be more fundamentally estimated; that is, they should be estimated from differences among color-mixture functions of individual observers. Such data became available in 1959. This paper describes how this fundamental estimation of the between-observer variances and covariances may be made for the 10°-field color-mixture data of the 53 observers of Stiles-Burch [4] and the 27 observers of Speranskaya [5], and gives estimates of the average within-observer variances and covariances of two observers, one with 4 and the other with 5 replications. The estimates of covariances are developed on the basis of the data of the 53 observers of Stiles-Burch.

2. Statistical Model

Fundamental color-matching data are obtained on a device where an observer is presented two fields which he is asked to color-match, by adjusting the

¹ Figures in brackets indicate the literature references at the end of this paper.

amounts of three primary colors. In one field, there is a fixed amount, E_λ , of a given spectral color of wavelength λ . The three adjustable primaries having wavelengths 645.2, 526.3, and 444.4 m μ may be denoted R , G , B , respectively. One is added to the field containing the given color, and the other two are mixed in the second field. If R_λ is the amount of the Red (R) primary (in energy units) used in matching E_λ , and G_λ and B_λ are similarly defined, then the condition of color-matching may be expressed:

$$E_\lambda \stackrel{m}{=} R_\lambda + G_\lambda + B_\lambda, \quad (1)$$

where the primary which was mixed with the given color is represented by a negative quantity on the right-hand side of this symbolic equation.

The complete set of color-mixture data for a single observer consists of the amounts of the three primaries used in matching a series of spectrum colors sampling the entire visible portion. These data are adjusted so that, in suitable units, (R_λ , G_λ , B_λ) represent the amounts used in matching a unit amount (energy) of the spectrum color at wavelength λ .

The empirical results for the i th observer may be denoted ($R_{\lambda i}$, $G_{\lambda i}$, $B_{\lambda i}$). Omitting the subscript λ we consider the following model:

$$\begin{aligned} R_i &= R + b_{Ri} + \epsilon_{Ri} \\ G_i &= G + b_{Gi} + \epsilon_{Gi} \\ B_i &= B + b_{Bi} + \epsilon_{Bi}, \end{aligned} \quad (2)$$

where each of R_i , G_i , B_i is represented as the sum of three terms: R , the average amount which would be used in color-matching in a population of observers with normal color vision; the "bias" b_{Ri} of the i th observer; and an "error" ϵ_{Ri} . The bias b_{Ri} is the amount by which the i th observer's "true" average differs from the population average R . The error ϵ_{Ri} is the difference between a given observation R_i and the i th observer's true average which is ($R + b_{Ri}$). The terms of the second and third eqs of (2) are similarly defined. The expected value (true average) of error ϵ_i is assumed to be zero; thus, for the i th observer,

$$\begin{aligned} E_i[R_i] &= R + b_{Ri} \\ E_i[G_i] &= G + b_{Gi} \\ E_i[B_i] &= B + b_{Bi} \end{aligned}$$

where E_i denotes the operation of averaging (taking the expected value) over the hypothetical set of repeated observations by the i th observer.

The "within" variance for the i th observer (for the red primary, say) is the average squared deviation of R_i from $E_i[R_i]$,

$$V_i(R_i) = E_i[R_i - E_i[R_i]]^2 = E_i[\epsilon_{Ri}^2].$$

If all observers have the same "within" variance, then this common variance is denoted by ${}_w\sigma_R^2$; when it is desired to specify the dependence of this variance on the wavelength of the spectrum color being matched, the subscript λ is restored and the within variance is denoted by ${}_w\sigma_{R\lambda}^2$.

Now, referring again to eq(2), the average or expected value of R_i over the whole population of observers is R ; $E[R_i] = R$, where E (without subscript i) denotes the operation of averaging over the population of observers. Therefore, it follows that $E[b_{Ri}] = 0$.

The "between" variance, in the population of observers, is the average squared deviation of the i th observer's average ($R + b_{Ri}$) from the population average R ,

$${}_b\sigma_R^2 = E[b_{Ri}^2].$$

It is assumed that the error ϵ_{Ri} is statistically independent of the observer bias b_{Ri} . Thus, the total variance σ_R^2 of the observed value R_i for one observation by one observer is the sum of the within and between variances, and similarly for the green and blue primaries:

$$\begin{aligned} \sigma_R^2 &= {}_b\sigma_R^2 + {}_w\sigma_R^2 \\ \sigma_G^2 &= {}_b\sigma_G^2 + {}_w\sigma_G^2 \\ \sigma_B^2 &= {}_b\sigma_B^2 + {}_w\sigma_B^2. \end{aligned} \quad (3)$$

Although it is perhaps reasonable to assume that an observer's "errors" are not correlated with his "biases", the analyses of empirical data below will show that it cannot be assumed that the three observations R_i , G_i , B_i for the i th observer are independent. Accordingly, it is necessary to know the covariances. The "between" covariances are:

$$E[b_{Ri}b_{Gi}], E[b_{Ri}b_{Bi}], E[b_{Gi}b_{Bi}].$$

The "within" covariances are:

$$C_i(R_i, G_i) = E_i[(R_i - E_i[R_i])(G_i - E_i[G_i])] = E_i[\epsilon_{Ri}\epsilon_{Gi}].$$

When we assume that the within covariances are equal for all observers, we may then write the total covariances as:

$$\begin{aligned} C(R, G) &= {}_b\sigma_{RG} + {}_w\sigma_{RG}, \\ C(R, B) &= {}_b\sigma_{RB} + {}_w\sigma_{RB}, \\ C(G, B) &= {}_b\sigma_{GB} + {}_w\sigma_{GB}, \end{aligned} \quad (4)$$

where, for example,

$${}_b\sigma_{RG} = E[b_{Ri}b_{Gi}]$$

and

$${}_w\sigma_{RG} = E_i[\epsilon_{Ri}\epsilon_{Gi}].$$

No attempt will be made in this paper to estimate the within and between covariances separately; estimates are given for the total correlations such as ρ_{RG} , defined by eqs (3), (4), and

$$C(R, G) = \rho_{RG}\sigma_R\sigma_G \quad (5)$$

The total variances and covariances are estimated in the usual way. For example, the estimate for σ_R^2 is

$$\hat{\sigma}_R^2 = \sum_{i=1}^n (R_i - \bar{R})^2 / (n-1)$$

where n is the number of observers and \bar{R} is the average of the R_i for the n observers.

3. Rigorous Transformation to a System Analogous to CIE

Spectral tristimulus values, \bar{r} , \bar{g} , \bar{b} , have been derived from measured color-matching data of the primaries, wavelengths 645.2, 526.3, and 444.4 m μ . An empirically-based system of spectral tristimulus values, \bar{x} , \bar{y} , \bar{z} , analogous to the present CIE system, may be derived from these tristimulus values, \bar{r} , \bar{g} , \bar{b} , by means of the eqs (6) below. If we wish to locate the chromaticity coordinates of the equal-energy point (x_n , y_n , z_n) at the point (1/3, 1/3, 1/3) we must set a restriction that $\Sigma \bar{x}_\lambda = \Sigma \bar{y}_\lambda = \Sigma \bar{z}_\lambda$. This is the purpose of the adjustment factors, f_x , f_y , and f_z , respectively.

$$\begin{aligned}\bar{x}_\lambda &= f_x(k_1 \bar{r}_\lambda + k_2 \bar{g}_\lambda + k_3 \bar{b}_\lambda) = f_x \bar{x}'_\lambda \\ \bar{y}_\lambda &= f_y(k_4 \bar{r}_\lambda + k_5 \bar{g}_\lambda + k_6 \bar{b}_\lambda) = f_y \bar{y}'_\lambda \\ \bar{z}_\lambda &= f_z(k_7 \bar{r}_\lambda + k_8 \bar{g}_\lambda + k_9 \bar{b}_\lambda) = f_z \bar{z}'_\lambda.\end{aligned}\quad (6)$$

We may impose two additional restrictions on the system:

1. That the tristimulus values be not less than zero.
2. That no one of the functions, \bar{x} , \bar{y} , \bar{z} , be equivalent to a linear combination of the other two. This restriction is established if the determinant of coefficients k_i does not vanish, that is,

$$\begin{vmatrix} k_1 & k_2 & k_3 \\ k_4 & k_5 & k_6 \\ k_7 & k_8 & k_9 \end{vmatrix} \neq 0. \quad (6a)$$

If all the luminosity, V_λ , is placed in the \bar{y}_λ primary, and if f_y is set equal to 1, then k_4 , k_5 , and k_6 are determined by

$$V_\lambda = \bar{y}'_\lambda = L_r \bar{r}_\lambda + L_g \bar{g}_\lambda + L_b \bar{b}_\lambda = k_4 \bar{r}_\lambda + k_5 \bar{g}_\lambda + k_6 \bar{b}_\lambda \quad (7)$$

thereby making $k_4 = L_r$, $k_5 = L_g$, and $k_6 = L_b$. The symbols L_r , L_g , and L_b are the luminous units of the \bar{r} \bar{g} \bar{b} system.

The solutions for the k_i 's in the \bar{x} and \bar{z} spectral tristimulus functions are not obtained as directly as those for the k_i 's in the \bar{y} function. Equations (8) to (8e) below show how a solution of the k_i 's in the \bar{x} function may be obtained.

The \bar{x}' equation (eq 8) contains three k_i 's to be determined. A unique solution for k_1 , k_2 , and k_3 requires three simultaneous equations. By assigning, tentatively, a value of unity to k_1 (eq 8a), no loss of gen-

erality is encountered that cannot be accounted for by the adjustment factor, f_x , in eq (6).

$$\bar{x}'_\lambda = k_1 \bar{r}_\lambda + k_2 \bar{g}_\lambda + k_3 \bar{b}_\lambda \quad (8)$$

$$\bar{x}'_\lambda = \bar{r}_\lambda + k_2 \bar{g}_\lambda + k_3 \bar{b}_\lambda. \quad (8a)$$

When this is done, only two simultaneous equations are required to solve for the remaining two constants. The equations are set up by selecting, at two wavelengths (λ_1 and λ_2), numerical values for say, \bar{x}_1 and \bar{x}_2 , that will satisfy the requirement that all the tristimulus values be positive. Equation (8a) then becomes

$$\bar{x}'_1 = r_1 + k_2 \bar{g}_1 + k_3 \bar{b}_1 \quad (8b)$$

$$\bar{x}'_2 = r_2 + k_2 \bar{g}_2 + k_3 \bar{b}_2 \quad (8c)$$

and the solutions for k_2 and k_3 are given by:

$$k_2 = [(\bar{x}'_1 - r_1) \bar{b}_2 - (\bar{x}'_2 - r_2) \bar{b}_1] / (\bar{g}_1 \bar{b}_2 - \bar{g}_2 \bar{b}_1) \quad (8d)$$

$$k_3 = [(\bar{x}'_2 - r_2) \bar{g}_1 - (\bar{x}'_1 - r_1) \bar{g}_2] / (\bar{g}_1 \bar{b}_2 - \bar{g}_2 \bar{b}_1). \quad (8e)$$

The constants k_i of the \bar{z} function are solved in a similar manner as shown by eqs (9) to (9e):

$$\bar{z}'_\lambda = k_7 \bar{r}_\lambda + k_8 \bar{g}_\lambda + k_9 \bar{b}_\lambda \quad (9)$$

$$\bar{z}'_\lambda = k_7 \bar{r}_\lambda + k_8 \bar{g}_\lambda + \bar{b}_\lambda \quad (9a)$$

$$\bar{z}'_3 = k_7 \bar{r}_3 + k_8 \bar{g}_3 + \bar{b}_3 \quad (9b)$$

$$\bar{z}'_4 = k_7 \bar{r}_4 + k_8 \bar{g}_4 + \bar{b}_4 \quad (9c)$$

$$k_7 = [(\bar{z}'_3 - \bar{b}_3) \bar{g}_4 - (\bar{z}'_4 - \bar{b}_4) \bar{g}_3] / (\bar{r}_3 \bar{g}_4 - \bar{r}_4 \bar{g}_3) \quad (9d)$$

$$k_8 = [(\bar{z}'_4 - \bar{b}_4) \bar{r}_3 - (\bar{z}'_3 - \bar{b}_3) \bar{r}_4] / (\bar{r}_3 \bar{g}_4 - \bar{r}_4 \bar{g}_3). \quad (9e)$$

4. Variances and Covariances in a Rigorous Transformation

In general, variances and covariances based on propagation of error theory are:

$$\sigma_U^2 = \sum_{n=1}^k \left(\frac{\partial U}{\partial v_n} \right)^2 \sigma_{v_n}^2 + \sum_{m \neq n} \left(\frac{\partial U}{\partial v_n} \right) \left(\frac{\partial U}{\partial v_m} \right) \sigma_{v_m v_n} \quad (10)$$

$$\sigma_{UV} = \sum_{n=1}^k \left(\frac{\partial U}{\partial v_n} \right) \left(\frac{\partial V}{\partial v_n} \right) \sigma_{v_n}^2 + \sum_{m \neq n} \left(\frac{\partial U}{\partial v_m} \right) \left(\frac{\partial V}{\partial v_n} \right) \sigma_{v_m v_n} \quad (11)$$

where U and V are functions of (v_1, \dots, v_k), and the partial derivatives are understood to be evaluated at the average values of these variables. The variances of v_1, \dots, v_k are $\sigma_{v_1}^2, \dots, \sigma_{v_k}^2$ and $\sigma_{v_m v_n}$ denotes the covariance of v_n, v_m .

As \bar{x} is a function of 9 variables ($\bar{r}, \bar{g}, \bar{b}, \bar{r}_1, \bar{g}_1, \bar{b}_1, \bar{r}_2, \bar{g}_2, \bar{b}_2$), \bar{y} is a function of 6 variables ($\bar{r}, \bar{g}, \bar{b}, L_r, L_g, L_b$), and \bar{z} is a function of 9 variables ($\bar{r}, \bar{g}, \bar{b}, \bar{r}_3, \bar{g}_3,$

$\bar{b}_3, \bar{r}_4, \bar{g}_4, \bar{b}_4$), the numbers of terms required by our problem can be counted readily. The numbers of terms in the variance equations are shown in table 1, and the numbers of terms in the covariance equations are shown in table 2.

TABLE 1. Number of terms in the variance equations

Variables	Variance terms	Covariance terms	Total
6	6	15	21
9	9	36	45

TABLE 2. Number of terms in the covariance equations

Variables		Terms
n	m	
9	6	54
9	9	81

Thus, for every wavelength there are required in a rigorous transformation from $\bar{r}\bar{g}\bar{b}$ to $\bar{x}\bar{y}\bar{z}$, $45+21+45=11$ terms for the variances and $54+81+54=189$ terms for the covariances, or a total of 300 terms for each wavelength. If the computation of the variances and covariances is performed at every $1\text{ m}\mu$ step, as the $\bar{x}, \bar{y}, \bar{z}$ functions have been derived by Judd [6], from 360 to 830 $\text{m}\mu$ or at 471 wavelengths, some 141,300 terms would need to be computed.

Observe further that eqs (10) and (11) would include covariances between observations belonging to different spectral colors. That is, the possible correlation between (for example) \bar{g}_3 and \bar{g}_4 would be included in eq (10) and between \bar{b}_1 and \bar{b}_3 in eq (11).

5. Variances and Covariances in an Arbitrary Transformation

The foregoing procedure is completely general and rigorous for the conditions where empirical data are asked to indicate the choice of transformation constants, k_i , for a transformation to a system analogous to the present CIE system. Where one wishes to make an arbitrary choice of transformation constants he indicates that the constants are without error or correlation with other variables.

In an arbitrary transformation the variances and covariances involving k_i are chosen equal to zero thus:

$$V(k_i) = C(k_i, k_j) = C(k_i, v_i) = 0 \quad (12)$$

where v_i denotes any of the variables, $\bar{r}, \bar{g}, \bar{b}, L$, with various subscripts. Thus in the transformation,

$$\left. \begin{aligned} \bar{x}_\lambda &= k_1 \bar{r}_\lambda + k_2 \bar{g}_\lambda + k_3 \bar{b}_\lambda \\ \bar{y}_\lambda &= k_4 \bar{r}_\lambda + k_5 \bar{g}_\lambda + k_6 \bar{b}_\lambda \\ \bar{z}_\lambda &= k_7 \bar{r}_\lambda + k_8 \bar{g}_\lambda + k_9 \bar{b}_\lambda \end{aligned} \right\} \quad (13)$$

the variance equations are:

$$\left. \begin{aligned} V(\bar{x}) &= k_1^2 V(\bar{r}) + k_2^2 V(\bar{g}) + k_3^2 V(\bar{b}) \\ &\quad + 2[k_1 k_2 C(\bar{r}, \bar{g}) + k_1 k_3 C(\bar{r}, \bar{b}) + k_2 k_3 C(\bar{g}, \bar{b})] \\ V(\bar{y}) &= k_4^2 V(\bar{r}) + k_5^2 V(\bar{g}) + k_6^2 V(\bar{b}) \\ &\quad + 2[k_4 k_5 C(\bar{r}, \bar{g}) + k_4 k_6 C(\bar{r}, \bar{b}) + k_5 k_6 C(\bar{g}, \bar{b})] \\ V(\bar{z}) &= k_7^2 V(\bar{r}) + k_8^2 V(\bar{g}) + k_9^2 V(\bar{b}) \\ &\quad + 2[k_7 k_8 C(\bar{r}, \bar{g}) + k_7 k_9 C(\bar{r}, \bar{b}) + k_8 k_9 C(\bar{g}, \bar{b})] \end{aligned} \right\} \quad (14)$$

$$\left. \begin{aligned} C(\bar{x}, \bar{y}) &= k_1 k_4 V(\bar{r}) + k_2 k_5 V(\bar{g}) + k_3 k_6 V(\bar{b}) + (k_1 k_5 + k_2 k_4) C(\bar{r}, \bar{g}) + (k_1 k_6 + k_3 k_4) C(\bar{r}, \bar{b}) + (k_2 k_6 + k_3 k_5) C(\bar{g}, \bar{b}) \\ C(\bar{x}, \bar{z}) &= k_1 k_7 V(\bar{r}) + k_2 k_8 V(\bar{g}) + k_3 k_9 V(\bar{b}) + (k_1 k_8 + k_2 k_7) C(\bar{r}, \bar{g}) + (k_1 k_9 + k_3 k_7) C(\bar{r}, \bar{b}) + (k_2 k_9 + k_3 k_8) C(\bar{g}, \bar{b}) \\ C(\bar{y}, \bar{z}) &= k_4 k_7 V(\bar{r}) + k_5 k_8 V(\bar{g}) + k_6 k_9 V(\bar{b}) + (k_4 k_8 + k_5 k_7) C(\bar{r}, \bar{g}) + (k_4 k_9 + k_6 k_7) C(\bar{r}, \bar{b}) + (k_5 k_9 + k_6 k_8) C(\bar{g}, \bar{b}) \end{aligned} \right\} \quad (15)$$

To estimate these variances and covariances from the data obtained in color-matching by n observers, the following formulas are used:

$$\left. \begin{aligned} \hat{V}(\bar{r}) &= \sum_{i=1}^n (\bar{r}_i - \hat{\mu}_r)^2 / (n-1) = \hat{\sigma}_r^2 \\ \hat{V}(\bar{g}) &= \sum_{i=1}^n (\bar{g}_i - \hat{\mu}_g)^2 / (n-1) = \hat{\sigma}_g^2 \\ \hat{V}(\bar{b}) &= \sum_{i=1}^n (\bar{b}_i - \hat{\mu}_b)^2 / (n-1) = \hat{\sigma}_b^2 \end{aligned} \right\} \quad (16)$$

$$\left. \begin{aligned} \hat{C}(\bar{r}, \bar{g}) &= \sum_{i=1}^n (\bar{r}_i - \hat{\mu}_r)(\bar{g}_i - \hat{\mu}_g) / (n-1) = \hat{\sigma}_{rg} \\ \hat{C}(\bar{r}, \bar{b}) &= \sum_{i=1}^n (\bar{r}_i - \hat{\mu}_r)(\bar{b}_i - \hat{\mu}_b) / (n-1) = \hat{\sigma}_{rb} \\ \hat{C}(\bar{g}, \bar{b}) &= \sum_{i=1}^n (\bar{g}_i - \hat{\mu}_g)(\bar{b}_i - \hat{\mu}_b) / (n-1) = \hat{\sigma}_{gb} \end{aligned} \right\} \quad (17)$$

and

$$\hat{\mu}_r = \sum_{i=1}^n \bar{r}_i / n, \quad \hat{\mu}_g = \sum_{i=1}^n \bar{g}_i / n, \quad \hat{\mu}_b = \sum_{i=1}^n \bar{b}_i / n.$$

The transformation equations derived by Kelly and Judd [6] and being considered for recommendation by the CIE for a 10°-field standard observer system for colorimetry are:

$$\left. \begin{aligned} \bar{x}_{10} &= 0.341080 \bar{r}_{10} + 0.189145 \bar{g}_{10} + 0.387529 \bar{b}_{10} \\ \bar{y}_{10} &= 0.139058 \bar{r}_{10} + 0.837460 \bar{g}_{10} + 0.073316 \bar{b}_{10} \\ \bar{z}_{10} &= 0.000000 \bar{r}_{10} + 0.039553 \bar{g}_{10} + 2.026200 \bar{b}_{10} \end{aligned} \right\} \quad (18)$$

As the constants, k_i , of these equations were derived in a very complex way, the transformation will be treated as an arbitrary one.

6. Variability of Color-Mixture Data

The results of a statistical analysis of the Stiles-Burch color-matching data may be useful in determining the variances and covariances for the transformation to the proposed new CIE standard observer system. The Stiles-Burch data were analyzed because in the development of the new system they are being given greater weight than those of other investigators and because they are available in terms of individual observer data. Such an analysis was reported by Nimeroff, Rosenblatt, and Danne-miller [7] at the October 1959 meeting of the Optical Society of America and the results are given below.

The Stiles-Burch data were obtained under two experimental conditions. For condition I, 24 observers were used, while for condition II, 29 observers were employed. These two conditions differ in that the blue instrumental primary of condition I was located at 445.4 mμ (22,450 cm⁻¹) while that of condition II was located at 470.6 mμ (21,250 cm⁻¹). The data taken under these two conditions were transformed by Stiles and Burch to an $\bar{r}\bar{g}\bar{b}$ system in which the blue primary is located at 444.4 mμ. To test whether these two sets of transformed data are significantly different from each other the Hotelling T^2 test [8] was used. The T^2 test provides a comparison between the two sets of averages \bar{p}_1 , representing $\bar{r}_1, \bar{g}_1, \bar{b}_1$, of condition I and \bar{p}_2 , representing $\bar{r}_2, \bar{g}_2, \bar{b}_2$, of condition II in which the three correlated differences between corresponding averages are evaluated simultaneously against the variability present in the data from which the averages were estimated. In this test a high value of T^2 implies that the differences between the averages compared are large. The T^2 statistic in quadratic form for our problem of three variables in each of two conditions for N_1 and N_2 observers is:

$$T^2 = \frac{N_1 N_2}{N_1 + N_2} \sum_{i,j=1}^3 S^{ij} (\bar{p}_{1i} - \bar{p}_{2i}) (\bar{p}_{1j} - \bar{p}_{2j}), \quad (19)$$

where S^{ij} is the (i,j) element of the inverse of the three by three matrix with elements

$$S_{ij} = \frac{1}{N_1 + N_2 - 2} \left[\sum_{k=1}^{N_1} (\bar{p}_{1ik} - \bar{p}_{1i}) (\bar{p}_{1jk} - \bar{p}_{1j}) + \sum_{k=1}^{N_2} (\bar{p}_{2ik} - \bar{p}_{2i}) (\bar{p}_{2jk} - \bar{p}_{2j}) \right] \quad (20)$$

The subscripts 1 and 2 refer to the two conditions, i and j refer to the three primaries compared two at a time, and k identifies the observer for each condition.

The results of this test are plotted on figure 1 and indicate that the data obtained by Stiles under the two experimental conditions are in general statistically different even at the 2.5 percent level of significance, the exceptions lying between 470 and 580 mμ. The difference may result from either an improper evaluation of the transformations of the two sets of data to a common primary system or an inherent dependence on the choice of instrumental primary. This latter supposition, if true, would invalidate any attempt to make the data obtained under the two sets of conditions comparable through such transformation. The differences between conditions I and II, while statistically significant, are in practical terms small.

In spite of these differences the CIE is proceeding to average these data as well as those of Speranskaya for 27 observers, to derive the 10°-field tristimulus functions, $\bar{x}\bar{y}\bar{z}$. Figure 2 shows the averaged \bar{r} function with its three lobes, two positive and one negative. Also shown are the estimates of between-observer standard deviations for the \bar{r} function, based on the Stiles-Burch combined data and the Speranskaya data. Note that the standard deviations of these independent investigations are essentially of the same order of magnitude and range approximately from 10 to 20 percent of the mean function.

Strictly, the estimates shown in figure 2 (likewise figs. 3, 4) are estimates of the total standard deviation (see eq (3)). As the within-observer variances are relatively small, the estimated between-observer variances are only slightly over-stated. A satisfactory estimate of between-observer variance alone can be made only when the data include repeated observations by many (if not all) observers.

The lower curve gives the estimated within-observer standard deviations for the \bar{r} function.

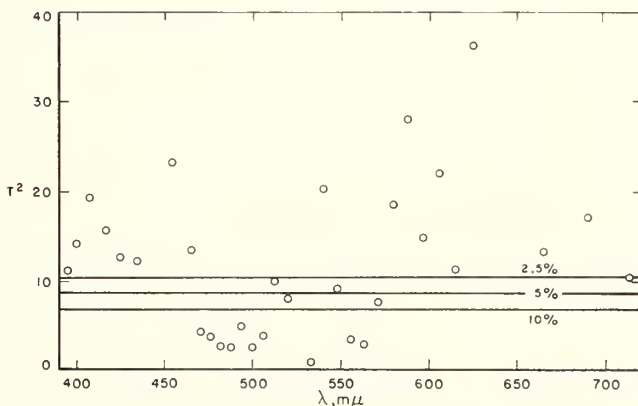


FIGURE 1. Values of the T^2 statistic for evaluating differences between Stiles' data obtained under condition I and under condition II.

Critical values of T^2 at three probability levels are indicated, showing that even at the 2.5% level the results obtained under the two conditions are for the most part significantly different.

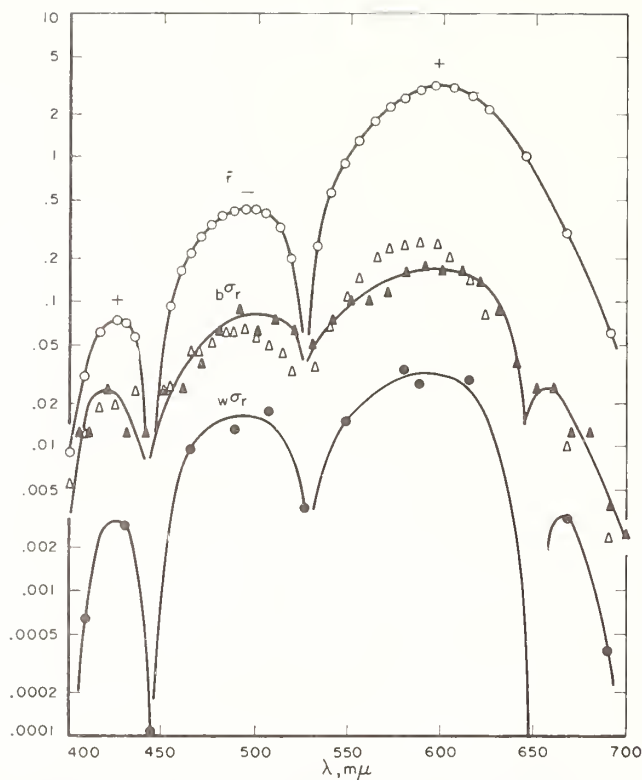


FIGURE 2. Means and variability of the red-primary data.
Means \circ ; standard deviations between observers ($b\sigma_r$): Stiles Δ , Speranskaya \blacktriangle ; standard deviations within observer ($w\sigma_r$).

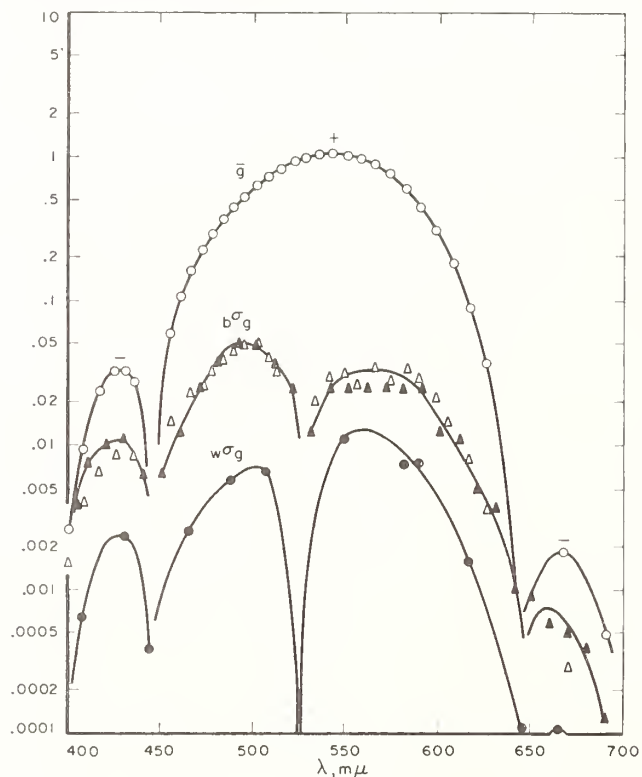


FIGURE 3. Means and variability of the green-primary data.
Means \circ ; standard deviations between observers ($b\sigma_g$): Stiles Δ , Speranskaya \blacktriangle ; standard deviations within observer ($w\sigma_g$).

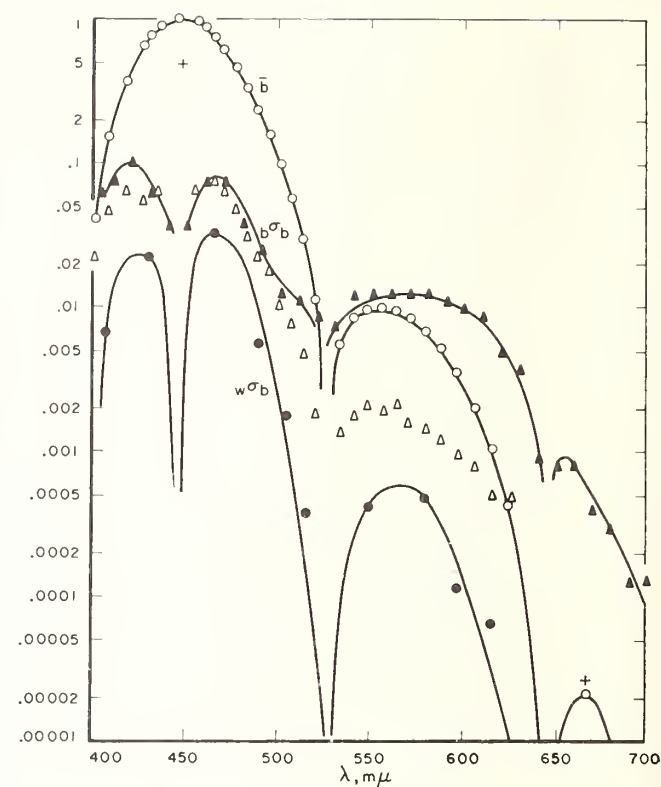


FIGURE 4. Means and variability of the blue-primary data.

Means \circ ; standard deviation between observers ($b\sigma_b$): Stiles Δ , Speranskaya \blacktriangle ; standard deviation within observer ($w\sigma_b$).

These data are based on the average of individual variability of two observers, one repeating his measurements four times and the other repeating his measurements five times. As might be expected, the within-observer standard deviations are approximately an order of magnitude lower than the between-observer standard deviations. Note that because settings in matches of the primaries themselves are assumed to be free from error the curves for the standard deviations are extended so as to approach zero at the location of the $\bar{r}\bar{g}\bar{b}$ primaries, that is at wavelengths 444.4, 526.3, and 645.2 $m\mu$.

Figure 3 shows the same kind of information for the \bar{g} function with its three lobes, two negative and one positive. Here also there is the same agreement between the Stiles-Burch data and the Speranskaya data; the within standard deviations are about an order of magnitude lower than the between standard deviations. The standard deviations approach zero at the location of the primaries.

Figure 4 shows the data for the \bar{b} function. Here there is a disparity between the Stiles-Burch and the Speranskaya data in the longwave region. This disparity may result from the transformation from the instrumental primaries used in the Speranskaya investigation to those of the $\bar{r}\bar{g}\bar{b}$ system. It is more likely, however, that the procedure used by Stiles, that of employing for longwave matches a yellow

primary at 588.2 rather than a green instrumental primary, is responsible for the lower variability of the \bar{b} function in this region.

On figure 5 are shown averages of the between standard deviations derived from the data of the 53 Stiles-Burch observers and the data of the 27 Speranskaya observers. In the longwave (640 m μ and beyond) region of the \bar{b} function the averages are based on the same weights used by Judd in deriving the mean \bar{b} function itself from the British and Russian data. The weights for the British data are greater than those for the Russian data and increase with wavelength.

It was noted above that the within standard deviations are about an order of magnitude lower than the between standard deviations. We are interested in learning whether the ratio of between to within standard deviations for the three functions can be represented by a single constant independent of wavelength and, if so, in determining the magnitude of such a constant. The computation of within-observer variability in a transformation to an $\bar{x}\bar{y}\bar{z}$ system would be simplified were this permissible. Figure 6 shows these ratios for the three color-mixture functions. The overall average ratio was derived from the geometric mean and is 5.7 in the spectral region 400 to 700 m μ . The geometric mean was used because of the wide disparity between the extreme values, namely 2.0 and 30.9. We consider this value reasonably representative of the ratio of between to within standard deviations, as better than 80 percent of the ratios lie between half and twice 5.7.

In the transformations from color-mixture functions, $\bar{r}\bar{g}\bar{b}$, to tristimulus values, $\bar{x}\bar{y}\bar{z}$, covariances as well as variances need to be known. The correlation coefficient, ρ_{ij} , relates the covariance, σ_{ij} , to the variances, σ_i^2, σ_j^2 , thus:

$$\rho_{ij} = \sigma_{ij} / \sigma_i \sigma_j.$$

On figure 7 are plotted the correlation coefficients, $\rho_{\bar{r}\bar{g}}, \rho_{\bar{r}\bar{b}}$, and $\rho_{\bar{g}\bar{b}}$. In these data, a positive correlation coefficient implies that if an observer uses more or less than average of one instrumental primary in making a match he uses more or less, respectively, than average of the other primary, also. Conversely, a negative correlation coefficient implies that if more or less than average of one primary is used, less or more, respectively, than average of the other primary is used. We note that $\rho_{\bar{r}\bar{g}}$ is always positive or zero, while $\rho_{\bar{r}\bar{b}}$ is almost always negative or zero. On the other hand, $\rho_{\bar{g}\bar{b}}$ is negative in the shortwave spectral region, 380 to 590 m μ , and positive in the longwave region, 600 to 720 m μ .

Approximately two-thirds of the correlation coefficients are significantly different from zero at the 5 percent level of significance. The general consistency of sign and magnitude among values of the correlation coefficients at adjacent wavelengths provides further confirmation for the general pattern of values shown. Although some of the correlation coefficients are small in magnitude they should not be

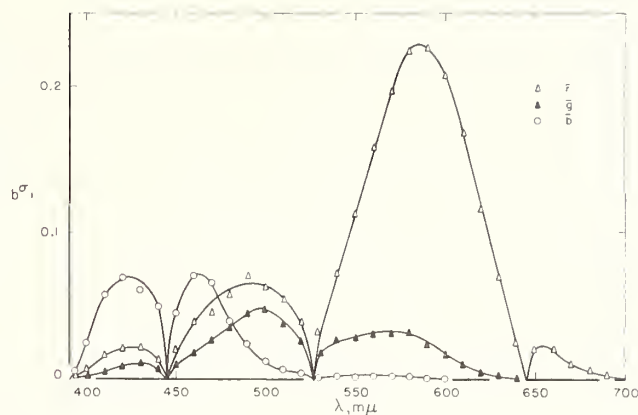


FIGURE 5. Mean standard deviation between observers for the Stiles-Burch and the Speranskaya data.

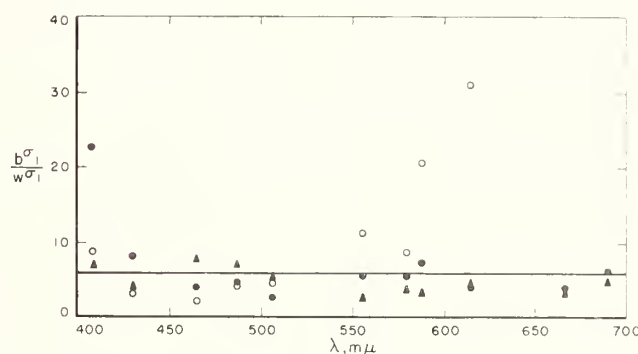


FIGURE 6. Ratio of the between to within standard deviations, for the three primaries, \bar{r} (●), \bar{g} (▲), and \bar{b} (○), showing overall average ratio at 5.7.

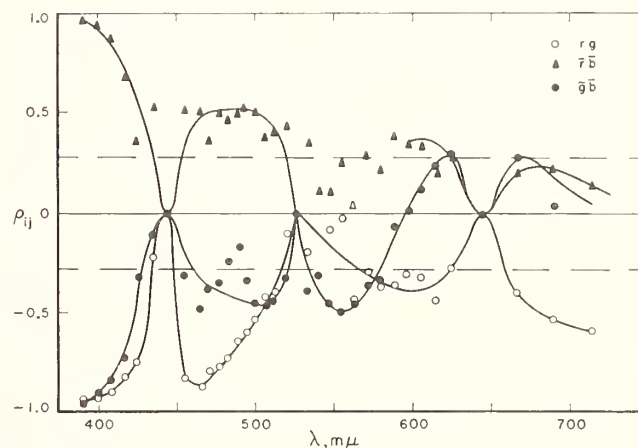


FIGURE 7. Correlation coefficients, ρ_{ij} , for ij equal to $\bar{r}\bar{g}$ (○), $\bar{r}\bar{b}$ (▲), $\bar{g}\bar{b}$ (●).

Dashed lines define the region within which these coefficients are not (statistically) significantly different from zero at the 5% level.

neglected in determining the variances and covariances in the $\bar{x}\bar{y}\bar{z}$ system transformed from the color-mixture data.

A few qualifications to the interpretation of the statistical analyses should be mentioned. First, statements concerning "statistical significance" are based on the assumption that the observations are normally distributed.

There has been reported, however, by Judd [9] for 7 observers and by Wyszecki [10] for 10 observers, some evidence that the population of observers with normal color vision may be bimodally distributed. A limited analysis of the Stiles-Burch data for the 53 observers was undertaken to see if bimodality is revealed in their individual observer data. This analysis showed (see for example fig. 8 for R_{500} against G_{500}) that although the distribution may be slightly skewed, there is insufficient data to conclude the existence of any bimodality. If more extensive data should lead to a reliable statement that the distribution of the population is bimodal, two standard observer systems might be required, one for each group.

A second qualification is required since possible correlations among observations for different spectral colors have been ignored because of the arbitrary choice of constants in the transformation. For example, the presence of correlation could account in part for the apparently consistent difference between conditions I and II, as shown in figure 1.

7. Complete Standard Observer System

A complete standard observer system should contain not only the mean spectral tristimulus functions, \bar{x}_λ , \bar{y}_λ , \bar{z}_λ , derived from the color mixture data, but should contain, also, the variances and covariances

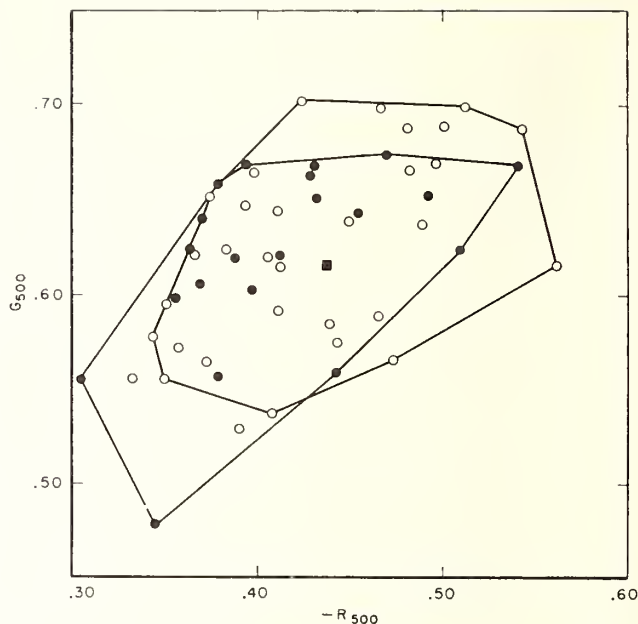


FIGURE 8. Plot of R_{500} against G_{500} for Stiles' individual observers indicating no bimodality and very little skewness.

Condition I (●), condition II (○), and mean (■). Straight lines join outlying values obtained under the two conditions.

of these functions as derived from the within- and between-observer variability of the color mixture data.

Table 3 lists, for every 10 mμ, the spectral tristimulus values of the proposed standard observer system for colorimetry and the estimated total vari-

TABLE 3. Proposed CIE tristimulus functions for 10°-field and between variances and covariances in $\bar{x}, \bar{y}, \bar{z}$

λ (mμ)	\bar{x}_{10}	\bar{y}_{10}	\bar{z}_{10}	$V(\bar{x})$	$V(\bar{y})$	$V(\bar{z})$	$C(\bar{x}, \bar{y})$	$C(\bar{x}, \bar{z})$	$C(\bar{y}, \bar{z})$
400	0.0191097	0.0020044	0.0860109	0.000126	0.00000118	0.00256	+0.0000104	+0.000568	+0.0000467
10	.084736	.008756	.389366	.000661	.0000110	.0132	+.0000708	+.00294	+.000321
20	.204492	.021391	.972542	.000937	.0000262	.0193	+.0000731	+.00421	+.000350
30	.314679	.028676	1.55348	.000737	.0000671	.0154	+.000106	+.00329	+.000509
40	.383734	.062077	1.96728	.000385	.0000448	.00978	+.0000816	+.00189	+.000355
450	.370702	.089456	1.99480	.000353	.0000561	.00815	+.0000413	+.00161	+.000185
60	.362273	.128201	1.74537	.00109	.0000995	.0206	+.0000151	+.00454	+.000136
70	.195618	.185190	1.31756	.00110	.000272	.0180	-.000101	+.00423	-.000131
80	.080507	.253589	0.772125	.000716	.000602	.00606	-.000108	+.00182	-.0000827
90	.016172	.339133	.415234	.000675	.00105	.00234	-.0000633	+.000973	+.0000759
500	.003816	.460777	.218502	.000414	.00129	.000593	+.000000553	+.000279	-.000245
10	.037465	.606741	.112044	.000325	.000829	.000190	+.0000884	+.000112	-.0000999
20	.117749	.761757	.060709	.000183	.000457	.0000649	+.000109	+.0000325	-.0000185
30	.236491	.875211	.030451	.000143	.000253	.0000271	+.000109	+.00000547	-.00000204
40	.376772	.961988	.013676	.000622	.000590	.0000431	+.000301	+.0000133	-.0000285
550	.529826	.991761	.003988	.00143	.000668	.000105	+.000528	+.0000531	-.0000700
60	.752224	.997340	.000000	.00283	.000847	.000100	+.000960	+.000102	-.0000513
70	.878655	.955552		.00437	.000998	.0000942	+.00142	+.000147	-.0000107
80	1.01416	.868934		.00113	.000778	.0000778	+.00181	+.000184	+.0000381
90	1.11852	.777405		.00588	.000947	.0000652	+.00194	+.000194	+.0000806
600	1.12399	.658341		.00493	.000731	.0000379	+.00170	+.000148	+.0000720
10	.103048	.527963		.00324	.000475	.0000248	+.00116	+.000102	+.0000518
20	0.856297	.398057		.00159	.000240	.0000123	+.000601	+.0000449	+.0000222
30	.647467	.283493		.000575	.0000918	.00000587	+.000226	+.0000120	+.00000591
40	.431567	.179828		.0000750	.0000127	.000000280	+.0000305	+.000000138	+.000000067
650	.268329	.107633		.0000470	.00000794	.000000050	+.0000191	+.000000052	+.000000032
60	.152568	.060281		.0000458	.00000713	.000000043	+.0000179	+.000000166	+.000000091
70	.0812606	.0318004		.0000129	.00000192	.000000016	+.00000492	+.000000084	+.000000044
80	.0408508	.0159051		.00000427	.000000619	.000000017	+.00000161	+.000000013	+.000000077
90	.0199413	.0077488		.000000894	.000000128	.0000000043	+.000000335	+.0000000030	+.000000016
700	.00957688	.00371774		.000000366	.000000052	.0000000016	+.000000137	+.0000000088	+.0000000049
10	.00455263	.00176847		.000000114	.000000016	.00000000039	+.000000042	+.0000000012	+.0000000086
20	.00217496	.00084619		.000000019	.0000000026	.000000000043	+.0000000069	+.000000000027	+.000000000051

ances, $V(\bar{x})$, $V(\bar{y})$, $V(\bar{z})$, and covariances, $C(\bar{x}, \bar{y})$, $C(\bar{x}, \bar{z})$, $C(\bar{y}, \bar{z})$, for these spectral tristimulus values derived on the basis of an arbitrary transformation. It should be noted that the variances in table 3 are $(5.7)^2$ or 32.5 times those in table 4.

TABLE 4. Within variances in \bar{x} , \bar{y} , \bar{z}

λ (m μ)	$v(\bar{x})$	$v(\bar{y})$	$v(\bar{z})$
400	0.00000390	0.000000364	0.0000758
10	.0000204	.000000338	.000407
20	.0000258	.000000806	.000595
30	.0000227	.000000207	.000475
40	.0000119	.00000138	.000301
450	.0000109	.00000173	.000251
60	.0000337	.00000306	.000635
70	.0000340	.00000837	.000556
80	.0000220	.0000185	.000187
90	.0000208	.0000322	.0000720
500	.0000128	.0000396	.0000183
10	.0000100	.0000255	.00000587
20	.00000562	.0000141	.00000200
30	.00000440	.00000779	.000000834
40	.0000192	.0000182	.00000133
550	.0000441	.0000206	.00000324
60	.0000871	.0000261	.00000309
70	.000135	.0000307	.00000290
80	.000172	.0000347	.00000240
90	.000181	.0000292	.00000201
600	.000152	.0000225	.00000117
10	.0000999	.0000146	.000000764
20	.0000489	.00000740	.000000380
30	.0000177	.00000283	.000000181
40	.00000231	.000000391	.0000000862
650	.00000145	.000000245	.00000000154
60	.00000141	.000000220	.00000000132
70	.000000398	.0000000591	.000000000493
80	.000000131	.0000000191	.0000000000524
90	.0000000275	.00000000394	.0000000000132
700	.0000000113	.00000000160	.00000000000493
10	.00000000351	.000000000493	.00000000000120
20	.000000000585	.0000000000801	.000000000000132

8. Applications of the Complete System

As the individual observer data, on which the 1931 system is based, have been lost in antiquity, this system can be used only to determine what color matches the standard observer would make. A comprehensive analysis of color-mixture data in terms of means, variances, and covariances permits use of the proposed system to determine, in addition, the region

of within and between uncertainties; that is, the extent to which a normal observer tends to make different matches on successive attempts, and the extent to which different normal observers vary one from another.

Some applications of the variances and covariances in the $\bar{x}\bar{y}\bar{z}$ system have been reported by Nimeroff [3] and by Wyszecki [11]. Further applications are sure to be suggested when a complete standard observer system for colorimetry, consisting of means, variances, and covariances, is established.

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Comparison of Uncertainty Ellipses Calculated from Two Spectrophotometric Colorimetry Methods by an Automatic-Computer Program

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The parameters and equations for calculating uncertainty of chromaticity coordinates in spectrophotometric tristimulus colorimetry have already been derived for the 10°-field standard-observer system. From the variability data for eleven optical filters obtained from two methods of spectrophotometric tristimulus colorimetry, the standard-observer-system method and the actual-observer method, two sets of chromaticity uncertainty ellipses are derived and intercompared. The similarity between corresponding ellipses of these two sets leads to the following conclusion: the system-derived uncertainty ellipses, based on the variances and covariances of the spectral tristimulus values of the large-field standard-observer system, can be used to estimate observer-derived chromaticity-uncertainty ellipses.

The equations are the basis of the automatic-computer program and are used to compute uncertainty ellipses; they are given to aid anyone who wishes to arrange such a program in his laboratory. The fixed combinations of the spectral tristimulus values of their variances and covariances can be readily prepared by researchers who may wish to use desk calculators to compute uncertainty ellipses for actual-observer data or for standard-observer-system data, respectively.

INDEX HEADINGS: Color vision; Colorimetry; Computers.

1. INTRODUCTION

THE problem of calculating measures of uncertainty for results obtained in spectrophotometric tristimulus colorimetry has been discussed previously.¹⁻³ Spectrophotometric tristimulus colorimetry may be defined as a computational procedure by which spectrophotometric data of the light source and the intervening medium are used, together with the spectral response characteristics of the observer's visual system, to compute numerical specification of colors. By this definition spectrophotometric tristimulus colorimetry can be achieved by either of two methods. In one, the actual-observer method, spectral distributions of flux, selected by actual observers from a combination of a number of primary light sources, are used with data that represents the trireceptor systems of these observers. In the other, the standard-observer method, a given spectral distribution of flux is used with data that represents a three-function standard-observer system.

In this paper chromaticity uncertainty, represented by an ellipse on the xy chromaticity diagram, will be presented as obtained by the actual-observer method in a study of color matches of eleven optical filters by eleven observers and compared with the corresponding ellipse obtained by the standard-observer method of spectrophotometric tristimulus colorimetry. The two sets of ellipses, computed for the two methods by an automatic-computer program described here, were intercompared to determine if corresponding pairs of ellipses are related. A close relationship is established between chromaticity-coordinate uncertainty obtained by the standard-observer method and that obtained by

the actual-observer method, thereby indicating that color-matching data by actual observers may no longer be required to obtain reliable chromaticity uncertainty data. The variability of spectral tristimulus values, reported by Nimeroff, Rosenblatt, and Dannemiller,³ can be used to estimate these uncertainties.

2. VARIABLES IN SPECTROPHOTOMETRIC TRISTIMULUS COLORIMETRY

In the standard-observer method the spectral responses of the standard observer are those derived from the responses of a sufficiently large number of observers and specified by the Commission Internationale de l'Eclairage (CIE) in 1931⁴ for 2° fields and in 1964⁵ for 10° fields. When data of a large number of actual observers are used, the spectral responses of the individual observers may be represented by the spectral responses of the standard observer.

Spectrophotometric tristimulus colorimetry by either actual-observer method or standard-observer method makes use of equations for computing tristimulus values and chromaticity coordinates. In the equations for tristimulus values,

$$\begin{aligned} X &= \int_0^\infty E_\lambda I_{x\lambda} d\lambda, \\ Y &= \int_0^\infty E_\lambda I_{y\lambda} d\lambda, \\ Z &= \int_0^\infty E_\lambda I_{z\lambda} d\lambda, \end{aligned} \quad (1)$$

¹ I. Nimeroff, *J. Opt. Soc. Am.* 43, 531 (1953).

² I. Nimeroff, *J. Opt. Soc. Am.* 47, 697 (1957).

³ I. Nimeroff, J. R. Rosenblatt, and M. C. Dannemiller, *J. Res. Natl. Bur. Std.* 65A, 475 (1961); also condensed account in *J. Opt. Soc. Am.* 52, 685 (1962).

⁴ Intern. Comm. Illum. Proc. 8th Session, Cambridge (CIE Bureau Central, Paris, 1931) 19.

⁵ Intern. Comm. Illum. Proc. 15th Session, Vienna (CIE Bureau Central, Paris, 1964) 11A, 35.

where E_λ is the spectral irradiance at the receptor. This irradiance is the product of the spectral radiance of the source N_λ and a factor $R(\lambda)$ that is related either to the reflectance or to the transmittance of the medium or media between the source and the receptor, thus $E_\lambda = N_\lambda R(\lambda)$. The symbols $I_{x\lambda}$, $I_{y\lambda}$, and $I_{z\lambda}$ represent the spectral tristimulus values of the colorimetric observer system of the receptor. In standard-observer spectrophotometric tristimulus colorimetry the receptor is represented by a set of three functions based on the response of the average color-normal human eye, while in actual-observer spectrophotometric tristimulus colorimetry the receptor is the eye of the actual observer. The equations for computing the chromaticity coordinates by either method are:

$$\begin{aligned}x &= X/S, \\y &= Y/S, \\z &= Z/S,\end{aligned}\quad (2)$$

where S is the sum of the tristimulus values $X+Y+Z$. When chromaticity coordinates of a test color are obtained by either method the results should be comparable.

The standard-observer method of spectrophotometric colorimetry is applicable directly to the evaluation of tristimulus values of the color of the test field. From the spectral radiance of the source N_λ , the spectral transmittance of the test medium $R(\lambda)$, and the spectral tristimulus values of the standard-observer system $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, the tristimulus values X , Y , and Z can be computed by Eq. (1), thus

$$\begin{aligned}X &= \Sigma_\lambda [N_\lambda R(\lambda) \bar{x}(\lambda)], \\Y &= \Sigma_\lambda [N_\lambda R(\lambda) \bar{y}(\lambda)], \\Z &= \Sigma_\lambda [N_\lambda R(\lambda) \bar{z}(\lambda)].\end{aligned}\quad (3)$$

The Donaldson colorimeter is an instrument from which colorimetric results may be obtained by application of the actual-observer method of spectrophotometric tristimulus colorimetry. This instrument presents to the observer two juxtaposed fields, a test field and a primary-mixture field. The j th observer sees the summation of the spectral irradiance from the test field E_λ equal to the summation of the products $N_\lambda R(\lambda)$ as evaluated by the spectral responses of his eye $I_{x\lambda j}$, $I_{y\lambda j}$, and $I_{z\lambda j}$. The observer then matches the color of this irradiance with the summation of the irradiance $E_{\lambda j}'$ from the instrumental primaries, which consist of a source of spectral radiance $N_{\lambda j}'$ and three shuttered optical filters of spectral factors ${}_bR(\lambda)_j$, ${}_gR(\lambda)_j$, ${}_rR(\lambda)_j$. From the spectral radiance of the primary source, the shuttered transmittance factors of the three primary optical filters selected by the observer, and the spectral response functions of the observer's eye, the tristimulus values may be computed by Eq. (1). When a match is

achieved, the following conditions hold simultaneously:

$$\begin{aligned}X_j &= \Sigma_\lambda [N_\lambda R(\lambda) I_{x\lambda j}]^m \Sigma_\lambda [N_{\lambda j}' ({}_bR(\lambda) \\&\quad + {}_gR(\lambda) + {}_rR(\lambda))_j I_{x\lambda j}], \\Y_j &= \Sigma_\lambda [N_\lambda R(\lambda) I_{y\lambda j}]^m \Sigma_\lambda [N_{\lambda j}' ({}_bR(\lambda) \\&\quad + {}_gR(\lambda) + {}_rR(\lambda))_j I_{y\lambda j}], \\Z_j &= \Sigma_\lambda [N_\lambda R(\lambda) I_{z\lambda j}]^m \Sigma_\lambda [N_{\lambda j}' ({}_bR(\lambda) \\&\quad + {}_gR(\lambda) + {}_rR(\lambda))_j I_{z\lambda j}],\end{aligned}\quad (4)$$

where the symbol m represents a color match and is read, "matches."

The spectral irradiances selected by n individual observers are replaced by the average spectral irradiances

$$\bar{E}_\lambda = \sum_{j=1}^n (E_{\lambda j}')/n,$$

and the spectral responses of the individual observers are replaced by the standard-observer's spectral responses $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$. The statement that the tristimulus values of the average observer's match are equal to the standard-observer's prediction may be written simply as:

$$\begin{aligned}\Sigma_\lambda [E_\lambda \bar{x}(\lambda)] &= \Sigma_\lambda [\bar{E}_\lambda \bar{x}(\lambda)], \\ \Sigma_\lambda [E_\lambda \bar{y}(\lambda)] &= \Sigma_\lambda [\bar{E}_\lambda \bar{y}(\lambda)], \\ \Sigma_\lambda [E_\lambda \bar{z}(\lambda)] &= \Sigma_\lambda [\bar{E}_\lambda \bar{z}(\lambda)].\end{aligned}\quad (5)$$

As the quantities represented by \bar{E}_λ in the actual-observer method of spectrophotometric tristimulus colorimetry and $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ in the standard-observer method are measured quantities, they are subject to measurement errors and uncertainties. The quantities \bar{E}_λ are intended to represent the average of all the observers who make the color matches, no matter how they differ among themselves. The quantities $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are intended to represent all observers with normal color vision, even though observers are known to differ among themselves. Variability of these measured quantities may be represented by their variances $\sigma_{\bar{E}_\lambda}^2$, $\sigma_{\bar{x}_\lambda}^2$, $\sigma_{\bar{y}_\lambda}^2$, $\sigma_{\bar{z}_\lambda}^2$.

3. PROPAGATION OF ERRORS

Nimeroff^{1,2} has applied the propagation-of-error theory to the problem of evaluating the uncertainties in chromaticity coordinates in terms of variances σ_U^2 and covariances σ_{UV} . These are computed from the variances and covariances of the measured quantities by use of equations of the form:

$$\sigma_U^2 = \sum_{n=1}^k \left(\frac{\partial U}{\partial v_n} \right)^2 \sigma_{v_n}^2 + \sum_{m \neq n} \left(\frac{\partial U}{\partial v_n} \right) \left(\frac{\partial U}{\partial v_m} \right) \sigma_{v_m v_n}, \quad (6)$$

$$\sigma_{UV} = \sum_{n=1}^k \left(\frac{\partial U}{\partial v_n} \right) \left(\frac{\partial V}{\partial v_n} \right) \sigma_{v_n}^2 + \sum_{m \neq n} \left(\frac{\partial U}{\partial v_m} \right) \left(\frac{\partial V}{\partial v_n} \right) \sigma_{v_m v_n}, \quad (7)$$

where U and V are functions of the variable measured quantities (v_1, \dots, v_k), and the partial derivatives are understood to be evaluated at the average values of these variable quantities. The variances of these quantities are $\sigma_{v_1}^2, \dots, \sigma_{v_k}^2$; the covariance of v_n and v_m is denoted by $\sigma_{v_m v_n}$.

4. SYSTEM UNCERTAINTY

As the color-matching data of the individual observers on which a standard-observer system is based were found to differ among the observers, the system of spectral tristimulus values derived from these data do have corresponding uncertainties associated with them. These uncertainties are here called "system uncertainties." Consequently, with any chromaticity coordinates that may be computed by the use of these spectral tristimulus values there is associated an uncertainty.

It should be pointed out that there are two kinds of variability, population variability and variance of a mean value. The first gives an estimate of the region of values within which any observation may lie, while the second gives a measure of the reliability of a mean value. The variances $\sigma_{\bar{x}}^2, \sigma_{\bar{y}}^2, \sigma_{\bar{z}}^2$ and the covariances

$\sigma_{\bar{x}\bar{y}}, \sigma_{\bar{x}\bar{z}}, \sigma_{\bar{y}\bar{z}}$ for the 10° CIE observer system⁶ developed by Nimeroff, Rosenblatt, and Dannemiller,³ reflect the variability of observer's color perceptions. These variances and covariances, divided by the number of sets of observations from which the mean values were derived, are the variances and covariances for the CIE system and show the uncertainty associated with the spectral tristimulus values published as estimates of values hypothetically obtainable from color-matching data on the whole population of normal observers. The average spectral tristimulus values are based on sets of observations of 53 British and 27 Russian observers.

In problems and experiments wherein we wish to know the uncertainty of the chromaticity coordinates computed from the standard-observer system for some particular colors, we may assume that the spectral characteristics of the colors are *not* uncertain. The chromaticity uncertainty will result only from the uncertainties in the spectral tristimulus values of the standard-observer system $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$. The system variances $\sigma_{\bar{x}}^2$ and $\sigma_{\bar{y}}^2$ in corresponding chromaticity coordinates x and y may be computed by the following equations derived from application of propagation-of-error theory [Eq. (6)]:

$$\begin{aligned} \sigma_x^2 &= \frac{(S-X)^2}{S^4} \Sigma_{\lambda} E_{\lambda}^2 [\sigma_{\bar{x}\lambda}^2] + \frac{(X)^2}{S^4} \Sigma_{\lambda} E_{\lambda}^2 [\sigma_{\bar{y}\lambda}^2 + \sigma_{\bar{z}\lambda}^2 + 2\sigma_{\bar{y}\bar{z}}] - 2 \frac{(S-X)(X)}{S^4} \Sigma_{\lambda} E_{\lambda} [\sigma_{\bar{x}\bar{y}\lambda} + \sigma_{\bar{x}\bar{z}\lambda}], \\ \sigma_y^2 &= \frac{(S-Y)^2}{S^4} \Sigma_{\lambda} E_{\lambda}^2 [\sigma_{\bar{y}\lambda}^2] + \frac{(Y)^2}{S^4} \Sigma_{\lambda} E_{\lambda}^2 [\sigma_{\bar{x}\lambda}^2 + \sigma_{\bar{z}\lambda}^2 + 2\sigma_{\bar{x}\bar{z}}] - 2 \frac{(S-Y)(Y)}{S^4} \Sigma_{\lambda} E_{\lambda} [\sigma_{\bar{x}\bar{y}\lambda} + \sigma_{\bar{y}\bar{z}\lambda}]. \end{aligned} \quad (8)$$

The covariance σ_{xy} may be computed by the following equation, similarly derived from Eq. (7):

$$\begin{aligned} \sigma_{xy} &= - \frac{(S-X)(Y)}{S^4} \Sigma_{\lambda} E_{\lambda}^2 [\sigma_{\bar{x}\lambda}^2 + \sigma_{\bar{z}\lambda}^2] - \frac{(S-Y)(X)}{S^4} \Sigma_{\lambda} E_{\lambda} [\sigma_{\bar{y}\lambda}^2 + \sigma_{\bar{y}\bar{z}\lambda}] \\ &\quad + \frac{(X)(Y)}{S^4} \Sigma_{\lambda} E_{\lambda}^2 [\sigma_{\bar{x}\lambda}^2 + \sigma_{\bar{x}\bar{y}\lambda} + \sigma_{\bar{x}\bar{z}\lambda} + \sigma_{\bar{y}\bar{z}\lambda}] + \frac{(S-X)(S-Y)}{S^4} \Sigma_{\lambda} E_{\lambda}^2 [\sigma_{\bar{x}\bar{y}\lambda}]. \end{aligned} \quad (9)$$

It should be noted that σ_x^2, σ_y^2 , and σ_{xy} measure the uncertainty in the chromaticity coordinates that might be obtained by individual observers. On the other hand, $\sigma_x^2/80, \sigma_y^2/80$, and $\sigma_{xy}/80$ measure the uncertainty of chromaticity coordinates as an estimate of the hypothetical true chromaticity coordinates.

5. OBSERVER VARIABILITY

As the color-matching data of the individual observers obtained on a visual tristimulus colorimeter differed among them, the chromaticity coordinates derived from these data also have associated uncertainties. These uncertainties are here called "observer variabilities."

In problems and experiments wherein several observers make a color match, settings of E_{λ} may vary among them. This variability among the observers may be represented by the variance $\sigma_{\bar{E}_{\lambda}}^2$, which is computed by:

$$\sigma_{\bar{E}_{\lambda}}^2 = \Sigma_j (E_{\lambda_j}' - \bar{E}_{\lambda})^2 / (n-1), \quad (10)$$

where E_{λ_j}' is the spectral irradiance at the j th observer's eye and \bar{E}_{λ} is the average spectral irradiance for the n observers. As indicated in Sec. 2 above, the spectral responses of individual observers are replaced by the spectral responses of the standard observer to compute chromaticity coordinates. In this computation, the response of the standard observer is given by definition and is *not* uncertain. The tristimulus values and

chromaticity coordinates computed from actual-observer data are thus estimates of the values that would be obtained by the standard-observer method of tristimulus colorimetry. Their uncertainty arises from the variability among observers in selecting spectral irradiances to make the matches. The chromaticity coordinates computed from actual-observer data may also, of course, be interpreted as estimates of the "true" chromaticity coordinates. When this interpretation is made, there is an added component of variance that arises from the system variances and covariances. Since the spectral tristimulus values of the standard-observer

system are based on a large number of observers, however, this component of variance is negligible. It should be pointed out that here also there are two kinds of variances. The variance $\sigma_{\bar{E}}^2$ measures the between-observer variability, while the variance $\sigma_{\bar{E}}^2 = \sigma_{\bar{E}}^2/n$ measures the uncertainty of the average spectral irradiance \bar{E} as an estimate of the "true" matching spectral irradiance.

The variances ${}_{00}\sigma_x^2$ and ${}_{00}\sigma_y^2$ in the corresponding chromaticity x and y , respectively, may be computed by the following equations derived from the application of propagation-of-error theory Eq. (6):

$$\begin{aligned} {}_{00}\sigma_x^2 &= \frac{(S-X)^2}{S^4} \Sigma_{\lambda} [\bar{x}^2(\lambda)] \sigma_{\bar{E}_{\lambda}}^2 + \frac{(X)^2}{S^4} \Sigma_{\lambda} [\bar{y}^2(\lambda) + \bar{z}^2(\lambda) + 2\bar{y}(\lambda)\bar{z}(\lambda)] \sigma_{\bar{E}_{\lambda}}^2 \\ &\quad - 2 \frac{(S-X)(X)}{S^4} \Sigma_{\lambda} [\bar{x}(\lambda)\bar{y}(\lambda) + \bar{x}(\lambda)\bar{z}(\lambda)] \sigma_{\bar{E}_{\lambda}}^2, \\ {}_{00}\sigma_y^2 &= \frac{(S-Y)^2}{S^4} \Sigma_{\lambda} [\bar{y}^2(\lambda)] \sigma_{\bar{E}_{\lambda}}^2 + \frac{(Y)^2}{S^4} \Sigma_{\lambda} [\bar{x}^2(\lambda) + \bar{z}^2(\lambda) + 2\bar{x}(\lambda)\bar{z}(\lambda)] \sigma_{\bar{E}_{\lambda}}^2 \\ &\quad - 2 \frac{(S-Y)(Y)}{S^4} \Sigma_{\lambda} [\bar{x}(\lambda)\bar{y}(\lambda) + \bar{y}(\lambda)\bar{z}(\lambda)] \sigma_{\bar{E}_{\lambda}}^2. \end{aligned} \quad (11)$$

The covariance ${}_{00}\sigma_{xy}$ may be computed by the following equation, similarly derived from Eq. (7):

$$\begin{aligned} {}_{00}\sigma_{xy} &= - \frac{(S-X)(Y)}{S^4} \Sigma_{\lambda} [\bar{x}^2(\lambda) + \bar{x}(\lambda)\bar{z}(\lambda)] \sigma_{\bar{E}_{\lambda}}^2 - \frac{(S-Y)(X)}{S^4} \Sigma_{\lambda} [\bar{y}^2(\lambda) + \bar{y}(\lambda)\bar{z}(\lambda)] \sigma_{\bar{E}_{\lambda}}^2 \\ &\quad + \frac{(X)(Y)}{S^4} \Sigma_{\lambda} [\bar{z}^2(\lambda) + \bar{x}(\lambda)\bar{y}(\lambda) + \bar{x}(\lambda)\bar{z}(\lambda) + \bar{y}(\lambda)\bar{z}(\lambda)] \sigma_{\bar{E}_{\lambda}}^2 + \frac{(S-X)(S-Y)}{S^4} \Sigma_{\lambda} [\bar{x}(\lambda)\bar{y}(\lambda)] \sigma_{\bar{E}_{\lambda}}^2. \end{aligned} \quad (12)$$

It should be noted also that ${}_{00}\sigma_x^2$, ${}_{00}\sigma_y^2$, ${}_{00}\sigma_{xy}$ measure the variability that might be found among individual observers. On the other hand, ${}_{00}\sigma_x^2/n$, ${}_{00}\sigma_y^2/n$, and ${}_{00}\sigma_{xy}/n$ measure the uncertainty of mean chromaticity coordinates as an estimate of the true chromaticity coordinates based on n observers.

In addition to the use of Eqs. (11) and (12) for observer variability, as described above, they may be and have been used in all cases where variability of spectral illuminance $E_{\lambda} = N_{\lambda} R(\lambda)$ is involved. Such involvement may be brought about by uncertainty of N_{λ} as in the repeated measurement of spectral radiance of a source; it may be brought about by uncertainty in $R(\lambda)$ as in the repeated measurement of the spectral transmittance of optical filters; or it may be brought about by uncertainty in both of these variables. Applications that involve $R(\lambda)$ and N_{λ} , separately, have been reported by Nimeroff.^{1,2}

6. UNCERTAINTY ELLIPSE

The variances and covariances for chromaticity coordinates may be represented graphically by an ellipse on the chromaticity diagram. Nimeroff¹ has shown that

if we form an ellipse of the general type

$$T_{\epsilon} = \frac{(x-x_0)^2 \sigma_y^2 - 2(x-x_0)(y-y_0) \sigma_{xy} + (y-y_0)^2 \sigma_x^2}{\sigma_x^2 \sigma_y^2 - \sigma_{xy}^2}, \quad (13)$$

for each specimen-mean chromaticity point x, y , then on the average an observed chromaticity point x_0, y_0 will lie inside the ellipse, so determined, a fraction ϵ of the number of times such observations are made. If x_0 and y_0 have a joint normal distribution, then T_{ϵ} and ϵ are related exactly by the equation $\epsilon = 1 - \exp(-T_{\epsilon}/2)$; otherwise they are related only approximately. For $\epsilon = 0.95$, $T_{\epsilon} = 6$.

The confidence region for each mean chromaticity point is obtained when the variances σ_x^2 and σ_y^2 and covariance σ_{xy} in Eq. (13) are given values that correspond to the variance of the mean, that is σ_x^2/n , σ_y^2/n , σ_{xy}/n , respectively.

It is important to note that even when x_0 and y_0 have a joint normal distribution, the confidence region is exact only when the covariance matrix,

$$\Sigma = \begin{vmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{vmatrix}$$

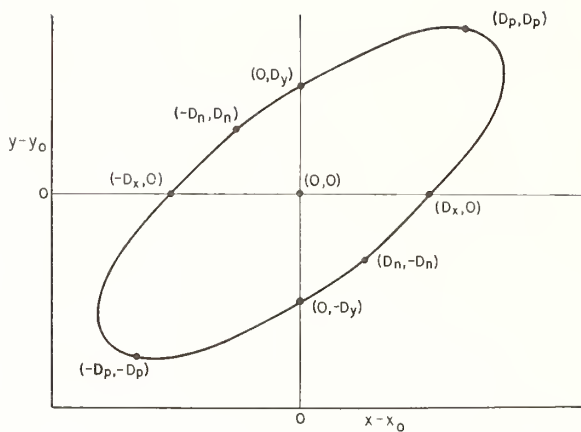


FIG. 1. Parameters useful in plotting an uncertainty ellipse.

for the chromaticity coordinates is known. When the variances and covariances are estimated from a sample of only moderate size, the probability level is less than ϵ .

The ellipse may be characterized by its size, location, and orientation. These may be stated, respectively, by the half-length of each of its mutually perpendicular axes (major and minor), the intersection of these axes, and the slope of the major axis, as has been done for the MacAdam perceptibility ellipses.⁶ For plotting an ellipse, however, it may be convenient sometimes to use four special cases in order to determine eight points on the ellipse that is described by Eq. (13). The

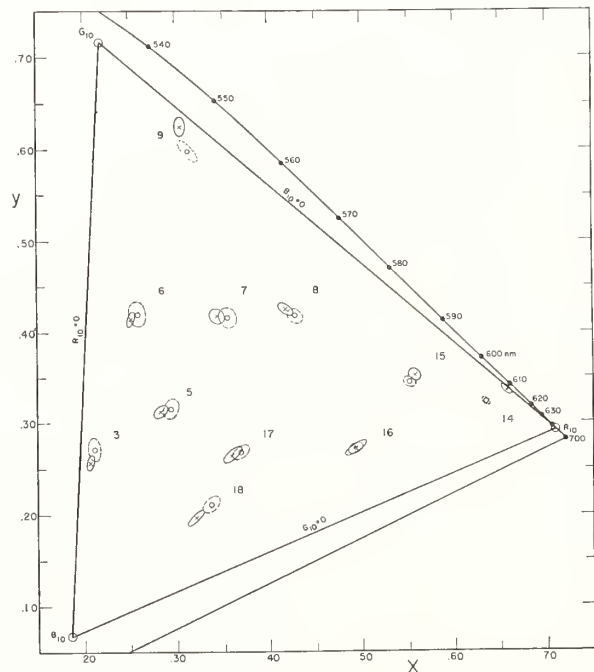


FIG. 2. Individual-variance ellipses resulting from observer variability (solid line) and system variability (dashed line) for the 10°-field, color-matching function, field-trial filters.

⁶ D. L. MacAdam, J. Opt. Soc. Am. 32, 247 (1942) and 33, 18 (1943).

four special cases are: $\Delta y=0$; $\Delta x=0$; $\Delta x=\Delta y$; $\Delta x=-\Delta y$, where $\Delta x=x-x_0$ and $\Delta y=y-y_0$. Other special convenient cases may be needed to plot the ellipse; these are those for which $\Delta=m(\Delta y)$, where m takes on positive and negative values. The four special cases are graphically demonstrated in Fig. 1. The solutions of Eq. (13) for these four special cases are given in Table I for the condition that $T_e=6$, with the symbols D_i for each of these four cases.

7. COMPARISON OF SYSTEM UNCERTAINTY WITH OBSERVER VARIABILITY

In a field trial of the 10°-field standard-observer system⁷ considerable amounts of data were obtained. Predictions of chromaticity by the system were compared with those obtained on a Donaldson visual tristimulus colorimeter by eleven observers who visually color-matched light from selected combinations of the three instrumental primaries to light from filters. There were sufficient data to serve as a good illustration of the applicability of computer program with which to compute the ellipse parameters in the two variability problems discussed above, the system uncertainty, and the observer variability. The uncertainty-ellipse parameters for the individual variances, obtained from the data for the field-trial filters, are listed in Table II for observer variability and system uncertainty. The uncertainty ellipses, computed from the individual variances for the observer variability and the system uncertainty, are plotted in Fig. 2 for the field-trial filters. The uncertainty-ellipse parameters for the variance of the mean chromaticity, obtained by dividing the ellipse parameters of Table II by $n^{\frac{1}{2}}$, are listed in Table III. For the system uncertainty $n=80$, while for the observer variability $n=11$, except for filters 15 and 16 ($n=8$) and for filters 8 and 17 ($n=9$).

As attempts have already been made,⁷⁻¹⁰ to understand and to explain the differences, on the basis of a functional relationship between the average observed chromaticities and the average system-predicted chrom-

TABLE I. Special-case solutions of the uncertainty-ellipse equation for $T_e=6$.

Case	Symbol	Solution
I. $\Delta y=0$	D_x	$(x-x_0)=\pm[6(\sigma_x^2\sigma_y^2-\sigma_{xy}^2)/\sigma_y^2]^{\frac{1}{2}}$
II. $\Delta x=0$	D_y	$(y-y_0)=\pm[6(\sigma_x^2\sigma_y^2-\sigma_{xy}^2)/\sigma_x^2]^{\frac{1}{2}}$
III. $\Delta x=\Delta y$	D_p	$(x-x_0)=(y-y_0)=\pm\left[\frac{6(\sigma_x^2\sigma_y^2-\sigma_{xy}^2)}{\sigma_y^2+\sigma_x^2-2\sigma_{xy}}\right]^{\frac{1}{2}}$
IV. $\Delta x=-\Delta y$	D_n	$(x-x_0)=(y_0-y)=\pm\left[\frac{6(\sigma_x^2\sigma_y^2-\sigma_{xy}^2)}{\sigma_y^2+\sigma_x^2+2\sigma_{xy}}\right]^{\frac{1}{2}}$

⁷ I. Nimeroff, J. Opt. Soc. Am. 54, 696 (1964).

TABLE II. Individual-variance uncertainty-ellipse parameters for field-trial filters.

Filter No.	Observer variability				System uncertainty			
	D_x	D_y	D_p	D_n	D_x	D_y	D_p	D_n
3	0.0041	0.0067	0.0044	0.0030	0.0068	0.0135	0.0059	0.0062
5	0.0067	0.0065	0.0057	0.0040	0.0082	0.0114	0.0075	0.0060
6	0.0039	0.0077	0.0043	0.0030	0.0088	0.0139	0.0069	0.0081
7	0.0079	0.0076	0.0057	0.0062	0.0092	0.0103	0.0064	0.0075
8	0.0075	0.0054	0.0036	0.0060	0.0083	0.0078	0.0048	0.0072
9	0.0066	0.0107	0.0056	0.0056	0.0074	0.0084	0.0042	0.0113
14	0.0025	0.0022	0.0012	0.0049	0.0036	0.0027	0.0017	0.0033
15	0.0070	0.0056	0.0040	0.0048	0.0065	0.0049	0.0034	0.0048
16	0.0066	0.0048	0.0084	0.0028	0.0081	0.0053	0.0060	0.0037
17	0.0062	0.0058	0.0086	0.0032	0.0086	0.0082	0.0080	0.0049
18	0.0045	0.0040	0.0080	0.0022	0.0074	0.0077	0.0080	0.0043

aticities for these filters no such attempt will be made here. We are, however, concerned with the statistical similarities both of the variabilities and of the average chromaticities. With regard to the similarities in the average chromaticities, Fig. 2 shows that the uncertainty ellipses either overlap or are in fairly close proximity. We should note from Fig. 2 also that most of the pairs of ellipses are approximately the same size and have major axes that are sloped at approximately the same angle.

Strictly, uncertainty in the CIE system "true" chromaticity point arises from the simultaneous uncertainty of the spectral irradiance E_λ and the spectral tristimulus value $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$. Only if, as we have assumed here, E_λ is known with negligible error, may the scatter of individual color perceptions logically be represented by the system-based ellipses computed on an individual basis. This scatter may then be represented by the variances and covariances of the spectral tristimulus values.

If the two sets of ellipses are compared on the basis of individual variances, then their corresponding sizes and orientations should be comparable, assuming that the observers employed in devising the system and those employed in performing the field trial are satisfactory normal-observer representatives. The observer-based ellipses should be expected to overlap the system-

based chromaticity points about 90% of the time for $T_\epsilon=6$. The high degree of metamerism between the two matched fields of the colorimeter¹¹ discussed here, however, represent a small percentage of color-matching problems generally encountered in practice. The results of the analysis made in this paper for these highly metameric matches, therefore, tend to support use of the system variances and covariances as approximations to observer-based variances and covariances.

It should be noted also that the system-uncertainty ellipses, whose parameters are listed in Table III, computed with σ^2/n ($n=80$) measure the uncertainty of the mean chromaticity point, derived from the 10^2 -field CIE spectral tristimulus values, to estimate the true point for the population of normal observers. The observer-variability ellipses, whose parameters are listed in Table III, with σ^2/n ($n=11$, except for the exceptions listed above) measure the uncertainty of the mean chromaticity point derived from the color-matching data of the field trial, together with the 10^2 -field spectral tristimulus values of the CIE, to evaluate the true point for the same population.

8. THE AUTOMATIC ELECTRONIC COMPUTER PROGRAM

It is readily evident from the complexity of Eqs. (8), (9), and (11)–(13) that to perform by hand all

TABLE III. Mean-variability uncertainty-ellipse parameters for the field-trial filters.

Filter No.	Observer variability				System uncertainty			
	D_x	D_y	D_p	D_n	D_x	D_y	D_p	D_n
3	0.0012	0.0020	0.0013	0.0009	0.0009	0.0019	0.0008	0.0009
5	0.0020	0.0020	0.0017	0.0012	0.0011	0.0016	0.0010	0.0008
6	0.0012	0.0013	0.0013	0.0009	0.0012	0.0019	0.0009	0.0011
7	0.0024	0.0023	0.0017	0.0019	0.0013	0.0014	0.0009	0.0010
8	0.0025	0.0018	0.0012	0.0020	0.0011	0.0011	0.0007	0.0010
9	0.0020	0.0032	0.0017	0.0017	0.0010	0.0012	0.0006	0.0015
14	0.0008	0.0007	0.0004	0.0015	0.0005	0.0004	0.0002	0.0004
15	0.0025	0.0020	0.0014	0.0017	0.0009	0.0007	0.0005	0.0007
16	0.0023	0.0017	0.0030	0.0010	0.0011	0.0007	0.0008	0.0005
17	0.0021	0.0020	0.0029	0.0011	0.0012	0.0011	0.0011	0.0007
18	0.0014	0.0012	0.0024	0.0007	0.0010	0.0011	0.0011	0.0006

⁸ W. S. Stiles and G. Wyszecki, J. Opt. Soc. Am. **52**, 58 (1962).

¹⁰ I. Nimeroff, J. Opt. Soc. Am. **54**, 833 (1964).

⁹ I. Nimeroff, J. Opt. Soc. Am. **54**, 824 (1964).

¹¹ I. Nimeroff, J. Opt. Soc. Am. **55**, 185 (1965).

the indicated computations for wavelengths at 10-nm intervals from 380 to 770 nm for every specimen for which variability or uncertainty information is required, would indeed be a very formidable, tedious, and time-consuming task. To alleviate this strain on time and patience we use the automatic electronic computing system currently available at the National Bureau of Standards. This system has already been used to compute tristimulus values and chromaticity coordinates from spectrophotometric data.¹² To make this program readily operable, in addition, for our problem of computing system uncertainty and observer variability ellipses from Eqs. (8)–(13), requires some simplifying substitutions. While these equations are essential in the computation of uncertainty ellipses, their form may be simplified to help anyone who wishes to arrange such a program for convenience in his laboratory. The simplified equations are given below. In the equations there are some fixed combinations of the spectral tristimulus values for computing

the observer variability and of their variances and covariances for computing the system uncertainty. These combinations are readily computable even by the use of desk calculators.

Table I lists the equations that will yield the uncertainty-ellipse parameters if the chromaticity variances and covariances are known. In the discussions below the simplified chromaticity variance and covariance equation will be developed.

8a. System Variability

It will be noted that in the equations for computing system variability, Eqs. (8) and (9), there are fixed combinations of the between-observer variances $\sigma_{\bar{x}_\lambda}^2$, $\sigma_{\bar{y}_\lambda}^2$, $\sigma_{\bar{z}_\lambda}^2$ and covariances $\sigma_{\bar{x}_\lambda\bar{y}_\lambda}$, $\sigma_{\bar{x}_\lambda\bar{z}_\lambda}$, $\sigma_{\bar{y}_\lambda\bar{z}_\lambda}$ of the spectral tristimulus values $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$. By means of the following substitutions these combinations can be made a permanent part of the program for computing system variability:

$$\begin{aligned} P_{x_\lambda} &= \sigma_{\bar{x}_\lambda}^2; & Q_{x_\lambda} &= \sigma_{\bar{y}_\lambda}^2 + \sigma_{\bar{z}_\lambda}^2 + 2\sigma_{\bar{y}_\lambda\bar{z}_\lambda}; & R_{x_\lambda} &= 2(\sigma_{\bar{x}_\lambda\bar{y}_\lambda} + \sigma_{\bar{x}_\lambda\bar{z}_\lambda}); \\ P_{y_\lambda} &= \sigma_{\bar{y}_\lambda}^2; & Q_{y_\lambda} &= \sigma_{\bar{x}_\lambda}^2 + \sigma_{\bar{z}_\lambda}^2 + 2\sigma_{\bar{x}_\lambda\bar{z}_\lambda}; & R_{y_\lambda} &= 2(\sigma_{\bar{x}_\lambda\bar{y}_\lambda} + \sigma_{\bar{y}_\lambda\bar{z}_\lambda}); \\ P_{x_\lambda y_\lambda} &= \sigma_{\bar{x}_\lambda}^2 + \sigma_{\bar{x}_\lambda\bar{z}_\lambda}; & Q_{x_\lambda y_\lambda} &= \sigma_{\bar{y}_\lambda}^2 + \sigma_{\bar{y}_\lambda\bar{z}_\lambda}; & R_{x_\lambda y_\lambda} &= (\sigma_{\bar{x}_\lambda}^2 + \sigma_{\bar{x}_\lambda\bar{y}_\lambda} + \sigma_{\bar{x}_\lambda\bar{z}_\lambda} + \sigma_{\bar{y}_\lambda\bar{z}_\lambda}); \text{ and } T_{x_\lambda y_\lambda} = \sigma_{\bar{x}_\lambda\bar{y}_\lambda}. \end{aligned} \quad (14)$$

A program similar to this for computing uncertainty ellipses in the u , v chromaticity diagram has been reported by Mori and associates.¹³ On substitution, the system variance equation Eq. (8) becomes simply

$$\begin{aligned} \sigma_x^2 &= \frac{(S-X)^2}{S^4} \Sigma_\lambda E_\lambda^2 P_{x_\lambda} + \frac{(X)^2}{S^4} \Sigma_\lambda E_\lambda^2 Q_{x_\lambda} - \frac{(S-X)(X)}{S^4} \Sigma_\lambda E_\lambda^2 R_{x_\lambda} \\ \sigma_y^2 &= \frac{(S-Y)^2}{S^4} \Sigma_\lambda E_\lambda^2 P_{y_\lambda} + \frac{(Y)^2}{S^4} \Sigma_\lambda E_\lambda^2 Q_{y_\lambda} - \frac{(S-Y)(Y)}{S^4} \Sigma_\lambda E_\lambda^2 R_{y_\lambda} \end{aligned} \quad (8a)$$

and the covariance equation, Eq. (9), becomes

$$\sigma_{xy} = \left[\frac{(S-X)(Y)}{S^4} \Sigma_\lambda E_\lambda^2 P_{x_\lambda y_\lambda} + \frac{(S-Y)(X)}{S^4} \Sigma_\lambda E_\lambda^2 Q_{x_\lambda y_\lambda} \right] + \left[\frac{(X)(Y)}{S^4} \Sigma_\lambda E_\lambda^2 R_{x_\lambda y_\lambda} + \frac{(S-X)(S-Y)}{S^4} \Sigma_\lambda E_\lambda^2 T_{x_\lambda y_\lambda} \right]. \quad (9a)$$

For observations in a 10° field the values of the between-observer system variances $\sigma_{\bar{x}_\lambda}^2$, $\sigma_{\bar{y}_\lambda}^2$, $\sigma_{\bar{z}_\lambda}^2$ and the covariances $\sigma_{\bar{x}_\lambda\bar{y}_\lambda}$, $\sigma_{\bar{x}_\lambda\bar{z}_\lambda}$, $\sigma_{\bar{y}_\lambda\bar{z}_\lambda}$ have already been derived by Nimeroff, Rosenblatt, and Dannemiller;³ they are denoted by $V(\bar{x})$, $V(\bar{y})$, $V(\bar{z})$, $C(\bar{x},\bar{y})$, $C(\bar{x},\bar{z})$, and $C(\bar{y},\bar{z})$, respectively.

To make the program useful for small-field data, we need similar information for the 2°-field observer system. But, because the individual-observer color-matching functions are not readily available, the covariances of the spectral tristimulus values cannot be derived to any satisfactory degree of reliability. From the published data on the 2°-field color-matching functions, on which the CIE 1931 2°-field observer system is based, only a rough estimate of the variances has been made.²

8b. Observer Variability

The equations for computing observer variability, Eqs. (11) and (12), contain fixed combinations of the spectral tristimulus values $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$. By means of the following substitutions these combinations can be made a

¹² H. J. Keegan, W. C. Reinboldt, J. C. Schleter, J. P. Menard, and D. B. Judd, *J. Opt. Soc. Am.* **48**, 863A (1958).

¹³ L. Mori, H. Sugiyama, and N. Kambe, *J. Illum. Eng. (Inst. Japan)* **48**, No. 8, 4 (1964).

part of the program for computing observer variability:

$$\begin{aligned} U_{x\lambda} &= \bar{x}^2(\lambda); & V_{x\lambda} &= \bar{y}^2(\lambda) + \bar{z}^2(\lambda) + 2\bar{y}(\lambda)\bar{z}(\lambda); & W_{x\lambda} &= 2[\bar{x}(\lambda)\bar{y}(\lambda) + \bar{x}(\lambda)\bar{z}(\lambda)]; \\ U_{y\lambda} &= \bar{y}^2(\lambda); & V_{y\lambda} &= \bar{x}^2(\lambda) + \bar{z}^2(\lambda) + 2\bar{x}(\lambda)\bar{z}(\lambda); & W_{y\lambda} &= 2[\bar{x}(\lambda)\bar{y}(\lambda) + \bar{y}(\lambda)\bar{z}(\lambda)]; \\ U_{x\lambda y\lambda} &= \bar{x}^2(\lambda) + \bar{x}(\lambda)\bar{z}(\lambda); & V_{x\lambda y\lambda} &= \bar{y}^2(\lambda) + \bar{y}(\lambda)\bar{z}(\lambda); & W_{x\lambda y\lambda} &= [\bar{z}^2(\lambda) + \bar{x}(\lambda)\bar{y}(\lambda) + \bar{x}(\lambda)\bar{z}(\lambda) + \bar{y}(\lambda)\bar{z}(\lambda)]; \end{aligned}$$

and

$$X_{x\lambda y\lambda} = \bar{x}(\lambda)\bar{y}(\lambda). \quad (15)$$

The variance $\sigma_{\bar{E}_\lambda}^2$ may be computed from a simplification of Eq. (10):

$$\sigma_{\bar{E}_\lambda}^2 = [\Sigma_i (E_{\lambda_i}')^2 / n - 1] - [(\Sigma E_{\lambda_i}')^2 / n(n-1)]. \quad (10a)$$

On substitution, the observer variance equations, Eqs. (11) and (12), become simply:

$$\begin{aligned} \sigma_x^2 &= \frac{(S-X)^2}{S^4} \Sigma_\lambda U_{x\lambda} \sigma_{\bar{E}_\lambda}^2 + \frac{(X)^2}{S^4} \Sigma_\lambda V_{x\lambda} \sigma_{\bar{E}_\lambda}^2 - \frac{(S-X)(X)}{S^4} \Sigma_\lambda W_{x\lambda} \sigma_{\bar{E}_\lambda}^2, \\ \sigma_y^2 &= \frac{(S-Y)^2}{S^4} \Sigma_\lambda U_{y\lambda} \sigma_{\bar{E}_\lambda}^2 + \frac{(Y)^2}{S^4} \Sigma_\lambda V_{y\lambda} \sigma_{\bar{E}_\lambda}^2 - \frac{(S-Y)(Y)}{S^4} \Sigma_\lambda W_{y\lambda} \sigma_{\bar{E}_\lambda}^2, \end{aligned} \quad (11a)$$

and

$$\sigma_{xy} = -\frac{(S-X)(Y)}{S^4} \Sigma_\lambda U_{x\lambda y\lambda} \sigma_{\bar{E}_\lambda}^2 - \frac{(S-Y)(X)}{S^4} \Sigma_\lambda V_{x\lambda y\lambda} \sigma_{\bar{E}_\lambda}^2 + \frac{(X)(Y)}{S^4} \Sigma_\lambda W_{x\lambda y\lambda} \sigma_{\bar{E}_\lambda}^2 + \frac{(S-X)(S-Y)}{S^4} \Sigma_\lambda X_{x\lambda y\lambda} \sigma_{\bar{E}_\lambda}^2, \quad (12a)$$

9. SUMMARY

In computation of chromaticity coordinates from spectral data two methods are possible, an actual-observer method and a standard-observer method. The source of the spectral information, obtained by measurement, determines which method is to be considered in evaluating the uncertainty of the computed results. By application of the theory of propagation of errors in computational procedures, system uncertainty of the standard-observer method and observer variability of the actual-observer method may be evaluated. A distinction was made between two kinds of variability, population variability and variance of a mean, or an average, value. When we are interested in knowing the area within which any color-normal individual observer may be expected to place the chromaticity of a color, then we should use the population variability. On the other hand, when we are interested in stating with what reliance a mean chromaticity point has been

determined, then we should use the variance of the mean value.

Because of the complexity of the equations and the large amount of data analyzed, an automatic computer program was devised to compute the chromaticity-coordinate variances and covariances and the useful parameters of their associated uncertainty ellipses. The simplified equations for the observer-based and system-based chromaticity variances and covariances, developed for use in the program, are given.

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The Variability of Color Measurement

by Isadore Nimeroff

Because all the components in the equations for computing chromaticity coordinates in spectrophotometric colorimetry are measured, these components are subject to measurement uncertainty. The problem of determining the chromaticity uncertainty can be treated most simply by the application of the theory of propagation of errors in a computed result. The ellipse is chosen as the most convenient closed plane curve by which to represent the area of uncertainty in the chromaticity diagram within which the true chromaticity point can be expected to lie a specified fraction of the time. The theory has been applied to determining chromaticity uncertainty resulting separately from uncertainties in spectral irradiance of a source, spectral remittance of objects and spectral response of normal human observers. In actual practice the effects of these uncertainties should be computed coincidentally. On the basis of experience it is suggested that a complete standard observer system, consisting of means, variances, and covariances of the spectral tristimulus values, in xyz and uvw coordinates, be recommended by Commission Internationale de l'Eclairage (CIE) for use in color measurement.

Sources of Variability

SPECTROPHOTOMETRIC tristimulus colorimetry stems from two basic facts of color vision. First, the color of objects depends on the spectral character of at least three components: 1. the source, 2. the object, and, 3. the receptor response. Second, the spectral response of the receptor (the human eye) can be described by a minimum of three sets of parameters. Let us represent the spectral characteristics of the source by spectral radiance N_λ , the object by spectralmittance $T(\lambda)$, and the receptor response by the average normal human spectral tristimulus values $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$. Spectralmittance is used here as a generic term for spectral emittance, spectral reflectance, or spectral transmittance relative to a suitable standard on a suitable instrument.

At the 1931 meeting of the International Commission on Illumination (CIE), use of a Standard Observer System for Colorimetry was recommended. This system defines the manner in which spectral data are reduced to tristimulus values that describe colors of emitted, reflected, or transmitted lights. The defining equations by which to compute these values are

$$\begin{aligned} X &= \sum_{\lambda} \bar{x}(\lambda) N_{\lambda} T(\lambda) \\ Y &= \sum_{\lambda} \bar{y}(\lambda) N_{\lambda} T(\lambda) \\ Z &= \sum_{\lambda} \bar{z}(\lambda) N_{\lambda} T(\lambda). \end{aligned} \quad (1)$$

From these values are computed the chromaticity coordinates, $x = X/S$, $y = Y/S$, and $z = Z/S$, where $S = X + Y + Z$.

At the 1963 meeting of the CIE, use of a supplementary observer system for colorimetry was recommended for viewing large fields. Now we are able, by the choice of a suitable set of spectral tristimulus values, to apply the colorimetry system either to small fields (2°) or to large fields (10°).

Because all of the quantities in these equations, N_λ , $T(\lambda)$, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ are measured quantities, they are subject to measurement uncertainty. The uncertainty in these measured quantities leads us to the variability of spectrophotometric colorimetry. In the following discussion we will characterize uncertainties either by variance of individual measurements or by their square root, the standard deviation of individual measurements. The variance of sample means is obtained by dividing the variance of individual measurements by the number of measurements in a sample.

This is a complex problem. It consists of a system of three functions, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$, which vary over a wavelength range from 380 to 760 nm, each multiplied by two factors N_λ and $T(\lambda)$ which vary over the same wavelength range. This problem can be treated most simply, however, by the application of the theory of propagation of errors in a computed result. This theory (1) may be explained as follows:

If we have two different functions, U and V , of the variates, v_n , they can be written:

$$U = f_1(v_1, \dots, v_n, \dots, v_k) \text{ and } V = f_2(v_1, \dots, v_n, \dots, v_k).$$

The variances of these functions consist of variances and covariances of the variates and may be expressed as:

$$\sigma_U^2 = \sum_{n=1}^k \left(\frac{\partial U}{\partial v_n} \right)^2 \sigma_{v_n}^2 + \sum_{m \neq n} \left(\frac{\partial U}{\partial v_n} \right) \left(\frac{\partial U}{\partial v_m} \right) \sigma_{v_n v_m} \quad (2)$$

$$\sigma_V^2 = \sum_{n=1}^h \left(\frac{\partial V}{\partial v_n} \right)^2 \sigma_{v_n}^2 + \sum_{m \neq n} \left(\frac{\partial V}{\partial v_n} \right) \left(\frac{\partial V}{\partial v_m} \right) \sigma_{v_n v_m},$$

where $\sigma_{v_n}^2$ is the variance of the variate v_n and $\sigma_{v_n v_m}$ is the covariance of variates v_n and v_m . The covariances of the functions U and V consist of linear combinations of the variances and covariances of the variates and may be expressed as:

$$\sigma_{UV} = \sum \left(\frac{\partial U}{\partial v_n} \right) \left(\frac{\partial V}{\partial v_n} \right) \sigma_{v_n}^2 + \sum_{m \neq n} \left(\frac{\partial U}{\partial v_n} \right) \left(\frac{\partial V}{\partial v_m} \right) \sigma_{v_n v_m}. \quad (3)$$

Even with this simplification of the problem, we proceeded in a logical manner to treat one source of variability at a time to arrive at our present concept of the problem. It should be possible to treat all sources of variability simultaneously.

Uncertainties in Spectrophotometric Data

In 1953, we treated the problem of determining the uncertainty in chromaticity coordinates that results from

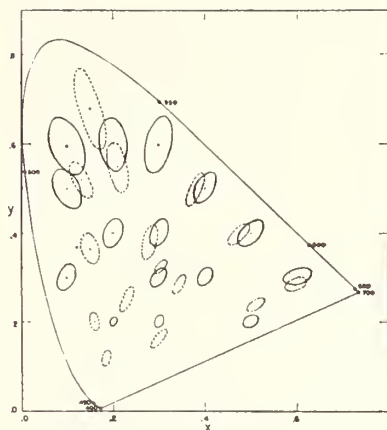


Figure 1—Ellipses derived from spectrophotometric uncertainty $\sigma_{T(\lambda)} = 0.001$ for colors with daylight luminous reflectance of 0.10, compared with MacAdam perceptibility ellipses, dotted lines. (All ellipses are plotted to 10 times scale of the diagram.)

uncertainties in $T(\lambda)$ (2). We assumed that $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, and N_λ are free from uncertainty so we called their products $A(\lambda)$, $B(\lambda)$, and $C(\lambda)$, respectively. From the application of the simple theory of propagation of errors in a computed result, eqs. (2) and (3), we were able to derive the equations of uncertainty in the chromaticity coordinates x and y ,

$$\sigma_x^2 = \sum_\lambda [SA(\lambda) - Xs(\lambda)]^2 \sigma_{T(\lambda)}^2 / S^4$$

$$\sigma_y^2 = \sum_\lambda [SB(\lambda) - Ys(\lambda)]^2 \sigma_{T(\lambda)}^2 / S^4 \quad (4)$$

$$\sigma_{xy} = \sum_\lambda [SA(\lambda) - XA(\lambda)][SB(\lambda) - YA(\lambda)] \sigma_{T(\lambda)}^2 / S^4 \quad (5)$$

where $s(\lambda) = [A(\lambda) + B(\lambda) + C(\lambda)]$. We wish to form in the chromaticity diagram plane a closed figure, namely the ellipse,

$$T_e = \frac{(x - x_n)^2 \sigma_y^2 - 2(x - x_n)(y - y_n) \sigma_{xy} + (y - y_n)^2 \sigma_x^2}{\sigma_x^2 \sigma_y^2 - \sigma_{xy}^2} \quad (6)$$

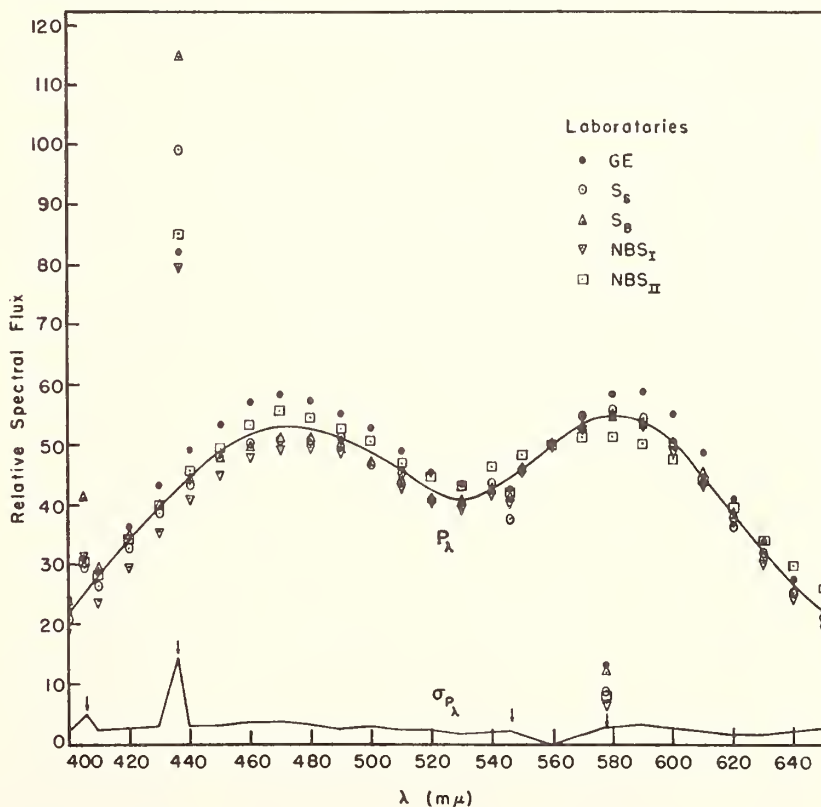


Figure 2—Relative spectral flux data $P(\lambda)$ of a daylight fluorescent lamp as measured on 5 spectroradiometers. Standard deviation data $\sigma_{P(\lambda)}$ are also shown.

for each specimen mean chromaticity point x_n, y_n to represent the area of uncertainty for a specified confidence level, ϵ (3). It has been shown that, for a confidence level $\epsilon = 0.95$, $T_\epsilon = 6$ (2). Figure 1 shows these ellipses for a number of colors of luminous reflectance 10% for which the spectral data were assumed to be uncertain by $\sigma_T = 0.1\%$. The dashed ellipses shown are MacAdam's perceptibility ellipses (4) for one observer's repeating his matches several times. Both sets of ellipses are plotted ten times the scale of the diagram.

Uncertainties in Spectroradiometric Data

In 1957, we were led to apply the error-propagation procedure to the problem of evaluating the uncertainty in chromaticity coordinates resulting from uncertainty in spectroradiometric data for a source, N_λ (3). On Figure 2 we show spectral data and uncertainty data in terms of relative spectral flux P_λ (analogous to spectral radiance N_λ) obtained from a fluorescent lamp measured at several laboratories. In

this problem we treated $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, and $T(\lambda)$ as free from uncertainty. Figure 3 shows the uncertainty ellipse derived from these data. This ellipse is considerably larger than that obtained for several observers who made visual measurements of the chromaticity as shown by the smaller ellipse.

Uncertainties in Spectral Tristimulus Data

We next applied the procedure to evaluating the uncertainties in spectral tristimulus values, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$. We first attempted to estimate these uncertainties from the published mean values of several independent investigations as summarized by Ishak (5), but as the data of individual observers were not available the covariances could not be taken into account for the 2° system. Later, when the individual-observer data of the recent CIE investigation became available, we were able to take them into account for the 10° observer system (6). The CIE was then considering the recommendation of a new Standard Observer System for Colorimetry eval-

uated for a 10°-field of view. This system is based on more extensive color-matching data than were used in developing the 1931 system. These data were assembled by Stiles and Burch in Great Britain (7) and by Speranskaya in Russia (8) for 80 observers. Because in the development of the new system the Stiles-Burch color-matching data were given greater weight than those of Speranskaya, and because the Stiles-Burch data for individual observers had been made available, the Stiles-Burch data were given a rather complete statistical review.

53 Observers

The CIE has proceeded to average the Stiles-Burch data for 53 observers and the Speranskaya data for 27 observers to derive the 10°-field tristimulus functions, \bar{x} , \bar{y} , and \bar{z} . Figure 4 shows the \bar{b} function with its three lobes, two positive and one negative. (The data for the other two functions \bar{g} and \bar{r} are similar to these.) Also shown are the between-observer deviations, ${}_b\sigma_b$, of the Stiles-Burch data and the Speranskaya data. The results of

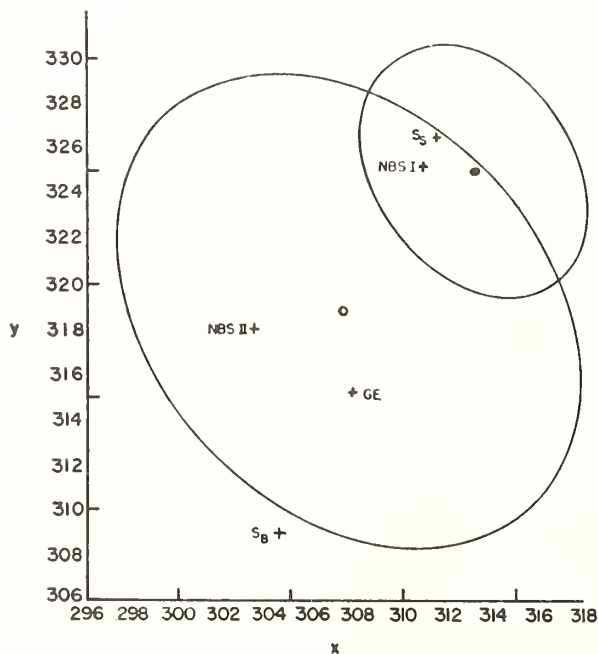


Figure 3—Uncertainty ellipse about mean computed chromaticity point (●) for data of figure 3. Smaller ellipse shows observer variability for this lamp about the average visually observed chromaticity point (●).

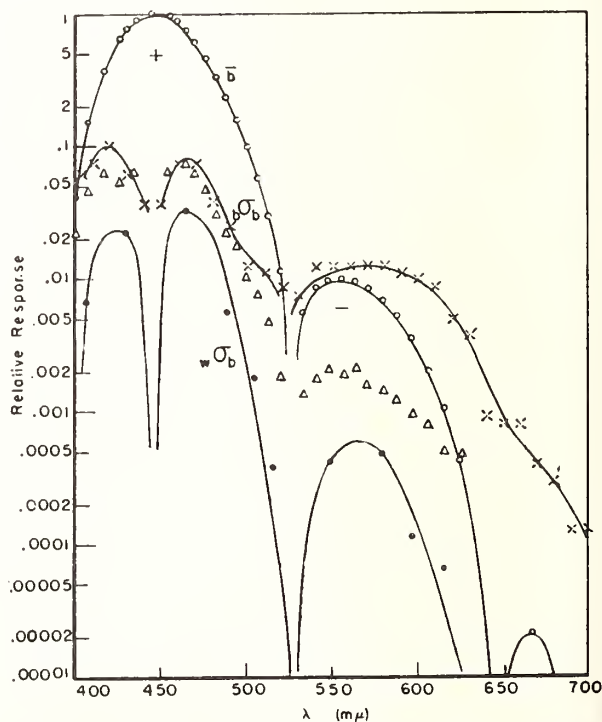


Figure 4—Means and variabilities of observer data for the blue primary: Means (●); ${}_b\sigma_b$ standard deviation between observers, Stiles (Δ), Speranskaya (x); $w\sigma_b$ standard deviation within observers (•).

these two independent investigations are essentially of the same order of magnitude and range from approximately 10 to 20% of the mean function.

The lowermost curve shows the within-observer standard deviations, $w\sigma_{\bar{b}}$, for the \bar{b} function. These data are based on the average of the individual variability of two observers at the National Physical Laboratory, one repeating his measurements five times, the other four times. As might be expected, these standard deviations are approximately one order of magnitude lower than the between-observer standard deviations. (We should note that the standard deviations approach zero at the location of the primaries.)

We were interested in learning the answer to two questions about the ratio of between-observer standard deviations to within-observer standard deviations: 1—Can the ratio be represented by a constant independent of wavelength? 2—What is the magnitude of such a constant? The computation of within-observer variability in an xyz system would be simplified were this feasible. Figure 5 shows these ratios for the three color-matching functions. The overall average ratio, neglecting one extreme value in each function, is 5.7 in the spectral region 410 to 630 nm. As practically all the points lie between $\frac{1}{2}$ and 2 times 5.7, we consider this value reasonably representative of the ratio.

Interesting Facts

In the transformation from color-matching functions, rgb, to CIE tristimulus values, $\bar{x}, \bar{y}, \bar{z}$, covariances as well as variances need to be known to compute the variabilities in the transformed system. To evaluate the covariances we computed the correlation coefficients, ρ_{ij} . From a study of these correlation coefficients we learned some interesting facts, although their significance relative to the visual mechanism is not fully understood. A positive correlation coefficient implies that if an observer uses more or less than average of one instrumental primary in making a match he compensates by using respectively, more or less than average of the other primary also. Conversely, a negative coefficient implies that if more or less than average of one primary is used respectively, less or more than average is

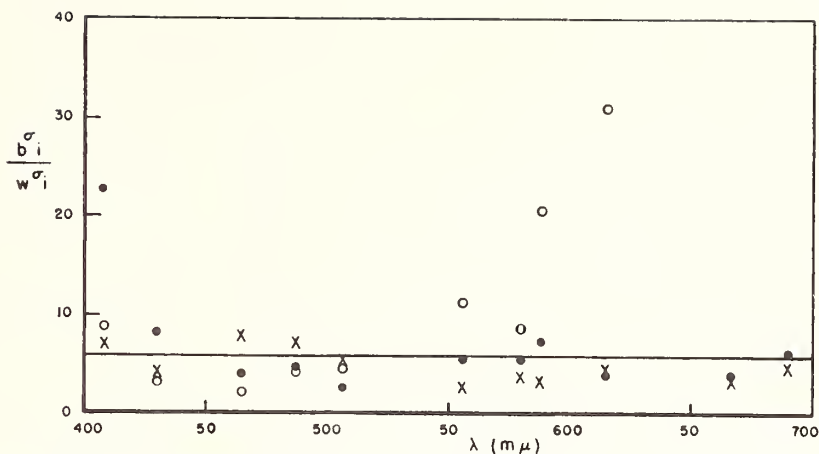


Figure 5—Ratio of between-observer to within-observer standard deviations for the three primaries, red, green, and blue: \bar{r} (•), \bar{g} (x), and \bar{b} (•) showing average ratio of 5.7.

used of the other primary. On Figure 6 are plotted the correlation coefficients ρ_{rg} , ρ_{rb} , and ρ_{gb} . We note that ρ_{rg} is always negative (or zero) and ρ_{rb} is always positive (or zero). On the other hand, ρ_{gb} is negative in the short-wave region (400 to 590 nm) and positive in the long-wave region, that is, greater than 590 nm.

As the determination of linear transformation coefficients is rather complex, we proposed to treat the transformation from rgb system to xyz system as though it were an arbitrary one. What is meant by an arbitrary transformation system may be explained as follows: In a derived transformation system based on experimental data all components, variates, v , and constants, k , are treated as variates and are subject to uncertainty. In other words, the variances $V(v)$ and the covariances $C(uv)$ are different from zero, that is $V(v)$, $C(uv) \neq 0$. In an arbitrarily transformed system, on the other hand, contributions to the variances and covariances in the transformed system arise only from variances and covariances in the original variates, that is, $V(k) = C(kv) = C(k,k_v) = 0$.

The derived values of the variances and covariances in the x,y,z system are listed in Table 1 for between-observer variabilities. Table 2 gives similar values for the MacAdam system (9), recommended for use by the International Commission on Illumina-

tion (10) when uniform chromaticity-coordinate spacing is required. The spectral tristimulus values \bar{u}_λ , \bar{v}_λ , \bar{w}_λ for this system were derived by Nimeroff (11).

Correlation Between System and Observer Variances

While we considered that we had correctly analyzed the basic data and had correctly applied the propagation of error theory to our problem, we needed some empirical proof that the derived variances and covariances of the spectral tristimulus values computed for the standard observer system (system variability) reasonably represent the variances and covariances for actual observer color matches (observer variability) (12). The data obtained in a field trial of the 10°-field observer system afforded a reasonable test of such a representation. Here, on the one hand, we had test colors for which the system of spectral tristimulus values, variances and covariances could be used to compute the chromaticity coordinates as well as the uncertainty ellipses. These ellipses reveal the system variability. On the other hand, we have the several spectral distributions that actual observers chose as color matches for each of the test colors. These individually selected distributions provide the means for computing chromaticity coordinates as well as uncertainty el-

TABLE I—CIE TRISTIMULUS FUNCTIONS FOR 10°-FIELD AND BETWEEN-OBSERVER
VARIANCES AND COVARIANCES IN \bar{x} , \bar{y} , \bar{z} .

λ (nm)	\bar{x}_{10}	\bar{y}_{10}	\bar{z}_{10}	$V(\bar{x})$	$V(\bar{y})$	$V(\bar{z})$	$C(\bar{x}, \bar{y})$	$C(\bar{x}, \bar{z})$	$C(\bar{y}, \bar{z})$
400	0.0191097	0.0020044	0.0860109	0.000126	0.00000118	0.00256	+0.0000104	+0.000568	+0.0000467
10	.084736	.008756	.389366	.000661	.0000110	.0132	+ .0000708	+ .00294	+ .000321
20	.204492	.021391	.972542	.000937	.0000262	.0193	+ .0000731	+ .00421	+ .000350
30	.314679	.038676	1.55348	.000737	.0000671	.0154	+ .000106	+ .00329	+ .000509
40	.383734	.062077	1.96728	.000385	.0000448	.00978	+ .0000816	+ .00189	+ .000355
450	.370702	.080456	1.99480	.000353	.0000561	.00815	+ .0000413	+ .00161	+ .000185
60	.302273	.128201	1.74537	.00109	.0000995	.0206	+ .0000151	+ .00454	+ .000136
70	.195618	.185190	1.31756	.00110	.000272	.0180	+ .000101	+ .00423	+ .000131
80	.080507	.253589	0.772125	.000716	.000602	.0606	+ .00108	+ .00182	+ .0000827
90	.016172	.339133	.415254	.000675	.00105	.00234	+ .0000633	+ .000973	+ .0000759
500	.003816	.460777	.218502	.000414	.00129	.000593	+ .00000553	+ .000279	+ .000245
10	.037465	.606741	.112044	.000325	.000829	.000190	+ .0000884	+ .000112	+ .0000999
20	.117749	.761757	.060709	.000183	.000457	.000649	+ .000109	+ .0000325	+ .0000185
30	.236491	.875211	.030451	.000143	.000253	.000271	+ .000100	+ .00000547	+ .00000204
40	.376772	.961988	.013676	.000622	.000590	.0000431	+ .000301	+ .0000133	+ .0000285
550	.529826	.991761	.003988	.00143	.000668	.000105	+ .000528	+ .0000531	+ .0000700
60	.705224	.997340	.000000	.00283	.000847	.000100	+ .000960	+ .000102	+ .0000513
70	.878655	.955552	.00437	.000998	.000942	.0000942	+ .00142	+ .000147	+ .000107
80	1.01416	.868934	.00569	.00113	.000078	.000078	+ .00131	+ .000184	+ .0000381
90	1.11852	.777405	.00588	.000947	.000652	.000194	+ .00194	+ .000194	+ .0000806
600	1.12399	.658341	.00493	.000731	.0000379	.000379	+ .00170	+ .000148	+ .0000720
10	1.03048	.527963	.00324	.000475	.0000248	.000248	+ .00116	+ .000102	+ .0000518
20	0.856297	.399057	.00159	.000240	.0000123	.0000123	+ .000601	+ .0000449	+ .0000222
30	.647467	.283493	.000575	.0000918	.00000587	.00000587	+ .000226	+ .0000120	+ .00000591
40	.431567	.179828	.0000750	.0000127	.00000280	.00000280	+ .0000305	+ .000000138	+ .000000067
650	.268329	.107633	.0000470	.00000794	.00000050	.00000050	+ .0000191	+ .000000052	+ .000000032
60	.152568	.060281	.0000458	.00000713	.00000043	.00000043	+ .0000179	+ .000000166	+ .000000091
70	.0812606	.0318004	.0000129	.00000192	.00000016	.00000016	+ .00000492	+ .000000084	+ .000000044
80	.0408508	.0159051	.00000427	.000000619	.000000017	.000000017	+ .00000161	+ .000000013	+ .000000007
90	.0199413	.0077488	.000000894	.000000128	.0000000043	.0000000043	+ .000000335	+ .000000030	+ .000000016
700	.00957688	.00371774	.000000366	.000000052	.0000000016	.0000000016	+ .000000137	+ .0000000088	+ .0000000049
10	.00455263	.00176847	.000000114	.000000016	.00000000039	.00000000039	+ .000000042	+ .0000000012	+ .00000000086
20	.00217496	.00084619	.000000019	.0000000026	.000000000043	.000000000043	+ .0000000069	+ .000000000027	+ .000000000051

TABLE II—CIE UNIFORM TRISTIMULUS FUNCTIONS FOR 10°-FIELD AND BETWEEN-OBSERVER
VARIANCES AND COVARIANCES IN \bar{u} , \bar{v} , \bar{w} .

λ (nm)	\bar{u}_{10}	\bar{v}_{10}	\bar{w}_{10}	$V(\bar{u})$	$V(\bar{v})$	$V(\bar{w})$	$C(\bar{u}, \bar{v})$	$C(\bar{u}, \bar{w})$	$C(\bar{v}, \bar{w})$
400	0.0127	0.0020	0.0364	0.0000560	0.00000118	0.000444	+0.00000694	+0.000157	+0.0000200
10	.0565	.0088	.1655	.000294	.0000110	.00240	+ .0000472	+ .000830	+ .000141
20	.1364	.0214	.4161	.000417	.0000262	.00343	+ .0000487	+ .00116	+ .000177
30	.2099	.0387	.6774	.000328	.0000671	.00315	+ .0000707	+ .000965	+ .000302
40	.2559	.0621	.8847	.000171	.0000448	.00211	+ .0000544	+ .000584	+ .000204
450	.2473	.0895	.9463	.000157	.0000561	.00166	+ .0000275	+ .000460	+ .000156
60	.2016	.1282	.9138	.000485	.0000995	.00359	+ .0000101	+ .000227	+ .000225
70	.1305	.1852	.8388	.000489	.000272	.00322	+ .0000674	+ .0000943	+ .000392
80	.0537	.2536	.7262	.000318	.000602	.00229	+ .0000720	+ .000260	+ .000953
90	.0108	.3391	.7082	.000300	.00105	.00283	+ .0000422	+ .0000360	+ .00184
500	.0025	.4608	.7985	.000184	.00129	.00264	+ .000000369	+ .0000444	+ .00181
10	.0250	.6067	.9473	.000144	.000829	.00166	+ .0000589	+ .0000180	+ .00115
20	.0785	.7618	1.1142	.0000814	.000457	.000882	+ .0000727	+ .0000590	+ .000622
30	.1577	.8752	1.2098	.0000636	.000253	.000454	+ .0000667	+ .0000543	+ .000329
40	.2513	.9620	1.2614	.000276	.000590	.00100	+ .000201	+ .0000980	+ .000721
550	.3534	.9918	1.2248	.000636	.000668	.000956	+ .000352	+ .0000690	+ .000701
60	.4707	.9973	1.1433	.00126	.000847	.00107	+ .000640	+ .0000510	+ .000760
70	.5861	.9555	0.9939	.00210	.000998	.00123	+ .000947	+ .000111	+ .000780
80	.6765	.8689	.7962	.00253	.00113	.00174	+ .00121	+ .0000300	+ .000985
90	.7460	.7774	.6068	.00261	.000947	.000720	+ .00129	+ .0000400	+ .000590
600	.7497	.6583	.4254	.00219	.000731	.000350	+ .00113	+ .000110	+ .000290
10	.6873	.5280	.2767	.00144	.000475	.000180	+ .000773	+ .000110	+ .000159
20	.5712	.3981	.1690	.000707	.000240	.0000510	+ .000401	+ .0000860	+ .0000710
30	.4319	.2835	.1015	.000256	.0000918	.0000140	+ .000151	+ .0000380	+ .0000280
40	.2879	.1798	.0539	.000333	.0000127	.00000190	+ .0000203	+ .00000550	+ .00000380
650	.1790	.1076	.0272	.0000209	.00000794	.00000110	+ .0000127	+ .00000340	+ .00000235
60	.1018	.0603	.0141	.0000204	.00000713	.000000900	+ .0000119	+ .00000270	+ .00000175
70	.0542	.0318	.0071	.00000574	.00000192	.000000180	+ .00000328	+ .000000650	+ .000000440
80	.0273	.0159	.0034	.00000190	.000000619	.000000060	+ .00000107	+ .000000190	+ .000000127
90	.0133	.0077	.0016	.000000397	.000000128	.000000010	+ .000000223	+ .000000038	+ .000000025
700	.0064	.0037	.0007	.000000163	.000000052	.000000003	+ .000000091	+ .000000015	+ .000000010
10	.0031	.0018	.0004	.000000051	.000000016	.000000002	+ .000000028	+ .000000004	+ .000000003
20	.0015	.0008	.0001	.000000008	.000000003	.000000001	+ .000000005	+ .000000001	+ .000000000

lipes. The ellipses reveal the observer variability. Figure 7 shows observer variability (solid-line ellipses) and system variability (dashed-line ellipses).

Applications

We have reviewed a series of papers that deal with various phases of variability in the total spectrophotometric colorimetry problem. We have included the variabilities that are attributed to uncertainties in each of the components of the standard observer system for colorimetry: the spectral characteristics of a source, the spectral characteristics of an intervening medium, and the spectral characteristics of observers. For the last mentioned component two types of variability were described, namely the variability within the measurements of one observer, and the variability among measurements of a number of observers. A distinction was also made between the variability of individual measurements and the variability of sample means.

Several examples were given to show how we may compute separately variances and covariances of chromaticity coordinates attributed to each of the components of the standard observer system. There may be applications in which an over-all analysis

of the effects of any combination of variate uncertainty is required. Such an analysis may be performed by computing the variances and by the use of equations (2) and (3), respectively, with the proper components assumed to be uncertain.

Conclusion

From this discussion we conclude that a comprehensive analysis of color-matching data in terms of means, variances, and covariances permits use of the standard colorimetry systems to determine not only how the average observer sees color, but also how individual observers may depart from the standard observer. Although some applications of variances and covariances have already been reported, further applications may be suggested when a complete standard observer system is established. Such a system should consist of mean spectral tristimulus values, together with their variances and covariances. These should be given in the xyz system and in the uvw system as well. We suggest that the use of such a complete system be recommended by the International Commission on Illumination (CIE) as strongly as it recommends the use of the present system of mean spectral tristimulus values alone.

Epilogue

Most of the attempts to add to the knowledge of color, color vision, and color measurement have been concerned with how alike are normal observations of color. The main thesis presented here is that we should know how different observers are and how we may use this information obtained from past experience to make predictions about such differences in future problems. If this thesis leaves workers in the field of color measurement without a feeling of reassurance that all is right with colorimetry, then they should be referred to the concluding remarks of J. W. M. DuMond in a paper entitled *Pilgrims Progress in Search of the Fundamental Constants* (13):

"I think the spirit of our times presents a grave danger that we may attach so much undeserved importance to mere standardization and conformity of usage that we lose sight of the fact that scientific truth in this field is only to be reached by looking for discrepancies and disagreements. It is a free battlefield, no place for worshippers of the cult of Dale Carnegie, I assure you."

(Continued on page 41)

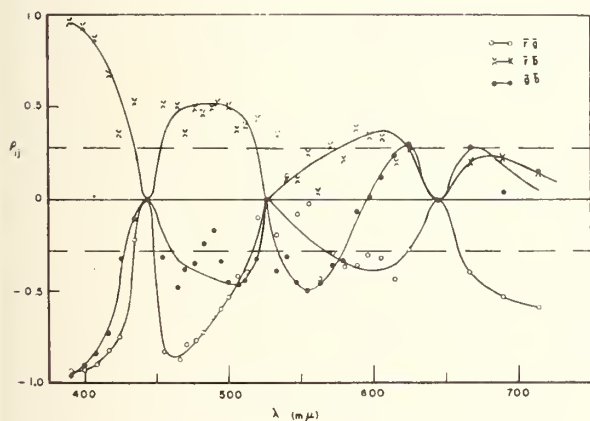
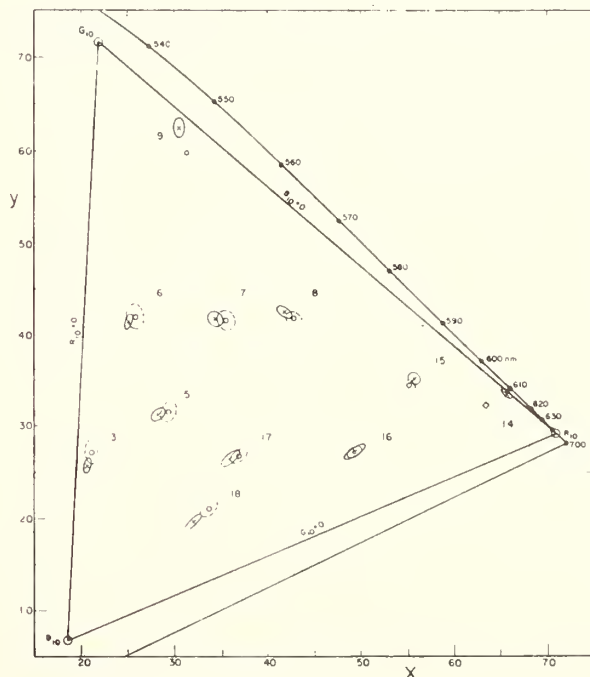


Figure 6—Correlation coefficients ρ_{ij} for $ij = rg$ (\circ), rb (\times), gb (\bullet). (Dashed lines define the region within which there is no significant statistical difference from zero at the 5% level.)

Figure 7—Comparison of individual-variance ellipses (solid line) with system-variance ellipses (dashed line) for the 10°-field color-matching functions.



The Variability of Color Measurement

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Foreword

Color, its measurement, and its specification, are a part of a larger field of visual optics, namely visual appearance. A collection of works on colorimetry would not be complete if it did not include the work that was done at the National Bureau of Standards on the other attributes of appearance. Probably the most important of these is gloss, because the color appearance of surfaces is strongly influenced by the gloss.

This section contains five papers on gloss and appearance evaluation. The first, by Nimeroff, is a survey of the ASTM methods and standards for the evaluation of appearance by visual and instrumental means. These appearance attributes are evaluated in terms of color, gloss, haze, turbidity, and opacity.

The second paper, by Judd, develops a five-attribute system for describing visual appearance, which is classified into four modes: aperture, illuminant, volume, and surface. The most complex of these, the surface mode, has five attributes: hue, saturation, lightness, transparency, and glossiness. The relationships between the attributes of the various modes are discussed in this paper.

The next paper, by Nimeroff, Hammond, Richmond, and Crandall, while using ceramic materials on which to make gloss measurements, discusses the instrumental factors that determine gloss scales. Several different kinds of instrumental geometry are treated in attempting to duplicate the scale that resulted from the indeterminable geometry of an outmoded instrument with a new instrument of determinable geometry. The success of the new instrument in duplicating the scale of the old instrument reflects the depth of the knowledge of the effects of the instrumental factors.

The next paper, by Nimeroff, describes two-parameter methods of gloss measurement. By such methods, wide-angle, intermediate-angle, and narrow-angle scatterers can be distinguished. The two-parameter techniques are correlated with subjective evaluation of gloss appearance.

The last paper, by Hammond, is the most recent review of gloss standards and glossmeter standardization. It emphasizes the use of gloss standards with glossmeters built to meet the close geometric tolerances specified for these meters.

STATUS OF ASTM METHODS AND STANDARDS FOR APPEARANCE EVALUATION

BY I. NIMEROFF¹

SYNOPSIS

A survey of the ASTM methods and standards for appearance evaluation is presented. Three classifications of these methods are distinguished: casual visual inspection, controlled visual inspection, and instrumental measurement. The number of published methods of each class is, respectively, 25, 19, and 31. The underlying principles and limitations of these methods are discussed. Instrumental measurement of appearance characteristics is divided into two groups: spectral evaluation and spacial evaluation of emitted, transmitted, or reflected light. The measurement of color involves spectral evaluation, while the measurement of gloss, haze, turbidity, and hiding involves spacial evaluation. A suggestion is made that ASTM Committee E-12 on Appearance emphasize unification, simplification, and generalization in the development of future methods for appearance measurement.

Our own experience teaches us the fallibility of human senses in making reliable estimates of absolute or relative magnitudes for such fundamental phenomena as time, length, mass, and temperature. In these fields we have come to rely almost entirely on physical measurements for accurate evaluations because the instruments that have been devised are more consistent and reliable than are human observers. The human eye, however, which is the primary instrument for appearance evaluation, can discriminate smaller appearance differences than most instruments developed to date. This discriminating ability of the human eye is especially effective

when a specimen under examination is in juxtaposition with a standard and only a relative evaluation is desired.

VISUAL TESTS

Sight, which is involved in all appearance evaluations, is the most reliable of our senses. Visual methods of test for appearance are simple and direct, often requiring no special apparatus but only instructions about how to look at specimens. Also, since the customer uses his eyes in the final inspection, the supplier favors using his own eyes during manufacture.

For these reasons, as a survey of ASTM methods reveals, there are a greater number of methods which use visual tests—some casual and some controlled—than those which use instrumental tests.

As an example of what is meant by a

¹ Physicist, In charge of research on gonio-photometry of materials, Photometry and Colorimetry Section, Optics and Metrology Division, National Bureau of Standards, Washington, D. C.

casual visual test we may cite the following from ASTM Method D 333 - 40²:

"A convenient method of determining apparent gloss is to hold the test panel at an angle of incidence approaching 180 deg to a source of artificial light and to note the comparative sharpness of the definition of the image (for example, the lamp filament)."

This description presumably means that the angle between the direction of

TABLE I.—NUMBER OF TEST METHODS OF EACH CLASS OF APPEARANCE.

Appearance Characteristic	Visual Inspection		Instrumental Measurement
	Casual	Controlled	
Color.....	21	19	9
Gloss.....	1	0	7
Reflectance.....	2	0	10
Transmittance.....	0	0	4
Clarity.....	1	0	0
Haze.....	0	0	1
Opacity.....	0	0	1
Totals	25	19	32

illumination and the direction of view is nearly 180 deg. In this test there is no control of geometry and no comparison scale.

An example of a partially controlled visual test is found in ASTM Method D 509 - 55.² In this method evaluations of "lighter or darker" are required to be given for test specimens relative to United States Rosin Standards whose specifications are given in terms of CIE chromaticity coordinates (2)³ x and y , luminous transmittance T , dominant wavelength λ , and colorimetric purity p . This method has a comparison scale, but exercises no control of the illuminating and viewing conditions.

The number of test methods of each

TABLE II.—ASTM VISUAL TEST METHODS FOR APPEARANCE EVALUATION.

ASTM Designation	Material
COLOR TESTS	
D 29 - 40	Shellac
D 29 - 54 T	Shellac
D 49 - 44	Dry red lead
D 154 - 56	Varnishes
D 268 - 53	Lacquer, solvents and diluents
D 365 - 39	Nicrocellulose base solutions
D 387 - 52 T	Color pigments
D 509 - 55	Rosin
D 555 - 54	Drying oils
D 564 - 47	Liquid dryers
D 716 - 45	Mica pigment
D 717 - 45	Magnesium silicate pigment
D 718 - 45	Aluminum silicate pigment
D 719 - 51	Diatomaceous silica pigment
D 1209 - 54	Clear liquids
D 1535 - 58	Paint
D 155 - 45 T	Lubricating oil and petrolatum
D 156 - 53 T	Petroleum products (Saybolt color)
D 565 - 45	White mineral oil
D 612 - 56 T	Paraffin wax
D 130 - 55 T	Petroleum products
D 664 - 54	Petroleum products
D 910 - 56 T	Aviation gasolines
D 976 - 48 T	U. S. military gasoline
D 1219 - 56	Jet fuels
D 94 - 56 T	Petroleum products
D 117 - 54 T	Electrical insulating oils
D 351 - 53 T	Mica
D 901 - 56	Askarels
D 848 - 47	Benzene, toluene, xylene, etc.
D 853 - 47	Industrial aromatic hydrocarbons
D 233 - 51	Turpentine
D 801 - 48	Dipentene
D 802 - 49	Pine oil
D 803 - 55 T	Tall oil
D 856 - 49	Pine tars and pine tar oils
D 1131 - 53	Rosin oils
D 620 - 49	Plastics
D 1045 - 55 T	Plastics
GLOSS TESTS	
D 333 - 40	Lacquers and enamels
REFLECTANCE TESTS	
D 332 - 55 T	White pigments
D 725 - 55	Paper
CLARITY TESTS	
D 555 - 54	Drying oils

type for seven classes of appearance characteristics found in ASTM methods is shown in Table I. Table II, indicates

² 1958 Book of ASTM Standards, Part 8.

³ The boldface numbers in parentheses refer to the list of references appended to this paper, see p. 8.

the variety of materials for which visual appearance evaluation methods are used.

There are several limitations to visual methods. Two of these involve: (1) the differences that exist between observers, and (2) the variation that one observer may have with himself from day to day. Another is that observers are unable to assign reliable absolute or relative numbers to an appearance sensation. He may be able to state that color A is red or that color B is bluer than color C. The observer cannot, with satisfactory pre-

except when viewed edgewise they shall possess not more than a slight greenish tint, the depth of which shall not affect the transmission of light rays, optics, or interfere in any manner with the use of the slide for its intended purpose."

It is difficult, if not impossible, for a reader of this specification to determine precisely what is meant by the phrase "a slight greenish tint."

INSTRUMENTAL METHODS

Although sight is the most reliable of

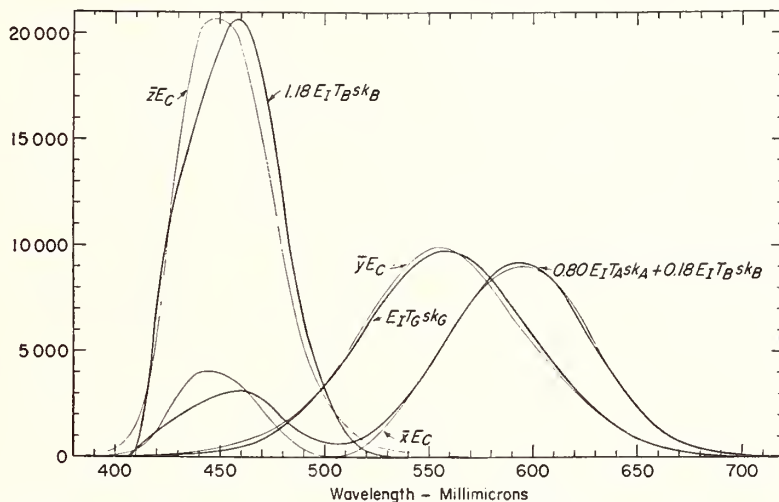


FIG. 1.—CIE Standard Observer Functions and Their Approximation by a Three-Filter Colorimeter.

cision, state how red color A is or how much bluer is color B than is color C.

Still another disadvantage of visual methods is that word descriptions of visual appearance may mean different things for different applications or to different people. An example of this difficulty may be found in a federal specification (Fed. Spec. GS-446a, Oct. 9, 1956) for microscope slides that reads:

"**Material.**—Unless otherwise specified, material shall be colorless, transparent glass.

"**1 Color.** Microscopical slides shall be colorless, as judged by the unaided eye,

our senses, the differences among observers and the difficulties of language interpretation are sufficient to lead us to seek refined instrumental procedures. These procedures should correlate with visual judgments and yield reproducible results that cannot be misunderstood—that is, the results must be in terms of numbers. Although repeatability and reproducibility may not be perfect, indexes of the differences found on one instrument from day to day and of the differences found among several instru-

ments of the same type can be obtained easily.

Practically all appearance evaluations made by instruments involve either a spectral or a spatial delimiting of light emanating from a primary source (that is, by emission) or from a secondary source (that is, by reflection or transmission). Spectral measurements are useful in color evaluations, while spatial measurements are useful in evaluating gloss, haze, turbidity, opacity, diffusion, and the like. Often both spectral and spatial measurements are required in an appearance evaluation. To make such an evaluation, two methods, say one for color and one for gloss, would be employed.

COLOR

In 1931, the International Commission on Illumination (CIE) recommended a standard observer and coordinate system for colorimetry (2). This system has as its basis three primary spectral functions which together represent the color response of the average human observer with normal color vision. Figure 1 shows these CIE functions. This system permits the analysis of spectral data so that the color of light emitted, transmitted, or reflected can be represented by three numbers. ASTM Method D 307 - 44⁴ is based directly on the spectrophotometric analysis of this system.

As early as 1930, when adoption of the CIE system appeared imminent, a photoelectric colorimeter with three filters that match the CIE functions was suggested in a British patent (2). Since then a number of such colorimeters that give results equivalent to daylight illumination have been developed. Difficulties have been experienced in making a simple filter for the two-lobed \bar{x} function. Some colorimeters use a portion of the \bar{z} -filter

reading to represent the blue lobe of the \bar{x} function. One of the earliest of this type is that described by Hunter (3). A comparison of the response functions of his colorimeter with the CIE functions, shown in Fig. 1, reveals the error introduced by this technique. Others, such as those of Barnes (4), and of Nimeroff and Wilson (5), have made use of a filter that matches the blue lobe. The recently adopted ASTM standard filter method for colorimetry, E 97 - 55⁵, and other methods refer to photoelectric colorimeters of the former kind, although those of the latter kind are not specifically excluded.

The filters of these colorimeters fail to duplicate exactly the CIE functions. Consequently, users of colorimeters are cautioned by instructions in some methods to attempt only colorimetry of specimens having reflectance spectrally similar to the standard plaques used to calibrate the colorimeter. These standard plaques are generally calibrated by means of spectrophotometric colorimetry.

Further use of these colorimeters has been made in color-difference measurement. There have been developed and adopted in the past four years some five color-difference methods, ASTM Method D 1260 - 55 T, D 1365 - 55 T, D 1482 - 57 T, D 1495 - 57 T, and D 1536 - 58 T.² These methods refer respectively to the Hunter multipurpose reflectometer, the Hunter color difference meter, and GE spectrophotometer, the "color eye," and the "colormaster" differential colorimeter. The general color difference formula of these methods in vector notation is

$$\bar{E} = (\Delta L)\bar{L} + (\Delta a)\bar{a} + (\Delta b)\bar{b} \dots (1)$$

where \bar{L} , \bar{a} , and \bar{b} are mutually perpendicular unit vectors in color space, ΔL is a lightness-index difference, and

⁴ 1958 Book of ASTM Standards, Part 9.

⁵ 1958 Book of ASTM Standards, Part 6.

Δa and Δb are chromaticity-index differences. The magnitude of the vector sum, or resultant, of these differences is computed from

$$E = \sqrt{(\Delta L)^2 + (\Delta a)^2 + (\Delta b)^2} \dots (2)$$

The space defined by the three unit vectors approximates psychologically-

tion, and lightness. Because of the established correlation, this controlled visual method is a powerful and useful tool for colorimetry.

In each of these methods, a serious attempt has been made to specify the geometry of illuminating and viewing the specimen.

TABLE III.—FORMULAS USED IN COMPUTING COLOR DIFFERENCE.

ASTM Method	Instrument	L	a	b
D 1260 - 55 T...	Hunter reflectometer	$10 \sqrt{G}$	$\frac{70\sqrt{G}(A - G)}{B + A + 2G}$	$\frac{28\sqrt{G}(G - B)}{B + A + 2G}$
D 1365 - 55 T...	Hunter color difference meter	$10\sqrt{R_d}$	$175f_y^* (1.02X - Y)$	$70f_y^* (Y - 0.847Z)$
D 1482 - 57 T...	GE spectrophotometer	$9.2 V_Y^{**}$	$40 (V_X - V_Y)$	$16 (V_Y - V_Z)$
D 1495 - 57 T...	"Color eye"	$9.2 V_Y^{**}$	$40 (V_x - V_y)$	$16 (V_y - V_z)$
D 1536 - 58 T...	"Colormaster" differential colorimeter	$10.0 V(G)$	$41.86 [V(G) - V(R')]**$	$16.74 [V(G) - V(B)]$

$$*f_y = \frac{0.51(21 + 20Y)}{1 + 20Y}$$

$$** \frac{Y_s}{Y_{M90}} = a_1 V_Y - a_2 V_Y^2 + a_3 V_Y^3 - a_4 V_Y^4 + a_5 V_Y^5$$

$$*** R' = 0.8R + 0.2B$$

perceived uniform color space. For computing the differences, ΔL , Δa , and Δb , each of the methods in the preceding paragraph uses formulas that are convenient to the data obtained from the device used. These formulas are shown in Table III.

An elaborate, controlled visual method of evaluating color by the Munsell System has been published recently as ASTM Method D 1535 - 58 T². This system is correlated with the CIE system and with the concept of perceptually-uniform color spacing. The "Munsell Book of Color", on which this method is based, is composed of painted chips representing uniform variations in the three parameters of color perception, hue, saturation,

GLOSS

The appearance of an object is determined by its gloss as well as by its color. Whereas the color of an object depends on light that has penetrated into the body of the object, the gloss depends on the light reflected by its surface. It has long been observed that the glossier an object is, the greater is the amount of incident light reflected in the specular direction. To evaluate completely this spacially preferential reflectance a multiplane goniophotometer is required. Generally, however, use of such an instrument, or even a monoplane goniophotometer, would be too costly in time and effort to permit universal applica-

tion to all problems of gloss. Analysis of data thus obtained would further complicate its applicability. In 1952 Nimeroff (6) suggested a rather simple way in which monoplanar goniophotometric data could be analyzed in terms of dispersion index and peakedness index. Although successful application of this analysis has been reported in private communication, no ASTM method embodying this analysis has been proposed, primarily because no general goniophotometric

related to the angular size of the source as well as to that of the receptor and since there are no infinitesimally small light sources or photoelectric receptors, fractional reflectance is defined as "the ratio of the luminous flux reflected from, to that incident on, a specimen for specified solid angles." Rigid specification of these angles and of the angles of incidence and view is mandatory. This method, which is designed for 20-, 60-, and 85-deg incidence angles, does rigor-

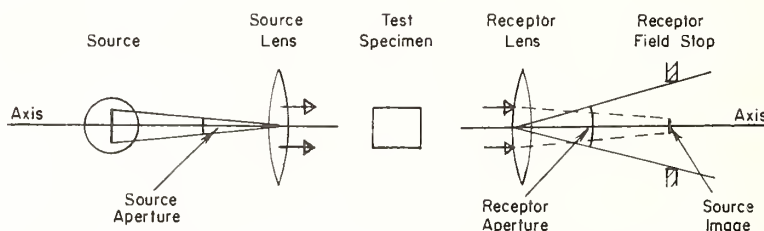


FIG. 2.—Generalized Glossmeter, Showing Apertures and Source Image Formation for a Collimated-Beam Type Instrument.

method has been adopted. Committee E-12 is now preparing a method for goniophotometry.

There is, however, a special ASTM goniophotometric method, D 636 - 43⁴, in which the data are analyzed by simple integration of critical areas under the spatial distribution curve obtained on a monoplanar goniophotometer. The geometry of this instrument is clearly outlined in the method. This goniophotometer is not, however, a device of high geometric resolution.

Glossiness may be evaluated in several ways but the simplest is to determine how much of the light incident on a surface is reflected in the specular direction. For this reason gloss methods measure specular gloss. In particular, ASTM Method D 523 - 53 T², has defined this quantity as "the luminous fractional reflectance of a specimen at the specular angle." But, since the reflected light measured by the receptor is strongly

ously specify instrumental geometry. A schematic drawing of the generalized glossmeter is shown in Fig. 2. Other gloss methods such as ASTM Method C 346 - 55 T (45 deg)⁶ for porcelain enamels and ASTM Method D 1223 - 52 T⁵ (75 deg) for paper, follow the rigorous geometry tolerances of ASTM Method D 523. These tolerances are based in part on work reported by Hammond and Nimeroff (7).

Recently there was adopted by ASTM a method, D 1471 - 57 T² that overcomes some of the difficulties encountered by the one-parameter methods described above. The difficulties arise because a single-number description of a phenomenon as complex as the scattering of surface-reflected light is inadequate. Nimeroff (8) showed the existence (not revealed by a one-parameter method) of at least three distinct kinds of scatterers

⁶ 1958 Book of ASTM Standards, Part 5.

among a group of 26 varnish applications on wood. These were called narrow-angle, wide-angle, and intermediate scatterers. Separation of the kinds of scatterers requires a two-parameter method, the parameters being specular gloss readings with a large- and a small-receptor aperture. The geometry of a receiver-aperture, two-parameter glossmeter is shown in Fig. 3.

a surface-scattering phenomenon, are body-scattering phenomena. The names that have been associated with these characteristics are "haze," "turbidity," and "opacity." Haze and turbidity are applied to nearly transparent media, while opacity is applied to nearly opaque media. In haze and turbidity measurements, as in gloss measurements, rigorous specification of instrumental

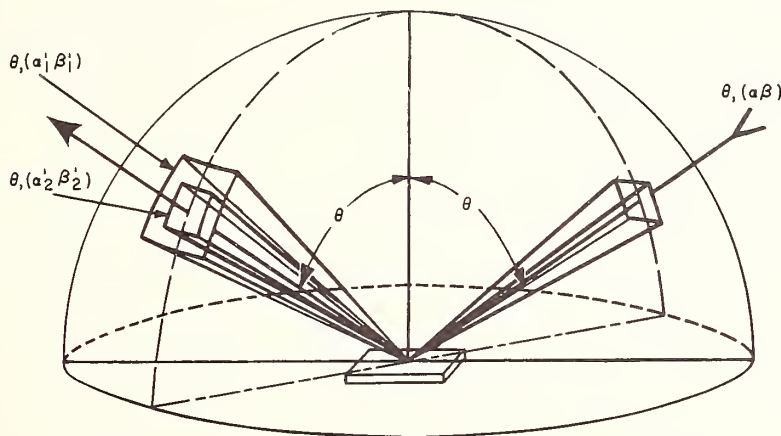


FIG. 3.—Geometry of a Receiver-Aperture, Two-Parameter Glossmeter.

In the same paper, Nimeroff showed also that some sheen problems could be solved by a two-parameter gloss method. Here the two parameters are specular gloss at two different angles of incidence with the same source and receptor apertures. No method of this kind has been formulated as yet. ASTM Method D 673 - 44⁴, however, requires a 45-deg specular gloss measurement and a measurement with the receptor displaced 15 deg toward the perpendicular to the test surface. This is a two-parameter method although only one parameter is a specular gloss measurement.

OTHER APPEARANCE CHARACTERISTICS

Several other appearance characteristics of materials are based on their light-scattering properties. These, in contrast to gloss, which is predominantly

geometry is required, while in opacity, as in colorimetry, such specification is not required.

ASTM Method D 1003 - 49 T⁴ compares the light that is rectilinearly transmitted by the test material with that which has been scattered out of the beam by more than 2.5 deg. Instrumental geometry and tolerances are described in this method.

There is as yet no ASTM method for turbidity. Work on the turbidity of water is proceeding, however. It is hoped that there will be an ASTM method in the not-too-distant future.

Opacity is specified in ASTM Method D 589 - 44⁶, on paper, in terms of a contrast ratio. The ratio in this method is,

$$C = 100 \frac{R_b}{R_w} \dots \dots \dots (3)$$

where:

R_b = the reflectance of the test specimen when backed by a black material, and

R_w = its reflectance when backed by a white material.

This ratio indicates the relative scattering of light in the specimen material because the more the material scatters light the closer is the ratio to 100.

A body-scattering characteristic related to opacity is "hiding power." This use of the term "power" is not technically rigorous because it does not refer to time-rate of energy flow. Hiding power is described in a method proposed in ASTM Committee D-1 on Paint, Varnish, Laquer, and Related Products as the area covered by a gallon of paint to give a contrast ratio of 0.98. Here the contrast ratio equals R_b/R_w .

RECAPITULATION

We have reviewed the kinds of appearance tests that are now in the form of published ASTM methods. It was found that these can be classed into visual tests—casual and controlled—and instrumental tests. We have noted that appearance evaluation can be grouped into spectral and spacial delimiting of reflected, transmitted, and emitted light. The color of an object has been shown to depend mostly on the spectral distribution of the light, while gloss, haze, turbidity, and opacity depend mostly on the spacial distribution of the light.

Filter colorimeters have been developed that will analyze light approxi-

mately in accord with the CIE Standard Observer. Because of the approximation, the utility of these instruments has been limited to measurement of color close to the standard and to the evaluation of small color differences.

Glossmeters and hazemeters have been developed as abridgments of the more involved goniophotometry technique needed to measure the spatial distribution of reflected and transmitted light. Where single-parameter techniques have proven unsuccessful, two-parameter techniques have been devised to supplement or replace them.

THE ROAD AHEAD

What lies ahead? As indicated above there are some problems currently being given attention that have not reached the formal method stage. Certainly this work needs to be carried forward.

There have been developed a large number of similar methods, each designed for special materials, that could well be placed in one unified method. For the sake of simplicity we need to develop methods that will apply to a class of problems rather than to a class of materials. Perhaps if there were general methods for appearance evaluation they could be adapted, with only minor variation, to fit the special problems. Then there might not be a need for so many special methods for each material class.

We may sum up the goal for the road ahead in three words: unification, simplification, and generalization.

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A FIVE-ATTRIBUTE SYSTEM OF DESCRIBING VISUAL APPEARANCE

By
DEANE B. JUDD



Reg. U.S. Pat. Off.

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FOREWORD

Among the active groups of ASTM Committee E-12 on Appearance is its Task Group IV on Aluminum Finishes. The objective of this task group is to develop methods for measuring and specifying the appearance of aluminum finishes.

Structural and decorative aluminum is a major factor in our economy. Aluminum surfaces finished mechanically, chemically, or anodically are increasingly evident in automotive and appliance trim, commercial and residential architecture, and truck and aircraft construction.

The technical aspect of this project is equally impressive. It soon became apparent that the conceptual framework for handling the interrelated dimensions of color, gloss, and transparency is not to be found in conventional treatments of these factors.¹ From the collective experience of the membership of Committee E-12 it was recalled that Deane B. Judd, of the Photometry and Colorimetry Section of the National Bureau of Standards, had some twenty-six years earlier developed a rationale for consolidating these parameters of appearance. For want of a pertinent application, this concept had not been published.

This initiation of substantial effort on a technical problem of such difficulty and importance seemed to Task Group IV an appropriate reason for publishing Dr. Judd's paper. Accordingly, this thought was presented to Dr. Judd, who agreed to review and reshape his presentation to take into account some changes in understanding of appearance problems since his paper was first written. Now entitled "A Five-Attribute System of Describing Visual Appearance," this paper is published as a Special Technical Publication to reflect its stature as a major contribution to the science of describing appearance of engineering materials.

G. W. INGLE,
Chairman, Committee E-12.

¹See R. V. Paulson, "Problem of Measuring Appearance of Aluminum Surfaces," ASTM BULLETIN, No. 242, Dec., 1959, p. 33.

A FIVE-ATTRIBUTE SYSTEM OF DESCRIBING VISUAL APPEARANCE

BY DEANE B. JUDD¹

SYNOPSIS

This is an extension of the three-attribute system of color description based on the perceptual studies of Katz. The nonshape, nontextural aspects of visual appearance are classified according to four modes of appearance: aperture, illuminant, volume, and surface modes. The aperture mode has three attributes: hue, saturation, and brightness; the illuminant mode, four: hue, saturation, brightness, and transparency; the volume mode, four: hue, saturation, lightness, and transparency; and the surface mode, five: hue, saturation, lightness, transparency, and glossiness. The relations between the attributes are discussed, and a short list of appearance descriptions is included as a demonstration of the convenience and inclusiveness of the system.

A common usage of the term, color, is one that distinguishes between colors and grays; but the distinction is seldom rigidly kept. No one says that a man wears a colorless suit because it is a black, white, or gray suit. No one says that a light red is the same color as a dark red, even though the two may be identical in hue and saturation; instead, color names (such as pink and maroon) identifying them as different colors are used. It has been recognized accordingly that colors perceived in the nonself-luminous object mode have the three attributes, hue, saturation, and lightness [1].² The corresponding attributes of color perceptions in the self-luminous modes are hue, saturation, and brightness.

Common parlance also recognizes appearance distinctions not to be de-

scribed in terms of these triads of color-perception attributes. It is said, for example, that the appearance of one solution is a transparent blue as distinguished from a murky blue. Likewise, the appearance of a painted surface is commonly described as a glossy black as distinguished from a mat black. Those who refuse to class white, black, and grays as colors are likely to consider that water is colorless. This position is not particularly helpful, however, because a mirror surface, and air, which may equally well be called colorless, plainly are both different in appearance from water. The three-attribute system of color-appearance description is, of course, insufficient to deal with these distinctions because they are not color distinctions.

As a final example of the inadequacy of the three-attribute system of color-appearance description may be cited the distinction between gold and yellow, and that between copper and red-

¹ Photometry and Colorimetry Section, National Bureau of Standards, Washington, D. C.

² The italic numbers in brackets refer to the list of references appended to this paper.

dish brown. These distinctions are not wholly a matter of gloss, because a gold surface may be depolished until it has the same gloss as that of a glossy yellow paint of the same hue, saturation, and lightness, and yet the appearance of the two surfaces will be very different—it will be said that they have different colors. It is equally impossible to duplicate the color of copper by means of glossy reddish-brown paint even though the three attributes of color-appearance are duplicated.

These difficulties have long been recognized and investigated by psychologists [2-8]. One result of these investigations is the drawing of the distinction, already mentioned, between the various modes of visual appearance. A classification of these modes and the nonshape, nontextural attributes possessed by each mode are listed below.

Modes of Visual Appearance:

1. Aperture (nonlocated) mode
2. Located modes
 - (a) Illuminant (self-luminous object) mode
 - (b) Nonself-luminous object modes
 - (1) Volume mode
 - (2) Surface mode

Nonshape, Nontextural Attributes of Visual Appearance:

1. Magnitude attributes: Possessed by all modes
 - (a) Brightness: Possessed by the aperture and illuminant modes, varies from very dim to very bright
 - (b) Lightness: Possessed by the object modes (volume and surface) and varies from black to white (surface), or from black to perfectly clear (volume)
2. Quality attributes: Possessed by all modes
 - (a) Hue
 - (b) Saturation
3. Transparency: Possessed by all located modes
4. Glossiness: Possessed by the surface mode

SYSTEMATIC CLASSIFICATION OF APPEARANCE

Classification is achieved by listing for each of the modes of appearance its nonshape, nontextural attributes:

1. Aperture (nonlocated) mode
Attributes: Hue, saturation, brightness
2. Located (nonaperture) modes
 - (a) Illuminant (self-luminous, volume) mode
Attributes: Hue, saturation, brightness, transparency
 - (b) Object (nonself-luminous) modes
 - (1) Volume (nonsurface) mode
Attributes: Hue, saturation, lightness, transparency
 - (2) Surface (nonvolume) mode
Attributes: Hue, saturation, lightness, transparency, glossiness

DEFINITIONS

Aperture mode (of color appearance) refers to color perceived simply as filling a hole in a screen.

A color perceived in the aperture mode has no definite location, because it may be either in the plane of the screen or indefinitely far behind it; such a color is perceived neither as concentrated in a single plane nor spread into a volume, but is nonlocated in depth. Viewing a surface at a distance of several feet through a hole in a screen a few inches in front of the surface often results in the perception of color in the aperture mode, and the screen, on this account, is sometimes called a *reduction* screen, because it may succeed in reducing the color perception from the surface mode to the aperture mode. If, because of the presence of such microstructure as brush marks, or for no determinable reason at all, the color is perceived as belonging to the surface, the reduction screen has failed of its purpose; that is, it has failed to reduce the mode of appearance from surface to aperture. The distinction between surface mode and aperture mode is thus not depend-

ent on the particular devices in front of the observer, but is dependent on the facts of his visual perception of those devices at the moment.

Illuminant mode (of color appearance) refers to color perceived to belong to a volume or bulk that seems to be emitting light (distinction from a volume that seems to be scattering or transmitting light). A color perceived in the illuminant mode is always volumic, never confined to a surface [6].

Volume mode (of color appearance) refers to color perceived to belong to a volume or bulk.

Surface mode (of color appearance) refers to color perceived to belong to a surface.

Object mode (of color appearance) refers to color perceived to belong to an object, that is, to a definite location in visual space with definite boundaries.

The object mode is either the volume mode or the surface mode. Although the view of a scene from a single direction often results in the perception of colors in the object mode, the use of two sets of angular conditions of viewing the scene, such as, for example, is provided by viewing binocularly, greatly enhances the possibility of perceptions in the object mode and makes them less ambiguous. The color perceived to belong to any one nonself-luminous object is often found to be nearly constant, regardless of whether the spectral distribution of the source varies within rather wide limits. The term "color constancy" refers to this approximate constancy of the color perceptions of any given object viewed under different amounts and kinds of light.

Brightness is the attribute of a color perceived in the aperture or illuminant mode by which the observer judges whether the corresponding area is

emitting light at a greater or lesser rate. Brightness varies from very dim to very bright.

Lightness is the attribute of a color perceived in the object mode by which the observer judges whether the object is transmitting or reflecting a greater or lesser fraction of the incident light. Lightness varies from black to white for colors perceived in the surface mode. It varies from black to perfectly transparent (or colorless) for nonself-luminous object colors perceived in the volume mode.

Hue is the attribute of certain color perceptions that permits them to be

TABLE I.—END POINTS OF TRANSPARENCY.

Mode of Appearance	Lower Limit	Upper Limit
Illuminant....	just greater than zero	just less than perfect
Volume.....	just greater than zero	perfectly transparent
Surface.....	zero	just less than perfect

classified as reddish, yellowish, greenish, bluish, or their intermediates.

(An) *achromatic color perception* is one having no hue. White, gray, black, the color perceived to belong to air, to water, or to a nonselectively reflecting mirror, are examples of achromatic color perceptions.

(A) *chromatic color perception* is one having a hue.

Saturation is the attribute of any chromatic color perception that determines the degree of its difference from the achromatic color perception most nearly like it. The achromatic color perception most nearly like a given chromatic color perception will be the one perceived in the same mode of appearance and neither lighter nor darker (nonself-luminous modes), or

neither brighter nor dimmer (self-luminous modes).

Transparency is the attribute of appearance in the volume mode which determines the degree to which colors

per unit solid angle leaving an infinitesimal area containing the point under consideration to the product of the infinitesimal area by the cosine of the angle between the line of sight and the

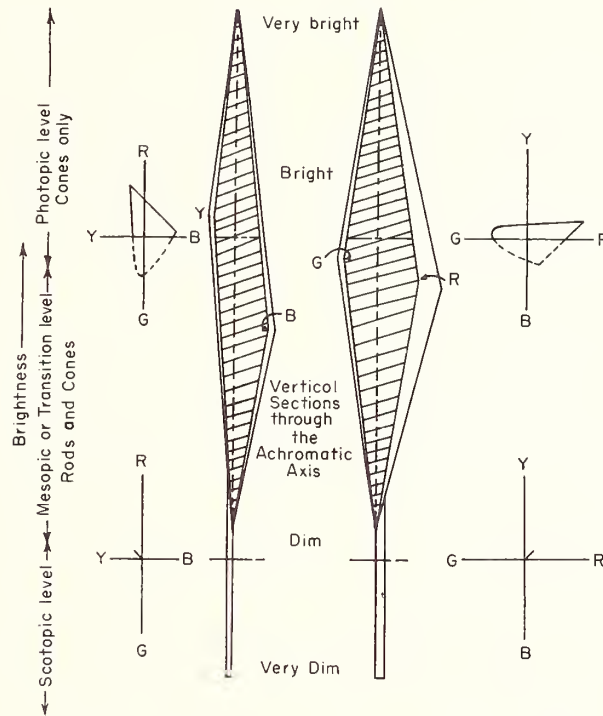


FIG. 1.—Sections of the Solid Representing Color Perceived at 4-deg Subtense in the Aperture and Illuminant Modes with Dark Background.

The two orthogonal vertical sections through the axis, representing achromatic color perceptions, are cross-hatched. One is a section through the plane representing colors perceived as having yellow and blue hues; the other, through the plane representing colors perceived as having red and green hues. There are four horizontal sections, two at moderately high brightness, two at low brightness. The brightnesses at which the sections are taken are indicated by dot-dash lines on the vertical sections. The part of the solid cut away in each vertical section is indicated by a dotted line on the corresponding horizontal section at the higher brightness. Most of the color perceptions in the scotopic level of brightness result from rod action and are achromatic, but the yellowish red hues experienced from threshold and near-threshold spectral stimuli of wavelength $620 \text{ m}\mu$, and higher, result from cone action. On this account the horizontal cross-section at low brightness is simply a short straight line.

are seen behind this volume. The end points of transparency for the three modes of appearance possessing this attribute are indicated in Table I.

Luminance (at a point on an object or aperture and for a given line of sight) is the ratio of the luminous flux

normal to the infinitesimal area. Luminance is the stimulus correlate for the brightness of the color perception.

Luminous directional reflectance (of a light-scattering object) is the ratio of the luminous flux reflected from the object in the direction of view to the

luminous flux reflected in the same direction from the ideal, perfectly reflecting, perfectly diffusing surface illuminated in the same way as the object. Luminous directional reflectance is the stimulus correlate for the lightness of the color perception of a light-scattering object.

Luminous transmittance (of a non-light-scattering object) is the ratio of the luminous flux transmitted by the object to the luminous flux incident upon it. Luminous transmittance is the stimulus correlate for the lightness of the color perception of a nonlight-diffusing object.

Glossiness is the attribute of appearance in the surface mode that determines the degree of its difference from the appearance of a mat surface perceived to have the same hue, saturation, lightness, and transparency. Glossiness determines the degree to which colors in front of a surface may be seen as if behind it.

Gray is a general name for achromatic colors perceived in the object mode, and is also sometimes less aptly used to designate achromatic colors of not too high brightness perceived in the aperture and illuminant mode. For the object mode, saturation is the attribute of any chromatic color perception that determines the degree of its difference from the nearest gray.

INTERRELATIONS AMONG THE ATTRIBUTES

Aperture and Illuminant Modes:

Some general relations among the attributes, hue, saturation, and brightness of colors of moderate angular subtense (say 4 deg) perceived in the aperture and illuminant modes with a dark surround have long been recognized; they have been expressed by drawing the color "pyramid" or color

solid.³ In this solid, hue and saturation are shown in polar coordinates, hues being arranged according to angle about the vertical axis, and saturation being proportional to distance from the achromatic axis of the solid. The boundaries of the solid are irregular and approximate somewhat a long, relatively thin double pyramid or cone (Fig. 1) converging to a vertex toward high brightness, and to a short line toward low brightness. These boundaries represent the limits beyond which our experience of colors in the aperture and illuminant modes cannot extend; they express the limitations of the visual mechanism. The axis shown in each vertical section of the figure, for example, represents achromatic color perceptions; it extends from very dim (absolute threshold) to very bright (dazzling). Its extent is finite because the number of distinguishable brightness steps is finite. Although the luminance of the stimulus may be indefinitely increased, the higher luminances fail to produce higher brightnesses because the visual mechanism has a certain limited capacity for perceiving brightness that cannot be exceeded. There are indicated in Fig. 1, along with the sketch of the various cross-sections of the color solid, the approximate extents of the photopic (cone) state and the mesopic (rod and cone) state according to brightness of the color perception. The lowest brightnesses correspond chiefly to the rod state. These are the brightnesses of the photochromatic interval between the absolute threshold and the chromatic threshold [9-11].

³ The explanatory value of the color solid has frequently been impaired because the terms *white* and *black* have been used to describe the colors represented at the upper and lower extremities of the achromatic axis. These terms apply strictly to colors perceived in the surface mode only and, as will appear presently, the color solid referring to surface colors has a very different shape.

If it were not for the fact that there is no photochromatic interval for spectrum stimuli of wavelength greater than $620\text{ m}\mu$, the color solid would come to a point at the bottom of the mesopic range and would extend throughout a purely scotopic range simply as an extension of the achromatic axis representing the hueless colors perceived purely by action of the retinal rods. For long-wave spectrum stimuli, however, the color perceived at the absolute threshold of light is a yellowish red of threshold saturation [11]; so the lower terminal of the color solid is a line extending in the direction corresponding to a yellowish-red hue.

The surface of the hue-saturation-brightness solid shows the colors perceived to belong to self-luminous areas either sending single-frequency light to the eye, or sending mixtures in various proportions of light from the two extremes of the visible spectrum. The part of the surface representing greenish-yellow color perceptions of maximum saturation is closer to the axis of the figure throughout a medium range of brightnesses than that of other hues because the number of distinguishable saturation steps from daylight to greenish yellow spectrum light ($570\text{ m}\mu$) is less than that obtainable from spectrum stimuli and mixtures thereof yielding color perceptions of other hues [12]. The tapering of the figure both toward higher and lower brightnesses expresses the fact that the chromatic part of the visual mechanism works best with stimuli of intermediate luminance. As the luminance is lowered the chromatic response of the cone mechanism is progressively swamped by increased rod action and by increased importance of the self-light of the retina, both of which yield achromatic perceptions. As the lumi-

nance is increased, the chromatic response is swamped by achromatic excitation from the cones themselves [13, 14]. The consequence of these two saturation-decreasing influences, one for increasing luminance, the other for decreasing luminance, is that there is a luminance at which for increasing luminance the increase in saturation with brightness gives way to a decrease. This turning point of saturation [13] occurs at a higher brightness for color perceptions of yellow hue than for other hues [9, 15]. All of these experimental facts are shown schematically in Fig. 1.

Surface Mode:

The solid representing appearances of zero transparency and glossiness belonging to surfaces of large angular extent (say 4 to 10 deg) illuminated by daylight and viewed against a light background (middle gray to white) has boundaries that are determined not only by the limitations of the visual mechanism but also by the limitations arising from the physical surface and its illumination by daylight. One of these limitations is the fact that a nonfluorescing surface cannot reflect more radiant energy of a given wavelength than falls upon it. The axis of this figure represents grays, ranging in lightness from black to white. Instead of being like a double cone or pyramid sharply pointed at the ends, this figure may be approximately described as a rounded rhomboid; the height is not much different from the greatest horizontal thickness (Fig. 2).

Since this solid represents the color perceptions of surfaces illuminated by daylight, it refers to the photopic range of brightness only and depends for its shape on action of the retinal cones as opposed to the retinal rods. There is no swamping of the chromatic

responses by rod action or by the self-light of the retina near zero lightness (black), nor is there any achromatic response of the cones due to overstimulation to swamp the chromatic responses near maximum lightness (white).

There is a ridge extending from the white point toward the area representing saturated surface-color perceptions of yellow hue, and this ridge bends to pass through orange and red hues as it progresses to higher saturations and lower lightnesses [16-20]. This ridge

(say the \bar{z} -function of the CIE system) can be confined to the short-wave portion of the visible spectrum where the luminous-efficiency is relatively low. The blue ridge is partly due to this cause also, and partly due to the antichromatic influence of the bright surrounding field, which prevents the chromatic aspect of the response to areas of much lower luminance from being perceived. In this surface-color-perception solid, all of the color perceptions of surfaces whose reflectances are confined to a single

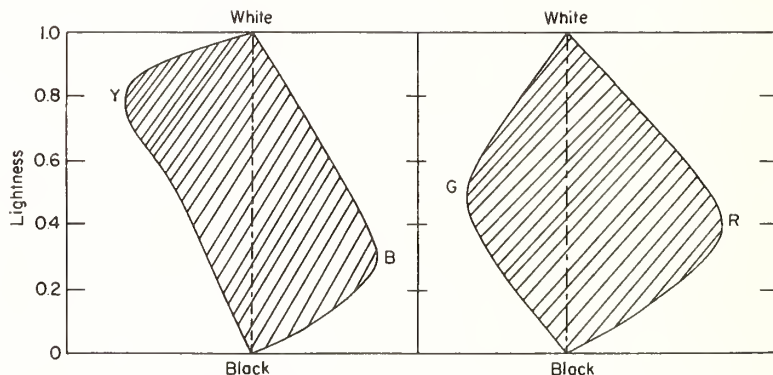


FIG. 2.—Vertical Orthogonal Sections of the Solid Representing Colors Perceived in the Surface Mode as at Zero Glossiness and Transparency with White Background.

expresses the fact that there are surfaces perceived to have saturated yellow colors whose lightness is nearly as high as that of white.

It is also true that saturated blue surface colors exist whose lightness is very low, almost as low as that of black. The possibility of these surface-color perceptions is represented in the lower boundary of the surface color solid, which has an inverted ridge roughly symmetrical to the upper ridge representing yellow color perceptions [18].

As an explanation for the yellow ridge may be cited the fact that one of the three color-matching functions

waveband of infinitesimal width are represented at precisely one point, the black point.

The shape of this surface-color solid is not very different from that which just encloses the Munsell color tree, and the charts of constant hue and those of constant value in the "Munsell Book of Color" [21] represent approximately vertical half-sections and horizontal sections, respectively, of the surface-color-perception solid. Attempts have been made [17,18] to determine by computation the exact shape of the limiting surface of this solid. The series of Ostwald "Vollfarben" [22] are represented by a

closed curve on the surface of this solid [18]; that is, the "Vollfarben" or "full colors" yield surface-color perceptions whose saturation is close to the maximum derivable from real surfaces illuminated by daylight for the given hue and lightness. Other surface-

surface appearances of zero transparency and glossiness in accord with the attributes of hue, saturation, and lightness, there must also be a solid representing achromatic appearances of surfaces in terms of lightness, glossiness, and transparency. In this case Cartesian coordinates rather than cylindrical coordinates are convenient. The vertical coordinate, as in Fig. 2, is lightness, one horizontal coordinate is glossiness, and the other horizontal coordinate is transparency. The contours of this three-dimensional figure are likewise determined both by the limitations of the visual mechanism and by the limitations arising from the fact that we are dealing with the appearances of physical surfaces viewed in daylight. Unlike the previous case, however, these contours have not been worked out even approximately. There is shown in Fig. 3, however, a rough sketch of one face of this solid, that at zero transparency.

The vertical line at the left of the figure represents grays, ranging in lightness from black to white; it is coincident with the axis of the hue-saturation-lightness solid, just discussed, which represents mat, opaque appearances. The sloping line at the top of the figure is taken to represent the appearances of perfectly reflecting surfaces; these appearances may be arranged in a series of light grays extending from an opaque white appearance to a perfectly glossy, gray appearance. Whether this line is straight is not definitely known, but that it slopes downward cannot be doubted because the appearance of a perfect mirror is not as light as the white color of the ideal perfectly reflecting and perfectly diffusing surface.

The vertical line at the right of the figure represents the appearances of perfect mirror surfaces of various re-

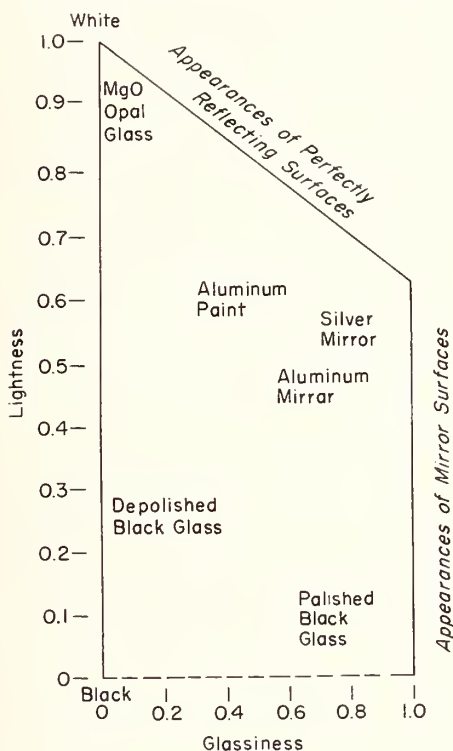


FIG. 3.—Diagram Representing Opaque Achromatic Appearances in the Surface Mode.

color perceptions derivable from real surfaces and having the same hue as, but a different lightness from, a given full color would likewise be represented on the boundary of this surface-color-perception solid; but whether the saturations of such perceptions are necessarily less than those yielded by the full color has yet to be determined.

Just as there is a solid representing

flectances from unity down to, but not including, zero. Such appearances are approximately equal (precisely equal according to Jones [23]) in glossiness; they are accordingly represented as equally distant from the black-white axis which refers to zero glossiness, though, perhaps, the line should incline somewhat toward black.

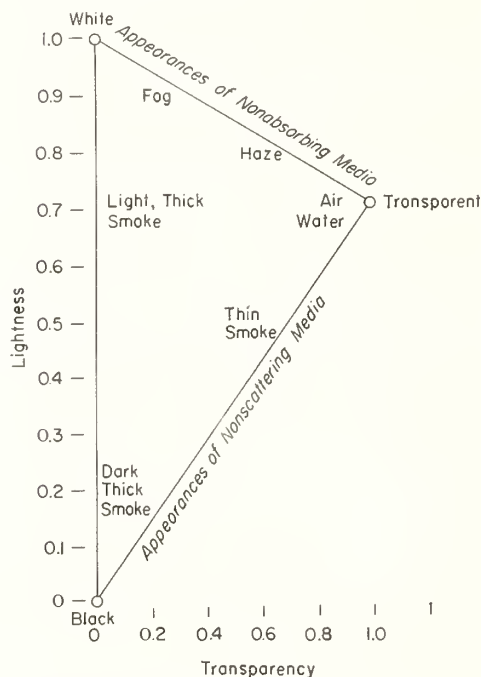


FIG. 4.—Diagram of Achromatic Appearance in the Volume Mode.

The dotted line at the bottom of the figure expresses the fact that there is no series of appearances extending from mat black to the appearance of a perfect mirror of zero reflectance. As soon as the reflectance of the mirror becomes zero, visually its surface ceases to exist. There is, however, a series of appearances extending from mat black to perfectly glossy black, which would correspond to a perfect mirror of reflectance just sufficiently

different from zero to yield a visible image of a small light source. The appearance of polished black glass would be represented at glossiness somewhat less than 1.0 (perfect), and lightness somewhat greater than zero. The lightness and glossiness of the appearances of various other opaque surfaces have been estimated and are indicated in Fig. 3. It is to be noted that the appearances of surfaces of constant reflectance may be represented along lines roughly parallel to the top line of the figure.

Because of their transparency, it is possible to see either surfaces, volumes, apertures, or lights through surfaces. The case of the appearance of a volume of one color seen through a somewhat transparent surface of another is a frequent one; it is dealt with under object mode.

Volume Mode:

Since appearances perceived to belong to volumes possess only four attributes, the relations are more easily represented than for surface appearances, which possess five. All achromatic object appearances (except for shape and texture) may be represented on a plane diagram (Fig. 4) by plotting lightness against transparency. Since we are dealing with the appearances of actual physical media viewed in daylight, the boundaries of this diagram are determined partly by physical limitations and partly by limitations introduced by the visual mechanism. Figure 4 shows a rough sketch of the boundaries.

The vertical line at the left of the figure represents zero transparency. The colors here represented are not in the volume mode of appearance, but rather in the surface mode, because only surface appearances may have zero transparency. But we have vol-

ume appearances approaching zero transparency, hence this line is a boundary of the area representing achromatic volume appearances, but no point on it represents a volume appearance.

The sloping line extending from white to transparent presents the appearances of media that do not absorb light. Whether the line is straight is not known, but that it slopes downward is certain because the volume appearance of a transparent, nonabsorbing medium (vacuum, air, water) is unmistakably darker than white.

The sloping line at the bottom of the figure from transparent to black represents the colors of nonlight-scattering volumes. It expresses the fact that a medium of dark color cannot be seen through as easily as one of lighter color even though both media transmit light in a rectilinear way. Whether this line is straight or curved is not known.

The appearances of various media having achromatic colors have been estimated in terms of lightness and transparency; these are indicated in Fig. 4. Some of these media (smoke, water) yield colors in the surface mode as well as in the volume mode; but, of course, only the appearance in the volume mode is represented by a point in this figure.

Object Modes:

Some nonself-luminous objects yield color perceptions in the surface mode only; these are objects which the incident light seems not to penetrate. It is also possible to experience color in the volume mode alone, although in these cases the possessor of the color is termed a medium rather than an object (fog, haze, water to a diver). A more complicated case, however, is the class of objects that yield color perceptions both in the volume and

in the surface mode (water, glass, transparent and translucent objects). This case deserves some separate discussion.

Consider first the case of a piece of nonlight-absorbing, nonlight-scattering optical glass with polished surfaces immersed in nonlight-absorbing, nonlight-scattering media of various indices of refraction. The appearance of the glass in the volume mode remains constant independent of the medium; it is a perfectly transparent, achromatic appearance of moderate lightness (see appearance of air or water in Fig. 4). The appearance of the glass surface is always glossy but varies somewhat in lightness and transparency. The lightness decreases slightly and the transparency approaches perfection as the index of the medium approaches that of the glass. When the two indices become precisely equal, the surface disappears; that is, there is no color in the surface mode because visually there is no surface.

Consider next the colors of a piece of nonlight-absorbing optical glass viewed in air as a function of the state of polish of the surfaces. As before, the volume appearance of the glass remains constant; it is perfectly transparent. The surface appearance varies in lightness, glossiness, and transparency. As the surface is depolished, the lightness of the color perceived to belong to the surface increases, the glossiness and transparency perceived to belong to the surface decrease. The relation of lightness to glossiness is approximately indicated in Fig. 3 by a line parallel to the top line of the figure. Large decreases in glossiness due to roughening are accompanied by perceptually somewhat smaller increases in lightness. The glossiness of the surface may be reduced to zero by completely depolishing the surface;

but the transparency does not fall completely to zero.

Consider, finally, the colors of polished, nonlight-absorbing blocks of glass whose only difference is the presence of small, uniformly distributed air bubbles of uniform size in varying numbers. The surface appearance is constant; it is a glossy, rather highly transparent appearance of achromatic color of medium lightness. As the number of air bubbles increases, the appearance of the glass in the volume mode varies both in lightness and transparency. The lightness increases nearly to that of white; the transparency decreases to zero, at which point the glass ceases to yield any appearance in the volume mode at all. Such a block is seen only by the bounding surfaces with the moderately glossy opaque appearance of near-white color characteristic of polished milk glass.

Transition States Between the Modes:

The division lines between the various modes of appearance have been made definite in the interest of logic, and it is true that our visual experience justifies these distinctions in a rather striking way and makes them useful. There are, however, transition states between the modes. They arise out of the complexity of our visual responses which have so far resisted complete classification and require further careful study. Many appearances in a transition state are due to visual fields whose structure is insufficient to yield a definite determination of appearance. Such fields are ambiguous, permitting two or more interpretations or perceptions; the appearances may fluctuate from one mode to another according to interpretation or observer attitude. Some of the transition states are sufficiently common to deserve discussion.

Volume-to-Surface Mode.—When an object yields a volume appearance of medium to low transparency behind a surface appearance of high transparency and glossiness, the surface and volume appearances frequently tend to merge into a single glossy surface appearance of zero transparency and changed lightness. Conversely, any very glossy surface appearance of zero transparency (metallic mirror surface) frequently tends to resolve into a glossy highly transparent surface appearance overlying a volume appearance.

Surface-to-Aperture Mode.—A surface appearance may be changed into an aperture color by a failure to fixate the surface accurately. This change is particularly likely to occur with uniform, mat, surface appearances, because the structure of the physical surface ordinarily furnishing many of the clues by means of which the appearance takes on surface character is imperceptible. All nontextured, very dark, mat, surface appearances have something of the indefinite localization of aperture colors. Conversely, a dim aperture color with a bright surrounding field has something of this kind of surface character, and for this case the surface-color term, black, is likely to seem appropriate.

Aperture-to-Illuminant Mode.—An aperture color with a dark surrounding field takes on the character of a somewhat imperfectly localized illuminant color. Such aperture colors, if dim, do not at all seem to be black, any more than a very dim source could be called black. Conversely, a source surrounded by simple structures often loses its localization and appears to be filling a hole in a screen; that is, the illuminant appearance frequently changes to an aperture color.

Surface to Illuminant Mode.—A

surface appearance is the more likely to change into the illuminant mode the closer the point representing it in the surface-color solid (Fig. 2) approaches the upper boundary. This high probability of the illuminant mode seems to relate to a judgment on the part of the observer that the color perceived is too luminous to result from a non-self-luminous object; so the perception is of a color belonging to a self-luminous object, and the brightness attribute applies instead of the lightness attribute. Conversely, a rather dim color in the illuminant mode may shift to the surface mode, as when a glowing coal illuminated by sunlight may shift as it cools down to the appearance of a surface coated by red pigment. Surface colors near the upper boundary of the surface-color solid are particularly easy to produce by fluorescent colorants. The conditions normally required to achieve the transition from surface to illuminant mode have been studied in detail recently by Evans [24], and his results indicate that this transition stage usually involves the judgment that the appearance corresponds to that of a fluorescent specimen. Accordingly he has proposed to call this transition stage the "fluorence" mode. In the fluorence mode, the attributes of appearance are hue, saturation, brightness, and transparency, but Evans' results indicate that luminance is a poor correlate for brightness in the fluorence mode, much poorer than has been indicated by experimental results with the illuminant mode. In some "fluorence" appearances luminance gives spectacularly false indications of brightness.

DESCRIPTIONS OF SOME COMMON APPEARANCES

Aperture Mode:

1. The color of the completely overcast sky in daytime is an achromatic

color of medium brightness in the aperture mode. At night the brightness is nearly zero.

2. The color of the clear sky in daytime is a color of blue hue, low to medium saturation, and medium brightness in the aperture mode.

Illuminant Mode:

1. The appearance of the sun is a very bright yellow of very low saturation and transparency in the illuminant mode.

2. The appearance of the neon glow tube is in three modes; it consists of a bright yellowish-red illuminant appearance of high saturation and low transparency surrounded by the volume and surface colors of the glass tube.

3. In the dark, the appearance of radium paint (such as once used on watch dials) is a greenish-yellow of low brightness, saturation, and transparency in the illuminant mode.

Volume Mode of Appearance of Objects Viewed by Daylight:

1. The appearance of the air in a room (or of a vacuum) is light, achromatic, and perfectly transparent.

2. The appearance of the air in a valley is often a light blue of low saturation and medium transparency.

3. The appearance of fog in a room is a light gray of medium transparency.

4. The appearance of cumulus clouds varies from near-white surface colors to gray of low transparency and a wide range of lightness in the volume mode.

Surface Mode of Appearance of Objects Viewed by Daylight:

1. The usual appearance of depolished chalk (magnesium oxide, magnesium carbonate) is white of zero glossiness and transparency.

2. The appearance of mercury (pol-

ished chromium, polished steel, polished nickel, polished aluminum, and so forth) is achromatic of medium lightness, zero transparency, and nearly perfect glossiness.

3. The appearance of aluminum paint differs from that of polished aluminum by being lighter and less glossy.

4. The appearance of polished gold (or brass) is moderately saturated yellow of medium lightness, zero transparency, and nearly perfect glossiness.



FIG. 5.—Ambiguous Grays.

This illustration may be seen: (1) as six areas, two white, two black, and two different grays, all opaque, or (2) as three areas, an opaque white and black behind a uniform transparent gray.

5. The appearance of polished copper is a dark yellowish-red of medium saturation, zero transparency, and nearly perfect glossiness.

6. The appearance of polished black glass is usually black, perfectly glossy, and perfectly opaque; sometimes it has the more complex appearance of a mat black behind a glossy, transparent, achromatic surface.

7. The appearance of depolished black glass is a dark gray of zero transparency and glossiness.

8. The middle band of Fig. 5 may

be seen in two ways; it may be a uniform gray of medium lightness, glossiness, and transparency in front of a near white and a near black; or it may consist of two grays of medium glossiness and zero transparency, the left-hand gray being dark between two near blacks, the right-hand gray being light between two near whites.

9. The appearance of tracing cloth is usually a blue surface of low saturation and of medium lightness, transparency, and glossiness.

10. The appearance of clear cellophane is a very glossy, very transparent surface whose color is achromatic of medium lightness.

11. The appearance of nonlight-absorbing waxed paper usually differs from clear cellophane simply by being lighter, less glossy, and less transparent.

12. The appearance of sheer nylon hosiery is frequently a dark yellowish-red of moderate saturation and transparency, and low glossiness.

Complex Object Appearances in Daylight:

1. The appearance of a block of clear optical glass (small bulks of water viewed in air, air bubbles in water) has two parts: it consists of a light, perfectly transparent, achromatic volume enclosed by a highly transparent, perfectly glossy, achromatic surface of medium lightness.

2. The appearance of pencil yellow (representative of glossy paints) has two parts: it consists of a light, saturated, mat, opaque, yellow surface behind a glossy, moderately light, transparent, gray surface.

3. The appearance of a polished surface of solid methyl violet (representative of highly absorptive pigments) has two parts: it consists of a dark, unsaturated, mat, opaque, reddish-blue surface behind a glossy, trans-

parent surface whose color is a light, saturated yellowish-green.

4. The appearance of a polished block of canary glass has three parts: it consists of a moderately light, glossy, transparent, gray surface overlying a transparent, light, saturated yellow volume and a moderately transparent volume of a saturated green color.

SUMMARY

It should be remarked in conclusion that our knowledge of visual experience is considerable but far from complete. The foregoing system of classifying the nonshape, nontextural aspects of appearance accordingly is adequate to the description of a considerable portion of visual experience, but not to all; furthermore, other systems are possible, each one having different, but equally unavoidable, arbitrary features. Since students of visual perception are not in perfect agreement, this system cannot be consistent with all views; and, as a matter of fact, it is not in perfect agreement with any published views. This system does not cover all aspects of visual appearance (size, shape, texture, mottling), nor, indeed, all modes of appearance. No mention has been made of the important, but little studied, and rather mysterious illumination mode of appearance. A color perception in the illumination mode always accompanies the perception of an object color, yet

it is not referred to a definite volume in the illuminant mode, nor is it the perception of the volume color of the space in which the object color is perceived. It is a color perceived to belong to the illumination of the object based on clues from the scene within which the object is perceived instead of being based on any view of the source itself.

Acknowledgments:

The author wishes to express his appreciation to H. Helson and D. McL. Purdy for an introduction to the psychological literature on modes of appearance and for constructive criticism, and especially to K. S. Gibson for aid in working out and testing the system and for the following comment:

When I was young and life was bright
I used to say the sun was white;
But now I'm told in terms emphatic,
The sun is really achromatic.

It used to be correct to say
The autumn skies were bleak and gray;
But now I've learned—Oh, thought ec-
static,
Gray skies are really achromatic!

And when it rained and made a mess
The drops to me were colorless;
But now in scenes so hydrostatic
Those drops are simply achromatic!

And thus it was, and thus it is,
I think by gosh and then gee whiz,
My mind is warped and quite erratic;
I'm sure my thoughts are achromatic!

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Specular-Gloss Measurement of Ceramic Materials

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National Bureau of Standards, Washington, D. C.

A standard specular-gloss method is required to achieve duplicability between laboratories for the wide range of gloss found in recently developed porcelain enamels. A decision was made to standardize on the gloss scale resulting from the geometry of the 45° gloss attachment of the Hunter multipurpose reflectometer. As the source and receiver apertures of this instrument were not readily measurable, a goniophotometer was adjusted so that its scale duplicated that of the Hunter instrument. The source and receiver apertures of the adjusted goniophotometer were measured and these were recommended to glossmeter manufacturers for inclusion in the design of their instruments. Significant improvement in the duplicability of specular-gloss measurements resulted, as evidenced by the measurement of 45° specular gloss of a large number of ceramic specimens having a wide variety of surface-appearance characteristics. The recommendations resulting from this investigation have been incorporated, with slight modification, into gloss tests for porcelain enamels proposed by the Porcelain Enamel Institute and the American Society for Testing Materials.

I. Introduction

ALTHOUGH the gloss of porcelain enamels has been measured for many years, only recently has there been a request for a standardized method of measurement. An explanation for this delay is that the instruments that had been developed have been used with varying degrees of suc-

cess without standardization. Gloss measurement in the past has been used largely as a means for evaluating resistance to abrasion or to chemical attack of high-gloss enamels. For this purpose specimens are usually measured, before and after attack, on one instrument in one laboratory. The data thus obtained are useful for determining the relative resistance to attack even though a standardized gloss scale is not used. However, if one attempts to duplicate in one laboratory an experiment performed in another laboratory using different, nonstandard glossmeters, discrepancies will result from differences in glossmeter geometry.

Porcelain enamels have been developed in recent years in a wide range of gloss primarily for architectural purposes. This development further emphasizes the necessity for a standard gloss method that will yield consistent measurements throughout the gloss range of these porcelain enamels.

II. Factors Determining Gloss Scales and Glossmeter Readings

A glossmeter reading is determined by several instrumental factors: (1) the angle the incident beam makes with the perpendicular to the specimen, (2) the size of the receptor aperture, and (3) the ratio of the angular size (aperture) of the receptor to that of the source.

Much of the material in this paper was presented at the Fifty-Third Annual Meeting, The American Ceramic Society, Chicago, Ill., April 25, 1951 (Enamel Division, No. 18). Since then there have been requests for a more complete account of the procedures used. Received May 12, 1955.

Mr. Nimeroff and Mr. Hammond are physicists, Photometry and Colorimetry Section, and Mr. Richmond and Mr. Crandall are ceramic engineers, Enamelled Metals Section, National Bureau of Standards.

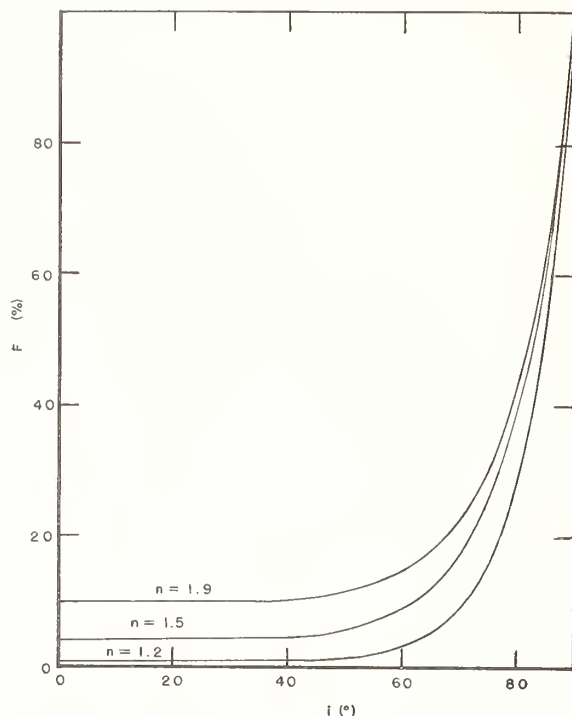


Fig. 1. Fresnel reflectance of unpolarized light, F , from polished surface dielectric as a function of incidence angle, i , for refractive indexes, n , equal to 1.2, 1.5, and 1.9.

The fraction of the incident light specularly reflected from the surface of a specimen of a material is determined by the angle of incidence, i , and the refractive index, n . The reflected fraction, F , from a perfectly polished surface can be computed from the Fresnel reflection equation¹

$$F = \frac{1}{2} \left[\frac{\sin^2(i - r)}{\sin^2(i + r)} + \frac{\tan^2(i - r)}{\tan^2(i + r)} \right] \quad (1)$$

r = angle of refraction.

As the angle of refraction cannot be measured in opaque materials, the Fresnel equation can be stated to advantage in terms of the angle of incidence and refractive index, thus:

$$F = \frac{1}{2} \left[\left(\frac{\cos i - \sqrt{n^2 - \sin^2 i}}{\cos i + \sqrt{n^2 - \sin^2 i}} \right)^2 + \left(\frac{n^2 \cos i - \sqrt{n^2 - \sin^2 i}}{n^2 \cos i + \sqrt{n^2 - \sin^2 i}} \right)^2 \right] \quad (1a)$$

This equation is used to compute the gloss of primary standards such as polished black glass. Figure 1 shows how the specular reflectance of a polished specimen varies with the angle of incidence for several values of index of refraction. Specular reflectance approaches unity, or 100%, as i approaches 90°. Specular gloss of a polished specimen is defined as an arbitrary multiple of specular reflectance. The arbitrary multiplying factor is selected not only to give convenient numerical values but to avoid readings in excess of 100.

The light flux entering a glossmeter receiver after reflection from a perfectly diffusing surface is a direct function of the size of the receptor entrance window. Thus gloss readings for low-gloss specimens are increased when the receptor window is made larger. Instrument readings for high-gloss specimens not perfectly polished will be a function of the size

of the source image formed at the receptor entrance window. If the source image formed by reflection from a polished specimen nearly fills the window, instrument readings will decrease rapidly as the gloss of a surface departs from perfect polish. On the other hand, if the size of the source image is small compared with that of the receiver entrance window, glossy specimens having large gloss differences may have nearly the same gloss reading.

There is another consideration that will influence the choice of the receiver-source aperture ratio, namely the flatness of specimens. Porcelain enameled specimens are frequently wavy or warped. For specimens having a wavy surface, the receiver-source aperture ratio should not be too small; otherwise specimens ranked by eye as having high gloss will be rated too low by the instrument. The gloss of warped specimens is always difficult to measure accurately because of the difficulty of ascertaining whether or not the image reflected from the specimen is centered on the receiver entrance window. A warped specimen distorts the reflected image so that the glossmeter is likely to provide erroneous gloss readings.

The standardization of specular-gloss measurements necessitates specification of the angles of illumination and reception and the size of the source and receptor field angles or apertures. The angles 45° and -45° have been employed for many years as the angles of illumination and reception for the measurement of gloss of porcelain enamels. For the present, the assumption is made that these angles are satisfactory. The sizes of source and receptor apertures, however, are the geometric parameters which must be carefully specified and controlled in order to provide duplicable glossmeter readings. Specification of these apertures has often been omitted from the apparatus requirements of a test method requiring gloss measurement. In many cases specification of instrument type alone has been used but has been found to be generally inadequate and unsatisfactory.

III. Hunter Multipurpose Reflectometer

The Hunter multipurpose reflectometer² was devised in 1936 for use with test methods for reflectance and for loss of gloss due to abrasion and to chemical attack. These methods were being developed at the National Bureau of Standards in cooperation with the Porcelain Enamel Institute. This instrument provided means for measuring both directional reflectance and gloss. The instrument has been and still is being widely used, although reflectometers and glossmeters of superior design have been developed.

The design of any multipurpose instrument introduces complexities which are not encountered in single-purpose designs. Figure 2 shows a schematic drawing of the arrangement of source, S ; lenses, L_m ; mirrors, M_m ; receiver entrance window, R ; photocell, P ; and test-specimen position, T , in the multipurpose reflectometer as used for gloss measurement. A diffusing mirror, M_s , is used to control and to make uniform the illumination on P .

This instrument when used as a glossmeter has two major shortcomings: First, the gloss scale must be carefully calibrated if measurements on the same instrument are to be reproduced from time to time; second, and more important, measurements made on one instrument will not in general agree with those made on another instrument. This second shortcoming results from the fact that the instrument was designed before accurate control of field angle was understood to play an important part in the reproduction of gloss measurements on different instruments. The receptor field angles of the multipurpose instrument cannot be controlled accurately because of the number of windows, or diaphragms,

¹ A. Fresnel, "Calcul des teintes que polarisation développe dans lames cristallisées," *Ann. chim. et phys.*, **17**, 194, 312 (1821).

² R. S. Hunter, "Multipurpose Photoelectric Reflectometer," *J. Research Natl. Bur. Standards*, **25** [5] 581-618 (1940); *RP* 1345; *Ceram. Abstr.*, **20** [7] 175 (1941).

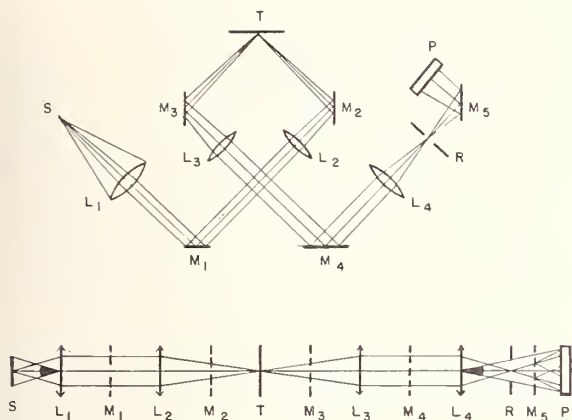


Fig. 2. Hunter multipurpose reflectometer showing projection and schematic views and components: source, S ; lenses, L_m ; mirrors, M_m ; test-specimen position, T ; receiver entrance window, R ; and photocell, P . Apertures are shown as filled angles. The centers of the components between M_1 and M_4 lie in a plane perpendicular to that of the centers of the components outside M_1 and M_4 . Control and uniformity of illumination on P is derived from a diffusing mirror, M_5 .

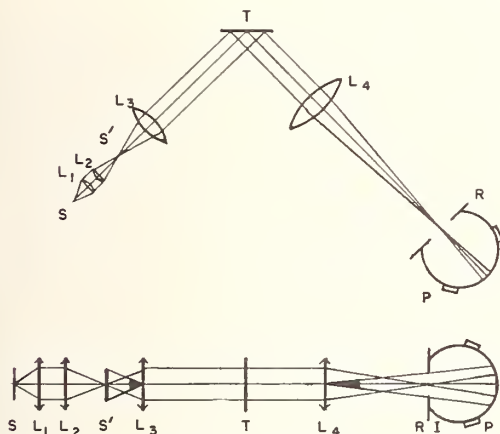


Fig. 3. Versatile glossmeter showing plan and schematic views and components: real source, S ; effective source, S' ; lenses, L_m ; test-specimen position, T ; receiver entrance window, R ; integrating enclosure, I ; and photocells, P . Apertures are shown as filled angles.

through which the gloss beam passes. Each of these beam-limiting components produces vignetting, or stopping, of some rays so that specification of the receptor aperture in terms of measurements of these components is not practical. In addition, since the location of these components varied slightly from instrument to instrument and no adjustment of them was provided in the design, adjustment of different instruments to obtain the same gloss readings is not possible.

On the other hand, because of the widespread use of the multipurpose instrument for measuring specular gloss of porcelain enamels, a gloss scale defined in terms of readings by this instrument has been established through usage. To avoid any major change in the gloss scale of the Hunter multipurpose reflectometer, a plan was formulated to determine the effective apertures of a representative instrument and to recommend these apertures for adoption as standard. This plan required obtaining enameled-metal specimens from manufacturers in as wide a gloss range as possible. These

specimens would then be measured both on a multipurpose reflectometer and on a goniophotometer or glossmeter having adjustable apertures. The apertures of the latter instrument would be adjusted by trial until gloss readings throughout the scale approximated those obtained with the multipurpose instrument.

The multipurpose reflectometer chosen for investigation was the one in use in the laboratory of the Enamelled Metals Section of the National Bureau of Standards. The gloss of nearly one hundred specimens was measured on this instrument. The instrument was set up in the usual manner with the gloss opening in a horizontal position and a sample of appropriate reflectance in position to furnish the comparison beam. The gloss scale was calibrated for linearity by inserting calibrated neutral filters in the gloss beam in front of the receptor window.

IV. The Versatile Glossmeter

The instrument used to duplicate the gloss scale of the multipurpose reflectometer was the versatile glossmeter described by Hammond and Nimeroff.³ This glossmeter is a modified goniophotometer and is shown diagrammatically in Fig. 3. It has provision for accurately controlling (1) the source and receiver apertures in and perpendicular to the plane of measurement, (2) the angles of illumination and view, (3) beam collimation, and (4) the position of the source image relative to the receiver window.

Control of the source aperture is accomplished by focusing, with a pair of condensing lenses, L_1 and L_2 , an image of the source, S , on an adjustable metallic aperture, S' , which then is the effective source and is controllable in size. S' is located in the focal plane of an achromatic collimating lens, L_3 , the focal length of which is accurately known. The specular angle is readily changed by rotating the receiver arm and specimen, T , about the axis of the turntable at T and can be set to any desired position to within $\pm 0.1^\circ$. The receiver collimating lens, L_4 , an achromat of known focal length, is of such aperture and so positioned that there is no vignetting of rays that should reach the receiver. As space conservation is not a problem, the instrument is made large enough to permit measurement of its components to high geometric accuracy.

V. Aperture Determination Procedure

The procedure for empirically determining apertures on the versatile glossmeter from gloss readings obtained with the multipurpose instrument for a series of specimens follows. First, compute the angular size of the receptor so that the readings for low-gloss, or highly diffusing, specimens duplicate those obtained with the multipurpose instrument (see Section V (1)). Second, adjust the angular size of the source so that the readings for high-gloss specimens are duplicated (see Section V (2)).

(1) Receiver Aperture

As the glossmeter reading for a perfect diffuser, perfect reflector depends on the aperture of the receptor, an equation relating the fraction, F_r , of the incident flux reflected into the receptor window to the angular size of that window is required to solve the first part of the problem. Such a relationship was derived by Hammond and Nimeroff³ and may be written

$$F_r = \frac{\alpha\beta \cos \theta'}{\pi} \quad (2)$$

where α and β are the angular dimensions in radians of the rectangular receptor in the plane and perpendicular to the

³ H. K. Hammond III and I. Nimeroff, "Measurement of Sixty-Degree Specular Gloss," *J. Research Natl. Bur. Standards*, **44** [6] 585-98 (1950); RP 2105.

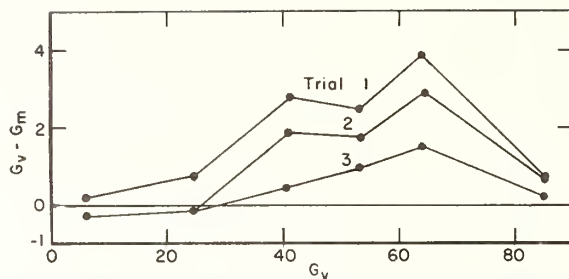


Fig. 4. Differences between readings, in parts per 1000 of incident flux, for six selected specimens on the versatile glossmeter, G_v , and multipurpose glossmeter, G_m . Trial 1: source aperture $2.7^\circ \times 0.5^\circ$, receiver aperture $12.2^\circ \times 7.3^\circ$; trial 2: $2.7^\circ \times 0.5^\circ$, $9.9^\circ \times 7.9^\circ$; trial 3: $2.7^\circ \times 1.8^\circ$, $9.9^\circ \times 7.9^\circ$.

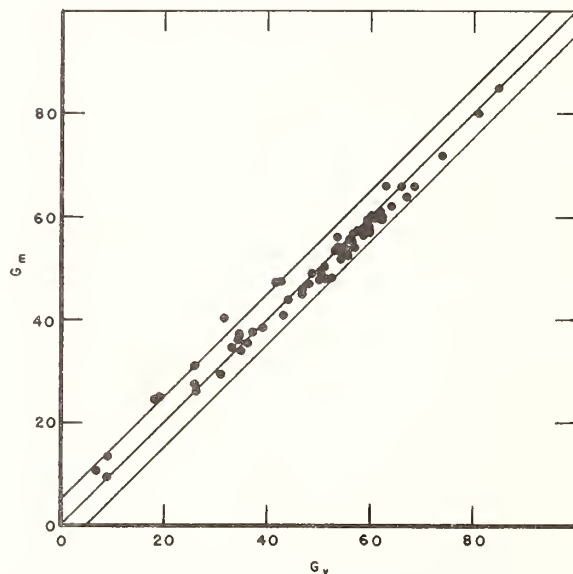


Fig. 5. Gloss readings on the multipurpose glossmeter, G_m , plotted against gloss readings on the versatile glossmeter, G_v .

plane of measurement,* respectively, and θ' is the specular angle measured from the perpendicular to the specimen. For a perfect diffuser of diffuse reflectance, ρ , equation (2) becomes

$$F_r = \frac{\rho \alpha \beta \cos \theta'}{\pi} \quad (3)$$

To obtain a first approximation of the angular size of the receiver aperture for the versatile instrument, a gloss reading of a good diffuser of known diffuse reflectance is taken on the multipurpose instrument. Equation (3) is then solved for the product $\alpha\beta$, by setting F_r equal to the multipurpose glossmeter reading for the diffuser and ρ equal to the reflectance of the diffuser. The low-gloss end of the gloss scale is determined by this receiver aperture.

Having obtained the product $\alpha\beta$, the question arises as to how to apportion these two dimensions, i.e., how to determine the aspect ratio. Consider a facet of the specimen displaced

from the gross plane of the specimen forming an angle γ with the α and β planes. The source image reflected from this facet will be displaced by an angle 2γ in the α plane and by an angle $2\gamma \cos \theta'$ in the β plane. Thus for $\theta' = 45^\circ$, β need be only 0.707α to stop rays deflected in the β plane at the same time that rays are stopped from equally displaced facets in the α plane. With only these considerations the aspect ratio, α/β , of the receptor window should be $1/0.707$. This result, however, is based on the assumption that the source image is a point. This assumption is not applicable to glossmeters in general as the source image has a rectangular cross section. As, on the average, the deflection of rays is greater in the α than in the β direction, the source is oriented with its smaller dimension in the α direction.

With an instrument like the multipurpose reflectometer, the gloss scale of which does not extend to zero, accurate reading of the "gloss" of a nearly perfectly reflecting, perfectly diffusing specimen is not possible. The instrument reading for a sample of depolished white structural glass was estimated by extrapolation of the scale (perfect, mirror-reflector gloss equals 1) to be approximately 0.008. This specimen has a $45^\circ 0'$ (45° incidence, perpendicular view) directional reflectance of about 0.89. Substituting the values $F_r = 0.008$, $\cos \theta' = 0.707$, $\rho = 0.89$, and $\beta = 0.707\alpha$ in equation (3) and solving for α , one obtains 0.24 radian or 13.6 degrees.

(2) Source Aperture

The second step involves consideration of the source aperture. The size of the source image formed in the plane of the receptor entrance window relative to the size of that window governs the magnitude of the instrument reading obtained for specimens with fairly high gloss. If the source is too large, diverging rays reflected from the specimen that should enter the receptor are blocked by the window frame and the reading obtained is too low. Conversely, if the source is too small, rays that should not enter the receptor are accepted for measurement and the reading is too high. By starting with an estimate of the angular size of the source, one can determine from the readings obtained whether the source size should be increased or decreased. The magnitude of the adjustment required is determined empirically.

The cross section of the source could be circular, square, elliptical, or rectangular. To obtain the maximal light flux as well as to utilize commercially available coiled-filament lamps, a rectangular aperture source is used. The considerations imposed by choice of this rectangular shape source have been treated in Section V (1).

(3) Duplication of Gloss Scale

(A) *Multipurpose Reflectometer:* The procedures and considerations outlined were applied to determine the source and receptor apertures required to duplicate the gloss scale of the multipurpose reflectometer. Six specimens were selected to represent types of appearance characteristics generally found for this material and to cover the range from high to low gloss.

The versatile glossmeter was arranged with apertures set to a first approximation of what they were expected to be. The apertures were: source, $2.7^\circ \times 0.5^\circ$; receiver, $12.2^\circ \times 7.3^\circ$. This receiver aperture differs from that computed in Section V (1) because $12.2^\circ \times 7.3^\circ$ is the largest rectangular receiver aperture practicable on the versatile glossmeter. When the readings at the lower end of the scale were found to be too large, the receiver aperture was reduced in size to $9.9^\circ \times 7.9^\circ$. The gloss readings were then found to be too high in the upper end of the gloss scale, so the source aperture was increased to $2.7^\circ \times 1.8^\circ$. Gloss readings obtained with this combination of source and receiver apertures were in agreement to within about one gloss unit with those obtained on the multipurpose reflectometer. The one gloss-unit criterion of agreement between instruments for gloss readings of

* The plane of measurement is the plane defined by the axes of the incident and reflected beams.

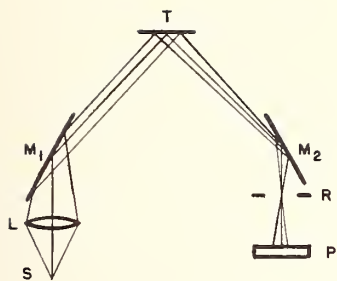


Fig. 6. Photovolt glossmeter showing plan and schematic views and components: source, S ; lens, L ; mirrors, M_1 ; test-specimen position, T ; receiver entrance window, R ; and photocell, P . Apertures are shown as filled angles.

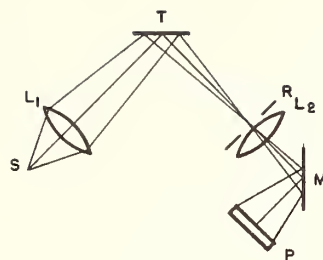


Fig. 8. Gardner glossmeter showing plan and schematic views and components: source, S ; lenses, L_1 ; test-specimen position, T ; receiver entrance window, R ; mirror, M ; and photocell, P . Apertures are shown as filled angles.

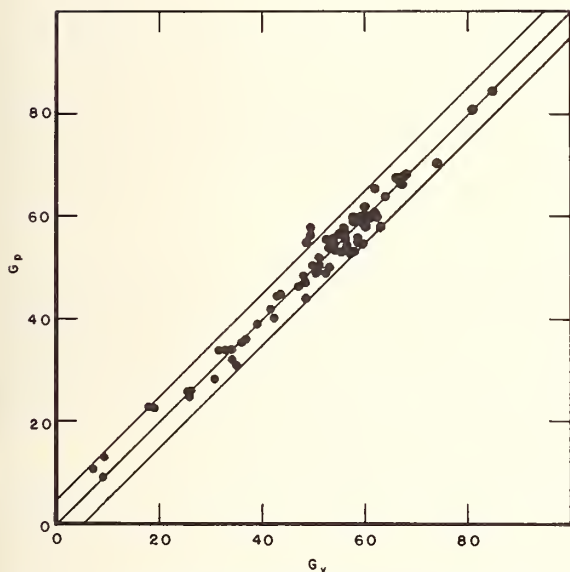


Fig. 7. Gloss readings on Photovolt glossmeter, G_p , plotted against gloss readings on the versatile glossmeter, G_v .

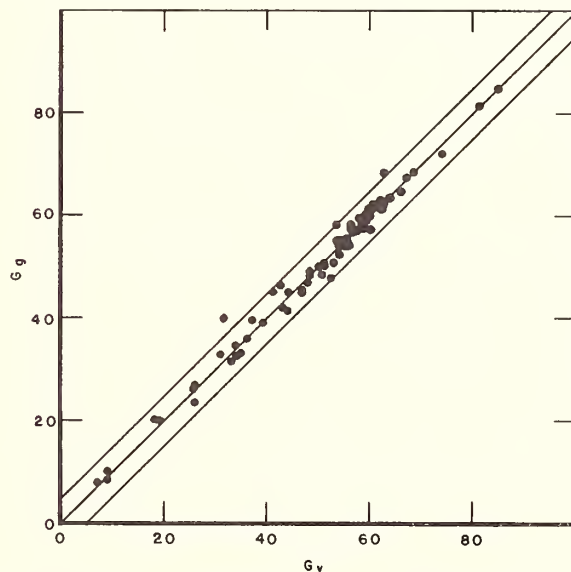


Fig. 9. Gloss readings on the Gardner glossmeter, G_g , plotted against gloss readings on the versatile glossmeter, G_v .

a few selected specimens was considered to be adequate, as the criterion of agreement of gloss readings of a large number of specimens is usually 5 gloss units. The results of these three trials are shown in Fig. 4.

After this degree of agreement between gloss readings on the multipurpose reflectometer and the versatile glossmeter was attained for the six specimens, gloss readings were obtained on both instruments for about 100 specimens. The gloss readings, shown on Fig. 5, indicate that the gloss scale of the multipurpose reflectometer was duplicated quite adequately. The central 45° line indicates the condition of perfect agreement; the other 45° lines indicate a ± 5 gloss-unit zone of scatter.

(B) *Photovolt Glossmeter*: A type of glossmeter made by the Photovolt Corporation has also been widely used for measuring specular gloss of porcelain enamels. A schematic drawing of this glossmeter is shown in Fig. 6. The components (source, S ; receiver entrance window, R ; and photocell, P) can be moved to give some adjustment to the gloss scale. The specimens previously measured on the multipurpose reflectometer and on the versatile glossmeter were again measured on the Photovolt instrument used by the Enamelled Metals Section of the National Bureau of Standards. The agreement between gloss readings obtained on this instrument and those obtained on the versatile glossmeter was not good. A new gloss head of the type currently

being supplied was furnished by the Photovolt Corporation. Gloss readings with this new gloss head after adjustment were in good agreement with the gloss readings obtained on the versatile glossmeter. These results are shown in Fig. 7.

(C) *Gardner Glossmeter*: The Enamelled Metals Section requested the Gardner Laboratory to build a gloss head for the photometer of the Gardner "photometric unit" instruments line. The aperture requirements determined with the versatile glossmeter were specified. With this information the Gardner Laboratory built a gloss exposure head. The geometry of this instrument (see Fig. 8) does not differ importantly from that of the Photovolt glossmeter. The major difference is the addition of a lens behind R used to focus the illuminated area of T onto the photocell. The gloss readings obtained with specimens measured on this instrument are plotted in Fig. 9 against those obtained with the versatile glossmeter. Note that although this instrument was constructed and designed to meet the aperture requirements determined on the versatile glossmeter, duplication of readings is not perfect, although agreement is not worse than with the other instruments investigated. The discrepancies for all instruments may be due to errors in positioning the specimens, especially the warped or wavy ones.

VI. Specular Gloss and Diffuse Reflectance

As specular (surface) and diffuse (body) reflected components cannot be separated readily, current ideas in specular-gloss measurement are based on the following definition⁴: Specular gloss is the ratio of the luminous flux reflected by a specimen in a specular direction within a specified solid angle to the luminous flux incident at a specified angle within a specified solid angle. This definition indicates that specular gloss includes flux reflected diffusely, as well as specularly, if that flux is included within the specified receptor aperture.

In the past, specular gloss has been defined for measurement purposes as the fraction of the incident flux picked up by an instrument with specified receptor aperture *minus* the diffusely reflected light flux. As many laboratories reported simply the instrument reading as specular gloss without making a correction for the diffuse component, the current definition was formulated.

There is now general agreement that although correction may be desirable, there is no convenient way to determine its magnitude accurately; at best, any estimate of its magnitude is only an approximation. Evaluation of the surface-reflected component of specular gloss is desirable when comparing specimens having low gloss and different diffuse reflectance. In approximating the diffusely reflected flux for a perfect diffuser of diffuse reflectance, ρ , equation (3) may be used. For any specimen, an approximation of the diffusely reflected flux may be made by using the expanded diffuse reflectance theory developed by Hammond and Nimeroff.³

VII. Correlation Between Specular-Gloss Methods

Occasionally the replacement of painted surfaces by porcelain enamel surfaces of similar gloss characteristics is desirable. The gloss of paints is generally measured at 60° incidence with prescribed source and receiver apertures whereas the gloss of porcelain enamels is generally measured at 45° incidence with a different set of prescribed source and receiver apertures. The question then arises: Is there any simple correlation between these different methods? Correlation can be established to a limited extent as shown in Fig. 10 for a series of five ceramic tile specimens. If there were linear correlations, the points representing measurements on both

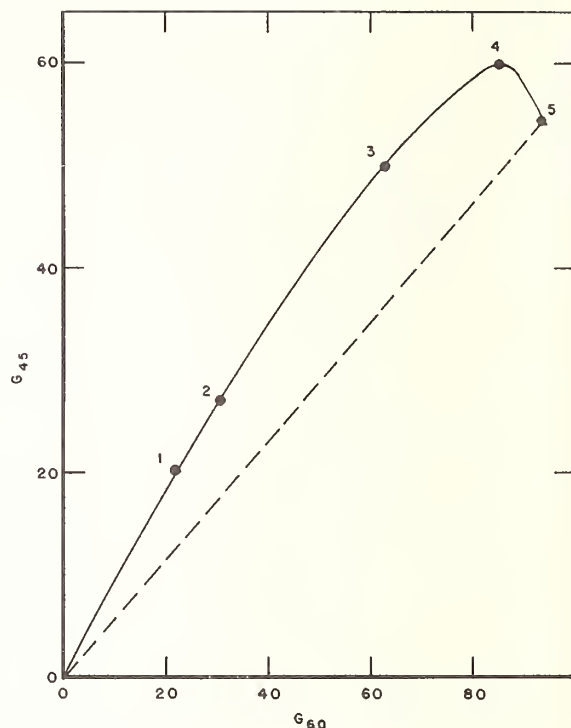


Fig. 10. Correlation between readings on a 45° glossmeter, G_{45} , and readings on a 60° glossmeter, G_{60} , for five ceramic tile specimens. Specimen 5 was highly polished.

scales might lie on a straight line, perhaps on the indicated dashed line. Correlation could be curvilinear also, but one would not expect to have reversals of order as indicated in Fig. 10 between specimens 4 and 5. This reversal is caused primarily by the fact that the receiver aperture of the 45° method is larger than that of the 60° method. The larger aperture permits scattered flux to reach the receiver that the smaller aperture would intercept.

A single measurement of gloss is not sufficient to characterize completely the glossy appearance of any specimen. Even when two specimens of porcelain enamel have the same gloss reading there is no assurance that they will look alike in all respects. There is need for a more complete specification of the appearance of a surface than can be given by a single gloss value. Nimeroff⁵ has advocated a two-parameter gloss method using either two different receptor apertures with one specular angle or two specular angles with one receptor aperture, as the problem requires. Goniophotometric curves give the most information about geometric distribution of reflected light flux, but here again two or more incidence angles may be required. Analysis of goniophotometric curves into single-number indexes of peakedness was shown by Nimeroff⁶ to require at least one other parameter, namely specular gloss, to separate specimens into groups consistent with appearance.

⁴ American Society for Testing Materials, "Tentative Method of Test for Specular Gloss," A.S.T.M. Designation D 523-53T, 1953 Supplement to Book of A.S.T.M. Standards, Part 4, p. 91.

⁵ I. Nimeroff, "A Two-Parameter Gloss Method," *J. Opt. Soc. Amer.*, **44**, 823 (1954).

⁶ Isadore Nimeroff, "Analysis of Goniophotometric Reflection Curves," *J. Research Natl. Bur. Standards*, **48** [6] 441-48 (1952); RP 2335.

VIII. Concluding Remarks

For a class of materials having similar gloss characteristics a single specular-gloss measurement may be adequate to specify the characteristics. Such a single gloss measurement has been found for ceramic and porcelain enameled materials. The 45° specular-gloss measurement, already proved to be satisfactory, was chosen for the following additional reasons: A gloss scale was desired ranging from approximately 0 to approximately 100 for the material whose gloss is to be evaluated by this method. As the specular reflectance of a polished specimen of this material may be as high as 100 parts per 1000 (refractive index, 1.9) for 45° incidence, the adopted gloss scale, G , is the specular reflectance multiplied by the factor 10. Thus, referring to equation (1), $G = 10F$ for polished specimens of negligible diffuse reflectance.

A glossmeter with geometry not readily measurable was already in use, and a gloss scale was already established. A goniophotometer was used as a versatile glossmeter, and source and receiver apertures were adjusted by trial to duplicate the desired gloss scale and thus establish the geometry requirements. The source and receiver apertures thus determined were $2.7^\circ \times 1.8^\circ$ and $9.9^\circ \times 7.9^\circ$, respectively. This glossmeter geometry with slight modification and addition of tolerances was ultimately recommended to the Porcelain Enamel Institute and the American Society for Testing Materials for incorporation in their test methods. The receiver aperture angles were rounded off to whole degrees.

With slight scale modification, the source aperture angle in the plane of measurement was reduced and that perpendicular to this plane increased to permit easier utilization of existing lamp filaments. Aperture tolerances were chosen partly on the basis of experience with similar tolerances for 60° specular-gloss geometry³ and partly on the basis of data ob-

Table I. Recommended Glossmeter Apertures and Tolerances for 45° Specular Gloss of Ceramic Materials (P.E.I. and A.S.T.M. Test Methods)

Measurement plane	Aperture and tolerance (degrees)	
	Source	Receiver
Parallel	1.4 ± 0.4	8.0 ± 0.1
Perpendicular	3.0 ± 1.0	10.0 ± 0.2

In May 1953, the Porcelain Enamel Institute published the recommendations in Bulletin T-18, "Gloss Test for Porcelain Enamels." In June 1954, the American Society for Testing Materials adopted the recommendations and published "Tentative Method of Test for 45° Specular Gloss of Porcelain Enamels," A.S.T.M. Designation C 346-54T, 1954 Supplement to Book of A.S.T.M. Standards, Part 3, p. 155.

tained in this investigation. The recommended glossmeter apertures and tolerances of the proposed methods of the Porcelain Enamel Institute and the American Society for Testing Materials are shown in Table I.

Two-Parameter Gloss Methods

I. Nimeroff

The concept of a parameter of glossiness, that is, measurement under one set of illuminating and viewing conditions, is developed. To describe completely the glossiness of a specimen requires multiparametric measurements on the geometric distribution of reflected light-flux. As one-parameter methods have been found to yield inadequate descriptions of specimen glossiness, two-parameter methods were investigated. For specimens with differing incidence-angle dependency, a two-parameter method using specular gloss measurements at two incidence angles is suggested. For specimens with differing flux-scattering characteristics, a two-parameter method using specular gloss measurements for two receptor apertures is suggested. Several existing standard two-parameter methods are reviewed. Results of a seven-laboratory round-robin test of the two-parameter (receptor aperture) method are reported and analyzed. To establish correlation between measurements obtained with the suggested two-parameter (receptor aperture) method and subjective evaluation of image brightness and image distinctness, a subjective test was formulated. The results of this test, here analyzed and discussed, indicate the usefulness of the two-parameter method.

1. Introduction

For many years American Society for Testing Materials committee deliberations on glossimetry definitions have been concerned with use of terms like "aspects," "criteria," "modes," and "types" of gloss in describing instrumental measurement techniques found to correlate with some glossiness characteristics. Glossiness, or glossy appearance, is a function of how an observer evaluates the geometric distribution of flux reflected by a specimen. To evaluate completely this appearance characteristic, called glossiness, observations under many conditions of illumination and view are required. A measurement under each illuminating and viewing condition may be called a parameter of glossiness.

Techniques involved in methods for evaluating of specimens have ranged from one-parameter to multiparameter observations. The multiparameter technique is complex, time consuming, and too costly to perform for every specimen. While a one-parameter technique can yield data simply, rapidly, and inexpensively, it is an oversimplified solution to the problem of instrumental evaluation of gloss characteristics. One-parameter data often lead to erroneous conclusions about the comparative appearance of specimens. To decrease the number of erroneous conclusions without materially increasing the complexity, time, and cost of the measurement, a compromise between a one-parameter and a multiparameter technique is required.

2. Multiparameter Technique

2.1. Multiplane Goniophotometry

Multiparametric observations can be taken on either multiplane or monoplane instruments, called goniophotometers. A multiplane goniophotometer would have many geometric variables at its disposal. Such an instrument was designed and constructed by McNicholas [1].¹ Figure 1 shows the geometric variables: θ , the angle between the axis of the incident

beam and the perpendicular to the specimen plane; ϕ , the azimuth angle (the angle between a reference line in the plane of the specimen and the projection of the incident axis on that plane); ω , the solid angular subtense, or aperture, of the incident beam; θ' , ϕ' , and ω' , the corresponding geometric variables of the reflected beam.

If the goniophotometric readings for a specimen under each set of illuminating and viewing conditions (parameters) are represented by $G_{i,r}$, where i refers to the incident beam, and r refers to the reflected beam, the complete gloss description of the specimen, G_ϕ , would be a function of these parameters. This may be symbolically stated:

$$G_\phi = F(G_{i,r}), \quad (1)$$

where i is dependent on θ , ϕ , and ω , and r is dependent on θ' , ϕ' , and ω' . This dependence may be stated:

$$\left. \begin{aligned} i &= f(\theta, \phi, \omega) \\ r &= g(\theta', \phi', \omega') \end{aligned} \right\} \quad (1a)$$

An analysis of goniophotometric data can be considered as a solution of eq (1) for the functional description, $F(G_{i,r})$. Thaler [2] published a rather complete set of parametric measurements for a series of MgO specimens, using constant ω and ω' , but did not develop a functional description of the appearance of the material.

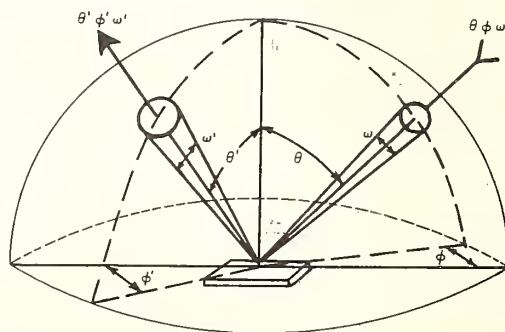


FIGURE 1. Geometric variables of a multiplane goniophotometer

¹ Figures in brackets indicate the literature references at the end of this paper.

2.2. Monoplane Goniophotometry

A monoplane instrument has one less variable at its disposal than a multiplane instrument. The variables of a monoplane goniophotometer are those of figure 1, except that $\phi' = \phi$. The variables are θ and ω for the incident beam, θ' and ω' for the reflected beam. If rectangular source and receptor are substituted for the circular ones, the angular sizes ω and ω' become $\alpha\beta$ and $\alpha'\beta'$, respectively. Goniophotometers of this type have been designed and constructed by Wetlaufer and Scott [3], Moon and Laurence [4], Hammond and Nimeroff [5], and Middleton and Mungall [6].

For this type of instrument, the gloss description of the specimen, G , could be symbolically represented by:

$$G = F(G_{i,r}),$$

where i is dependent on θ , α , and β , and r is dependent on θ' , α' , and β' . This dependence is stated:

$$\left. \begin{aligned} i &= f[\theta, (\alpha\beta)] \\ r &= g[\theta', (\alpha'\beta')] \end{aligned} \right\} \quad (2a)$$

Analyses of goniophotometric data obtained on this type of instrument have been made. Barkas [7] suggested a method for theoretically separating the specular and diffuse reflection components of a specimen by finding an equivalent theoretical surface that will have the same goniophotometric reflectance as the specimen. Middleton and Mungall [6] suggested a similar analysis for goniophotometric data. These analyses were confined to fairly matte surfaces, such as beaverboard and snow.

ASTM Method, D636-54 [8] recommends use of a Dows-Baumgartner reflectometer [9] and analysis of its goniophotometric data in terms of reflective diffusion indexes:

$$\left. \begin{aligned} D_{ra} &= \frac{\int_{3^\circ}^{90^\circ} R(\theta') d\theta'}{97R(3^\circ)} \\ D_{rb} &= \frac{\int_{15^\circ}^{90^\circ} R(\theta') d\theta'}{75R(15^\circ)} \end{aligned} \right\} \quad (3)$$

where $R(\theta')$ is the reflectance at viewing angle, θ' . The specimen is illuminated perpendicularly, $\theta = 0^\circ$.

Analysis of goniophotometric data for specimens ranging in gloss from matte to highly specular was made by Nimeroff [10]. The goniophotometric data were analyzed in terms of indexes of dispersion, skewness, and kurtosis (peakedness index), related to the second, third, and fourth moments, respectively.

3. One-Parameter Technique

3.1. Specular Glossmeters

One-parameter instruments that have been developed recently have been monoplanar with fixed source and receptor apertures and fixed incidence

and specular reflecting angles. Symbolic representation for the geometry of such a one-parameter instrument is

$$\left. \begin{aligned} i &= f[\theta, (\alpha\beta)] \\ r &= g[-\theta, (\alpha'\beta')] \end{aligned} \right\} \quad (4)$$

Specular glossmeters have been developed for at least five different incidence angles, 20° , 45° , 60° , 75° , and 85° , and are described in ASTM, PEI, and TAPPI gloss methods. These methods have proven satisfactory except for occasional confusion resulting from inadequacy of a one-parameter technique.

3.2. Inadequacies of the One-Parameter Technique

Specular reflectance of a specimen depends on the angle at which light flux is incident on the specimen. For some specimens the rate of increase of specular reflectance with increasing incidence angles exceeds the rate for other specimens. Thus two specimens can have the same gloss when illuminated at one incidence angle and have much different gloss when illuminated at another incidence angle. A one-parameter technique, specular reflectance measurement at one set of geometric conditions, will not reveal this incidence-angle dependency of the appearance of these specimens.

More frequently than in the above instance, inadequacy of a one-parameter technique is demonstrated when one specimen has a more peaked distribution of reflected flux than the other. Even though two such specimens may differ markedly in appearance, the flux reflected from one specimen and accepted by the receptor may equal the accepted flux reflected from the other specimen. The instrument, therefore, will indicate that both specimens have the same gloss in spite of their difference in appearance.

4. Two-Parameter Techniques

A two-parameter monoplane technique would provide for two variations of geometry. ASTM Method of Test for Mar Resistance of Plastics [11], Designation D673-44T, requires a two-parameter instrument that measures flux reflected on the specular angle, 45° , and at an angle displaced 15° from the specular angle, namely, 30° . The geometry of that instrument can be represented as

$$\left. \begin{aligned} i &= f[45^\circ, (\alpha\beta)] \\ r &= g[45^\circ, 30^\circ, (\alpha'\beta')] \end{aligned} \right\} \quad (5)$$

The data obtained by this method are analyzed in terms of percentage of gloss thus: Gloss, percent = $100[1 - (G_{45,30}/G_{45,45})]$, where G is the instrument reading for the geometry indicated by the subscripts.

Middleton and Mungall [12] described a distinctness-of-image glossmeter designed to use the maximum slope of the goniophotometric curve as the gloss index of a specimen. This instrument measures flux reflected from a specimen at the specular angle, 45° , and at an angle displaced $\Delta\theta$ from the specular

angle. The geometry of that instrument can be represented as

$$\left. \begin{aligned} i &= f[45^\circ, (\alpha\beta)], \\ r &= g[45^\circ, 45 + \Delta\theta, (\alpha'\beta')]. \end{aligned} \right\} \quad (6)$$

The instrument analyzes the data thus obtained in terms of a gloss index. Gloss index $= (G_{45, 45} - G_{45, 45 + \Delta\theta}) / \Delta\theta$, where G is the instrument reading for the geometry indicated by the subscripts.

For two-parameter glossmeters of the specular type the two variations of geometry would be variation of the specular angle or variation of the size of the receptor aperture. The symbolic representation for the geometry of a two-parameter instrument designed for measurements at two specular angles might be written

$$\left. \begin{aligned} i &= f[\theta_1, \theta_2, (\alpha\beta)], \\ r &= g[\theta_1, \theta_2, (\alpha'\beta')]. \end{aligned} \right\} \quad (7)$$

The geometry for such an instrument is shown in figure 2. The symbolic representation for the geometry of a two-parameter instrument designed for measurements with two receptor apertures might be

$$\left. \begin{aligned} i &= f[\theta, (\alpha_1\beta_1), (\alpha_2\beta_2)], \\ r &= g[\theta, (\alpha'_1\beta'_1), (\alpha'_2\beta'_2)]. \end{aligned} \right\} \quad (8)$$

The geometry for such an instrument is shown in figure 3.

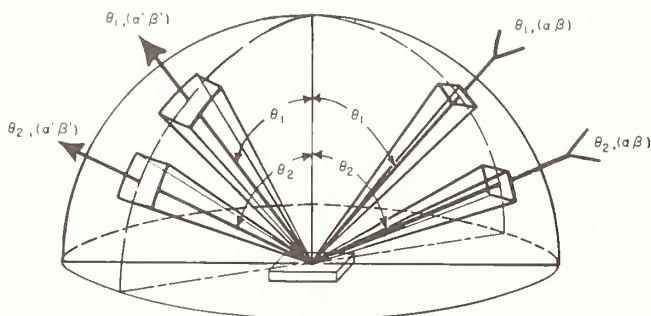


FIGURE 2. Geometry of an incidence angle two-parameter glossmeter.

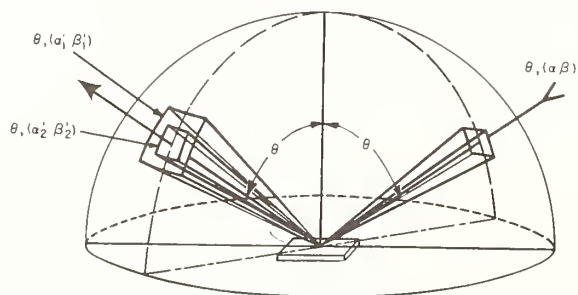


FIGURE 3. Geometry of a receiver aperture two-parameter glossmeter.

4.1. Specular Angle

To illustrate how a two-parameter instrument might be used to resolve problems where specular-angle dependence is important, two low-gloss specimens were selected. These specimens appear to have nearly equal gloss when viewed at angles ranging from perpendicular to about 65° . At angles greater than 65° one specimen appears considerably glossier than the other. Figure 4 shows plots of fractional reflectance, the ratio of flux reflected within a 0.6° cone to the incident flux, as a function of angle, d , between the viewing direction and that of mirror reflection as measured on a monoplane goniophotometer. The specimens are designated A and B. At 60° incidence both curves are flat (reflected flux is uniformly distributed) with A having a slightly higher fractional reflectance. At 80° incidence the curve for A remains reasonably flat, whereas that for B becomes quite peaked, having a peakedness index of 182 compared with 1.9 for A. Peakedness index for a unimodal curve was shown by Nimeroff [10] to be an index of the relative peak to shoulder heights.

A goniophotometer, arranged for the 60° geometry of ASTM Method D523-53T, [13] was used to measure the specular gloss of specimens A and B for 60°

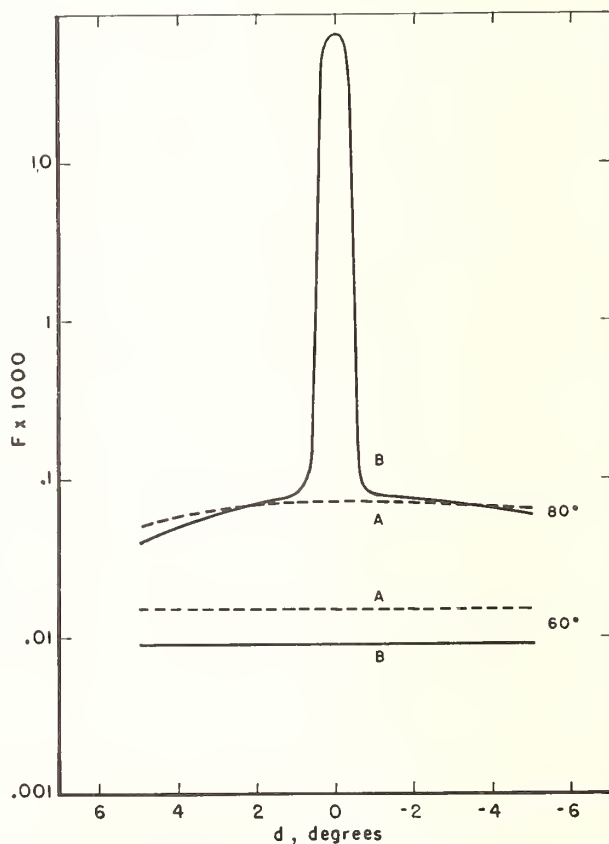


FIGURE 4. Fractional reflectance curves for two specimens exhibiting incidence-angle dependency at incidence angles of 60° and 80° .

and 80° incidence. At 60° incidence, the gloss reading for A is 6 and for B is 4, whereas at 80° incidence the gloss reading for A is 24 and for B is 109. These readings show the much stronger incidence-angle dependence for B than for A. This dependence is not suggested in a measurement of specular gloss at 60° incidence alone.

The solution to problems involving incidence-angle dependence is obvious. Measure the specular gloss at two incidence angles and relate the resulting data. The slope of the line joining plotted points of the gloss readings for a specimen obtained with two incidence angles may be used as an index of the dependence of gloss on incidence angle.

4.2. Receptor Aperture

Appearance characteristics ascribable to gloss may be classified into three distinct types, either in accord with appearance or in accord with flux-scattering properties. Appearance wise, the two extreme types are image-forming and nonimage forming; the third type exhibits intermediate characteristics. Goniophotometric distributions for specimens of the image-forming type are shown in figure 5. These curves are characterized by sharp peaks near the specular angle, with scattering at wide angles from the specular. As a considerable amount of scattered flux is found at wide angles, these specimens may be called "wide-angle scatterers." Goniophotometric distributions for specimens of the non-image-forming type are shown in figure 6. These

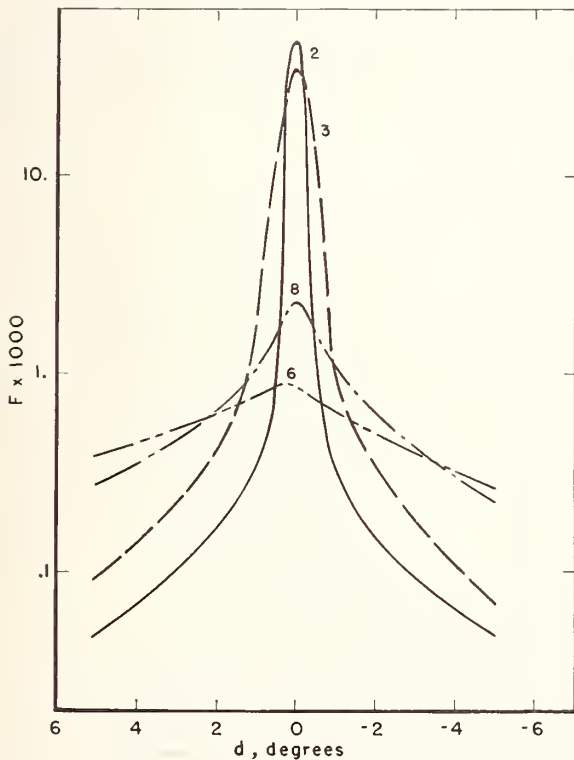


FIGURE 5.—Fractional reflectance curves for wide-angle scatterers.

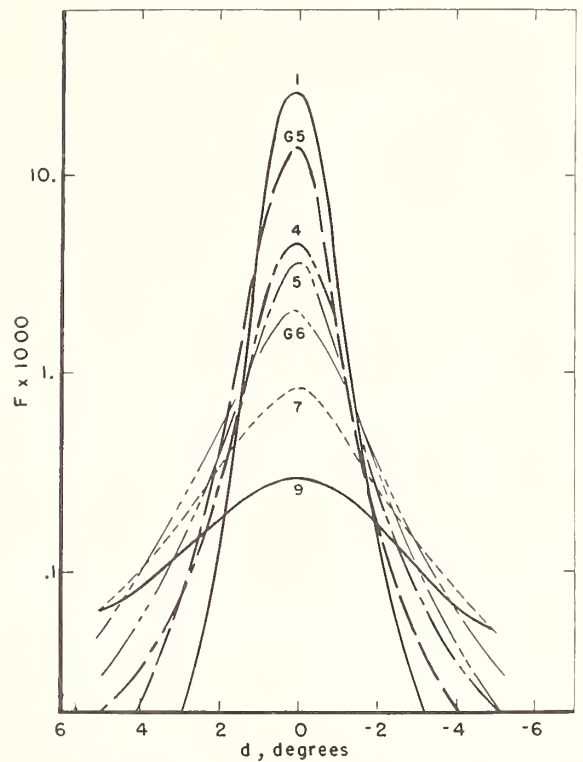


FIGURE 6.—Fractional reflectance curves for narrow-angle scatterers.

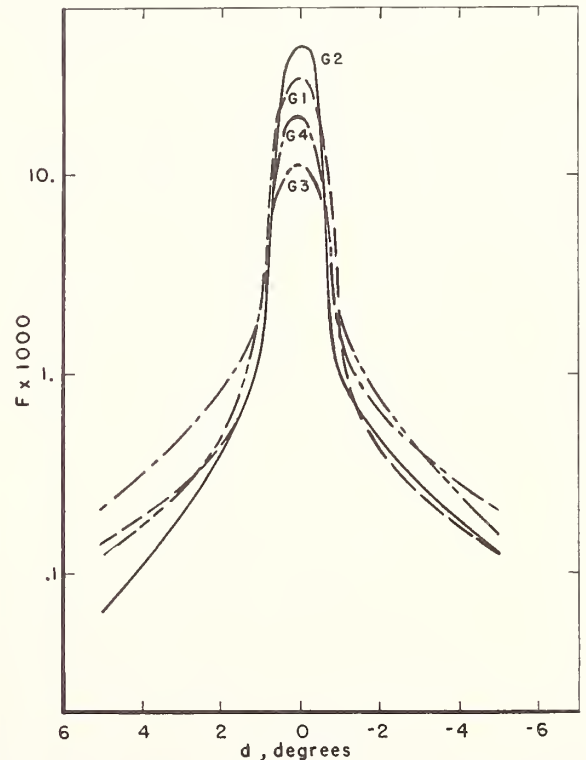


FIGURE 7.—Fractional reflectance curves for intermediate-angle scatterers.

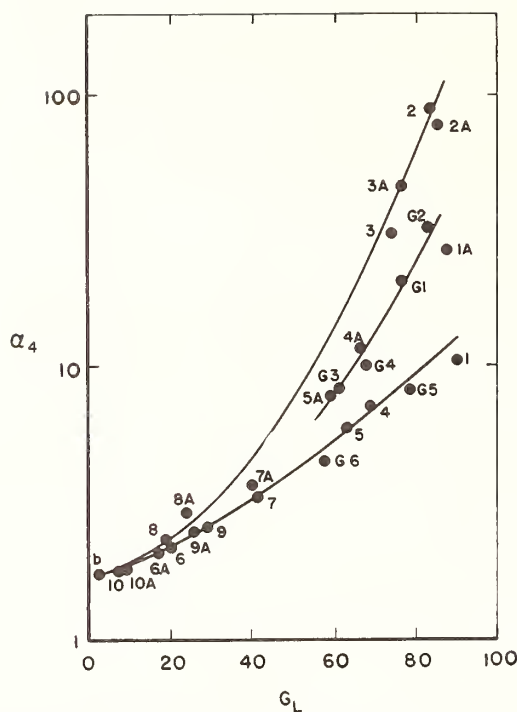


FIGURE 8. Plot of peakedness index, α_4 , against 60° specular glossmeter readings, G_L , showing separation into three groups.

curves are characterized by broad peaks with very little wide-angle scattering. As the scattering is confined to angles near the specular angle, these specimens may be called "narrow-angle scatterers," or "near-specular scatterers." Goniophotometric distributions for specimens of the intermediate type are shown in figure 7. These curves are characterized by peaks broader than those for the wide-angle scatterers but narrower than those for the narrow-angle scatterers. Such specimens might be called "intermediate-angle scatterers."

Nimeroff [10] showed that two parameters are required to reveal instrumentally the two extreme types of specimen appearance due to scattering. Figure 8 shows separation of the three types using the ASTM 60° specular gloss readings [7] and the peakedness indexes, α_4 , obtained from analysis of goniophotometric curves of the specimens, some of which are shown in figures 5, 6, and 7.

5. Gloss of Transparent Finishes

Recently a problem arose in correlating the measured gloss with the observed appearance of transparent finishes applied to wood substrates. Difficulty was encountered in the measurement of the gloss of these finishes in that one-parameter techniques failed to yield data consistent with visual evaluation of the glossy appearance. In seeking a technique that would correlate with visual evaluations, 26 specimen panels were prepared by the Sherwin-Williams Co. and the Grand Rapids Varnish

Co. Goniophotometric curves of representative panels are shown in figures 5, 6, and 7.

As stated above, a two-parameter technique was indicated as a possible solution to this problem. The slope of the monoplanar geometric distribution might be a useful representation of the gloss. An indication of the slope can be obtained from the ratio of a reading at the peak to a reading at the shoulder of a goniophotometric curve. The large receptor aperture used with the 60° geometry of ASTM Method D523 provides a measure of the sum of the peak and shoulders of the reflected flux distribution. A receptor aperture small enough to measure only the flux about the peak of a distribution was needed to supplement the reading with the larger aperture so that the slope of the flux distribution of a specimen can be evaluated:

$$\text{Slope} = \frac{2G_s - G_L}{\alpha'_1\beta'_1 - \alpha'_2\beta'_2}, \quad (9)$$

where G_s is the instrument reading for the small receptor aperture, $\alpha'_1\beta'_1$, and G_L is the instrument reading for the large receptor aperture, $\alpha'_2\beta'_2$. The small receptor aperture arbitrarily chosen is $\alpha'_1\beta'_1 = 2.0^\circ \times 4.5^\circ$. The larger aperture already prescribed by Method D523 is $\alpha'_2\beta'_2 = 4.4^\circ \times 11.7^\circ$.

Measurement of the slope by this procedure is useful over regions of a curve where the slope is constant. As the goniophotometric curves of most specimens do not have constant slope, other relationships between G_L and G_s were sought. Several graphical representations of data were considered.

Figure 9 shows specular gloss data taken with the large aperture, G_L , plotted against data taken with the small aperture, G_s , for the specimens whose

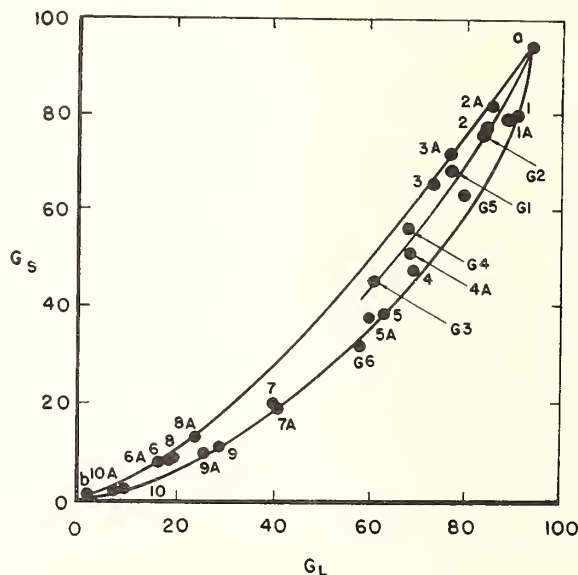


FIGURE 9. Plot of glossmeter readings with small aperture against those with large.

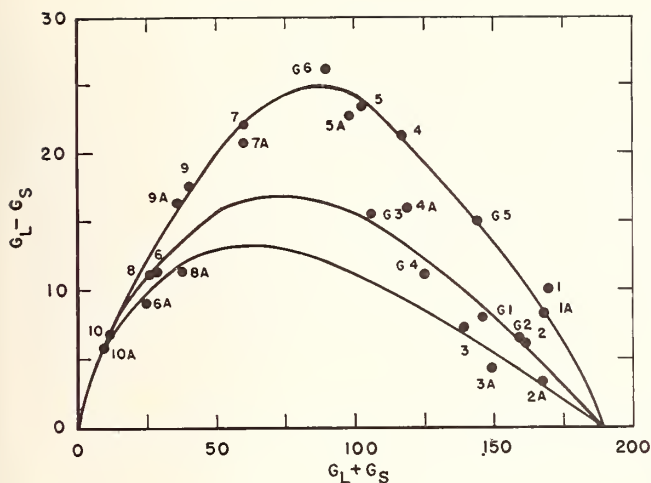


FIGURE 10. Plot of differences, $D = G_L - G_s$, against sums, $S = G_L + G_s$.

goniophotometric curves were shown earlier. The three branches that result are consistent with the separation into appearance types, as evidenced by the goniophotometric data. As aperture reduction stops more flux for narrow-angle scatterers than for wide-angle scatterers the data for the former lie in the lower branch. The plotted points in each branch lie in a sequence consistent with the order of the peaks of the goniophotometric curves for the specimens represented. The curve for a group of intermediate scatterers lies between those for the extreme types.

Terminal points *a* and *b* (fig. 10) occur at the indicated locations for the following reasons: If a receiver aperture is reduced to a size just larger than the source image the reading for a perfectly polished specimen is not varied because all the flux reflected from the surface of a polished specimen will enter the receiver if the receiver aperture is larger than the source image. Point *a* represents the specular gloss of a specimen of polished glass of index 1.527 and is plotted at 93.6 on both scales.

When a receiver aperture is reduced in area the glossmeter reading for a perfect diffuser will be reduced by the ratio of the areas. The glossmeter reading of a perfect diffuser for the receiver aperture of the 60° geometry of ASTM Method D523 was reported by Hammond and Nimeroff [5] to be 2.1. As the ratio of large to small receiver entrance window is 5.7, point *b* is plotted at G_L equal to 2.1 and G_s equal to 0.37.

6. Round-Robin Test

A round-robin test was undertaken on 26 specimens to evaluate data reproducibility, data repeatability, and specimen classification by the two-parameter (receptor aperture) method for specular gloss. By these terms are meant the following:

1. Data repeatability (within laboratory precision):

Repetition by each laboratory of data obtained at that laboratory to within a reasonably small tolerance.

2. Data reproducibility (between laboratory variability): Reproduction by each laboratory of data obtained by the other laboratories to within a reasonably small tolerance.

3. Specimen Classification: The existence of at least three classes of gloss characteristics that can be distinguished by a two-parameter gloss technique.

6.1. Analysis of Round-Robin Data

The data resulting from the round-robin are shown in table 1.

To test repeatability and reproducibility we select a suitable mathematical model. The model used here for the measurement of the *i*th specimen in the *j*th laboratory is

$$y_{ij} = t_i + L_j + \epsilon_{ij}, \quad (10)$$

where y_{ij} is the measurement, t_i is the population, or "true," value of the measurement, L_j is a random variable having an average value of zero and variance σ_L^2 , which reflects the variability between laboratories, and ϵ_{ij} is a random variable having an average value of zero and variance σ_j^2 , which reflects the precision for laboratory *j*.

a. Repeatability

A duplicate measurement is denoted by a prime, as y'_{ij} . Then the difference between duplicate measurements is a quantity d_{ij} , thus

$$d_{ij} = y_{ij} - y'_{ij} = \epsilon_{ij} - \epsilon'_{ij} \quad (10a)$$

having an average value of zero and variance $2\sigma^2$. Thus if there is no reason to believe that the within-laboratory error varies with the different specimens, the variance σ_j^2 of each laboratory for the 26 specimens can be estimated by means of the equation.

$$s_j^2 = \frac{\sum_i d_{ij}^2}{2n} = \frac{d_{1j}^2 + \dots + d_{26j}^2}{52}. \quad (11)$$

Systematic errors in duplicate measurements of G_L by laboratories 2 and 5 and of G_s by laboratory 2 were detected and eliminated from the estimates of precision for the determinations. Systematic errors in duplicate measurements for a laboratory are those in which d_{ij} is the same sign for all or most specimens. This error is evident when the average value for all specimens differs appreciably from zero. The results for each laboratory are given in table 2.

The variance of the within-laboratory data is of the same order of magnitude for all laboratories when systematic errors are eliminated from the estimates of variance. The average variance, s^2 , within laboratories is 0.217 for G_L when s_5^2 is eliminated, and 0.212 for G_s when s_2^2 is eliminated. Thus repeatability for G_L is approximately equal to that and G_s and both variances are reasonably small.

TABLE 1. Data for round-robin test

Specimen	G_L for laboratory—							G_S for laboratory—						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	90.7	90.5	89.8	90.0	90.0	89.5	88.4	79.4	77.3	77.0	79.0	81.0	83.3	73.1
1A	90.4	90.6	90.0	90	92	89.3	89.1	80.2	79.1	79.2	79	---	84.2	72.4
	89.5	86.1	87.6	88	91	86.7	86.8	74.3	71.1	83.1	81	77	84.5	81.5
	87.5	88.8	88.8	89	92	85.0	87.5	75.8	79.8	84.5	82	---	86.5	80.9
2	83.8	83.3	82.2	84	86	83.9	80.4	79.1	77.8	76.2	77	78	77.0	73.7
	83.6	84.1	83.0	83	87	84.5	80.9	79.3	79.5	76.1	77	---	78.8	73.1
2A	84.6	84.7	84.4	85	88	85.0	81.9	82.6	82.7	83.2	82	81	78.4	76.5
	85.5	85.7	85.0	85	88	85.1	82.1	82.9	82.3	83.1	82	---	77.6	76.6
3	73.6	73.6	73.1	73	74	76.0	69.8	66.3	65.2	66.2	67	66	60.1	59.5
	74.0	74.1	73.8	73	75	76.2	70.3	66.9	66.5	66.4	66	---	58.9	59.3
3A	75.6	75.8	76.0	76	80	77.8	71.3	73.1	72.9	73.7	74	72	71.4	66.2
	76.0	75.5	76.0	76	82	77.8	71.7	73.4	74.1	72.3	74	---	70.2	64.6
4	70.8	69.6	68.2	68	67	70.5	67.9	49.2	47.0	44.5	45	51	45.2	41.9
	71.4	71.6	68.1	68	67	70.8	68.2	50.2	48.1	44.9	46	---	45.8	41.6
4A	69.5	68.8	65.7	66	64	71.3	66.5	53.4	50.8	50.9	51	52	45.5	46.9
	69.8	69.6	66.3	66	64	70.7	66.9	53.3	54.0	50.1	50	---	46.5	46.7
5	63.7	62.6	61.3	62	62	63.0	59.4	40.6	38.9	37.6	38	42	37.5	33.6
	63.8	63.0	62.4	62	64	62.7	59.5	40.3	40.6	37.6	39	---	37.9	33.2
5A	60.4	60.1	60.1	60	60	59.8	55.6	38.5	35.3	37.2	38	40	35.1	29.8
	60.5	60.0	60.5	60	63	59.4	56.0	38.5	37.8	37.8	38	---	36.2	29.5
6	18.6	19.2	18.1	19	21	21.4	14.6	8.2	9.2	7.8	8.6	8.3	9.6	4.4
	18.6	19.5	18.1	18	23	21.2	14.5	8.3	9.5	7.5	8.3	---	9.7	4.3
6A	16.0	16.2	15.4	15	19	19.7	11.7	7.3	8.0	6.7	7.4	7.2	8.6	3.5
	15.9	16.5	15.8	16	20	18.4	11.9	7.3	8.3	6.5	7.3	---	8.7	3.4
7	41.8	41.5	39.5	41	40	41.6	36.5	19.0	19.3	17.5	19	20	19.0	12.9
	41.8	42.1	40.7	41	41	41.4	36.8	19.0	20.2	17.2	19	---	18.9	12.9
7A	40.3	40.4	39.3	40	41	41.9	35.6	19.9	20.4	19.0	20	21	20.4	14.1
	40.5	40.5	40.3	40	43	42.0	35.7	19.8	21.0	19.0	19	---	20.2	14.0
8	17.1	18.8	17.9	17	22	20.9	13.7	7.5	9.1	7.9	7.6	7.7	8.3	3.9
	17.7	19.3	17.8	18	23	20.0	13.6	7.7	9.1	7.4	8.2	---	9.4	3.8
8A	21.0	24.0	24.1	23	29	24.2	17.6	11.3	14.9	13.2	13	12	13.8	7.9
	22.5	24.8	22.0	23	29	25.1	17.5	12.7	14.6	12.1	13	---	15.0	7.6
9	29.3	29.7	28.2	29	30	29.9	24.4	10.9	11.8	9.9	11	12	11.9	6.3
	29.4	29.8	28.6	29	31	30.0	24.5	11.8	12.2	9.7	11	---	11.9	6.2
9A	26.2	26.4	25.2	26	27	27.0	21.4	9.6	10.4	8.4	9.8	9.7	11.0	5.4
	26.1	26.6	25.2	26	28	27.2	21.5	9.6	10.6	8.5	9.7	---	10.7	5.3
10	8.5	9.4	7.6	8.6	9.0	9.4	5.1	1.8	2.4	1.4	1.9	1.9	2.8	0.0
	8.5	9.4	7.7	8.6	10.0	9.3	5.1	1.8	2.5	1.4	1.9	---	2.8	0.0
10A	7.4	8.0	6.8	7.4	8.0	8.4	4.0	1.6	2.2	1.4	1.9	1.6	2.6	0.0
	7.4	8.2	6.8	7.4	8.0	8.4	4.0	1.6	2.2	1.3	1.8	---	2.6	0.0
G1	79.1	76.7	76.6	75	76	75.6	70.7	70.4	67.2	71.1	67	68	48.0	46.9
	79.1	77.7	78.8	76	76	75.3	70.8	70.7	70.7	70.1	67	---	46.7	48.3
G2	82.7	82.4	83.2	82	80	84.8	77.6	76.3	75.0	78.2	76	75	75.3	68.0
	83.7	83.7	83.3	83	84	85.0	78.3	78.0	77.2	78.2	76	---	77.1	68.1
G3	63.5	60.8	59.3	60	60	61.1	54.4	49.6	43.3	43.2	44	47	45.0	37.6
	62.6	58.7	58.7	60	64	61.6	54.6	48.7	44.4	43.1	45	---	44.4	37.0
G4	67.6	64.0	67.4	67	70	69.8	61.6	57.8	53.6	57.8	54	57	59.0	49.0
	67.6	66.6	68.3	66	74	68.6	62.4	57.7	56.3	57.2	54	---	58.6	47.4
G5	79.8	78.2	79.2	79	80	79.1	73.0	64.1	59.7	64.2	64	67	54.6	52.5
	79.9	78.3	80.9	79	80	78.9	73.7	65.4	63.4	64.5	64	---	55.5	52.3
G6	59.3	57.2	55.8	58	56	58.7	53.2	32.0	32.5	29.3	32	35	30.7	25.0
	59.4	57.6	57.4	58	59	58.4	53.7	32.0	33.5	29.6	31	---	30.8	24.7

TABLE 2. Variance of laboratories for G_L and G_S determinations

Laboratory	s_j^2	
	G_L	G_S
1 -----	0.196	0.152
2 -----	.330	.727
3 -----	.360	.179
4 -----	.154	.144
5 -----	.847	---
6 -----	.152	.396
7 -----	.108	.188

b. Reproducibility

The variance for the average of six laboratories, each laboratory result being made up of two duplicate readings, is

$$[\sigma_L^2 + (\sigma^2/2)]/6,$$

where σ_L^2 is the variability between laboratories and σ^2 is the variance within laboratories. The variability between laboratories for G_L is 2.07 and for G_S is 2.21. This analysis may be represented thus:

	$\sigma^2(G_L)$	$\sigma^2(G_S)$
Variance of a laboratory average of two readings -----	2.18	2.21
Variance of 6 laboratories average ---	0.36	0.37

This analysis shows that the reproducibility of G_L and G_S is approximately equal and the variance in both is reasonably small for the average of 6 laboratories, but is somewhat large for 1 laboratory average of 2 duplicate readings.

The data shown in figure 9 are the arithmetic mean values of readings obtained with the small aperture, G_S , plotted against arithmetic mean values of readings obtained with the large aperture, G_L . Because the errors in readings for G_L and G_S are correlated, simple functions of G_L and G_S are desired in which the errors are uncorrelated. These statistically uncorrelated functions would be useful for the final discrimination of gloss characteristics, that are, however, functionally related. Two simple functions, which seem to work well, are the sums, S , and differences, D :

$$S = G_L + G_S$$

$$D = G_L - G_S.$$

If only the determinations G_L and G_S are used for discrimination, the statistical correlations between them may obscure the discrimination between different gloss characteristics. Use of S and D accentuates the ability to discriminate gloss characteristics.

The fact that S and D are statistically uncorrelated can be demonstrated by the following simple argument. The correlation between S and D is given [14] by

$$\rho(S, D) = E[(S - \bar{S})(D - \bar{D})] / \sigma(S)\sigma(D), \quad (12)$$

where E is read as "the expected value of"; \bar{S} and \bar{D} are the mean values of S and D , respectively. When $G_L + G_S$ is substituted for S and $G_L - G_S$ is substituted for D in eq (12), the following expression for the correlation between S and D results:

$$\rho(S, D) = \frac{\sigma^2(G_L) - \sigma^2(G_S)}{\sigma(S)\sigma(D)}. \quad (12a)$$

Experimental results, discussed above, indicate that the variances, $\sigma^2(G_L)$ and $\sigma^2(G_S)$ are equal. Thus

$$\rho(S, D) = 0.$$

Figure 10 shows a plot of the D versus S values for the data of figure 9.

7. Two-Parameter Method and Subjective Evaluation

The two-parameter (receptor aperture) method was found to correlate with goniophotometric curve evaluation in that three groups are established by both methods, each containing essentially the same specimens in essentially the same order (see figs. 8 and 10). Correlation between subjective and instrumental evaluation is required to establish firmly the utility of the two-parameter method.

7.1. Image Distinctness and Constant Image Brightness

A test was arranged to determine whether image distinctness could be evaluated as the two-parameter method evaluates it, when variation in brightness is minimized. Twelve sets of three specimens, each set of approximately equal brightness (G_L), were selected from the 26 specimens and presented to the observers.

The source was a bank of fluorescent lamps. The standards were placed on a table so that images of the lamps were reflected at a specular angle of approximately 60° . The observers were asked to arrange the specimens in order of image distinctness, indicating ties where necessary. The selected sets of three specimens are listed in table 3, together with the corresponding G_L values. Image distinctness in each set of triplets increased from left to right and are given numbers 1, 2, and 3, respectively. In set 11, specimens 3 and 3A are from the same family of image distinctness and are both ranked 2.5. Similarly, specimens 4A and G3 of set 12 are ranked 2.5.

TABLE 3. Sets of three specimens for evaluation of I with constant G_L

Set	Specimens ^a			Corresponding G_L		
1	1	G2	2A	90	83	86
2	1A	2	2A	88	84	86
3	G5	2	2A	79	84	86
4	G5	G1	3	79	77	74
5	G5	G2	3A	79	83	77
6	G5	G1	3A	79	77	77
7	4	G1	3A	69	77	77
8	4	G1	3	69	77	74
9	4	G4	3	69	68	74
10	9A	6	8A	26	20	24
11	G1	b 3	b 3A	77	74	77
12	5	c 4A	c G3	63	67	61

^a Image distinctness increases from left to right in each set of triplets, and are given numbers 1, 2, 3, respectively.

^b Same image family.

^c Same image family.

The results of this test are listed in table 4. As a measure of agreement between ranking given by the m judges and ranking assigned by G_S of the two-parameter method, an average of m rank correlations was determined. The Spearman rank correlation coefficient was used, and the average is indicated by \bar{R}_s . If (X_1, X_2, X_3) is the assigned ranking of three objects and (Y_{i1}, Y_{i2}, Y_{i3}) , for $i=1$ to m , are the rankings given by the m judges, then,

$$\begin{aligned} \bar{R}_s &= \frac{1}{m} \sum_i \left\{ \sum_j (X_j Y_{ij} - 4) / 2 \right\} \\ &= \frac{1}{2m} \sum_j \left\{ X_j \left(\sum_i Y_{ij} \right) - 4m \right\}. \end{aligned} \quad (13)$$

The results of the computation of \bar{R}_s for these specimens show quite good agreement between the assigned and the given rankings.

TABLE 4. *Rankings of triplets by 10 judges*

Set	DBJ	WAH	KLK	MMB	LEB	MAB	HKH	IN	MZ	JRR	Average ranking	\bar{R}_s
1.....	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 3 2	1 2 3	1 2 3	0.95
2.....	1 2.5 2.5	1 2 3	1.5 1.5 3	1 2 3	1 2 3	1 2.5 2.5	1 2.5 2.5	1 2 3	1 2 3	1 2.5 2.5	1 2 3	.88
3.....	1 2 3	1 2.5 2.5	1 2 3	1 2 3	1 2 3	1 2 3	1 2.5 2.5	1 2 3	1 2 3	1 2 3	1 2 3	.95
4.....	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1.00
5.....	1 2 3	1 3 2	1 2 3	1 2 3	1 3 2	1 2 3	1 2 3	1 2 3	1 3 2	1 2 3	1 2 3	0.85
6.....	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1.00
7.....	2 1 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	0.95
8.....	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1.00
9.....	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1.00
10.....	1 2 3	2 1 3	2 1 3	1 2 3	2 1 3	1 2 3	1 2 3	1 2 3	2 1 3	1.5 1.5 3	1 2 3	0.78
11.....	1 3 2	1 3 2	1 2.5 2.5	1 2 3	1 3 2	1 2 3	1 3 2	1 3 2	1 3 2	1 2.5 2.5	1 3 2	.65
12.....	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	2 2 2	1 2 3	2 1 3	1.5 1.5 3	1 2 3	.83

To measure the agreement among judges without reference to the "correctness" of what they agree upon, Kendall's coefficient of concordance [15], W , was used.

$$W = \frac{12 \sum_j (\sum_i Y_{ij} - 20)^2}{2400 - 10 \sum_i (t_i^3 - t_i)} \quad (14)$$

where

$$t_i = \begin{cases} 0 & \text{for no ties} \\ 2 & \text{for two specimens tied} \\ 3 & \text{for three specimens tied.} \end{cases}$$

The results of the computation of W are listed in table 5. These results show a high concordance among the judges for the 12 sets selected.

We may therefore conclude that, where the image brightness range is not great, image distinctness is evaluated by the two-parameter method in accord with subjective evaluation. Thus the two-parameter method may be considered to be a successful technique for evaluating image distinctness when image brightness is relatively constant.

TABLE 5. *Measure of agreement among judges, Kendall's coefficient of concordance, W*

Set	W
1.....	0.91
2.....	.89
3.....	.96
4.....	1.00
5.....	0.79
6.....	1.00
7.....	0.91
8.....	1.00
9.....	1.00
10.....	0.77
11.....	.83
12.....	.80
Average.....	0.90

8. Summary

A complete description of the gloss characteristics of any specimen requires multiparameter data, obtained on either a multi plane or a monoplane goni-

photometer. For reasons of economy simpler techniques are sought that can adequately describe the gloss characteristics of any specimen. The simplest method, using one parameter, does not always provide a sufficient description.

Two-parameter gloss methods provide more adequate description of the gloss of specimens than do one-parameter methods. A two-parameter technique employing specular reflection measurement for two incidence angles reveal specimen incidence-angle dependency. A gloss index may state the difference of specular reflectance for a difference of specular angle, $dG/d\theta$.

A two-parameter technique employing specular reflection measurement with a large and a small receptor aperture will reveal the existence of more than one gloss type. A specification may state the ranges of G_L and G_S , or S and D , thereby indicating the desired surface-appearance type.

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Gloss Standards And Glossmeter Standardization

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Gloss standards provide very fine adjuncts for quickly checking the accuracy of geometric adjustment of glossmeters and the correctness of their readings. An instrument that is not in accurate geometric adjustment, however, cannot be calibrated by simply noting the scale corrections required to provide the assigned values of the standards. Standardization of specular-gloss measurements necessitates specification of the angles of illumination and reception and the sizes of the source and receptor field angles or apertures. The apertures, together with permissible tolerances, are specified in each ASTM method. The user of an instrument, however, may not wish to dismantle the instrument to check aperture dimensions or to ascertain if the image of the source is centered in and focused on the receptor entrance window. A simple check of the overall accuracy of instrument adjustment is provided by setting the instrument scale to read the gloss value of a polished opaque-glass standard, and then noting the readings for several standards of lower gloss. If the readings for these latter standards do not agree closely with the assigned values usually within a unit or two the instrument should be readjusted according to the manufacturer's instructions or returned to him for adjustment.

INTRODUCTION

More than 20 years have passed since Hunter and Judd (1939) made their initial report on a method of classifying paints according to gloss.¹ This was the work that led to the adoption of the first ASTM method of test for specular gloss.² For nearly the same length of time the National Bureau of Standards (NBS) has been issuing gloss standards. During the early years and until the end of World War II, these took the form of slips of heavy paper that were used a few times and then discarded. Since 1946, ceramic tile and opaque glass have been procured and calibrated as gloss standards. In 15 years, several thousand of these standards have been issued, many of them to the paint industry.

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But availability of calibrated standards, helpful though it may be, is not the panacea for accurate gloss evaluation. In an information sheet on gloss standards and their use, NBS has pointed out that accurate gloss readings can be obtained only from glossmeters that are in accurate geometric adjustment. Accurately calibrated standards provide a simple means by which to verify the geometric adjustment of an instrument, but they are no substitute for it. This is because gloss is a multidimensional geometric property.

MODERN METHODS OF TEST

Modern methods of test for specular gloss all specify the dimensions of source and receptor apertures together with tolerances for these apertures.² Such specifications limit the divergence of incident and reflected light rays. Glossmeters conforming to these specifications, even when of different manufacture, should reproduce measurements on the same specimens to within an acceptable tolerance, usually plus or minus one gloss unit.

Although each glossmeter manufacturer can be expected to design, manufacture, and adjust his instruments to provide gloss readings conforming to the ASTM methods, the instrument user who desires to make accurate measurements will find it advantageous to verify the geometric accuracy. One way to check geometric conformity is to remove the cover from the instrument, carefully measure the size of the receptor entrance window and its distance from the receptor lens, and then compute the receptor aperture. Likewise the image of the source can be checked to insure that it is focussed at and centered in the receptor entrance window. The source aperture may be computed from measurements of the dimensions of the source and the focal length of the collimating lens. Frequently it is more convenient to evaluate the source aperture on the receptor side of the instrument by measuring the size of the source image at the receptor entrance window.

CALIBRATED GLOSS STANDARDS

Experience has revealed that a more simple procedure for verifying the adequacy of geometric adjustment is to utilize calibrated gloss standards. The procedure is as follows: the glossmeter scale is adjusted to read correctly the gloss of a highly polished standard, usually opaque white or black glass. With this kind of standard all of the reflected light flux will enter the receptor window. Standards of lower gloss are then measured, and the glossmeter readings are compared with assigned values for the standards. If the instrument readings agree closely with the assigned values, say within one or two gloss units, the instrument is in satisfactory adjustment. If the readings are substantially different from

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the assigned values, readjustment is indicated, but one cannot state from this information alone just where the glossmeter adjustment is in error. If geometric adjustment instructions are not provided by the manufacturer, the instrument had best be returned to him for adjustment.

In times past, gloss test methods sometimes provided for use of "equivalent glossmeters," that is, instruments that did not conform strictly to the aperture specifications of the method but which were designed empirically to provide "equivalent results." Such instruments will provide equivalent results only when the light flux distributions of test specimen and calibration standard are virtually identical. The determination of the equivalence of reflected light flux distributions requires goniophotometric curves of the distributions. If one has the facility to obtain goniophotometric curves, he no longer needs glossmeter readings because a glossmeter is in reality an abridged goniophotometer just as a reflectometer is an abridged spectrophotometer. If one has the complete book of an author, he is seldom interested in a condensed version.

Scales for specular gloss have always been selected empirically. The upper end of the scale is adjusted so that a high-gloss specimen of a given class of material will have a glossmeter reading of nearly 100. A point at the high end of the scale is fixed by specifying the value to be assigned to polished black glass of specified refractive index. The near-100 readings for high-gloss specimens cause some people to assume that the scale value of 100 represents full gloss and cannot be exceeded. In reality the maximum gloss reading of a paint depends on three things; first, it depends on the degree to which the facets comprising the specimen surface are oriented in the plane of the surface; second, it depends on the absence of near forward scattering centers just below the surface (haze); and third it depends on the index of refraction of the paint film. The desirability of taking into account differences in refractive index when comparing specimens for gloss has been discussed several times by groups of gloss experts in ASTM. The view has usually been taken, however, that paints having large differences in refractive index will usually not be compared for gloss, so there has been no desire to complicate gloss

measurement methods by requiring that glossmeter readings be adjusted by a factor depending on the index of refraction of the paint.

USE OF SEVERAL GEOMETRIES

As has been indicated above, one cannot calibrate satisfactorily the scale of a glossmeter over a small range by simply noting the readings for two gloss standards and equating these to the gloss values assigned to the standards. If gloss were a one-dimensional variable like length or thickness, it would be meaningful to make a calibration in this way. But gloss, unfortunately, is multidimensional. Surface characteristics that affect specular gloss vary in a complicated manner. This is the reason why the ASTM Method of Test for Specular Gloss (D523) has provision for measurement under three geometric conditions. This method in reality contains three sub-methods, identified by the angles of incidence as the 20-, 60-, and 85-degree geometries. The angle of incidence is only one of the geometric factors which is changed. The size of the receptor aperture is also different for each geometry.

The philosophy behind the use of several geometries is easily explained. High gloss is best differentiated by observing visually and measuring instrumentally at an angle near the perpendicular to the specimen such as 20°. In the case of instrumental measurement, differentiation of high-gloss specimens is facilitated by using the smallest receptor aperture that will insure acceptance of the source image. When the receptor window is made only slightly larger than the image of the source at the receptor, excellent high-gloss differentiation is provided; however, specimen positioning and planarity must be held to very close tolerances. For best results, paint test specimens should be prepared by making draw-downs on plate glass.

Low-gloss specimens, on the other hand, are best differentiated by observing visually and measuring instrumentally at high angles of incidence, such as 85° from the perpendicular to the specimen. A relatively large receptor aperture is specified in order to obtain sufficient energy for measurements of high precision. The high angle of incidence is chosen because specimens that appear to be quite matte when illuminated and viewed at angles near the perpendicular will often appear quite shiny when examined at angles near grazing. This phenomenon of rapid increase in gloss as grazing angles are approached is referred to as sheen. Glossmeter readings at 85° are often called sheen values, but in reality any specimen, including a high-gloss automotive enamel, can be measured on an 85° glossmeter; however, one would not say that such a specimen possesses sheen.

For differentiating a wide range of specimens, some intermediate angle of incidence and some compromise receptor aperture must be used. This is why the 60° geometry was established first and why it has con-

tinued to be used so long. The receptor aperture was made longer than it is wide so that measurements can be made of specimens prepared by brushing without having the brush marks adversely affect the measurements. This aperture arrangement explains why brushed specimens are always measured with the brush marks parallel to the plane containing the axes of the illuminating and viewing beams.

The three geometries are designed to provide the most suitable measurement conditions for three categories of specimens. For the same reason, permanent materials for use as standards, such as tile and de-polished glass, must be chosen with appropriate flux distributions for each of the three geometries. This means that plaques regularly calibrated for use with one geometry are generally not suitable for use with another geometry because they do not have the desired range or type of reflected light flux distributions. The exception is polished glass which is the primary standard for each geometry. The specular gloss of the primary standard is computed from its index of refraction and the specified angle of incidence by using Fresnel's equation.³ A factor is then applied to produce the desired numerical scale.

Ceramic tiles are widely used as intermediate gloss standards because their reflected light flux distributions approximate those of typical paint specimens and because they are readily available. The planarity and gloss uniformity of commercial tiles leave something to be desired. One tile manufacturer, however, has assisted NBS in providing tile of better-than-usual planarity by glazing the tile, on both sides to minimize warping during firing. NBS endeavors to reduce gloss non-uniformity by inspecting tile for uniformity before calibration. The acceptance criterion is somewhat flexible in order to meet the demand for standards with the available materials.

The effect of non-uniformity is minimized by positioning the illuminated area at the center of the tile and by always illuminating in the same direction. To facilitate positioning in use, centerlines and an arrow to indicate direction of illumination are placed on the back face of each standard.

CONCLUSION

In conclusion, I wish to emphasize that calibrated gloss standards are a useful addition to a good glossmeter, that they provide a simple means of checking geometric accuracy, but that they are no substitute for accurate geometric adjustment of the instrument. ♦

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<p>16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)</p> <p>This volume is one of an extended series which brings together the previously published papers, monographs, abstracts, and bibliographies by NBS authors dealing with the precision measurement of specific physical quantities and the calibration of the related metrology equipment. The contents have been selected as being useful to the standards laboratories of the United States in tracing to the NBS standards the accuracies of measurement needed for research work, factory production, or field evaluation.</p> <p>Volume 9 treats the field of Colorimetry. It contains 37 reprints on Colorimetry and Spectrophotometry assembled in 8 sections: (1) Spectrophotometry, (2) Color Vision, (3) Color Specification, (4) Sources and Instrumentation, (5) Color Spacing and Metamerism, (6) Variabilities and Uncertainties, (7) Appearance and Gloss, (8) List of Publications, LP39 (in part), Colorimetry and Spectrophotometry Publications by the Staff of the National Bureau of Standards, January 1957 through August 1969.</p>			
<p>17. KEY WORDS (Alphabetical order, separated by semicolons)</p> <p>Appearance evaluation; color spacing; color vision; colorimetry; gloss; metamerism; spectrophotometry; standards; uncertainty; variability.</p>			
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Official SI Unit Names and Symbols

[For a complete statement of NBS practice, see
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Name	Symbol	Name	Symbol
meter_____	m	newton_____	N
kilogram_____	kg	joule_____	J
second_____	s	watt_____	W
ampere_____	A	coulomb_____	C
kelvin ¹ _____	K	volt_____	V
candela_____	cd	ohm_____	Ω
radian_____	rad	farad_____	F
steradian_____	sr	weber_____	Wb
hertz_____	Hz	henry_____	H
lumen_____	lm	tesla_____	T
lux_____	lx		

Additional Names and Symbols approved for NBS use

curie ² _____	Ci	mole_____	mol
degree Celsius ³ _____	°C	siemens ⁴ _____	S
gram_____	g		

¹ The same name and symbol are used for thermodynamic temperature and temperature interval.
(Adopted by the 13th General Conference on Weights & Measures, 1967.)

² Accepted by the General Conference on Weights & Measures for use with the SI.

³ For expressing "Celsius temperature"; may also be used for a temperature interval.

⁴ Adopted by IEC and ISO.

Table for Converting U.S. Customary Units to Those of the International System (SI)⁵

To relate various units customarily used in the United States to those of the International System, the National Bureau of Standards uses the conversion factors listed in the "ASTM Metric Practic Guide", designation: E 380-70. These are based on international agreements effective July 1, 1959, between the national standards laboratories of Australia, Canada, New Zealand, South Africa, the United Kingdom, and the United States.

To convert from:

- (1) inches to meters, multiply by 0.0254 exactly.
- (2) feet to meters, multiply by 0.3048 exactly.
- (3) feet (U.S. survey) to meters, multiply by 1200/3937 exactly.
- (4) yards to meters, multiply by 0.9144 exactly.
- (5) miles (U.S. statute) to meters, multiply by 1609.344 exactly.
- (6) miles (international nautical) to meters, multiply by 1852 exactly.
- (7) grains (1/7000 lbm avoirdupois) to grams, multiply by 0.064 798 91 exactly.
- (8) troy or apothecary ounces mass to grams, multiply by 31.103 48 . . .
- (9) pounds-force (lbf avoirdupois) to newtons, multiply by 4.448 222 . . .
- (10) pounds-mass (lbm avoirdupois) to kilograms, multiply by 0.453 592 . . .
- (11) fluid ounces (U.S.) to cubic centimeters, multiply by 29.57 . . .
- (12) gallons (U.S. liquid) to cubic meters, multiply by 0.003 785 . . .
- (13) torr (mm Hg at 0 °C) to newtons per square meter, multiply by 133.322 exactly.
- (14) millibars to newtons per square meter, multiply by 100 exactly.
- (15) psi to newtons per square meter, multiply by 6894.757 . . .
- (16) poise to newton-seconds per square meter, multiply by 0.1 exactly.
- (17) stokes to square meters per second, multiply by 0.0001 exactly.
- (18) degrees Fahrenheit to kelvins, use the relation $T_{es} = (t_F + 459.67) / 1.8$.
- (19) degrees Fahrenheit to degrees Celsius, use the relation $t_{es} = (t_F - 32) / 1.8$.
- (20) curies to disintegrations per second, multiply by 3.7×10^{10} exactly.
- (21) roentgens to coulombs per kilogram, multiply by $2.579\,760 \times 10^{-4}$ exactly.

⁵ Système International d'Unités (designated SI in all languages).

